Structure of the Nucleon with electroweak probes

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Outline

1. Introduction

   ⟹ QCD and the strong nuclear force

   ⟹ electron scattering

2. Quark distributions in the nucleon

   ⟹ sea quarks and flavour asymmetries

   ⟹ valence quarks at large x

   ⟹ nuclear effects

   ⟹ lattice QCD

Outline

3. Quark-hadron duality

   ⟹ structure functions in the resonance region

   ⟹ duality and QCD

   ⟹ global vs. local duality

4. Electromagnetic form factors

   ⟹ two-photon exchange

   ⟹ strangeness form factors

Thursday, February 2, 2006

U.S. Department of Energy Requests $4.1 Billion Investment
As Part of the American Competitiveness Initiative

Nuclear Physics Program ($454.1 million)

This is an $87 million increase over FY 2006. This funding supports research to provide new insights and knowledge of the structure and interaction of atomic nuclei and the primary forces of particles of nature in nuclear matter. The funding increase restores operations at both the Thomas Jefferson National Accelerator Facility (TJNAF) and the Relativistic Heavy Ion Collider (RHIC). In addition, new funding is requested for a TJNAF power upgrade and a new injector for RHIC.

High Energy Physics Program ($775.1 million)

This is a $58.4 million increase over FY 2006. This funding for grants and full experimental facility operations will be used to further explore basic research to explore the laws of nature governing the most basic constituents of matter and the forces binding them. These are fundamental principles at the heart of physics and the physical sciences. Project engineering and design funding of $10.3 million is requested for the new Electron Neutrino Appearance project.

"puts DOE's Office of Science on the path to doubling its budget by 2016"

1.
Introduction

- QCD and the strong nuclear force

Building Blocks of the Universe

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge	Flavor	Approx. Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0	u up	0.003	2/3
e electron	0.000511	−1	d down	0.006	−1/3
ν_μ muon neutrino	<0.0002	0	c charm	1.3	2/3
μ muon	0.106	−1	s strange	0.1	−1/3
ν_τ tau neutrino	<0.02	0	t top	175	2/3
τ tau	1.7771	−1	b bottom	4.3	−1/3

- **Each quark comes in 3 "colours": red, green and blue.**
- **Leptons do not carry color charge.**

Force Carriers of the Universe

BOSONS force carriers
spin = 0, 1, 2, ...

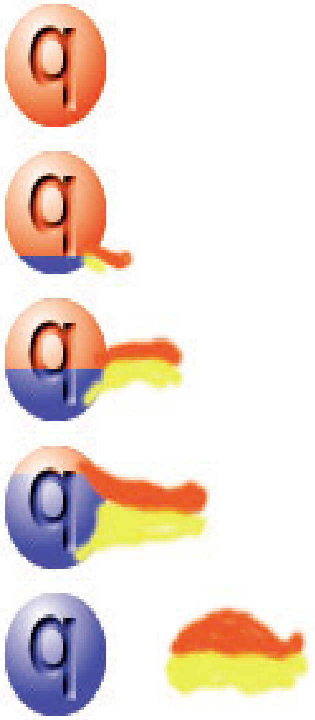
Unified Electroweak spin = 1		
Name	**Mass GeV/c^2**	**Electric charge**
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1		
Name	**Mass GeV/c^2**	**Electric charge**
g gluon	0	0

- **The massless photon mediates the long-range e.m. interactions.**
- **Gluons carry color and mediate the strong interaction.**
- **The very massive W^-, W^+, and Z^0 bosons mediate the weak interaction**

