Meson Radiative Transitions on the Lattice

hybrids and charmonium

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JLab, GlueX and photocouplings

- GlueX plans to photoproduce mesons
- especially exotic $J^{PC}$ mesons

\[ \langle \gamma M | X \rangle \]
photoproduction of exotics?

• exotic quantum numbers $1^{-+}, 0^{+-}, 2^{+-}$ may be explicable as hybrid meson states

• photoproduction an untested method

• relies upon reasonably large couplings
e.g. $\left\langle \gamma \frac{\rho}{a_2} | \pi_1 \right\rangle$

• large in some model estimations
Lattice QCD estimation?

• relatively straightforward in principle; evaluate three-point function with a vector current

• in practice, not so easy
  • truly light quarks unfeasible
  • transitions involve unstable states
  • experimental data is limited and imprecise (even for conventional meson transitions)
pragmatic approach

- try out an untested method in a region where approximations are controllable
- and where there is good experimental data to compare with charmonium
Charmonium - expt.

• multiple states below DD threshold have narrow widths

• radiative transitions are big branching fractions

• precision measurements
Charmonium - lattice

- states are small - small volumes OK
- quenched theory not sick* - just not expt.
- disconnected diagrams perturbatively suppressed
- need a fine lattice spacing $a \gtrsim 3$ GeV?

* "one heavy flavour QCD"; will only notice non-unitary up near 6 GeV
anisotropy

- charm quark mass scale requires a fine lattice
- but only in the temporal direction?
- spatial scale $\sim \| \vec{p} \| \sim 500 \text{ MeV}$
- so space direction can be more coarse
- introduce anisotropy param into fermion action and tune to get meson disp$^n$ rel$^{\text{ns}}$ right
our initial simulation

- anisotropic Wilson glue with $\xi = 3$ at $\beta = 6.1$
- $12^3 \times 48$ gives a 1.2 fm box
- Domain-Wall fermions ($L_5 = 16$)
  - Ginsparg-Wilson ensures $O(a)$ improvement
  - vector current only multiplicatively renorm$^d$

spectroscopic splittings came out reasonably - usual quenched problem of hyperfine too small

scale setting by Sommer parameter, but 1P-1S very similar
three-point functions

\[ \Gamma(t_f, t; \vec{p}_f, \vec{q}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_f \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \langle \varphi_f(\vec{x}, t_f) j^\mu(\vec{y}, t) \varphi_i(\vec{0}, 0) \rangle \]

• we use gaussian smeared fermion bilinears as interpolating fields

\[ \sum_{\vec{z}} F(\vec{z}) \bar{\psi}_{\vec{x}+\vec{z}, t} \Gamma \psi_{\vec{x}-\vec{z}, t} \]

• connected diagram constructed from forward propagator and sequential sink propagator with the simple point-like vector current

new inversion for each change of the sink, but all possible momenta inserted at the current
three-point functions and matrix elements

\[ \Gamma(t_f, t; \vec{p}_f, \vec{q}) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p}_f \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \langle \varphi_f(\vec{x}, t_f) j^\mu(\vec{y}, t) \varphi_i(\vec{0}, 0) \rangle \]

- inserting two complete sets of states

\[ \Gamma(t_f, t; \vec{p}_f, \vec{q}) = \frac{Z_i Z_f}{4E_i E_f} e^{-E_f (t_f - t)} e^{-E_i t} \langle f(\vec{p}_f) | j^\mu(0) | i(\vec{p}_i) \rangle \]

obtained from fits to two-point functions

to be extracted
$\eta_c$ ‘form-factor’

