Quark distributions
at large $x$

Wally Melnitchouk
Jefferson Lab
Significant advances in determination of quark and gluon distributions at small $x$ in recent years
$Q^2 = 25 \text{ GeV}^2$

Valence quarks: $u_V, d_V$

Sea quarks & gluons: $\bar{q} = \bar{u}, \bar{d}, \bar{s}...$
$q = u, d, s...$

$g$
Valence quarks

- Nucleon structure at intermediate & large $x$ dominated by *valence* quarks
- Most direct connection between quark distributions and models of the nucleon is through valence quarks
Valence quarks

At large $x$, valence $u$ and $d$ distributions extracted from $p$ and $n$ structure functions

\[
F_2^p \approx \frac{4}{9} u_v + \frac{1}{9} d_v
\]

\[
F_2^n \approx \frac{4}{9} d_v + \frac{1}{9} u_v
\]

$u$ quark distribution well determined from $p$

$d$ quark distribution requires $n$ structure function

\[
\frac{d}{u} \approx \frac{4 - F_2^n / F_2^p}{4F_2^n / F_2^p - 1}
\]
Valence quarks

- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon

- **SU(6) spin-flavour symmetry**

*Proton wave function*

$$p^\uparrow = -\frac{1}{3}d^\uparrow(uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow(uu)_1$$

$$+ \frac{\sqrt{2}}{6}u^\uparrow(ud)_1 - \frac{1}{3}u^\downarrow(ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow(ud)_0$$
Valence quarks

- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon
- **SU(6) spin-flavour symmetry**

**Proton wave function**

$$p^\uparrow = -\frac{1}{3} d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3} d^\downarrow (uu)_1$$

$$+ \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0$$

$$\Rightarrow u(x) = 2 \ d(x) \quad \text{for all } x$$

$$\frac{F_2^n}{F_2^p} = \frac{2}{3}$$
Valence quarks

- **scalar diquark dominance**

\[ M_\Delta > M_N \implies (qq)_1 \text{ has larger energy than } (qq)_0 \]

\[ \implies \text{ scalar diquark dominant in } x \to 1 \text{ limit} \]

since only \( u \) quarks couple to scalar diquarks

\[ \frac{d}{u} \to 0 \]

\[ \frac{F_2^n}{F_2^p} \to \frac{1}{4} \]

Valence quarks

- **hard gluon exchange**

at large $x$, helicity of struck quark = helicity of hadron

$\Rightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

\[ d \to 1 \quad 1 \to 5 \]
\[ u \to 5 \]
\[ \frac{F_n^2}{F_p^2} \to 3 \]
\[ \frac{F_n^p}{F_p^2} \to 7 \]
Valence quarks

- **BUT** no free neutron targets!
  (neutron half-life ~ 12 mins)

  - use deuteron as “effective neutron target”

- However: deuteron is a nucleus, and \( F_2^d \neq F_2^p + F_2^n \)
  - nuclear effects (nuclear binding, Fermi motion, shadowing)
    *obscure neutron structure information*

  - “nuclear EMC effect”
Nuclear “EMC effect”

![Graph showing the structure function ratio $F_2^A / F_2^d$ as a function of $x$. The graph includes data points for different elements, such as Ca, SLAC, Ca, NMC, Fe, SLAC, and Fe, BCDMS. The x-axis represents $x$, and the y-axis represents the ratio $F_2^A / F_2^d$. Arrows indicate regions of anti-shadowing and Fermi motion.]

- **Shadowing**: The decrease in the ratio at low $x$ values, indicating that the protons are shadowed by other nucleons.
- **Multiple Scattering**: The fluctuation of data points at higher $x$ values, indicating the effect of multiple scattering.
- **Anti-shadowing**: The increase in the ratio at intermediate $x$ values, suggesting a compensating effect.
- **Fermi Motion**: The region of $x$ where Fermi motion effects are significant, indicated by arrows.

Questions and Observations:

- What about $d / N$?
- Is there an effect from $N$ off-shell?