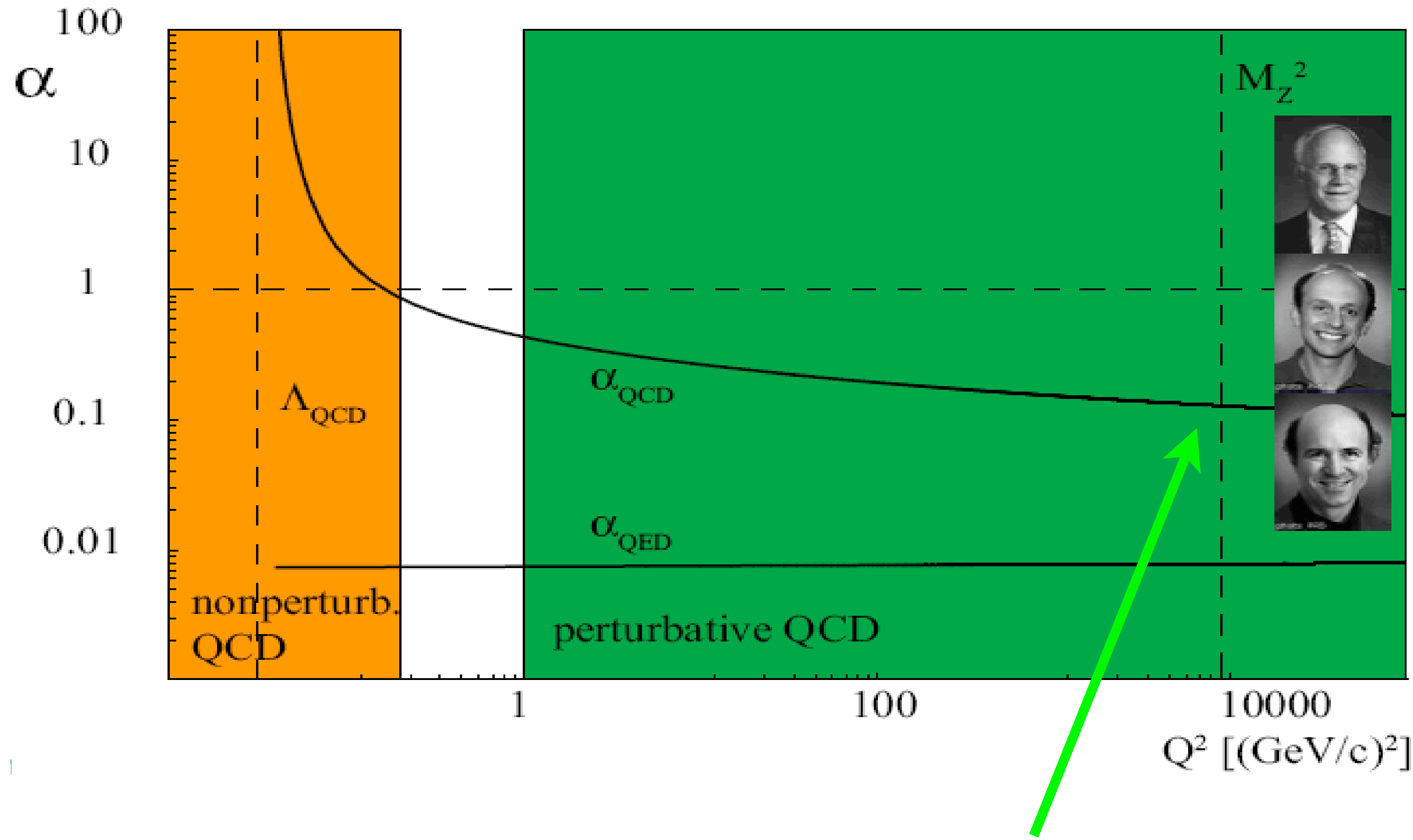
Quantum Chromodynamics (QCD)

- Photons do not carry electric charge.
- Gluons *do* carry colour charge!
- Gluons can directly interact with other gluons!
- This is new!



