

# Relativistic Aspects of Few Body Physics

Franz Gross -- Jlab and W&M

## Outline

- ★ Overview (3)
- ★ Progress with the 2 and 3 nucleon problem (15)
- ★ Convergence of the Bethe-Salpeter equation (5)
- ★ Relativistic treatment of the spin 3/2  $\Delta$  (3)
- ★ Current conservation with relativistic optical potentials (4)

## Acknowledgements

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  - Tobias Frederico
  - Vladimir Pascalutsa
  - Wayne Polyzou
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  - Alfred Stadler (recent)
  - Wally Van Orden (early)
  - Dick Arndt (SAID)
  - Karl Holinde and Ruprecht Machleidt (Bonn code)
- ★ Advice
  - Nijmegen group (phase shifts and fitting)



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# First -- why use a relativistic theory?

## ★ NOT because

- of size of  $(v/c)^2$  corrections (although they may be large in some applications)
- it is more accurate (it may not be)
- it is "better" than EFT (it complements EFT)

## ★ Use a covariant theory for the following reasons

- Intellectual: to preserve an exact symmetry (Poncare' invariance)
- Practical: to calculate boosts and Lorentz kinematics consistently to all orders (essential when energies are of the order of 1 GeV)

- Consistent: to use field theory for guidance in the construction of
  - ◆ forces ( $2 \leftrightarrow 3$  body consistency)
  - ◆ currents consistent with forces
- Conceptual: for "phenomenological economy", and to understand the non relativistic limit:
  - ◆ spin 1/2 particles (Dirac equation)
  - ◆ interpretation of  $L \bullet S$  forces (covariant scalar-vector theory of N matter)
  - ◆ efficient one boson exchange models of NN forces (?)



# Overview of relativistic methods for a fixed number of particles

## ★ Hamiltonian dynamics (Dirac classification)

demand a Hilbert space of positive energy states -- i.e. QM

discard antiparticles and lose manifest cluster separability

- front form light cone methods (Strikman, Sargsian, Miller, Pace, Salme, Frederico, Carbonell , and Karmanov,)
- instant form standard quantum mechanics - with relativity (Schiavilla and Arenhovel )
- point form kinematic Lorentz group; momentum not conserved (Klink)

## ★ Field dynamics (based on field theory)

demand manifest cluster separability

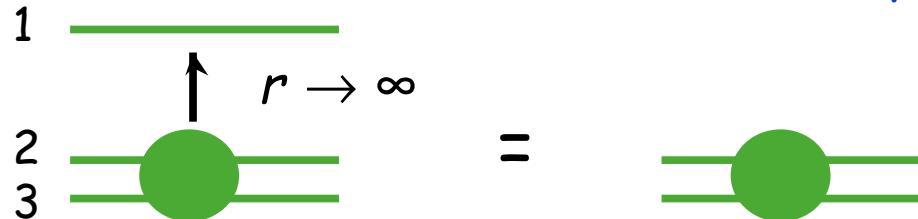
requires negative energy states and we lose the Hilbert space

- Bethe- Salpeter kinematic Poincaré group; 4-d ( Tjon )
- Spectator kinematic Poincaré group; 3-d ( Gross, Van Orden , Stadler )
- equal time integrate over  $x_0$ : ( Tjon , Pascalutsa, Wallace)
- front form BS integrate over  $x$  (Carbonell and Karmanov )



# Cluster separability -- 3-body example

- ★ Definition: when one particle is far away, the interaction between the other two is the same as it would be without the third particle



- ★ If  $P = p_1 + p_2 + p_3 = 0$ , and  $p_1 \neq 0$ , then the 23 amplitude is in a moving frame. The boost depends on the mass of the 2-body system.
- ★ Hamiltonian dynamics is **off-energy shell**,  $E_2 + E_3 \neq \sqrt{M_{23}^2 + \mathbf{p}_1^2}$ . The energies of particles and subsystems do not match the free particle energies, and under boosts the cluster property is not easy to implement.
- ★ Field dynamics is **off-mass shell**,  $p_0 \neq \sqrt{m^2 + \mathbf{p}^2}$ . Energy is conserved so boosts and cluster properties are easily satisfied, but off-mass shell  $\Rightarrow$  negative energy states.

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# Progress with 2 and 3 nucleon systems



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# Progress with the 2- and 3-nucleon problem -- 1

## Hamiltonian dynamics

- ★ Excellent fits to the 2-body data to 350 MeV Lab energy
  - $\chi^2 \sim 1/\text{datum}$
  - All relativistic corrections in the rest frame included phenomenologically
- ★ No solution of the full 3-nucleon problem (yet!)
  - S-wave Malfliet -Tjon potential: Glockle, Lee, and Coester, PRC **33**, 709 (1986)
  - V18 with linear boost corrections: J. Carlson, Pandharipande, and Schiavilla , PRC **47**, 484 (1993)
  - CD Bonn with minimal relativity: Sammarruca and Machleidt , Few Body Systems **24**, 87 (1998)
- ★ Three body forces needed to fit binding energy

Relativistic  
corrections to  
triton  
binding energy

0.2 (MeV)

$\begin{cases} 0.3 \text{ (boost)} \\ 0.1 \text{ (hamiltonian)} \end{cases}$   
-0.3



# Progress with the 2- and 3-nucleon problem -- 2

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## Hamiltonian dynamics

### Recent study of relativistic effects in 3-nucleon problem\*

\*Keister and Polyzou, PRC 73, 014005 (2006)

- ★ Supports the claim that effects (excluding pair terms) add positive correction to the triton binding
- ★ questions the transformation introduced by Kamada and Glockle [PRL 80, 2547 (1998)]. In both relativistic and nonrelativistic theory,  $p_{CM}^2 = \frac{1}{2} m E_{LAB}$ , and hence the CM momenta in both relativistic and nonrelativistic equations should be the same. KG assume the CM energies are the same.
- ★ emphasizes that relativistic corrections are not unique