Note: The graph shows a trend where the ratio $F_2^A / F_2^d$ deviates from unity, particularly at low $x$ values, indicating the EMC effect.
EMC effect in deuteron

Nuclear “impulse approximation”

\[ F^d_2(x) = \int dy \ f_{N/d}(y) \ F^N_2(x/y) + \delta^{\text{off}} F^d_2(x) \]

nucleon momentum distribution

off-shell correction
EMC effect in deuteron

Nucleon momentum distribution in deuteron

\[ f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}^2} dp^2 \frac{E_{p}}{p_0} |\Psi_d(p^2)|^2 \]

- relativistic \(dNN\) vertex function
- momentum fraction of deuteron carried by nucleon
EMC effect in deutron

Nucleon momentum distribution in deutron

relativistic $dNN$ vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}^2} dp^2 \frac{E_p}{p_0} |\Psi_d(p^2)|^2$$
EMC effect in deuteron

Nucleon momentum distribution in deuteron

\[ f_{N/d}(y) = \frac{1}{4} M_d \ y \left[ \int_{-\infty}^{P_{\text{max}}} dp^2 \frac{E_p}{p_0} \left| \Psi_d(p^2) \right|^2 \right] \]

Wave function dependence only at large \(|y-1/2|\)

- sensitive to large \(p\) components of wave function
- not very well known
EMC effect in deuteron

Nucleon momentum distribution in deuteron

\[ f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}} dp^2 \frac{E_p}{p_0} |\Psi_d(p^2)|^2 \]

Nucleon off-shell correction

\[ \delta^{(\text{off})} F_2^d \rightarrow \delta^{(\Psi)} F_2^d \]

negative energy components of \( d \) wave function

\[ \delta^{(p^2)} F_2^d \rightarrow \delta^{(p^2)} F_2^d \]

off-shell \( N \) structure function
Off-shell correction

\[ \leq 1 - 2 \% \] effect

\[ \delta(p^2) F_2^d \]

\[ \delta(\Psi) F_2^d \]

EMC effect in deuteron

![Graph showing EMC effect in deuteron](image)

Larger EMC effect (smaller $d/N$ ratio)

$F_2^n$ underestimated at large $x$
Unsmearing

Note: calculated $d/N$ ratio depends on input $F_2^n$

extracted $n$ depends on input $n$ ... cyclic argument

Solution: iteration procedure

0. subtract $\delta^{(\text{off})}F_2^d$ from $d$ data: $F_2^d \rightarrow F_2^d - \delta^{(\text{off})}F_2^d$

1. smear $F_2^p$ with $f_{N/d}$: $f_{N/d} \otimes F_2^p \equiv S_p F_2^p$

2. extract neutron via $F_2^n = S_n(F_2^d - F_2^p / S_p)$ starting with e.g. $S_n = S_p$

3. smear $F_2^n$ with $f_{N/d}$ to get new $S_n$

4. repeat 2-3 until convergence
Unsmearing

![Graph showing the ratio of $F_2^n/F_2$ vs. $x$ with three curves: exact result, 1 iteration, and 3 iterations.]

- good convergence after several iterations
- resulting $F_2^n$ independent of starting assumptions
- depends only on smearing function $f_{N/d}$

Effect on $n/p$ ratio

\[ F_{2}^{n}/F_{2}^{p} \]

- Fermi motion only
- with binding & off-shell

\[ x \]

- 2/3  SU(6) helicity retention
- 3/7  scalar diquarks
- 1/4

without EMC effect in $d$, $F_{2}^{n}$ underestimated at large $x$
Effect on $n/p$ ratio

\[
F_{2}^{\nu p} = 2x (d + \bar{u}) \quad xF_{3}^{\nu p} = x (d - \bar{u})
\]

\[
F_{2}^{\bar{\nu} p} = 2x (u + \bar{d}) \quad xF_{3}^{\bar{\nu} p} = x (u - \bar{d})
\]
Diquarks as Inspiration and as Objects

