The Origin of Nuclear Forces: QCD and the Structure of Hadrons

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GRC: Today’s Frontiers in Nuclear Physics
Bates College: July 10th, 2005
Special Mentions......
Outline

• The QCD vacuum
• Quarks to Hadrons
• Things we know about NN forces and nuclei…
• Inevitable consequences and important links…
• Nuclei emerging from QCD
• What needs measuring?
Major Challenges for Nuclear Physics

- Origin of Nuclear Saturation

- EOS ... as $\rho \uparrow$; as $T \uparrow$; as $S \uparrow$; as $N-Z \uparrow$

- Phase Transition to:
  - quark matter (QM), superconducting QM, strange condensate
  - related to nuclear astrophysics; n-stars....
QCD and the Origin of Mass

\[ u + u + d = \text{proton} \]

mass: \[ 0.003 + 0.003 + 0.006 \neq 0.938 \]

HOW does the rest of the proton mass arise?
Free space:

\[
\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = -(225 \pm 25 \text{ MeV})^3
\]

at a renormalization scale of about 1 GeV.

- commutator measures chiral symmetry breaking
  \( \approx \) valence + pion cloud +
  volume * (difference of condensate in & out of N)

and last term is as big as 20 MeV (or more)

i.e. presence of nucleon “cleans out” vacuum to some extent

Hence: Model independent LO term for in-medium condensate

\[
\frac{Q(\rho_B)}{Q_0} \approx 1 - \frac{\sigma_N}{f^2 \pi \rho_B}
\]

BUT this has no new physics at all!
Lattice QCD Simulation of Vacuum Structure

<\rho> = 0.16 \text{ fm}

Leinweber, Signal et al.
Strangeness Widely Believed to Play a Major Role – Does It?

• As much as 100 to 300 MeV of proton mass:

\[ M_N = \langle N(P) | -\frac{9\alpha_s}{4\pi} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d + m_s \bar{\psi}_s \psi_s | N(P) \rangle \]

\[ \Delta M_{N}^{s-\text{quarks}} = \frac{y m_s}{m_u + m_d} \sigma_N \]

\[ y = 0.2 \pm 0.2 \]

45 ± 8 MeV (or 70?)

Hence 110 ± 110 MeV (increasing to 180 for higher \( \sigma_N \))

• Through proton spin crisis:
  As much as 10% of the spin of the proton

• HOW MUCH OF THE MAGNETIC FORM FACTOR?
G0 Experiment at Jefferson Lab
World Data @ $Q^2 = 0.1$ GeV$^2$

$$G_E = -0.013 \pm 0.028$$

$$G_M = +0.62 \pm 0.31 \pm 0.62 \ 2\sigma$$

Contours

- $1\sigma$, $2\sigma$
- 68.3, 95.5% CL

Theories

1. Leinweber, et al.
   PRL 94 (05) 212001

2. Lyubovitskij, et al.
   PRC 66 (02) 055204

3. Lewis, et al.
   PRD 67 (03) 013003

   PRD 65 (01) 014016
Significance & Comparison with Lattice QCD

- Size and sign of the strange magnetic moment is astonishing!

- Experimental *isoscalar* nucleon moment is 0.88 $\mu_N$
  
c.f. this result which is (Beck) - 0.54 $\mu_N$ : i.e. - 60% !!

- Also remarkable versus lattice QCD which gives
  
  $+0.03 \pm 0.01 \, \mu_N$ (Leinweber et al., PRL 94 (2005) 212001)

- Sign would require violation of universality of
  
  valence quark moments by $\sim 70\%$ !
Magnetic Moments within QCD

\[ p = \frac{2}{3} \, u^p - \frac{1}{3} \, d^p + O_N \]
\[ n = -\frac{1}{3} \, u^p + 2/3 \, d^p + O_N \]

\[ \Sigma^+ = \frac{2}{3} \, u^\Sigma - \frac{1}{3} \, s^\Sigma + O_{\Sigma} \]
\[ \Sigma^- = -\frac{1}{3} \, u^\Sigma - 1/3 \, s^\Sigma + O_{\Sigma} \]