• strictly speaking this does not exist due to charge conjugation invariance

\[ 0^- \not\to 0^- + 1^- \]

\[ 0^- \to 0^- 1^- \neq 0 \]
$\eta_c$ ‘form-factor’

$$\langle \eta_c(\vec{p}_f)|j^\mu(0)|\eta_c(\vec{p}_i)\rangle = f(Q^2)(p_i + p_f)^\mu$$

$$\Gamma(t_f = 24, t)$$

$$\frac{Z_i Z_f}{4 E_i E_f} e^{-E_f(t_f - t) - E_i t} (p_f + p_i)^{\mu_{0.8}}$$

plateaux observed

$\vec{p}_f = (000)$
\(\eta_c \, \text{‘form-factor’}\)
\[ \eta_c \text{ ‘form-factor’} \]

\[ \left(1 + \frac{Q^2}{m_{\psi}^2}\right)^{-1} \sqrt{\langle r^2 \rangle} = 0.25 \text{ fm} \]

\[ \exp \left[ -\frac{Q^2}{16\beta^2} \left(1 + \alpha Q^2 \right) \right] \]
\( \eta_c \) ‘form-factor’
- not VMD?
J/ψ ‘form-factors’

• vector particle has three form-factors (c.f. deuteron)
  • charge
  • magnetic
  • quadrupole

\[
\langle V(\vec{p}_f, r_f) | j_\mu(0) | V(\vec{p}_i, r_i) \rangle \\
= -(p_f + p_i)\mu \left[ G_1(Q^2) \epsilon^*(\vec{p}_f, r_f) \cdot \epsilon(\vec{p}_i, r_i) \\
+ \frac{G_3(Q^2)}{2m_V^2} \epsilon^*(\vec{p}_f, r_i) \cdot p_i \epsilon(\vec{p}_i, r_i) \cdot p_f \right] \\
+ G_2(Q^2) \left[ \epsilon_\mu(\vec{p}_i, r_i) \epsilon^*(\vec{p}_f, r_f) \cdot p_i + \epsilon^{\mu*}(\vec{p}_f, r_f) \epsilon(\vec{p}_i, r_i) \cdot p_f \right]
\]
$J/\psi$ ‘form-factors’

$\sqrt{\langle r^2 \rangle} \sim 0.25$ fm

$\kappa_c \approx 0$

$a_D(J/\psi) \sim 10^{-3}$
\( \chi_{c0} \) ‘form-factors’

\[ \sqrt{\langle r^2 \rangle} \sim 0.3 \text{ fm} \]

larger radius due to centripetal barrier in P-wave meson
so much for unobservables, how about observables?
$J/\psi \rightarrow \eta_c \gamma$ transition

$$\langle \eta_c(p')|j^\mu(0)|J/\psi(p,r)\rangle = \frac{2V(Q^2)}{m_{\eta_c} + m_{\psi}} \epsilon^{\mu\alpha\beta\gamma} p'_\alpha p_\beta \epsilon_\gamma(p,r)$$

very sensitive to hyperfine

$m_{J/\psi} - m_{\eta_c}|_{\text{expt.}} = 117$ MeV

$m_{J/\psi} - m_{\eta_c}|_{\text{our lat.}} \sim 80$ MeV
\[ \frac{J/\psi \rightarrow \eta_c \gamma}{\text{transition}} \]

\[ \Gamma(J/\psi \rightarrow \eta_c \gamma) = \alpha \frac{|\vec{q}|^3}{(m_{\eta_c} + m_{\psi})^2} \frac{64}{27} \left| \hat{V}(0) \right|^2. \]

\[ V(Q^2) = V(0) e^{-\frac{Q^2}{16\beta^2}} \]

**phase space**

**physical**

**lattice**

beware - nobody gets this number ‘right’

new CLEO-c number soon!
P-wave to S-wave transitions

many good measurements
\( \chi_{c0} \to J/\psi \gamma \) transition

our ‘poster boy’

covariant multipole decomposition of matrix element

\[
\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V, r)\rangle =
\Omega^{-1}(Q^2) \left( E_1(Q^2) \left[ \Omega(Q^2) \epsilon^\mu(\vec{p}_V, r) - \epsilon(\vec{p}_V, r).p_S(p_V.p_S - m_V^2 p_S^\mu) \right] + \frac{C_1(Q^2)}{\sqrt{q^2}} m_V \epsilon(\vec{p}_V, r).p_S \left[ p_V.p_S(p_V + p_S)^\mu - m_S^2 p_V^\mu - m_V^2 p_S^\mu \right] \right).
\]