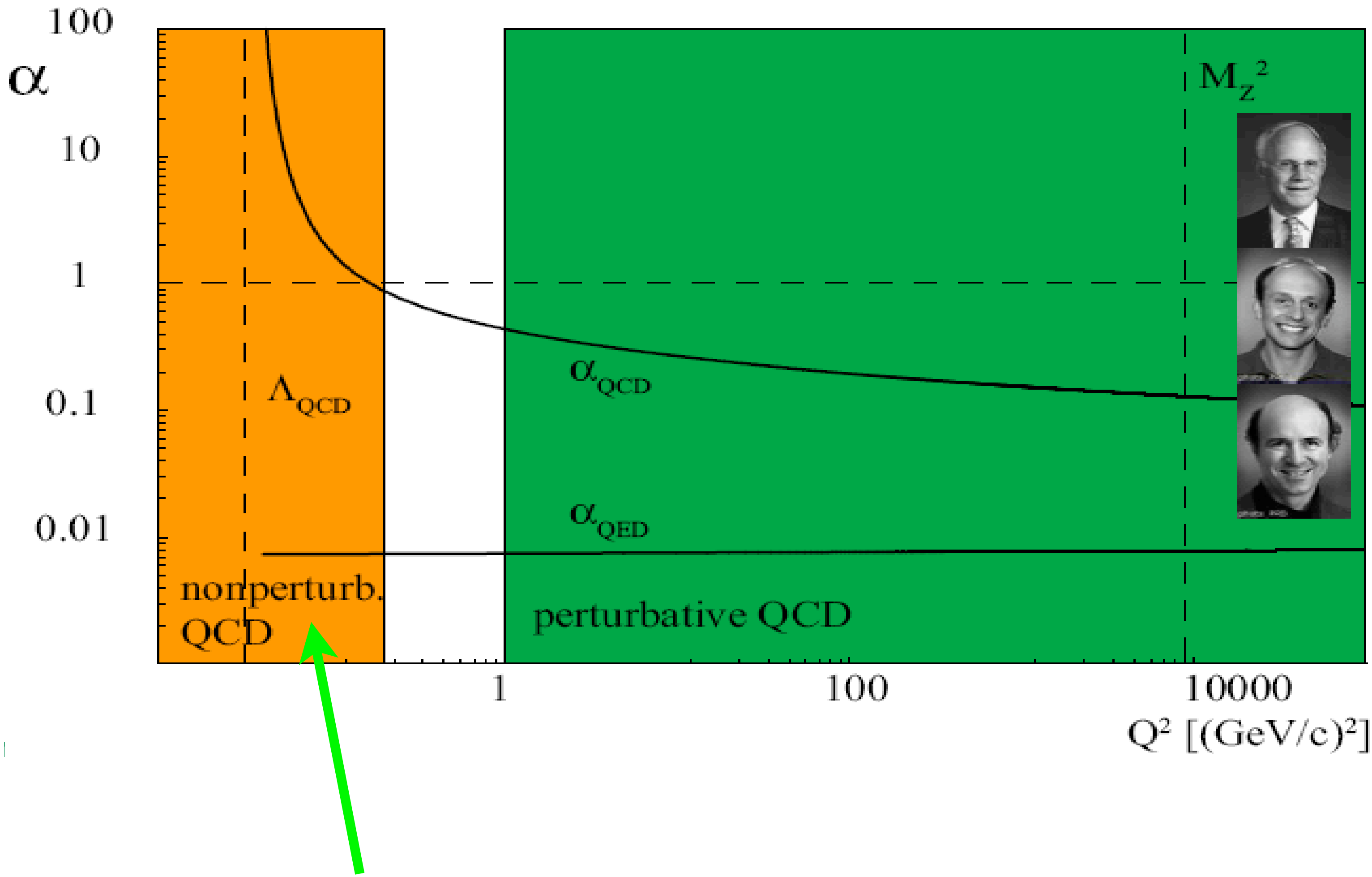
A red quark emitting a red anti-blue gluon to leave a blue quark.

Quark-quark force grows WEAKER as quarks come close
´ "Asymptotic Freedom"



2004 Nobel Prize for discovery of asymptotic freedom
(Gross, Politzer, Wilczek)

→ calculate observables using perturbation theory as power series in small expansion parameter α_s



BUT - only half of the story...

at low energy $\longrightarrow$ *confinement* !

➡ $\alpha_s \sim 1$ so cannot use perturbative expansion

➡ here QCD said to be "nonperturbative"

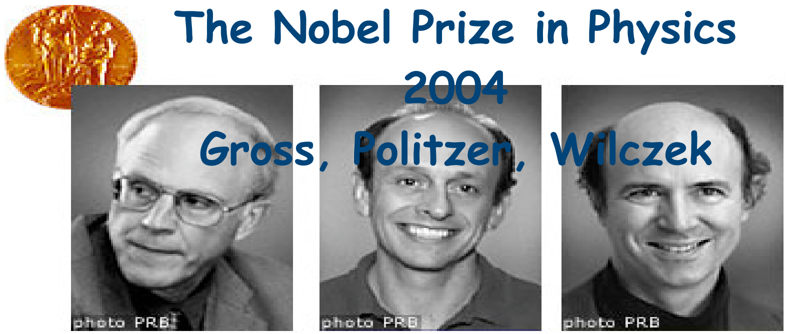
QCD and the Origin of Mass

$$\mathrm{u} \quad + \quad \mathrm{u} \quad + \quad \mathrm{d} \quad = \quad \mathrm{proton}$$

mass: $0.003 + 0.003 + 0.006 \neq 0.938$ MeV

HOW does the rest of the proton mass arise?