HENCE: \[ O_N = \frac{1}{3} \left[ 2p + n - \left( \frac{u^p}{u^\Sigma} \right) (\Sigma^+ - \Sigma^-) \right] \]

Just these ratios from Lattice QCD

OR \[ O_N = \frac{1}{3} \left[ n + 2p - \left( \frac{u^n}{u^\Sigma} \right) (\Xi^0 - \Xi^-) \right] \]
Constraint from Charge Symmetry

\[ O_N = \frac{2}{3} \ell G^u_M - \frac{1}{3} \ell G^d_M - \frac{1}{3} \ell G^s_M \]
\[ = \frac{1}{3} \left( \ell G^d_M - \ell G^s_M \right) , \]
\[ = \frac{\ell G^s_M}{3} \left( \frac{1 - \ell R^s_d}{\ell R^s_d} \right) , \]

\[ G^s_M = \left( \frac{\ell R^s_d}{1 - \ell R^s_d} \right) \left[ 3.673 - \frac{u_P}{u_{\Sigma^+}} (3.618) \right] \]

\[ G^s_M = \left( \frac{\ell R^s_d}{1 - \ell R^s_d} \right) \left[ -1.033 - \frac{u_n}{u_{\Xi^0}} (-0.599) \right] \]

$u^p_{\text{valence}} : \text{QQCD Data Corrected for Full QCD Chiral Coeff'\text{'s}}$

New lattice data from Zanotti et al. ; Chiral analysis Leinweber et al.

c.f. CQM
$\frac{2}{3} 940/540 \sim 1.18$

$\mu (\mu_R) = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \chi \text{ 'al loops}$
$u^\Sigma_{\text{valence}}$
Check: Octet Magnetic Moments

Leinweber et al., hep-lat/0406002
# State of the ART Magnetic Moments

<table>
<thead>
<tr>
<th></th>
<th>QQCD</th>
<th>Valence</th>
<th>Full QCD</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>2.69 (16)</td>
<td>2.94 (15)</td>
<td>2.86 (15)</td>
<td>2.79</td>
</tr>
<tr>
<td>n</td>
<td>-1.72 (10)</td>
<td>-1.83 (10)</td>
<td>-1.91 (10)</td>
<td>-1.91</td>
</tr>
<tr>
<td>Σ⁺</td>
<td>2.37 (11)</td>
<td>2.61 (10)</td>
<td>2.52 (10)</td>
<td>2.46 (10)</td>
</tr>
<tr>
<td>Σ⁻</td>
<td>-0.95 (05)</td>
<td>-1.08 (05)</td>
<td>-1.17 (05)</td>
<td>-1.16 (03)</td>
</tr>
<tr>
<td>Λ</td>
<td>-0.57 (03)</td>
<td>-0.61 (03)</td>
<td>-0.63 (03)</td>
<td>-0.613 (4)</td>
</tr>
<tr>
<td>Ξ⁰</td>
<td>-1.16 (04)</td>
<td>-1.26 (04)</td>
<td>-1.28 (04)</td>
<td>-1.25 (01)</td>
</tr>
<tr>
<td>Ξ⁻</td>
<td>-0.65 (02)</td>
<td>-0.68 (02)</td>
<td>-0.70 (02)</td>
<td>-0.651 (03)</td>
</tr>
<tr>
<td>uᵖ</td>
<td>1.66 (08)</td>
<td>1.85 (07)</td>
<td>1.85 (07)</td>
<td>1.81 (06)</td>
</tr>
<tr>
<td>u⁻</td>
<td>-0.51 (04)</td>
<td>-0.58 (04)</td>
<td>-0.58 (04)</td>
<td>-0.60 (01)</td>
</tr>
</tbody>
</table>
Accurate Final Result for $G_M^s$

Yields: $G_M^s = -0.046 \pm 0.019 \, \mu_N$

Leinweber et al., (PRL June '05) hep-lat/0406002
Parity Violating Studies on $^1$H and $^4$He

$3 \text{ GeV beam in Hall A}$ \quad $\theta_{lab} \sim 6^\circ$ \quad $Q^2 \sim 0.1 \text{ (GeV/c)}^2$