# Progress with the 2- and 3-nucleon problem -- 3

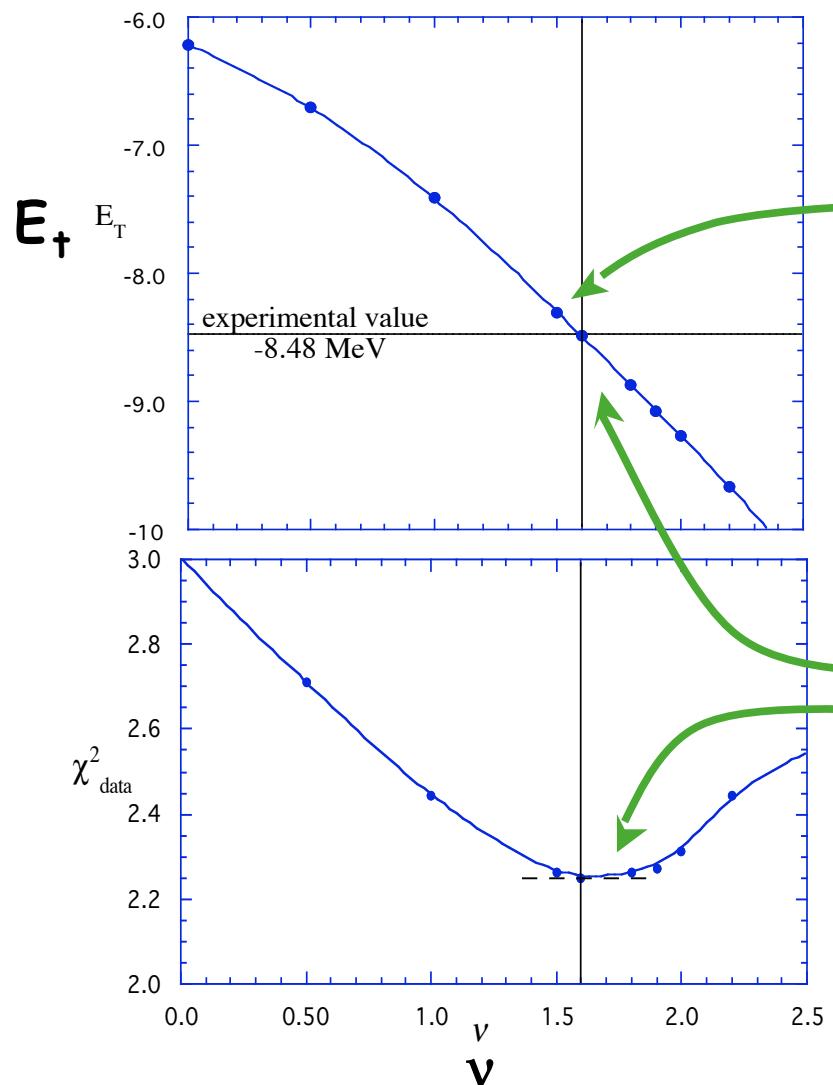
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## Field dynamics

- ★ New fits to the 2-body data to 350 MeV Lab energy [model WJC(2006)]
  - $\chi^2 \sim 1.6/\text{datum}$
  - Relativistic corrections pair terms and kinematics, even in the rest frame
- ★ solution for the triton using the spectator equation
  - Model W16 (1997) gave the best fit to the data and the correct binding without three body forces
  - New WJC(2006) also fits both the data and BE
  - corrections due to pair terms of three body origin = 0.26 MeV
- ★ OBE (or EBE) models predict NO three body forces



# Progress with the 2- and 3-nucleon problem\* -- 4



Results from earlier W16(1997) model

It turns out that the relativistic calculation of the three body binding energy is sensitive to a new, relativistic off-shell coupling (described by the parameter  $v$ ). Non-zero  $v$  is equivalent to effective three-body (and n-body forces).

The value of  $v$  that gives the correct binding energy is close to the value that gives the best fit to the two-body data!

\*three body calculations FG and Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

# Progress with the 2- and 3-nucleon problem -- 5

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## Field dynamics

Recent development of a realistic EBE model for the CS<sup>©</sup>

★ OBE: One Boson Exchange usually implies:

- only exchange *physical* bosons with masses less than or about one Gev, except for using the  $\sigma_0$  (isoscalar) and  $\sigma_1$  (isovector) to approximate TPE
- masses constrained to physical values

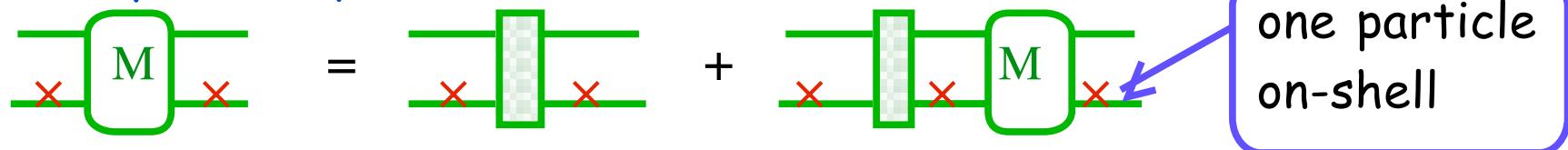
★ EBE: Effective Boson Exchange (defined today) differs:

- bosons are *effective* degrees of freedom only
- except for OPE, the masses, coupling constants, and quantum numbers are phenomenological
- general form constrained by relativistic field theory



# Equations of the Covariant Spectator theory\* (CS<sup>©</sup>)

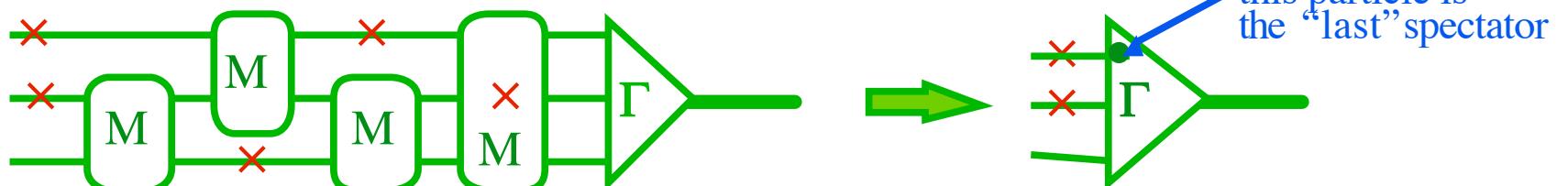
## ★ 2-body CS<sup>©</sup> equation



- ALL Poincare transformations are kinematic
- has a smooth one body limit

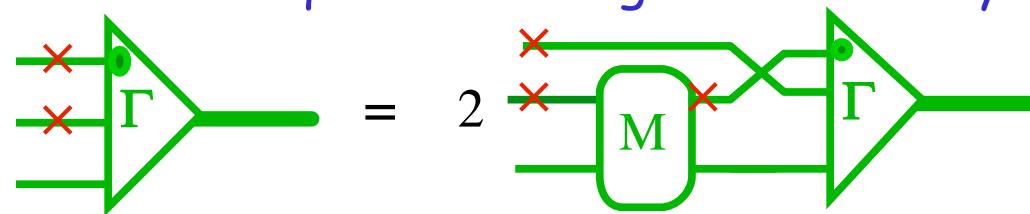
## ★ 3-body CS<sup>©</sup> equation

- Define three-body vertex functions for each possibility



- 3-body Faddeev-like equations emerge automatically:

Bound state  
equation for  
identical particles



\*FG, Phys.  
Rev. 186,  
1448 (1969)

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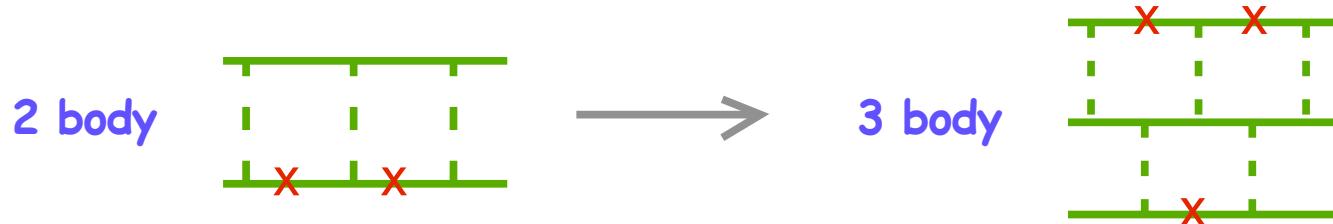
# Progress with the 2- and 3-nucleon problem -- 6