\( E_1 \) - electric dipole, \text{exptally} measured at \( Q^2 = 0 \)

\( C_1 \) - longitudinal, only non-zero at non-zero \( Q^2 \)
\[
\chi_{c0} \rightarrow J/\psi \gamma \quad E1 \text{ transition}
\]

**our ‘poster boy’**

Not used in the fit

\[
E_1(Q^2) = E_1(0) \left(1 + \frac{Q^2}{\rho^2}\right) e^{-\frac{Q^2}{16\beta^2}}
\]
$\chi_{c0} \rightarrow J/\psi \gamma$ CI transition
\[ \chi_{c1} \rightarrow J/\psi\gamma \] transition

covariant multipole decomposition of matrix element

\[
\langle A(\vec{p}_A, r_A) | j^\mu(0) | V(\vec{p}_V, r_V) \rangle = \frac{i}{4\sqrt{2}|Q^2|} \epsilon^{\mu\nu\rho\sigma} (p_A - p_V)_{\sigma} \times \\
\times \left[ E_1(Q^2)(p_A + p_V)_{\rho} \left( 2m_A [\epsilon^*(\vec{p}_A, r_A) \cdot p_V] \epsilon_\nu(p_V, r_V) + 2m_V [\epsilon(\vec{p}_V, r_V) \cdot p_A] \epsilon^*_\nu(\vec{p}_A, r_A) \right) \\
+ M_2(Q^2)(p_A + p_V)_{\rho} \left( 2m_A [\epsilon^*(\vec{p}_A, r_A) \cdot p_V] \epsilon_\nu(p_V, r_V) - 2m_V [\epsilon(\vec{p}_V, r_V) \cdot p_A] \epsilon^*_\nu(\vec{p}_A, r_A) \right) \\
+ \frac{C_1(Q^2)}{\sqrt{q^2}} \left( -4\Omega(Q^2) \epsilon^*_\nu(\vec{p}_A, r_A) \epsilon_\rho(\vec{p}_V, r_V) \right) \\
+ (p_A + p_V)_{\rho} \left( (m_A^2 - m_V^2 + q^2) [\epsilon^*(\vec{p}_A, r_A) \cdot p_V] \epsilon_\nu(p_V, r_V) + (m_A^2 - m_V^2 - q^2) [\epsilon(\vec{p}_V, r_V) \cdot p_A] \epsilon^*_\nu(\vec{p}_A, r_A) \right) \right] 
\]

\[ E_1 \] - electric dipole, expt\textsuperscript{ally} measured at \(Q^2 = 0\)

\[ M_2 \] - magnetic quadrupole, expt\textsuperscript{ally} measured at \(Q^2 = 0\)

\[ C_1 \] - longitudinal, only at non-zero \(Q^2\)
\[ \chi_{c1} \rightarrow J/\psi \gamma \] transition
$h_c \rightarrow \eta_c \gamma$ transition
quark potential model?

- our fitting form inspired by NR potential model with rel. corrections:

\[ E_1(Q^2) = E_1(0) \left( 1 + \frac{Q^2}{\rho^2} \right) e^{-\frac{Q^2}{16\beta^2}} \]

\[ \chi_{c0} \rightarrow J/\psi \gamma E_1 \]

\[ \beta = 542(35) \text{ MeV} \]

\[ \rho = 1.08(13) \text{ GeV} \]

\[ \chi_{c1} \rightarrow J/\psi \gamma E_1 \]

\[ \beta = 555(113) \text{ MeV} \]

\[ \rho = 1.65(59) \text{ GeV} \]

\[ h_c \rightarrow \eta_c \gamma E_1 \]

\[ \beta = 689(133) \text{ MeV} \]

\[ \rho \rightarrow \infty \]
what about $\chi_{c2} \rightarrow J/\psi \gamma$?