<table>
<thead>
<tr>
<th>target</th>
<th>$A_{PV}$ (G$^s = 0$ (ppm))</th>
<th>Stat. Error (ppm)</th>
<th>Syst. Error (ppm)</th>
<th>sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>-1.6</td>
<td>0.08</td>
<td>0.04</td>
<td>$\delta (G^s_E + 0.08G^s_M) = 0.010$</td>
</tr>
<tr>
<td>$^4$He</td>
<td>+7.8</td>
<td>0.18</td>
<td>0.18</td>
<td>$\delta (G^s_E) = 0.015$</td>
</tr>
</tbody>
</table>

Septum magnets (not shown)
High Resolution Spectrometers detectors

Brass-Quartz integrating detector

Elastic Rate:

$^1$H: 120 MHz
$^4$He: 12 MHz

Background $\leq 3\%$
χ’al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon

FRR give same answer to <<1% systematic error!

Leinweber et al., PRL 92 (2004) 242002

<table>
<thead>
<tr>
<th>Regulator</th>
<th>Bare Coefficients</th>
<th>Renormalized Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_0^\Lambda$</td>
<td>$a_2^\Lambda$</td>
</tr>
<tr>
<td>Monopole</td>
<td>1.74</td>
<td>1.64</td>
</tr>
<tr>
<td>Dipole</td>
<td>1.30</td>
<td>1.54</td>
</tr>
<tr>
<td>Gaussian</td>
<td>1.17</td>
<td>1.48</td>
</tr>
<tr>
<td>Sharp cutoff</td>
<td>1.06</td>
<td>1.47</td>
</tr>
<tr>
<td>Dim. Reg. (BP)</td>
<td>0.79</td>
<td>4.15</td>
</tr>
</tbody>
</table>
Comparison with $\chi$ QSM

Goeke et al., hep-lat/0505010
Analysis of pQQCD $\rho$ data from CP PACS

\[ \sqrt{(M_V^{\text{deg}})^2 - \Sigma_{TOT}} = (a_0^{\text{cont}} + X_1 a + X_2 a^2) + a_2(M_{PS}^{\text{deg}})^2 + a_4(M_{PS}^{\text{deg}})^4 + a_6(M_{PS}^{\text{deg}})^6 \]
Infinite Volume Unitary Results

All 80 data points drop onto single, well defined curve

Allton, Young et al., hep-lat/0504022
Nuclear Structure in Terms of QCD

Aim for:

• New physical insight into nuclear structure
• e.g. new mechanism for nuclear saturation!
• Precursors of the deconfinement transition?
• Guidance as to signals of deconfinement transition
• Signals of chiral restoration as density rises....
• Changes in hadron properties in-medium!
What do we know about Nuclear Environment?

• Walecka et al., (QHD): Lorentz structure of attraction and repulsion is crucial (\(\sigma\) and \(\omega\) respectively)

• NOT arbitrary – inspired by Paris potential, built on dispersion relations \(\Rightarrow I=0, J^\pi = 0^+\) channel dominates intermediate range attraction (origin two-pion \(\approx \sigma\) exchange)

• Modern version: Machleidt et al., RBHF \(\Rightarrow g_\sigma \sigma \approx 400\) MeV

   i.e. There are strong (\(\sim 0.4 \, M_N\)) Lorentz scalar fields in nuclei........

   so what?
What happens if we put an atom in a strong electric field?

Jackson ⇒

i.e. atom has a polarizability: its internal structure is rearranged in response to applied field

///'ly in applied magnetic field (indeed, in super strong field - e.g. n-star surface atoms & molecules essentially linear!)
Electric & Magnetic Polarizabilities of Nucleon are Measured

e.g. Compton scattering:

\[ 4\pi \alpha_E = 2 \sum_{I \neq N} \frac{|\langle I | d_z | N \rangle|^2}{E_I - E_N} \]

\[ \alpha_E^P = (12.1 \pm 1.3) \times 10^{-4} \text{ fm}^3, \]
\[ \beta_M^P = (2.1 \pm 1.3) \times 10^{-4} \text{ fm}^3. \]

Also Virtual Compton Scattering \( \Rightarrow \) GPs
So what?