## Advantages of an EBE or OBE model

★ Connection to field theory:

★ Consistency:

- 2-body  $\rightarrow$  3-body with NO relativistic three body forces



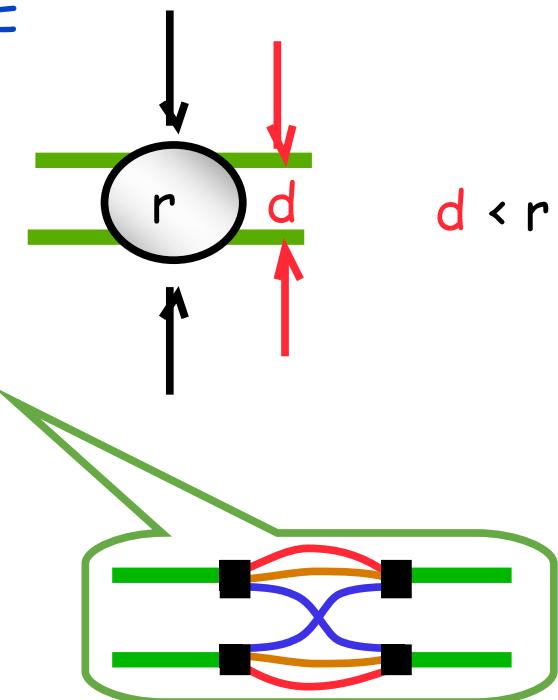
- hadronic  $\rightarrow$  electromagnetic (relativistic interaction currents)



# Progress with the 2- and 3-nucleon problem -- 7

## Problems with the traditional OBE model

- ★ TBE is neglected (cancellation theorem proved only for scalar exchanges)
- ★ TPE is certainly important: using  $\sigma_0$  and  $\sigma_1$  exchanges to approximate TPE violates the spirit of OBE
- ★ Isgur's arguments:
  - exchanging bosons over a distance small compared to their size make little sense
  - why isn't quark exchange more important?
- ★ AND, IT DOESN'T WORK (!)

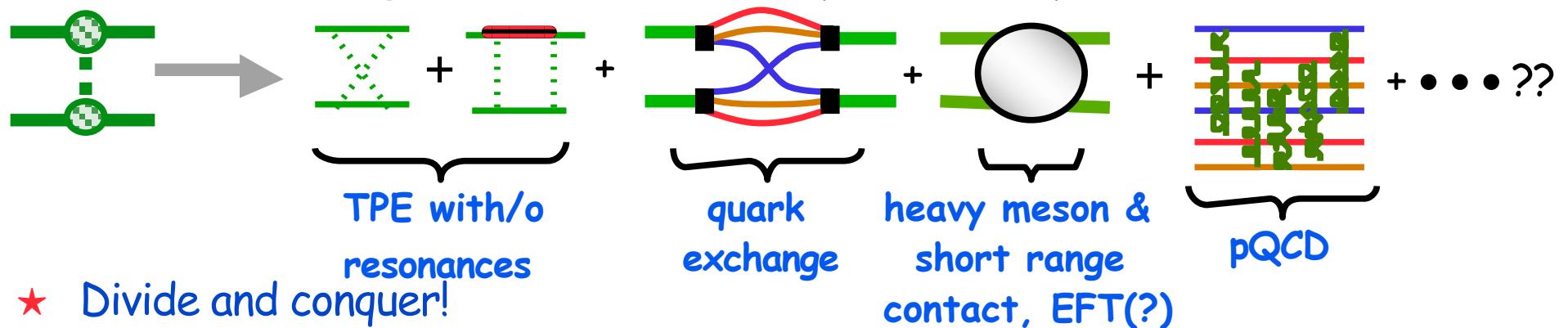


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# Progress with the 2- and 3-nucleon problem -- 8

## Advantages of the EBE model

- ★ mesons are “effective” and are not identified with physical mesons.  
NO crossed diagrams are needed; they are already included.



- ★ Divide and conquer!

- Part A: the effective bosons are determined phenomenologically and parameterize the most general interaction and include
  - TBE
  - quark exchange, etc., etc,
- Part B: properties of the bosons calculated from fundamental principals

- ★ AND, IT WORKS!



# Progress with the 2- and 3-nucleon problem -- 9

## Structure of the new EBE model

- ★ Most general on-shell kernel has 5 invariants for each *isospin*, written in terms of PS, S, V(g), V(f), A couplings
- ★ pion masses constrained, and mass of V(g) = V(f), leaving 20-4=16 parameters

16

- ★ off-shell coupling included so far

- pion (small admixture of  $\gamma_5 \neq \gamma_5 q$  off shell)
- scalar (addition of  $\mathbf{1}(m - p) + (m - p')\mathbf{1}$  term)
- vector (addition of  $\gamma^\mu(m - p) + (m - p')\gamma^\mu$  term)

2  
2  
2

- ★ 3 from factor masses (N,  $\pi$ , and all others)

3  
25



# Definitions of the EBE parameters

★ Only a few off-shell terms added to the kernel so far

★ Scalar:  $\sigma_0$  and  $\sigma_1$

$$\Lambda(p', p) = g_s + \frac{v_s}{2m} [2m - p' - p] \quad \text{zero on-shell}$$

★ Pseudoscalar:  $\pi$  and  $\eta$

$$\Lambda(p', p) = i g_P \left\{ \gamma^5 - \frac{1 - v_P}{2m} [(m - \not{p}') \gamma^5 + \gamma^5 (m - \not{p})] \right\}$$

★ Vector:  $\rho$  and  $\omega$

$$\Lambda(p', p) = g_V \left\{ \gamma^\mu + \frac{\kappa_V}{2m} i \sigma^{\mu\nu} (p' - p)_\nu - \frac{v_V}{2m} [(m - \not{p}') \gamma^\mu + \gamma^\mu (m - \not{p})] \right\}$$

★ Axial vector:  $H1$  and  $A1$

$$\Lambda(p', p) = g_A \{ \gamma^\mu \gamma^5 \}$$

Note: axial vector tensor couplings  
add no new structures



# Parameters from the new WJC (25)

as3.2.3 8/7/06

	$g^2/4\pi$	mass	f/g	off-shell $v$
p <sup>+</sup> and pi <sup>+</sup>	13.73	exp	---	0.01
eta	4.24	exp	---	1.72
sigma (I=0)	2.93	404	---	-4.65
sigma (I=1)	1.10	558	---	3.95
omega	3.80	508	0.06	0.56
rho	1.17	773	4.44	-1.82
axial vector (I=0)	-0.12	528	0.00	0.00
axial vector (I=1)	-0.17	513	0.00	0.00

form factor

masses:

N = 1717

$\pi$  = 2401

meson = 1329

Fit to the 2001

data:

$\chi^2/\text{datum} = 1.16$



# Three-nucleon bound state energy

as3.2.3 8/7/06

- ★  $^1S_0$  and  $^3S_1$ - $^3D_1$ ; (+) energy states only (5 channels)
- ★ all states to  $J=1$ ; (+) energy states only (14 channels)
- ★ all up to  $J=1$  (includes (-) energy; 28 channels)
- ★ up to  $J=2$  (52 channels)
- ★ up to  $J=3$  (76 channels)
- ★ up to  $J=4$  (100 channels)
- ★ up to  $J=5$  (124 channels)
- ★ up to  $J=6$  (148 channels)

