Two-photon exchange in elastic $e$ scattering

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Outline

- Two-photon exchange and nucleon structure
- Extraction of proton $G_E/G_M$ ratio
  - Rosenbluth separation and polarization transfer
- Excited state contributions
  - $\Delta$, $N^*(1/2^+)$, $N^*(1/2^-)$ contributions
- Effect on neutron form factors
- Summary
Proton $G_E/G_M$ Ratio

\[ \sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \]

\[ \tau = \frac{Q^2}{4M^2} \]

\[ \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \theta/2\right]^{-1} \]

\[ G_E/G_M \text{ from slope in } \varepsilon \text{ plot} \]

Rosenbluth (Longitudinal-Transverse) Separation

PT

\[ \frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon \frac{P_T}{P_L}}} \]

\[ P_{T,L} \text{ polarization of recoil proton} \]
Two-photon exchange
& nucleon structure
QED Radiative Corrections

cross section modified by $1\gamma$ loop effects

\[ d\sigma = d\sigma_0 (1 + \delta) \]

$\delta$ contains additional $\epsilon$ dependence

mostly from box (and crossed box) diagram

\[ \rightarrow \text{can modify } \epsilon \text{ dependence in } d\sigma_0 \]
Box diagram

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{N(k)}{D(k)} \]

where

\[ N(k) = \bar{u}(p_3) \gamma_{\mu}(\not{p}_1 - \not{k} + m_e)\gamma_{\nu} u(p_1) \]
\[ \times \bar{u}(p_4) \Gamma^\mu(q-k) (\not{p}_2 + \not{k} + M) \Gamma^\nu(k) u(p_2) \]

and

\[ D(k) = (k^2 - \lambda^2) ((k-q)^2 - \lambda^2) \]
\[ \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2) \]

with \( \lambda \) an IR regulator, and e.m. current is

\[ \Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \]
Various approximations to $M_{\gamma\gamma}$ used

- **Mo-Tsai**: soft $\gamma$ approximation
  - integrand most singular when $k = 0$ and $k = q$
  - replace $\gamma$ propagator which is not at pole by $1/q^2$
  - approximate numerator $N(k) \approx N(0)$
  - neglect all structure effects

- **Maximon-Tjon**: improved loop calculation
  - exact treatment of propagators
  - still evaluate $N(k)$ at $k = 0$
  - first study of form factor effects
  - additional $\varepsilon$ dependence

- **Blunden-WM-Tjon**: exact loop calculation
  - no approximation in $N(k)$ or $D(k)$
  - include form factors
Two-photon correction

\[ \delta^{(2\gamma)} \rightarrow \frac{2 \text{Re} \{ M_0^\dagger M_{\gamma\gamma} \}}{|M_0|^2} \]

\[ \delta^{(2\gamma)}_{\text{full}} - \delta^{(2\gamma)}_{\text{Mo-Tsai}} \]

\[ \Delta(\varepsilon, Q^2) \]

\[ \text{few \% magnitude} \]

\[ \text{positive slope} \]

\[ \text{non-linearity in } \varepsilon \]

Blunden, WM, Tjon

PRL 91 (2003) 142304;

PRC72 (2005) 034612
Two-photon correction

\[ \Delta (\varepsilon, Q^2) \]

\( Q^2 = 1 \text{ GeV}^2 \)

"realistic"

dipole

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC 72 (2005) 034612
Two-photon correction

\[ Q^2 = 6 \text{ GeV}^2 \]

\[ \Delta(\varepsilon, Q^2) \]

* different form factors

\{ Mergell, Meissner, Drechsel (1996) \\
Brash et al. (2002) \\
Arrington LT \( G_E^p \) fit (2004) \\
Arrington PT \( G_E^p \) fit (2004) \}

Blunden, WM, Tjon
PRL 91 (2003) 142304; 
PRC72 (2005) 034612
Effect on cross section

Born cross section with PT form factors

including TPE effects

* Super-Rosenbluth
Qattan et al.,
PRL 94, 142301 (2005)
$e^+ / e^-$ comparison

- 1γ exchange changes sign under $e^+ \leftrightarrow e^-$
- 2γ exchange invariant under $e^+ \leftrightarrow e^-$
- ratio of $e^+ p / e^- p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$

$$R^{e^+ e^-} = \frac{d\sigma^{e^+}}{d\sigma^{e^-}} \approx 1 - 2 \Delta$$

Simultaneous $e^- p / e^+ p$ measurement planned in Hall B (to $Q^2 \sim 1$ GeV$^2$)
Generalized form factors