• Atoms respond to external E and B fields

• Nucleons respond to external E and B fields

• Nucleons must respond large scalar fields in-medium! (scalar polarizability)

• Change of hadron mass $m \rightarrow m^*$ is accepted and studied

What about other properties?
Classic Illustration: The EMC effect

- Observation stunned and electrified the HEP community 20 years ago
- Nearly one thousand papers have been generated.....
- What is it that alters the quark momentum in the nucleus?

\[ \frac{\sigma(A)}{\sigma(D)} \]


Parton Distribution Functions of Nuclei

For those old enough to remember there were two responses to 1983 EMC discovery

a) Complete shock

b) So what: no reason for

\[ f_{q/N}(z) = p_- \int \frac{dw^-}{2\pi} e^{ip_z w^-} <N, p|\bar{\psi}(0)\gamma^+\psi(w^-)|N, p> \]

to be related to

\[ f_{q/A}(y_A) = \frac{P}{A^2} \int \frac{dw^-}{2\pi} e^{iP_y W^-/A} <A, P|\bar{\psi}(0)\gamma^+\psi(w^-)|A, P> \]

They are two different eigenstates of QCD Hamiltonian…

END of STORY!

i.e. NO derivation at all, within QCD (THE theory of the strong interaction) of a convolution of nucleon motion with free structure function!
Fundamental Question: “What is the Scalar Polarizability of the Nucleon?”

Nucleon response to a chiral invariant scalar field is then a nucleon property of great interest...

\[ M^* (\vec{R}) = M - g_\sigma \sigma(\vec{R}) + \frac{d}{2} \left( g_\sigma \sigma(\vec{R}) \right)^2 \]

Non-linear dependence \( \equiv \) scalar polarizability
\[ d \approx 0.22 \text{ R in original QMC (MIT bag)} \]

Indeed, in nuclear matter at mean-field level (e.g. QMC), this is the ONLY place the response of the internal structure of the nucleon enters.
Quark-Meson Coupling Model: QMC*

Intermediate step to full quark-gluon theory of $\infty$ nuclear matter

• Use successful model of hadron structure: MIT Bag

• Couple scalar ($\sigma: 0^+$) and vector ($\omega: 1^-$) mesons to confined quarks

• Confined quarks generate mean scalar and vector fields

• Scalar field changes confined quark wave function

$$[i\gamma^\mu \partial_\mu - (m_q - g_\sigma q\bar{\sigma}) - \gamma^0 g_\omega q\bar{\omega}]\psi = 0$$

• This changes source……..


Review: Saito et al., hep-ph/0506314
Numerical Results

Scalar mean-field \( \sim \frac{1}{3} \text{rd of QHD} \)

- Decrease of \( g_\sigma(\sigma) \) as density \( \uparrow \)
  - (effect of scalar polarizability)

\[
\begin{align*}
g_\sigma(\sigma) \text{ (MeV)} & \quad \rho_B / \rho_0 \\
& \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5
\end{align*}
\]

\[
\begin{align*}
g_\sigma(\sigma) \text{ (MeV)} & \quad 0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \quad 300 \quad 350 \quad 400
\end{align*}
\]
QMC: Generalization to Finite Nuclei*


- Use Born-Oppenheimer approximation…
  i.e. assume internal “nucleon” structure
  adjusts to local mean scalar field

  ⇒ 3% accuracy in typical nuclei

- I KNOW OF NO OTHER WAY TO DERIVE EXISTENCE
  OF NUCLEI WITHIN QCD

  ⇒ CHANGE IN PARADIGM

- everyone has heard of shell model

BUT: what occupies shell model orbits are NOT free nucleons
Application of QMC to Nuclear Density

$^{40}\text{Ca}$

- Walecka–Serot
- experiment
- this work

Linking QMC to Familiar Nuclear Theory

• Since early 70’s tremendous amount of work in nuclear theory is based upon effective force

• Used for everything from nuclear astrophysics to collective excitations of nuclei