- can’t get at spin 2 with point-like fermion bilinears
- we have to extend our interpolating field set
## extended interpolators

<table>
<thead>
<tr>
<th>Operator</th>
<th>$O_h$ rep.</th>
<th>lowest $J^{PC}$</th>
<th>name</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$A_1$</td>
<td>$0^{++}$</td>
<td>$a_0$</td>
<td>$^3P_0(\chi_{c0})$</td>
</tr>
<tr>
<td>$\gamma_5$</td>
<td>$A_1$</td>
<td>$0^{-+}$</td>
<td>$\pi$</td>
<td>$^1S_0(\eta_c)$</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>$T_1$</td>
<td>$1^{--}$</td>
<td>$\rho$</td>
<td>$^3S_1(J/\psi)$</td>
</tr>
<tr>
<td>$\gamma_5\gamma_i$</td>
<td>$T_1$</td>
<td>$1^{++}$</td>
<td>$a_1$</td>
<td>$^3P_1(\chi_{c1})$</td>
</tr>
<tr>
<td>$\gamma_i\gamma_j$</td>
<td>$T_1$</td>
<td>$1^{+-}$</td>
<td>$b_1$</td>
<td>$^1P_1(h_c)$</td>
</tr>
<tr>
<td>$\gamma_5\nabla_i$</td>
<td>$T_1$</td>
<td>$1^{+-}$</td>
<td>$\pi \times \nabla$</td>
<td></td>
</tr>
<tr>
<td>$\nabla_i$</td>
<td>$T_1$</td>
<td>$1^{++}$</td>
<td>$a_0 \times \nabla$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_i\nabla_i$</td>
<td>$A_1$</td>
<td>$1^{+-}$</td>
<td>$a_0 \times \nabla$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{ijk}\nabla_j \nabla_k$</td>
<td>$E$</td>
<td>$1^{+-}$</td>
<td>$\rho \times \nabla A_1$</td>
<td></td>
</tr>
<tr>
<td>$s_{ijk}\nabla_j \nabla_k$</td>
<td>$T_2$</td>
<td>$2^{++}$</td>
<td>$\rho \times \nabla T_1$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_5\gamma_i\nabla_i$</td>
<td>$A_1$</td>
<td>$2^{+-}$</td>
<td>$\rho \times \nabla T_2$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_5\gamma_i\nabla_j \nabla_k$</td>
<td>$T_2$</td>
<td>$2^{+-}$</td>
<td>$a_1 \times \nabla A_1$</td>
<td><strong>exotic</strong></td>
</tr>
<tr>
<td>$\gamma_5\gamma_i\gamma_j \nabla_k$</td>
<td>$T_2$</td>
<td>$2^{+-}$</td>
<td>$a_1 \times \nabla T_2$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_5\gamma_i\gamma_j \nabla_k$</td>
<td>$T_2$</td>
<td>$2^{+-}$</td>
<td>$a_1 \times \nabla E$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_4\gamma_5\epsilon_{ijk}\gamma_j \nabla_k$</td>
<td>$E$</td>
<td>$2^{+-}$</td>
<td>$b_1 \times \nabla T_1$</td>
<td><strong>exotic</strong></td>
</tr>
<tr>
<td>$\gamma_4\gamma_5\gamma_i\nabla_j \nabla_k$</td>
<td>$T_2$</td>
<td>$3^{+-}$</td>
<td>$b_1 \times \nabla T_1$</td>
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</tr>
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<td>$s_{ijk}\gamma_j \nabla_k$</td>
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<td>$2^{+-}$</td>
<td>$\rho \times \nabla T_2$</td>
<td></td>
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<td>$2^{+-}$</td>
<td>$\rho \times \nabla T_2$</td>
<td></td>
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<tr>
<td>$\gamma_4\gamma_5 s_{ijk}\nabla_j \nabla_k$</td>
<td>$E$</td>
<td>$2^{+-}$</td>
<td>$\pi \times \nabla T_2$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_5 B_i$</td>
<td>$T_1$</td>
<td>$1^{+-}$</td>
<td>$\pi \times B T_1$</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{ijk}\gamma_j B_k$</td>
<td>$T_1$</td>
<td>$1^{+-}$</td>
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</tr>
<tr>
<td>$\gamma_5\gamma_i B_i$</td>
<td>$A_1$</td>
<td>$0^{+-}$</td>
<td>$a_1 \times B_A_1$</td>
<td><strong>exotic</strong></td>
</tr>
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<td>$\gamma_5 s_{ijk}\gamma_j B_k$</td>
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<td><strong>exotic</strong></td>
</tr>
</tbody>
</table>