Generalized electromagnetic current

\[ \Gamma^{\mu} = \tilde{F}_1 \gamma^{\mu} + \tilde{F}_2 \frac{i\sigma^{\mu\nu} q_{\nu}}{2M} + \tilde{F}_3 \frac{\gamma \cdot K P^{\mu}}{M^2} \]

\[ K = (p_1 + p_3)/2, \quad P = (p_2 + p_4)/2 \]

\( \tilde{F}_i \) are complex functions of \( Q^2 \) and \( \varepsilon \)

In 1\( \gamma \) exchange limit

\[ \tilde{F}_{1,2}(Q^2, \varepsilon) \rightarrow F_{1,2}(Q^2) \]
\[ \tilde{F}_3(Q^2, \varepsilon) \rightarrow 0 \]

* Note: decomposition not unique
Generalized form factors

Generalized (complex) Sachs form factors

\[ \tilde{G}_E = G_E + \delta G_E, \quad \tilde{G}_M = G_M + \delta G_M, \quad Y_{2\gamma} = \tilde{\nu} \frac{\tilde{F}_3}{G_M} \]

\[ K \cdot P/M^2 = \sqrt{\tau (1+\tau) (1+\varepsilon)/(1-\varepsilon)} \]

\[ \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2 + 2 G_M^2 \text{Re} \left\{ \frac{\delta G_M}{G_M} + Y_{2\gamma} \right\} + \frac{2 \varepsilon}{\tau} G_M^2 \text{Re} \left\{ \frac{\delta G_E}{G_E} + \frac{G_M}{G_E} Y_{2\gamma} \right\} \]

cannot assume all TPE effects reside in \( Y_{2\gamma} \)
Extraction of proton $G_E/G_M$ ratio
\[ \frac{G_E^p}{G_M^p} \text{ ratio} \]

- estimate effect of TPE on \( \varepsilon \) dependence
- approximate correction by linear function of \( \varepsilon \)

\[
1 + \Delta \approx a + b \varepsilon
\]

reduced cross section is then

\[
\sigma_R \approx a G_M^2 \left[ 1 + \frac{\varepsilon}{\mu^2 \tau} \left( R^2 (1 + \varepsilon \frac{b}{a}) + \mu^2 \tau \frac{b}{a} \right) \right]
\]

where “true” ratio is

\[
R^2 = \frac{\bar{R}^2 - \mu^2 \tau \frac{b}{a}}{1 + \bar{\varepsilon} \frac{b}{a}}
\]

“effective” ratio contaminated by TPE

average value of \( \varepsilon \) over range fitted
resolves much of the form factor discrepancy
how does TPE affect polarization transfer ratio?

\[ \tilde{R} = R \left( \frac{1 + \Delta_T}{1 + \Delta_L} \right) \]

where \( \Delta_{L,T} = \delta_{L,T}^{\text{full}} - \delta_{\text{IR}}^{\text{Mo-Tsai}} \) is finite part of \( 2\gamma \) contribution relative to IR part of Mo-Tsai

experimentally measure ratio of polarized to unpolarized cross sections

\[ \frac{P^{1\gamma+2\gamma}_{L,T}}{P^{1\gamma}_{L,T}} = \frac{1 + \Delta_{L,T}}{1 + \Delta} \]
Longitudinal & transverse polarizations

* Note scales!

(a) $Q^2 = 6 \text{ GeV}^2$

- Small effect on $P_L$

(b) $Q^2 = 6 \text{ GeV}^2$

- Large effect on $P_T$
$G_E^P / G_M^P$ ratio

Rosenbluth separation

Polarization transfer corrected for $2\gamma$ exchange

large $Q^2$ data typically at large $\varepsilon$

< 3\% suppression at large $Q^2$
Normal polarization

\[ Q^2 = 6 \text{ GeV}^2 \]

\[ \Delta_N \]

\[ \varepsilon \]

\[ Q^2 = 6 \text{ GeV}^2 \]
Normal polarization

FIG. 10: Ratio of the $2\gamma$ contribution to the transverse spin asymmetry as a function of the center of mass scattering angle, $\Theta_{cm}$, for $Q^2 = 1$ (dotted), $3$ (dashed) and $6$ GeV$^2$ (solid).