• Skyrme Force: Vautherin and Brink

• Systematic phenomenology analogous to phase shifts connecting data and deeper derivation of NN force

Effective Interaction is of Skyrme III Type:

\[ V = t_3 \sum_{i<j<k} \delta(\vec{R}_{ij})\delta(\vec{R}_{jk}) + t_0 \sum_{i<j} (1 + x_0 P_\sigma)\delta(\vec{R}_{ij}) \]

\[ + \frac{1}{4} t_2 \vec{\nabla}_{ij} \delta(\vec{R}_{ij})\vec{\nabla}_{ij} - \frac{1}{8} t_1 \left[ \delta(\vec{R}_{ij})\vec{\nabla}_{ij}^2 + \vec{\nabla}_{ij}^2 \delta(\vec{R}_{ij}) \right] \]

\[ + \frac{i}{4} \vec{W}_0 (\vec{\sigma}_i + \vec{\sigma}_j) \cdot \vec{\nabla}_{ij} \times \delta(\vec{R}_{ij})\vec{\nabla}_{ij}, \]

and from comparison with QMC:

\[ t_0 = -G_\sigma + G_\omega - \frac{G_\rho}{4}, \]

\[ t_3 = 3dG_\sigma^2, \]

\[ x_0 = -\frac{G_\rho}{2t_0}. \]
## Comparison Between Skyrme III and QMC

<table>
<thead>
<tr>
<th></th>
<th>QMC</th>
<th>QMC</th>
<th>SkIII</th>
<th>QMC(N=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_\sigma (MeV)$</td>
<td>500</td>
<td>600</td>
<td>600</td>
<td>-1047</td>
</tr>
<tr>
<td>$t_0 (MeV fm^3)$</td>
<td>-1071</td>
<td>-1082</td>
<td>-1129</td>
<td>-1047</td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.89</td>
<td>0.59</td>
<td>0.45</td>
<td>0.61</td>
</tr>
<tr>
<td>$t_3 (MeV fm^6)$</td>
<td>16620</td>
<td>14926</td>
<td>14000</td>
<td>12996</td>
</tr>
<tr>
<td>$M_{eff}/M$</td>
<td>.915</td>
<td>.814</td>
<td>.763</td>
<td>.821</td>
</tr>
<tr>
<td>$5t_2 - 9t_1 (MeV fm^5)$</td>
<td>-7622</td>
<td>-4330</td>
<td>-4030</td>
<td>-4036</td>
</tr>
<tr>
<td>$W_0 (MeV fm^5)$</td>
<td>118</td>
<td>97</td>
<td>120</td>
<td>91</td>
</tr>
</tbody>
</table>

\[
\frac{M_{eff}}{M} = \left(1 + \frac{(3t_1 + 5t_2)M\rho_0}{8}\right)^{-1}
\]
Great Start: What’s Next

- Remove zero-range approximation
- Derive density-dependent forms
- Add the pion
- Derive $\Lambda N$, $\Sigma N$, $\Lambda \Lambda$ ... effective forces in-medium with no additional free parameters!
- Hence attack dense hadronic matter, n-stars, transition from NM to QM or SQM with more confidence
Experimental consequences

• Form factors: $G_{E, M, A}$

• Parton Distribution Functions

• Generalized Parton Distribution Functions

• more masses…..

e.g. Experiments on both $\rho$ and $\omega$ in finite nuclei* show predicted mass shifts

Further tests planned for $\eta$ and $\eta'$

*Ttnka et al., PRL 94 (2005) 192303
Huber et al., PR C68 (2003) 065202
Properties of quark systems in-medium: Origin of the EMC effect

• Observation that structure functions are altered in nuclei stunned and electrified much of the HEP community 30 years ago

• What is it that alters the quark momentum in the nucleus?