QCD: Unsolved in Nonperturbative Regime



- **2004 Nobel Prize awarded for "asymptotic freedom"**
- **BUT in nonperturbative regime QCD is still unsolved**
- **One of the top 10 challenges for physics!**
- **Is it right/complete?**
- **Do glueballs, exotics and other apparent predictions of QCD in this regime agree with experiment?**

→ central to answering these questions is the need to understand *how quarks form hadrons*

Looking for quarks in the nucleon
is like looking for the Mafia in Sicily -
everybody *knows* they're there,
but it's hard to find the evidence!

Anonymous

"QUARKS. NEUTRINOS. MESONS. ALL THOSE DAMN PARTICLES YOU CAN'T SEE. THAT'S WHAT DROVE ME TO DRINK. BUT NOW I CAN SEE THEM."

How to probe the structure of hadrons?

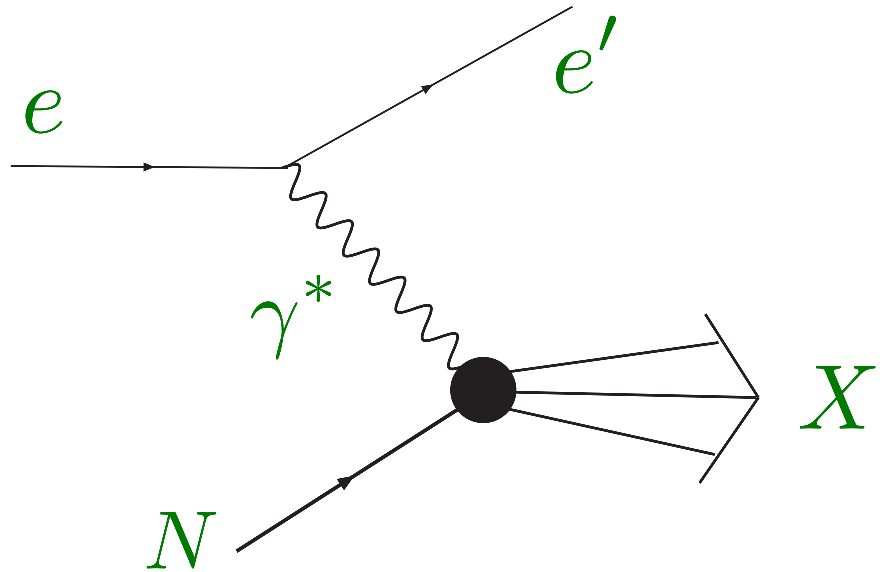
collide hadrons

probe with leptons

1.
Introduction
- electron scattering

Electron scattering

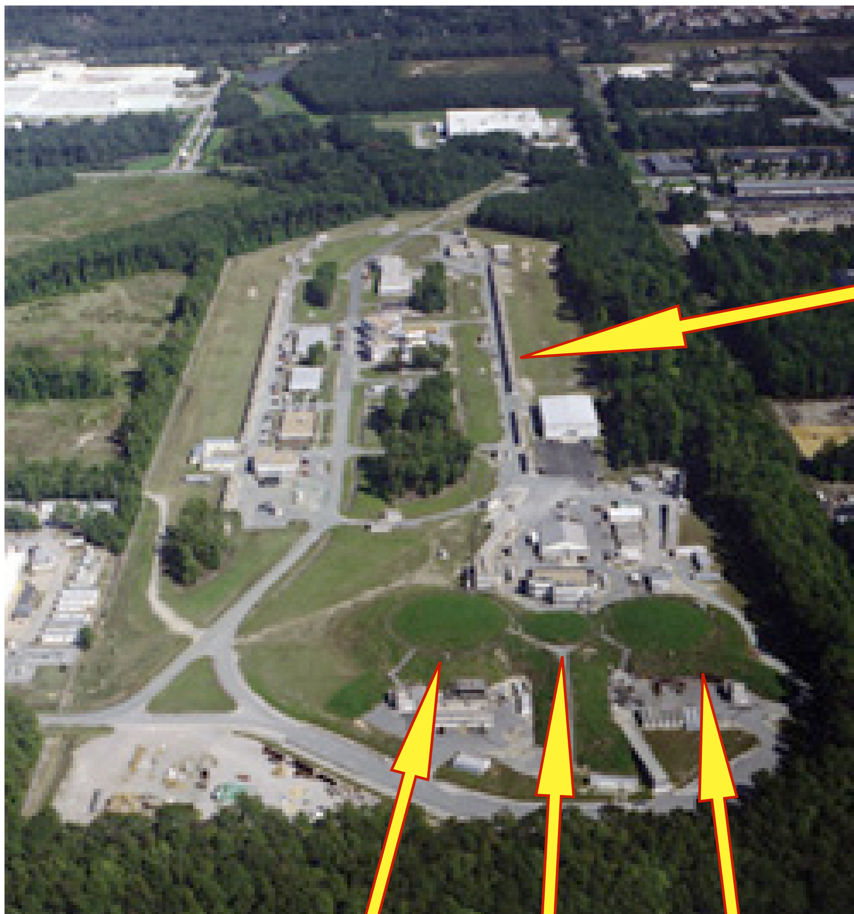
Electron Scattering Provides an Ideal Microscope for Nuclear Physics