$Q^2 = 1$ GeV$^2$
Excited intermediate states
Lowest mass excitation is $P_{33} \Delta$ resonance

relativistic $\gamma^* N\Delta$ vertex

\[
\Gamma_{\gamma\Delta \to N}^{\nu\alpha}(p, q) \equiv iV_{\Delta in}^{\nu\alpha}(p, q) = eF_\Delta(q^2) \frac{M_\Delta}{2M_\Delta} \left\{ g_1 [ g^{\nu\alpha}p^\mu q^\mu - p^\nu \gamma^\alpha q^\mu - \gamma^\nu \gamma^\alpha p \cdot q + \gamma^\nu p q^\alpha ] \\
+ g_2 [ p^\nu q^\alpha - g^{\nu\alpha} p \cdot q ] + (g_3/M_\Delta) [ q^2 (p^\nu \gamma^\alpha - g^{\nu\alpha} q^\mu) + q^\nu (q^\alpha q^\mu - \gamma^\alpha p \cdot q) ] \right\} \gamma_5 T_3
\]

form factor \[ \frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2} \]

coupling constants

$g_1$ magnetic $\rightarrow$ 7

$g_2 - g_1$ electric $\rightarrow$ 9

$g_3$ Coulomb $\rightarrow$ -2 ... 0
Two-photon exchange amplitude with $\Delta$ intermediate state

\[
\mathcal{M}^{\gamma\gamma} = -e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{N^\Delta_{box}(k)}{D^\Delta_{box}(k)} - e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{N^\Delta_{x-box}(k)}{D^\Delta_{x-box}(k)}
\]

**Numerators**

\[
N^\Delta_{box}(k) = \overline{U}(p_4)V^{\mu\alpha}_{\Delta in}(p_2 + k, q - k) [\not \! p_2 + \not \! k' + M_\Delta] \mathcal{P}_{\alpha\beta}^{3/2}(p_2 + k) V^\beta\nu_{\Delta out}(p_2 + k, k) U(p_2) \\
\times \overline{u}(p_3) \gamma_\mu [\not \! p_1 - \not \! k' + m_e] \gamma_\nu u(p_1)
\]

\[
N^\Delta_{x-box}(k) = \overline{U}(p_4)V^{\mu\alpha}_{\Delta in}(p_2 + k, q - k) [\not \! p_2 + \not \! k' + M_\Delta] \mathcal{P}_{\alpha\beta}^{3/2}(p_2 + k) V^\beta\nu_{\Delta out}(p_2 + k, k) U(p_2) \\
\times \overline{u}(p_3) \gamma_\nu [\not \! p_3 + \not \! k' + m_e] \gamma_\mu u(p_1)
\]

spin-3/2 projection operator

\[
\mathcal{P}_{\alpha\beta}^{3/2}(p) = g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{1}{3p^2} (\not \! p \gamma_\alpha p_{\beta} + p_{\alpha} \gamma_\beta \not \! p)
\]
\[ \Delta \text{ has opposite slope to } N \]

\[ \text{cancels some of TPE correction from } N \]
weaker $\varepsilon$ dependence than with $N$ alone

better fit to JLab data!
\[ J^P = \frac{1^+}{2} , \frac{1^-}{2} \] excited \( N^* \) states

\[ Q^2 \sim 3 \text{ GeV}^2 \]

higher mass resonance contributions small

enhance nucleon elastic contribution
Effect on neutron form factors
Neutron correction

since $G_E^n$ is small, effect may be relatively large

sign opposite to proton (since $\kappa_n < 0$)
Effect on neutron LT form factors

large effect at high $Q^2$ for LT-separation method

LT method unreliable for neutron

Blunden, WM, Tjon
Effect on neutron PT form factors

\[ \frac{\mu_n G^n_E}{G_M} \]

\( Q^2 \) (GeV\(^2\))

- uncorrected
- corrected \( \varepsilon = 0.3 \)
- corrected \( \varepsilon = 0.8 \)

Blunden, WM, Tjon

small correction for PT

4% (3%) suppression at \( \varepsilon = 0.3 \) (0.8) for \( Q^2 = 3 \) GeV\(^2\)

10% (5%) suppression at \( \varepsilon = 0.3 \) (0.8) for \( Q^2 = 6 \) GeV\(^2\)
Summary

- First explicit calculation of TPE taking into account nucleon structure

- Nucleon elastic intermediate states resolves most of LT/PT $G_E^p/G_M^p$ discrepancy

- $\Delta$ excited state opposite sign cf. nucleon, but smaller $P_{11}(1440)$ and $S_{11}(1535)$ contributions small

- Effect on neutron form factors large for LT method, small for PT method
The End