Table 1: Meson operators, names and quantum numbers. $s_{ijk} = |\epsilon_{ijk}|$ and $S_{ijk} = 0(j \neq k), S_{111} = 1, S_{122} = -1, S_{222} = 1, S_{233} = -1.D_1 = s_{ijk}\nabla_j \nabla_k, B_i = \epsilon_{ijk} \nabla_j \nabla_k$

higher spins and the $J^{PC}$ exotics
extended interpolators

non-exotics

exotics

Extended interpolators

Effective mass (0+−)

Effective mass (1++)

Effective mass (2++)

Effective mass (2−+)

Effective mass (3−−)

Effective mass (1−−)

Effective mass (2+−)

Effective mass (3+−}

Effective mass (2+−)
next up?

• radiative transitions with this extended set
• think we can do two-photon decays
• charmonium for now
• dynamical lattices for precision & maybe multi-particle (DD) states
• start turning down the quark mass if it all ‘works’ to get at JLab physics
extra slides for the inquisitive
$\chi_{c0} \rightarrow J/\psi \gamma \ E1 \ transition$

$\overline{a_t} E_1(Q^2)$ vs. $Q^2 (GeV^2)$

Graph showing the transition $\chi_{c0} \rightarrow J/\psi \gamma$ with various data points and labels.

Data points include:
- spat. $p_f = (000) J/\psi_{snk}$
- spat. $p_f = (100) J/\psi_{snk}$
- spat. $p_f = (000) \chi_{c0 snk}$
- spat. $p_f = (100) \chi_{c0 snk}$
- PDG phys. mass
- PDG lat. mass
- CLEO phys. mass
- CLEO lat. mass
some two-point functions
multiple form-factors?

• pick out the three-point functions with the same $Q^2$ - various momentum and Lorentz index combinations

\[
\begin{bmatrix}
\Gamma(a; t) \\
\Gamma(b; t) \\
\Gamma(c; t) \\
\vdots
\end{bmatrix}
= 
\begin{bmatrix}
P(a; t)K_1(a) & P(a; t)K_2(a) & \cdots \\
P(b; t)K_1(b) & P(b; t)K_2(b) & \cdots \\
P(c; t)K_1(c) & P(c; t)K_2(c) & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}
\begin{bmatrix}
f_1(Q^2)[t] \\
f_2(Q^2)[t] \\
\vdots
\end{bmatrix},
\]

• invert this system with SVD

$P \cdot K$ are known quantities
$Z_V$

- set using meson form-factors at zero $Q^2$
quenched?