- Electrons are point-like
- The interaction (QED) is well-known
- The interaction is weak

Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab (JLab)

Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab (JLab)



0.6 GeV electrons / linac

x 10 → 6 GeV

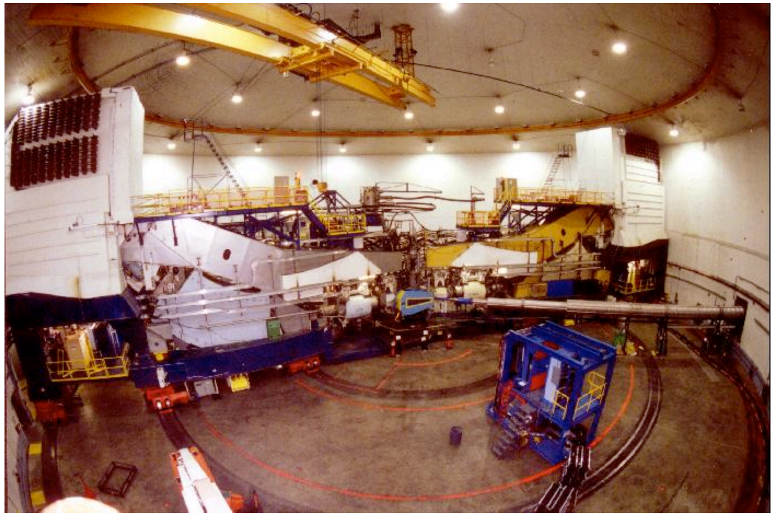
Hall A

Hall B

Hall C

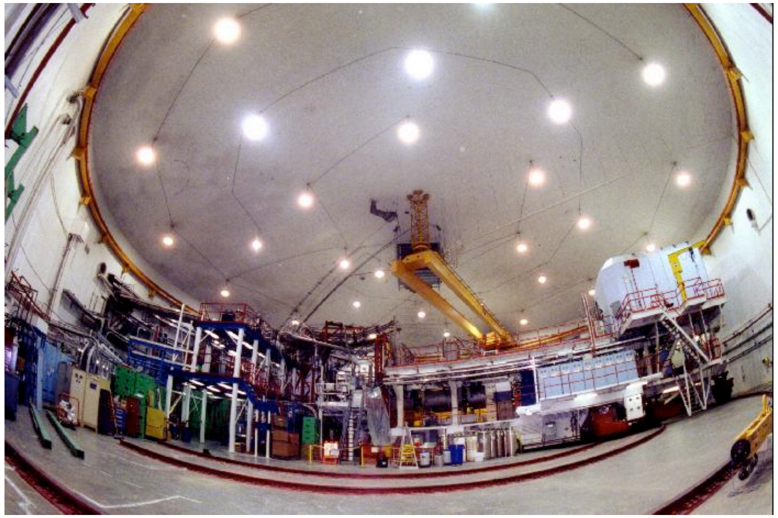
Experimental Halls

Hall A

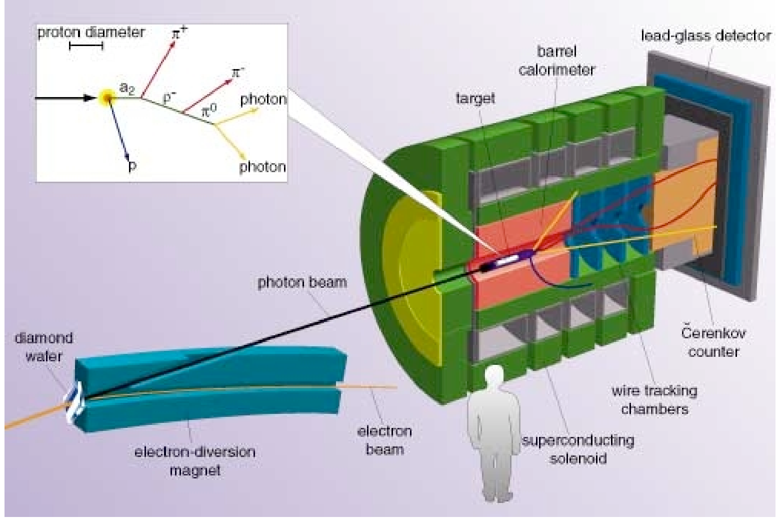


Hall B

Hall C

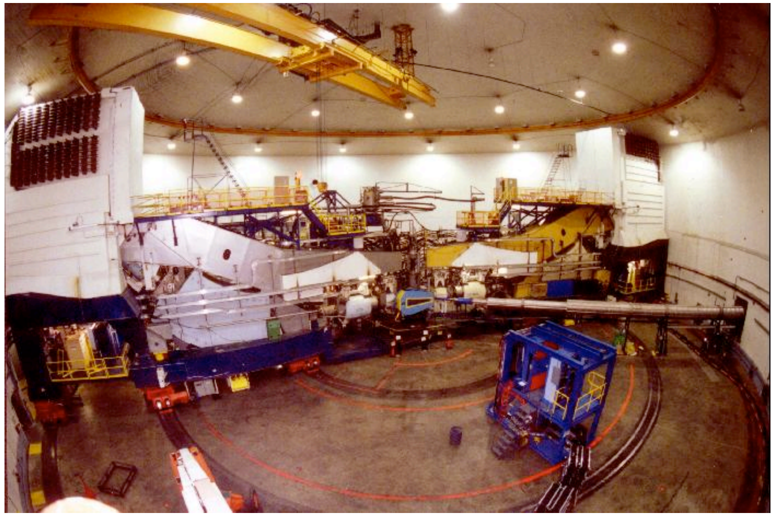


Hall D



Experimental Halls

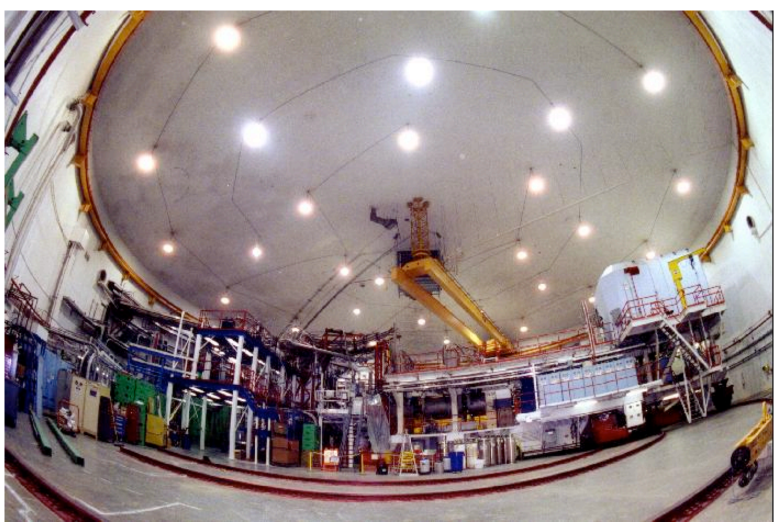
Hall A



high luminosity
$> 10^{38}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

very high precision
measurements

Hall C



high Q^2 form factors,
parity-violating e scattering,
precision structure functions,
...

Experimental Halls

Hall B

large acceptance

lower luminosity

$\sim 10^{35}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

CLAS

(CEBAF Large Acceptance Spectrometer)

collect all data "at once"

N^* spectroscopy
(multi-hadron final states),
structure function moments,
...