-9.058	negative energy contribution
-8.364	
-8.064	0.300
-8.222	0.295
-8.316	0.238
-8.362	0.245
-8.390	0.258
-8.390	0.261

- ★ experimental value -8.48



# Comments on WJC(25)

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- ★ The  $g_\pi$  that emerges from the fit agrees with Nijmegen
- ★ the off-shell pion coupling ( $v_\pi$ ) is very small in agreement with chiral symmetry
- ★ the meson masses that were adjusted are all near 500 MeV as expected if a dispersion integral is saturated by a mass near the  $2\pi = 280$  MeV threshold. Only the rho is larger.
- ★ the  $g_A^2$  couplings are negative! What does this mean? (Results are not final!)
- ★ the I=0 off-shell sigma coupling ( $v_\sigma$ ) can be adjusted to give the exact 3-body binding energy without any significant change in the  $\chi^2/\text{datum}$
- ★ 3-body binding energies will become part of the fitting procedure!



# Progress with the 2- and 3-nucleon problem -- conclusion

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## ★ Hamiltonian dynamics

- Excellent fits to NN data with  $\chi^2=1/\text{datum}$
- 3 -body bound state calculations can be made relativistic with uncertainties of  $\sim 0.2$  MeV; 3-body forces are several times larger
- There are uncertainties in how to go from nonrelativistic to relativistic

## ★ Field Dynamics

- Exchange of effective bosons, not real ones (except for the pion) better approximates the physics. Removes several long-standing issues.
- Axial vector mesons needed for the most general expansion of the kernel
- New fits to the NN data are a dramatic improvement. With 25 parameters, relativistic model gives a  $\chi^2=1.16/\text{datum}$ , competitive with the best nonrelativistic models.
- 3-body calculations give accurate binding energies without 3-body forces

## ★ Field Dynamics provides an economy and an effective theory of forces



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# Recent progress in Field Dynamics

- ★ Convergence of the BS equation
- ★ Relativistic treatment of the spin 3/2  $\Delta$
- ★ Current conservation with relativistic optical potentials



# Convergence of the Bethe-Salpeter equation -- 1

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- ★ Exact BS kernel is the sum of ALL 2-nucleon irreducible processes. The ladder sum is only the simplest approximation:



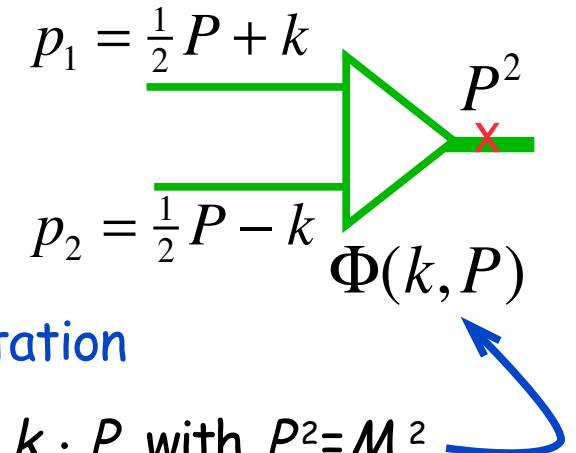
- ★ at 4th order, we must add the crossed ladder



- ★ The BS equation in ladder approximation **converges** only if the ladder is close to the exact result and the crossed ladder is small

# Convergence of the BS equation -- 2\*

\*Karmanov and Carbonell, Eur.Phys.J.A27:1, 2006;  
Carbonell and Karmanov, Eur.Phys.J.C



- ★ Carbonell and Karmanov use the Nakanishi representation
- ★ The BS amplitude,  $\Phi$ , depends on 2 variables:  $k^2$  and  $k \cdot P$ , with  $P^2=M^2$
- ★ Brief derivation: Starting from Feynman parameterization of the propagators

$$\frac{1}{A_+ A_-} = \frac{1}{2} \int_{-1}^1 \frac{dz}{\left( A_+ \frac{1}{2}(1+z) + \frac{1}{2} A_- (1-z) \right)^2} = \frac{1}{2} \int_{-1}^1 \frac{dz}{\left( m^2 - \frac{1}{4} M^2 - k^2 - z P \cdot k - i\epsilon \right)^2}$$

the Nakanishi representation includes additional the singularities that arise from the exchange of mesons:

$$\Phi(k, P) = \frac{1}{2} \int_{-1}^1 dz \int_0^\infty d\gamma \frac{g(\gamma, z)}{\left( \gamma + m^2 - \frac{1}{4} M^2 - k^2 - z P \cdot k - i\epsilon \right)^3}$$



## Convergence of the BS equation -- 3

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- ★ C&K Solve the BS equation in Minkowski space by inserting the Nakanishi representation and integrating over the light cone using the projection

$$\int_{-\infty}^{\infty} \Phi(k + \beta\omega, P) d\beta$$

where  $\omega$  is a light-like vector:  $\omega^2=0$ .

- ★ The equation for the spectral function becomes

$$\int_0^{\infty} \frac{d\gamma' g(\gamma', z)}{\left(\gamma + \gamma' + m^2 - \frac{1}{4}(1 - z^2)M^2\right)^2} = \int_0^{\infty} d\gamma' \int_{-1}^1 dz' V(\gamma z, \gamma' z') g(\gamma', z')$$

where  $V$  is related to the kernel of the BS equation. It has no singularities and can be solved numerically.

- ★ What do we find?