• scale setting ambiguity - running coupling
• non-unitarity a negligible issue
• above threshold states rendered stable - they were narrow anyway
anisotropy - $\text{disp}^n \text{ rel}^n$

$\sim 6\%$ deviation, could easily be reduced

\[ c^2(|\vec{p}|^2) \equiv \frac{E^2(|\vec{p}|^2) - m^2}{|\vec{p}|^2} \]
spectrum
‘wrap-around’ pollution

\[ \langle 0 | \varphi_f(t_f = 24) | f \rangle \langle f | j^\mu(t) | i \rangle \langle i | \varphi_i(t_i = 0) | 0 \rangle \]
\[ \sim Z_i Z_f \langle f | j^\mu(0) | i \rangle e^{-E_f (24-t) - E_i t} \]

\[ \langle 0 | j^\mu(t) | V \rangle \langle V | \varphi_i(0) | f \rangle \langle f | \varphi_f(t_i = -24) | 0 \rangle \]
\[ \sim Z_V Z_f \langle V | \varphi_i(0) | f \rangle e^{-E_v t - E_f 24} \]
‘wrap-around’ pollution

\[ Z_V \langle V | \varphi_i(0) | f \rangle e^{-(E_V - \delta E_{if})t} \]

\[ \frac{Z_V \langle V | \varphi_i(0) | f \rangle}{Z_i \langle f | j^\mu(0) | i \rangle} \]

\[ E_V \sim m_{J/\psi} \sim 3 \text{ GeV} \]

\[ \delta E_{if} \sim m_\chi - m_\psi \sim 600 \text{ MeV} \]

so wrap around should fall off relatively sharply.

if amplitude is large this will be a nasty pollution (prevents excited state extraction)
‘wrap-around’ pollution

rapid fall-off near $t=0$ indicative of the wrap-around pollution

we resorted to fitting the pollution with a single exponential

$$f_n(Q^2)[t] = f_n(Q^2) + f_i e^{-m_i t} + f_f e^{-m_f (24-t)}$$

$$\hat{V}(Q^2) = -1.55(1), f_i = 1.45(3), f_f = -0.42(14), m_i = 0.41(1), m_f = 0.27(7)$$
finite-size effects?

- previous charmonium spectrum studies saw no significant finite volume effects with $L_s \gtrsim 1.1$ fm

QCD-TARO collabn.

Table 6: Pseudoscalar mass and hyperfine splitting from non-perturbatively improved clover Dirac operator. The lattice spacing is fixed to 0.093 fm ($\beta = 6.0$) and the number of lattice points $L$, hence the physical volume $L_s$, is varied as indicated in the table. Results, averaged over 100 configurations (190 for $L = 8$), are given in physical units (MeV) with the scale set by $r_0$.

<table>
<thead>
<tr>
<th>$L$</th>
<th>$L_s$ (fm)</th>
<th>$^1S_0$ (MeV)</th>
<th>$^3S_1$ (MeV)</th>
<th>$^3S_1 - ^1S_0$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>2958(10)</td>
<td>3019(12)</td>
<td>61.4(4.4)</td>
</tr>
<tr>
<td>10</td>
<td>0.93</td>
<td>2953(5)</td>
<td>3023(6)</td>
<td>70.6(2.5)</td>
</tr>
<tr>
<td>12</td>
<td>1.12</td>
<td>2957(3)</td>
<td>3032(5)</td>
<td>75.4(2.7)</td>
</tr>
<tr>
<td>14</td>
<td>1.30</td>
<td>2947(3)</td>
<td>3020(4)</td>
<td>72.6(1.9)</td>
</tr>
<tr>
<td>16</td>
<td>1.49</td>
<td>2952(3)</td>
<td>3025(4)</td>
<td>74.9(2.1)</td>
</tr>
<tr>
<td>18</td>
<td>1.68</td>
<td>2949(2)</td>
<td>3021(3)</td>
<td>72.5(1.5)</td>
</tr>
</tbody>
</table>

- we extracted from the form-factors that radius of charmonium states is $\sim 0.2 \rightarrow 0.3$ fm. finite-size should be no problem for us @ $L_s = 1.2$ fm