Frank Wilczek*

September 17, 2004

hep-ph/0409168

One of the oldest observations in deep inelastic scattering is that the ratio of neutron to proton structure functions approaches $\frac{1}{4}$ in the limit $x \to 1$

$$\lim_{x \to 1} \frac{F_2^n(x)}{F_2^p(x)} \to \frac{1}{4} \quad (1.1)$$

Folklore that experiment gives $1/4$ limiting ratio...
uncertainty due to nuclear effects in neutron (full range of nuclear models)

d distribution poorly known beyond $x \sim 0.5$
“Cleaner” methods of determining $d/u$

\[ e^\pm p \rightarrow \nu(\bar{\nu})X \quad \text{need high luminosity} \]

\[ \nu(\bar{\nu}) p \rightarrow l^\mp X \quad \text{low statistics} \]

\[ p p(\bar{p}) \rightarrow W^\pm X \quad \text{need large lepton rapidity} \]

\[ \vec{e}_L(\vec{e}_R) p \rightarrow e X \quad \text{low count rate} \]

\[ e p \rightarrow e \pi^\pm X \quad \text{need } z \sim 1, \text{ factorization} \]

\[ e^{\text{3He}}(\text{3H}) \rightarrow e X \quad \text{tritium target} \]
“Cleaner” methods of determining $d/u$

$e\ d \rightarrow e\ p\ X$

- target $d$
- recoil $p$
- slow backward $p$
- neutron nearly on-shell
- minimize rescattering

JLab Hall B experiment (“BoNuS”) completed run Dec. 2005
Issues at large $x$

- **Target mass corrections**
  - finite $M^2/Q^2$ effects (but leading twist!)
  - Georgi, Politzer, PRD14 (1976) 1829
  - Kretzer, Reno, PRD69 (2004) 034002

- **Higher twists**
  - dynamical quark-gluon correlations, $1/Q^2$ suppressed

- **Quark-hadron duality**
  - low-$W$ resonances conspire to produce scaling function

- **Large-$x$ resummation**
  - extend validity of pQCD by resumming large-$x$ logs arising from soft & collinear gluons
  - Sterman, NPB281 (1987) 310
  - Catani, Trentadue, NPB327 (1989) 323
  - Corcella, Magnea, hep-ph/050742
**Issues at large $x$**

- **Target mass corrections**

  - **Georgi-Politzer (GP) prescription**

  \[
  F_{2}^{\text{GP}}(x, Q^2) = \frac{x^2}{r^3} F(\xi) + 6 \frac{M^2 x^3}{Q^2} \frac{1}{r^4} \int_{\xi}^{1} d\xi' F(\xi') + 12 \frac{M^4 x^4}{Q^4} \frac{1}{r^5} \int_{\xi}^{1} d\xi' \int_{\xi'}^{1} d\xi'' F(\xi'')
  \]

  \[
  \xi = \frac{2x}{1 + r}
  \]

  \[
  r = \sqrt{1 + 4x^2 M^2 / Q^2}
  \]

  “quark distribution function”

  \[
  F(y) = \frac{F_2(y)}{y^2}
  \]

  ... and similar for other structure functions
numerically...

Christy et al. (2005)

TMCs significant at large $x^2/Q^2$, especially for $F_L$
Threshold problem

if $F(y) \sim (1 - y)^\beta$ at large $y$

then since $\xi_0 \equiv \xi(x = 1) < 1$

$F(\xi_0) > 0$

$F_{i}^{TMC}(x = 1, Q^2) > 0$

is this physical?

problem with GP formulation?
Possible solutions

- Johnson/Tung - modified threshold factor

**Nachtmann moment**

\[
\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left( \frac{3 + 3(n + 1)r + n(n + 2)r^2}{(n + 2)(n + 3)} \right) F_2(x, Q^2)
\]

- \(n\) fixed, \(Q^2 \to \infty\)

\[
\mu_2^n(Q^2) \to (\ln Q^2/\Lambda^2)^{-\lambda_n} A_n
\]

\[A_n = \int_0^1 d\xi \xi^n F(\xi)\]

- \(n \to \infty\), \(Q^2\) fixed

\[
\mu_2^n(Q^2) \to \xi_0^n(Q^2) \tilde{\mu}_2^n(Q^2)
\]

“regularized” amplitudes (threshold-independent)
**Possible solutions**

- **Johnson/Tung - modified threshold factor**

**Nachtmann moment**

\[
\mu_n^2(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left( \frac{3 + 3(n + 1)r + n(n + 2)r^2}{(n + 2)(n + 3)} \right) F_2(x, Q^2)
\]

**Ansatz**

\[
\mu_n^2(Q^2) = \xi_0^n(Q^2) \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\lambda_n} A_n
\]

- consistent with asymptotic pQCD behavior

- not unique!