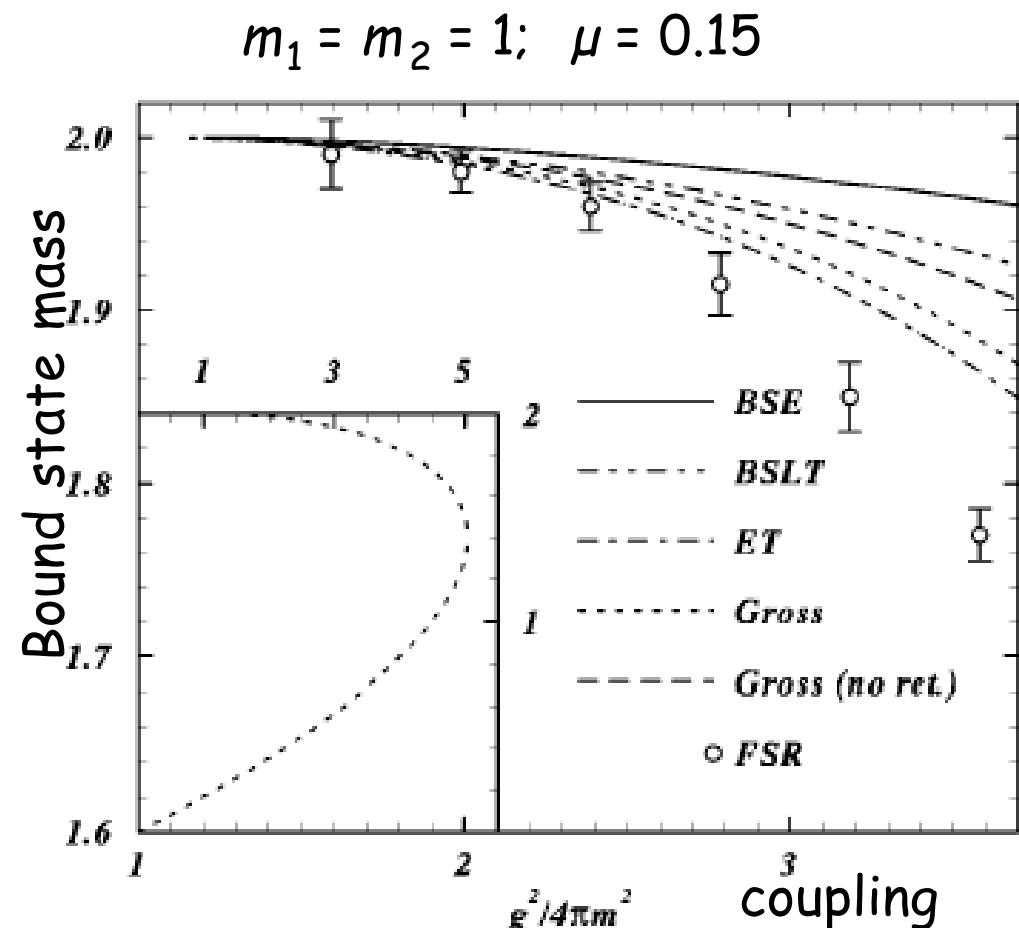
# Convergence of the BS equation -- 4

- ★ For  $\chi^2\phi$  theory BE vs.  $g^2$ 
  - Exact sum of ladders and crossed ladders from the Feynman-Schwinger method\*
  - BS equation in (in ladder approximation) ladder fails!
  - Quasipotential equations best

- ★ Crossed ladder contribution too small

\*Nieuwenhuis and Tjon, PRL 77, 814 (1996),

\*Cetin Savkli, FG, and John Tjon, Phys. Atom. Nucl. 68:842, 2005, Yad. Fiz. 68:874, 2005 and unpublished