Experimental Halls

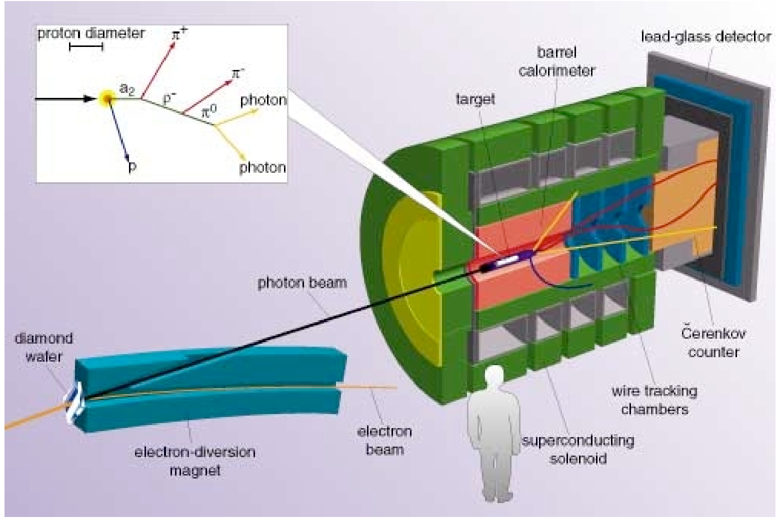
proposed new Hall
as part of 12 GeV upgrade

4π acceptance

photon beam

exotic meson spectroscopy
(GlueX Collaboration)
"origins of confinement"