*Bitar, Johnson, Tung*

*PLB 83B (1979) 114*
Possible solutions

- Johnson/Tung - modified threshold factor

\[ A_n \text{ with } M_2^n = \int_0^1 dx \ x^{n-2} F_2(x) \]

\[ \mu_2^n(Q^2) = \xi_0^n(Q^2) \ M_2^n(Q^2) \]

\[ M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \cdots \]

\textit{cf.} exact expression

\[ M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n + 2} \frac{M^2}{Q^2} M_2^{n+2} + \cdots \]

\[ \text{inconsistency at low } Q^2 ? \]
Possible solutions

Kulagin/Petti - expand expressions in $1/Q^2$

$$F_{2}^{\text{TMC}}(x, Q^2) = \left(1 - \frac{4x^2 M^2}{Q^2}\right) F_{2}^{\text{LT}}(x, Q^2)$$

$$+ \frac{x^3 M^2}{Q^2} \left(6 \int_x^1 \frac{dz}{z^2} F_{2}^{\text{LT}}(z, Q^2) - \frac{\partial}{\partial x} F_{2}^{\text{LT}}(x, Q^2)\right)$$

_Kulagin, Petti, NPA765 (2006) 126_

has correct threshold behavior
Alternative solution

work with $\xi_0$ dependent PDFs

$n$-th moment $A_n$ of distribution function

$$A_n = \int_0^{\xi_{\text{max}}} d\xi \, \xi^n \, F(\xi)$$

what is $\xi_{\text{max}}$?

- GP use $\xi_{\text{max}} = 1$, $\xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\text{max}} = \xi_0$

Steffens, WM
Alternative solution

- What is effect on phenomenology?
  - Try several “toy distributions”

**Standard TMC** ("sTMC")

\[ q(\xi) = \mathcal{N} \xi^{-1/2} (1 - \xi)^3 , \quad \xi_{\text{max}} = 1 \]

**Modified TMC** ("mTMC")

\[ q(\xi) = \mathcal{N} \xi^{-1/2} (1 - \xi)^3 \Theta(\xi - \xi_0), \quad \xi_{\text{max}} = \xi_0 \]

**Threshold dependent** ("TD")

\[ q^{TD}(\xi) = \mathcal{N} \xi^{-1/2} (\xi_0 - \xi)^3 , \quad \xi_{\text{max}} = \xi_0 \]
TMCs in $F_2$

\[ Q^2 = 1 \text{ GeV}^2 \]

- scaling
- sTMC
- mTMC
- $\xi q^{TD}$
- TD

$\rightarrow$ correct threshold behavior for “TD” correction
TMCs in $F_2$

$Q^2 = 5 \text{ GeV}^2$

- Scaling
- sTMC
- mTMC
- $\xi q^{TD}$
- TD

Effect small at higher $Q^2$
TMCs in $F_L$

$Q^2 = 1 \text{ GeV}^2$

- $\cdots \cdot \cdot$ sTMC
- $\cdots \cdot \cdot$ mTMC
- $\cdot \cdot \cdot \cdot \cdot$ TD

$\rightarrow$ correct threshold behavior for “TD” correction

$\rightarrow$ reduced TMC effect \textit{cf.} sTMC and mTMC
Nachtmann $F_2$ moments

moment of structure function agrees with moment of PDF to 1% down to very low $Q^2$
Nachtmann $F_2$ moments

$\mu^2 / A_n$ vs. $Q^2 (\text{GeV}^2)$

- $n=4$
- $n=6$

- $\cdots$ sTMC
- $\ldots$ mTMC
- $-$ TD

higher moments show much weaker $Q^2$ dependence than sTMC & mTMC prescriptions
Nachtmann $F_2$ moments

\[
\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}
\]

extract PDFs from structure function data at lower $Q^2$
Nachtmann $F_L$ moments

$\rightarrow$ weaker $Q^2$ dependence for TD prescription
Summary

- $d$ quark distribution poorly known at large $x$

- (anti)neutrino data can help determine $d/u$ ratio at large $x$
  → complement $e$ scattering data (e.g. BONUS)

- alternative formulation of TMC in GP approach 
  without threshold problem
  → much faster approach to scaling for $\xi_0$ dependent PDF