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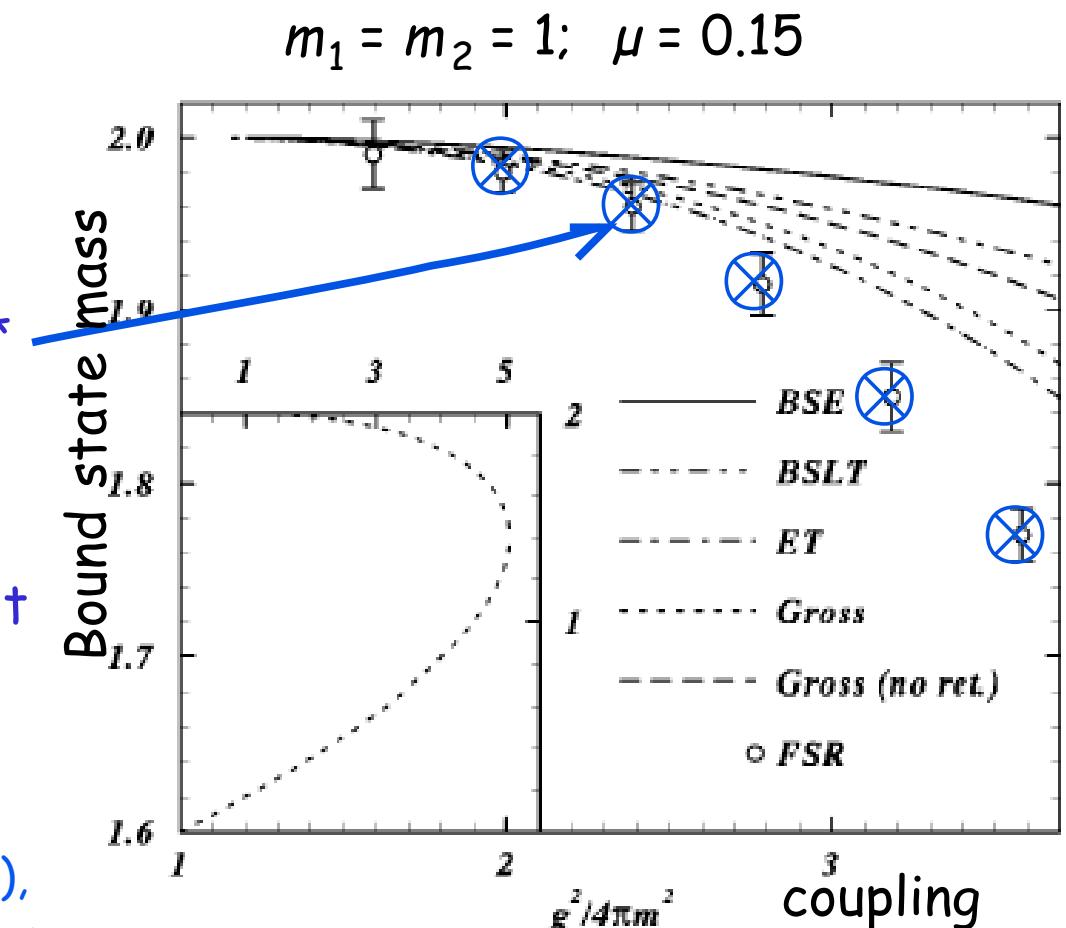
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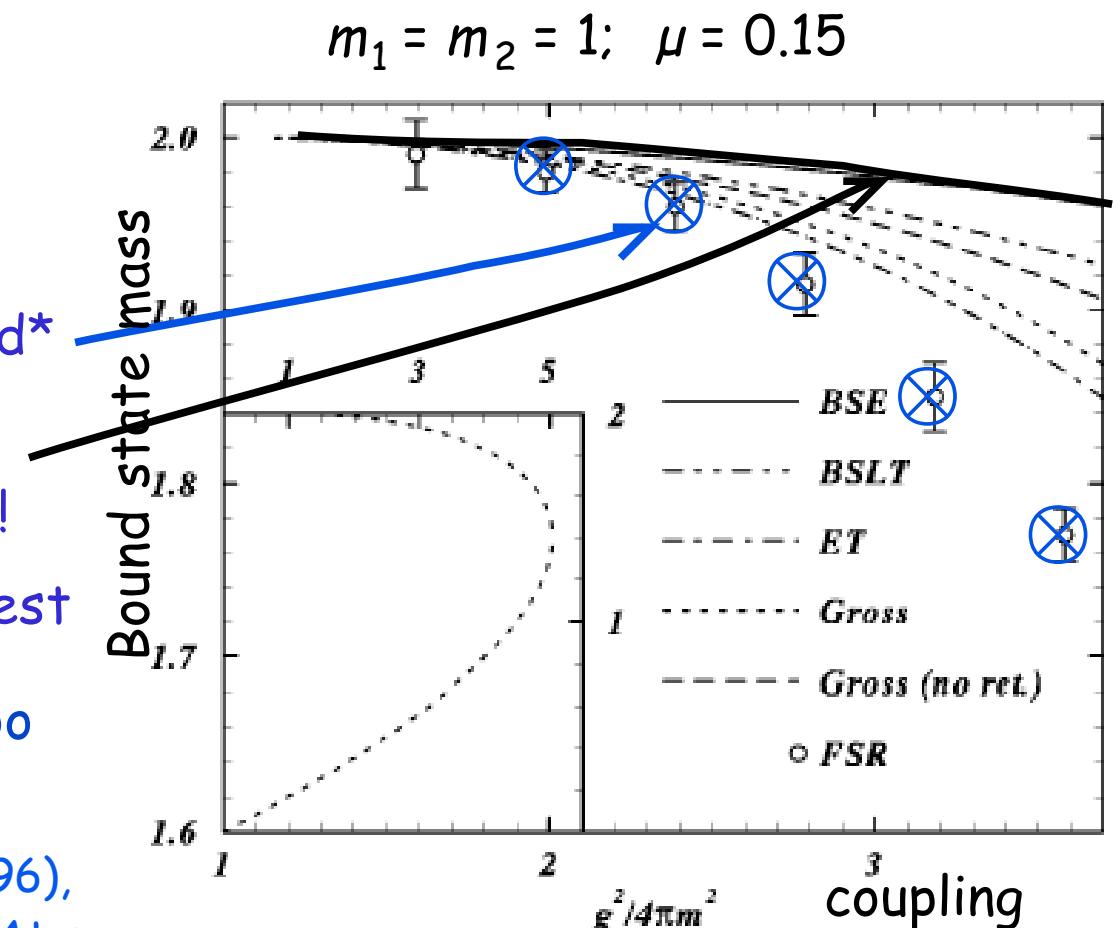
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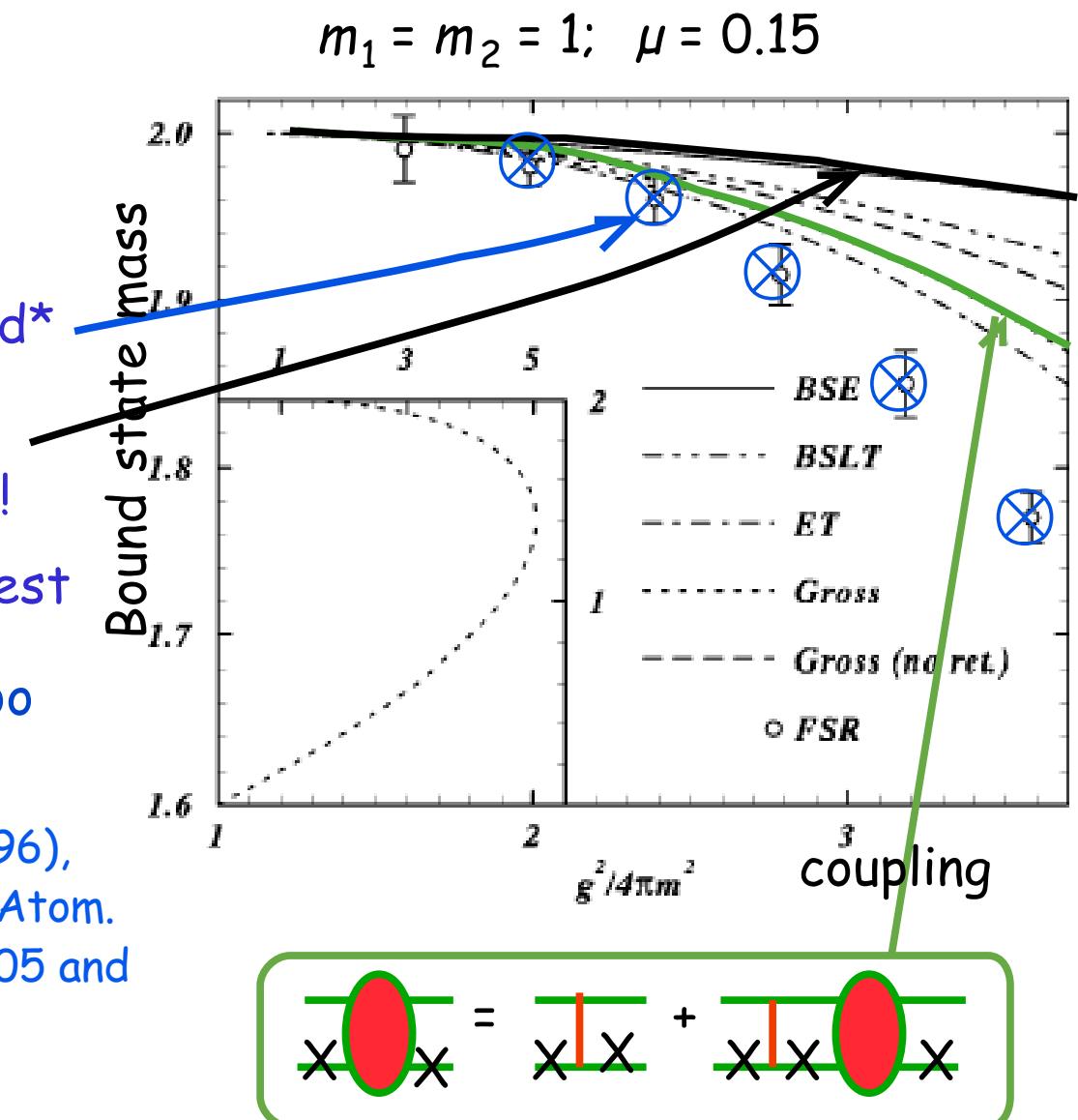
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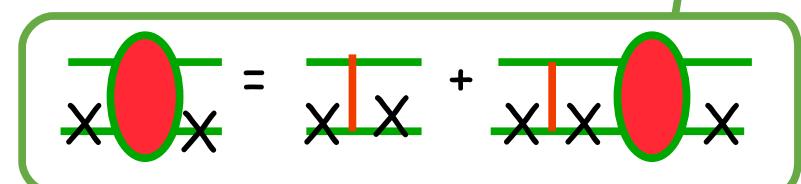
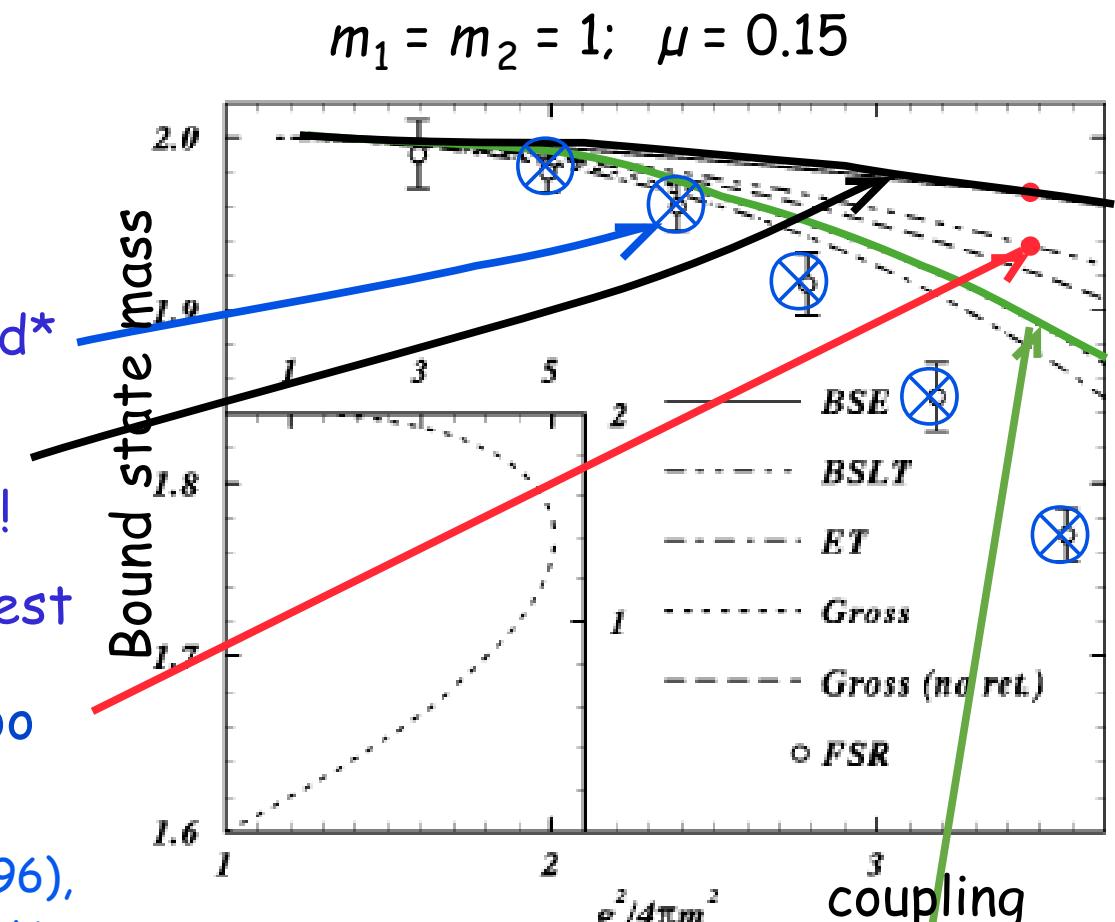
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# Convergence of the BS equation -- conclusion