Quark Model of QMC type: NJL with Confinement & Saturation of Nuclear Matter

• Similar to QMC of Guichon and collaborators

• Advantage that NJL is completely covariant

• Confinement modeled through “proper time regularization*” (Ebert et al., Phys Lett 388 (1996) 154)

• Saturation of nuclear matter as consequence of scalar polarizability of nucleon (response of light quarks to an applied scalar field)

Mineo et al., Nucl Phys A735 (2004) 482

* After Feynman parameters and Wick rotation:

\[ \frac{1}{A^n} \rightarrow \frac{1}{(n-1)!} \int_{1/\Lambda_{IR}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau A} \quad (n \geq 1) \]
$g_1(A)$ – “Polarized EMC Effect”

- New calculations indicate larger effect for polarized structure than unpolarized: scalar field modifies lower cpts of Dirac wave function
  
  (Cloet, Bentz, AWT, Phys Rev Lett, to appear: nucl-th/0504019)
- Spin-dependent parton distribution functions for nuclei unknown

![Graph showing $R_{A/N}(x)$ with $Q^2 = 10.0 \text{ GeV}^2$ and $\rho = 0.17 \text{ fm}^{-3}$]
Proton and Neutron Asymmetry

Covariant, NJL model with confinement and QMC mechanism for saturation of nuclear matter

Lawley, Bentz, AWT, nucl-th/0504020
Conclusion

- Quark level approach to nuclear structure leads naturally to effective force of Skyrme type
- Numerical agreement between QMC and Skyrme III ~10%
- Key ingredient is scalar polarizability of nucleon
- Dramatic change to how we view shell model
- Experimental consequences just now being explored

and…. lattice QCD naturally yields scalar polarizability of correct sign and magnitude (Nishinomiya, to be published)
Big Picture

• QCD \Rightarrow \text{phase transition at high density (and T)}

• Is dense matter (n-star) nuclear/strange/QM/superconducting QM/color condensate?

• Changes at low density are precursors of what happens under more extreme conditions

• Crucial part of our understanding of these phenomena

• Theoretical and experimental studies of these kinds are the only systematic way to make progress
Rapid Convergence from LNA to NLNA

<table>
<thead>
<tr>
<th>Regulator</th>
<th>LNA</th>
<th>NLNA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dim. regulator</td>
<td>0.784</td>
<td>0.884 ± 0.103</td>
</tr>
<tr>
<td>Dim. regulator (BP)</td>
<td>0.784</td>
<td>0.923 ± 0.103</td>
</tr>
<tr>
<td>Sharp cutoff</td>
<td>0.968</td>
<td>0.961 ± 0.116</td>
</tr>
<tr>
<td>Monopole</td>
<td>0.964</td>
<td>0.960 ± 0.116</td>
</tr>
<tr>
<td>Dipole</td>
<td>0.963</td>
<td>0.959 ± 0.116</td>
</tr>
<tr>
<td>Gaussian</td>
<td>0.966</td>
<td>0.960 ± 0.116</td>
</tr>
</tbody>
</table>
FRR Mass well determined by data

\[ \sqrt{(M_{V}^{\text{deg}})^2 - \Sigma_{TOT}} = (a_0^{\text{cont}} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{\text{deg}})^2 + a_4 (M_{PS}^{\text{deg}})^4 + a_6 (M_{PS}^{\text{deg}})^6 \]
Neutron Star Composition

Hyperons enter at just 2-3 $\rho_0$

Hence need effective $\Sigma$-N and $\Lambda$-N forces in this density region!

Hypernuclear data is important input – but not enough

Wang, Lawley et al., nucl-th/0506014
Spin Dependent PDFs

Data: Hermes & JLab

Cloet, Bentz, AWT, hep-ph/0504229
Free Spin Independent PDFs

Cloet, Bentz, AWT, hep-ph/0504229
Excited States are more Difficult

Crouch et al., QQCD unquenched using chiral coefficients from quark model of Capstick
Predicted “EMC” Effect Bigger for Spin

First “estimate” in $^3$He of medium effects in spin dependent PDFs in QMC model

Flavor Dependence of EMC Effect

Cloet, Bentz, AWT, nucl-th/0504019
Flavor Dependence of PDFs

\[ Q^2 = 5.0 \text{ GeV}^2 \]

- Model ratio
- Empirical ratio

\[ \Delta d_v/\Delta u_v \text{ and } d_v/u_v \]

_Cloet, Bentz, AWT, hep-ph/0504229_