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The BS equation, in summing all ladders and crossed ladders,

- ★ does not do as well as the spectator equation
- ★ converges slowly!



# Relativistic treatment of the spin 3/2 $\Delta^*$ -- 1

\*Pascalutsa, Phys. Rev. D **58**, 096002 (1998)

Pascalutsa and Timmermans, Phys. Rev. C **60**, 042201 (1999)

Pascalutsa and Phillips, Phys. Rev. C **68**, 055205 (2003)

Pascalutsa and Vanderhaeghen, Phys.Lett. **B63**, 31 (2006)

★ Old treatment of the  $\Delta$  included spurious spin 1/2 components

$$\begin{aligned} S_{\mu\nu}(P) &= \frac{1}{M - P - i\varepsilon} \left( g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2}{3M^2} P_\mu P_\nu - \frac{1}{3M} (\gamma_\mu P_\nu - P_\mu \gamma_\nu) \right) \\ &= \frac{1}{M - P - i\varepsilon} P_{\mu\nu}^{(3/2)} + \frac{2}{3M^2} (M + P) P_{22,\mu\nu}^{(1/2)} + \frac{1}{\sqrt{3}M} (P_{12,\mu\nu}^{(1/2)} + P_{21,\mu\nu}^{(1/2)}) \end{aligned}$$

where  $P_{\mu\nu}^{(3/2)} = g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3P^2} (P \gamma_\mu P_\nu + P_\mu \gamma_\nu P)$

$$P_{22,\mu\nu}^{(1/2)} = \frac{P_\mu P_\nu}{P^2}; \quad P_{12,\mu\nu}^{(1/2)} = \frac{P^\rho i\sigma_{\mu\rho} P_\nu}{\sqrt{3}P^2}; \quad P_{21,\mu\nu}^{(1/2)} = \frac{P_\mu P^\rho i\sigma_{\rho\nu}}{\sqrt{3}P^2}$$

Note that  
spin 1/2 parts  
are all linear  
in  $P_\mu$  or  $P_\nu$



# Relativistic treatment of the spin 3/2 $\Delta -- 2$

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- ★ Pascalutsa considers the strong gauge invariance of the spin 3/2 field (needed to reduce the number of degrees of freedom to  $4 \times 2 = 8$  to 4)
- ★ Conclusion is that strong gauge invariant couplings are needed

- the couplings often used in the past were
$$\bar{\psi}_N \Theta^{\mu\nu} \Psi_{\Delta\mu} \partial_\nu \phi_\pi \quad \text{with} \quad \Theta^{\mu\nu} = g^{\mu\nu} - (z + \frac{1}{2}) \gamma_\mu \gamma_\nu$$
where  $z$  is the "off-shell" parameter
- strong gauge invariance requires  $\Theta^{\mu\nu} P_\nu = 0$ . This constraint insures that all spin 1/2 parts of the propagator vanish, and is not satisfied by previous couplings
- Pascalutsa uses

$$\begin{aligned}\bar{\psi}_N \Theta^{\mu\nu} \Psi_{\Delta\mu} \partial_\nu \phi_\pi &= \bar{\psi}_N \gamma_5 \gamma_\mu \epsilon^{\mu\nu\rho\sigma} (\partial_\rho \Psi_{\Delta\sigma} - \partial_\sigma \Psi_{\Delta\rho}) \partial_\nu \phi_\pi \\ &\Rightarrow \bar{\psi}_N \gamma_5 \gamma_\mu k_{\pi\nu} \epsilon^{\mu\nu\rho\sigma} (P_\rho \Psi_{\Delta\sigma} - P_\sigma \Psi_{\Delta\rho})\end{aligned}$$



# Relativistic treatment of the spin 3/2 $\Delta$ -- Conclusion

- ★ The strong gauge invariant treatment solves a long standing problem -- the  $\Delta$  can now be treated as a pure spin 3/2 particle.
- ★ Technical simplifications abound. The bubble sum can be computed easily:

$$\begin{aligned} & \text{---} + \text{---} \text{---} + \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \\ P_{\mu\nu} &+ \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'v} + \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'\sigma} B g^{\sigma\sigma'} \Theta_{\sigma'v} + \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'\sigma} B g^{\sigma\sigma'} \Theta_{\sigma'\omega} B g^{\omega\omega'} \Theta_{\omega'v} + \dots \\ &= \frac{\Theta_{\mu\nu}}{1 - B} \end{aligned}$$

See: FG and Surya,  
Phys. Rev. C 47, 703 (1993)

- ★ A relativistic Effective Field Theory can be (and has been) developed



# Current conservation with relativistic optical potentials<sup>\*</sup> -- 1

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\*J. W. Van Orden, nucl-th/0605031

- ★ Long standing problem -- how to do a gauge invariant calculation of  $A(e, e'p)X$  when  $A$  is large. The usual matrix element is

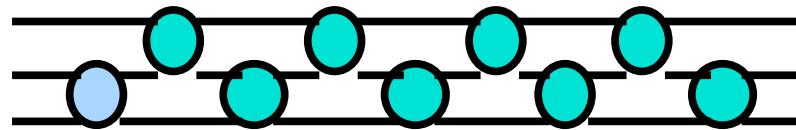
$$J^\mu = \langle \psi_{p', s'} | \Gamma^\mu(q) | \psi_{n, s} \rangle$$

- ★ Problem is that the optical potential used in the Hamiltonian is not the same for initial and final states.
- ★ For this reason it does not conserve current.
- ★ Problem was solved for  $A=3$  some time ago.
- ★ New result is to construct optical potential from exact result
- ★  $A=3$  result will be generalized to all  $A$ .



## Current conservation with relativistic optical potentials -- 2

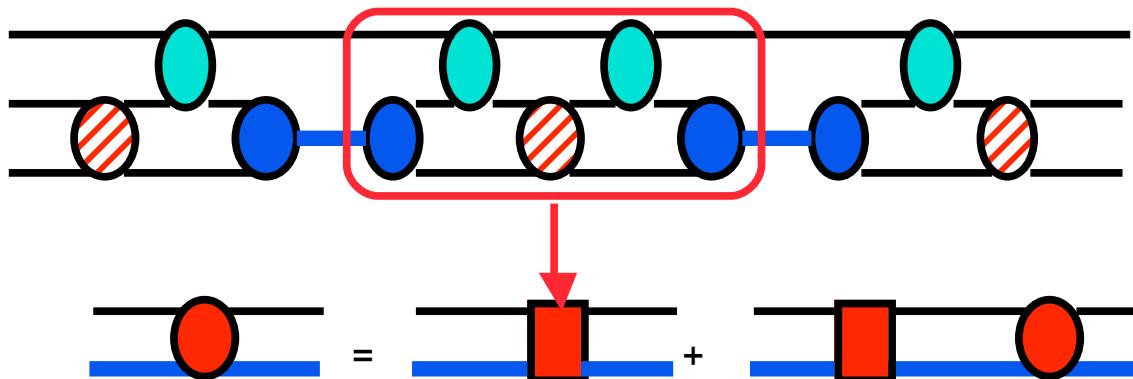
- ★ To illustrate the problem, consider non-identical nucleons with 12 and 23 interactions only. The most general Feynman diagram is



- ★ Isolate the deuteron (23) bound state contribution

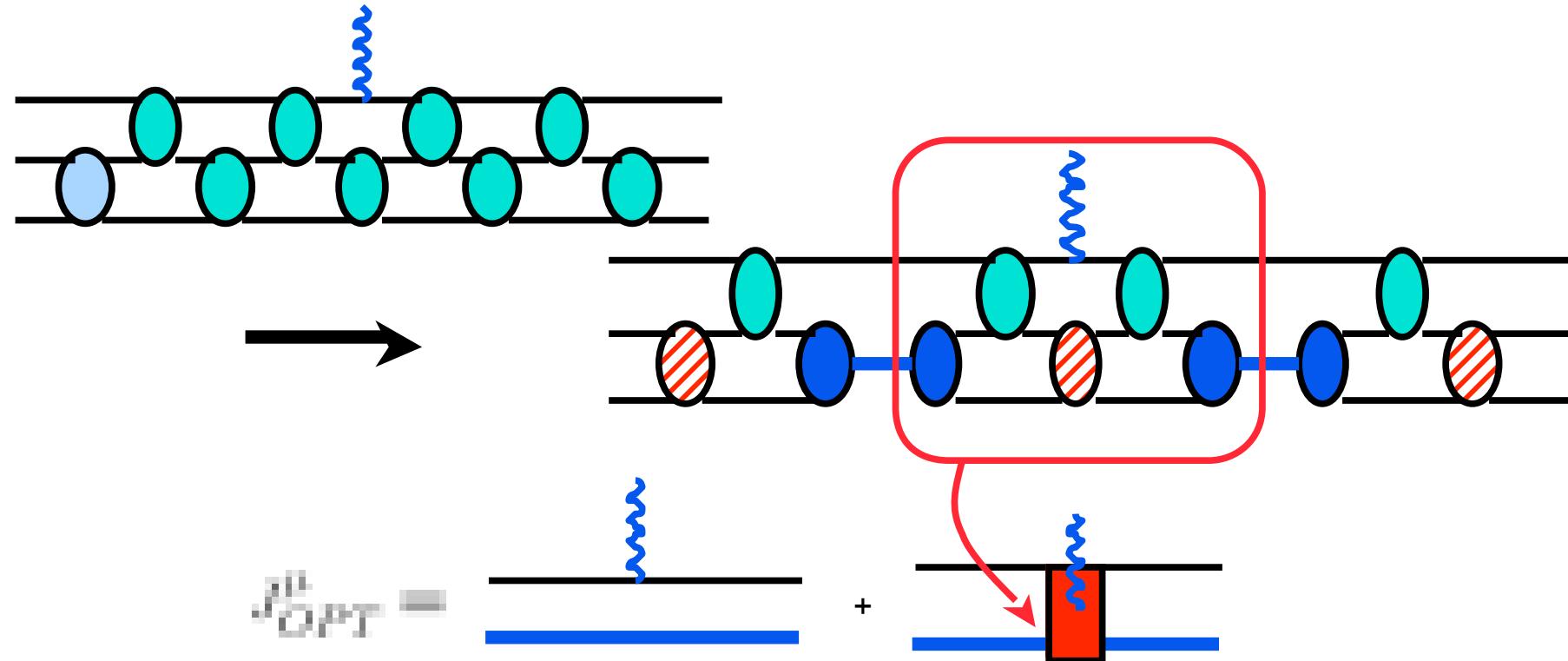
$$\text{Diagram with one teal circle} = \text{Diagram with two blue circles connected by a horizontal line} + \text{Diagram with one red-hatched circle}$$

- ★ Then the optical potential for 1 scattering from the 23 bound state is



## Current conservation with relativistic optical potentials -- 3

- ★ The optical potential for particle 1 +(23) is a 3-body T matrix with
  - no bound state poles for particles 2 and 3
  - first and last interactions cannot be a 2-body scattering between 2 & 3
- ★ Add current interaction with particle 1



# Current conservation with relativistic optical potentials -conclusion

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- ★ Combining this with the construction of conserved currents introduced in 1987\* and proved for three body interactions\*\* we can show that this construction conserves current.
- ★ This will provide a systematic basis for optical model approximations.

\*FG and Riska , PRC 36 , 1928 (1987)

\*\*Kvinikhidze & Blankleider , PRC 56, 2973 (1997)  
Adam & Van Orden , PRC 71: 034003 (2005)  
FG, A. Stadler , & T. Pena, PRC 69: 034007 (2004)



# Overall Conclusions

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- ★ Progress with the 2 and 3 nucleon systems using CS© Field Dynamics
  - new accurate fit to the NN data with  $\chi^2 \sim 1.16/\text{datum}$
  - correct 3-body binding energy without 3-body forces
  - manifestly covariant with cluster separability and all spin effects included
- ★ BS equation with 4th order crossed ladder kernel has been solved
  - uses Nakanishi representation and a new light-cone projection technique
  - convergence is not good.
- ★ Major advance in the relativistic treatment of spin 3/2 states
  - strong gauge invariance limits the number of degrees of freedom
  - propagators reduce to spin 3/2 projection operators with spurious spin 1/2 degrees of freedom decoupled from the physics
  - relativistic EFT of N and  $\Delta$  coupled system
- ★ Development of a current conserving optical model for  $(e, e'p)$  applications



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★ END



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# Phase shifts -- comparison between EBE-C and Nijmegen

A large discrepancy!