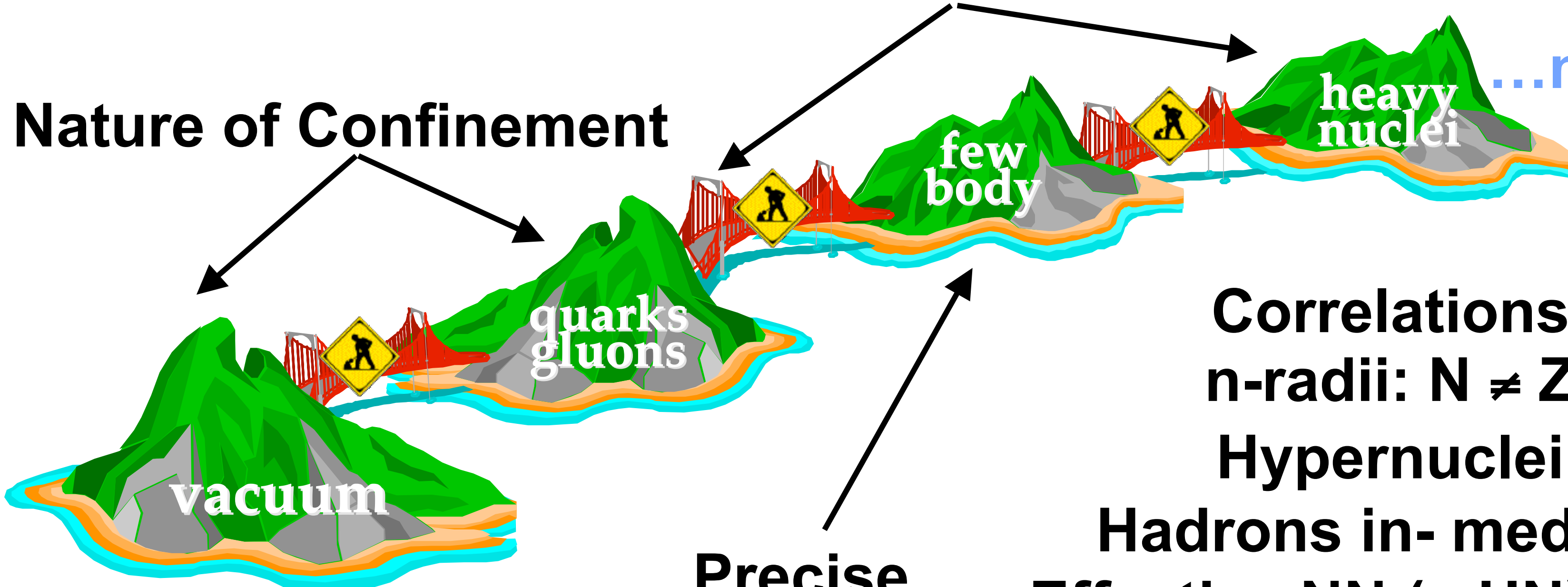
Hall D



JLab Central to *all* of Nuclear Science

Quark-Gluon Structure
Of Nucleons and Nuclei

Nature of Confinement



…n-stars

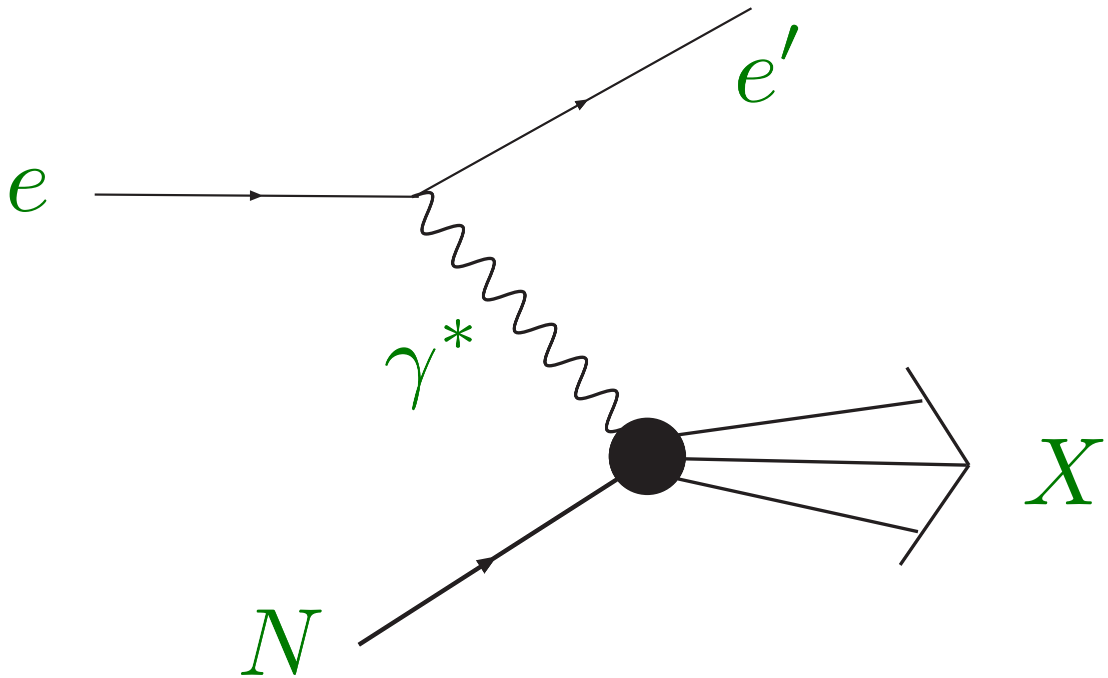
Correlations
n-radii: N ≠ Z
Hypernuclei
Hadrons in- medium
Effective NN (+ HN) force

Precise
few-nucleon
calculations

Exotic mesons
and baryons

Electron scattering

Inclusive cross section for $eN \to eX$



one-photon exchange approximation

Electron scattering

Inclusive cross section for $eN \to eX$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

F_1 , F_2 "structure functions"

$\longrightarrow$ contain all information about structure of nucleon

$\longrightarrow$ functions of x, Q^2 in general

Electron scattering

Inclusive cross section for $eN \to eX$

leptonic tensor

$$\frac{d^2\sigma}{d\Omega dE'} \sim L^{\mu\nu} \, W_{\mu\nu}$$

Hadronic tensor

$$W_{\mu\nu} = \sum_X \langle X|J_\mu(z)|N\rangle \, \langle N|J_\nu(0)|X\rangle \, \delta^4(p+q-p_X)$$
$$= \int d^4z \; e^{iq\cdot z} \, \langle N|J_\mu(z)J_\nu(0)|N\rangle$$

using completeness (sum over ALL states X)

$$\sum_X |X\rangle\langle X| = 1$$

"duality"

⟹ in general, $N \to X$ transition matrix element very complicated

⟹ at large Q^2 and large ν ("Bjorken limit") things simplify ...

- Wilson Operator Product Expansion

  Expand product of currents $J(z)J(0)$ in a series of (nonperturbative) local operators $\widehat{\mathcal{O}}$ and (perturbative) coefficient functions C_n

$$J(z)J(0) \;\sim\; \sum_n C_n(z^2)\; z^{\mu_1} z^{\mu_2} \dots z^{\mu_n}\; \widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$$

- Matrix elements of $\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$

M^2/Q^2 corrections

$$\langle N|\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}|N\rangle \;=\; \mathcal{A}_n(\mu^2)\; p_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} \;-\; \text{traces}$$

- Moments of structure function F_2

$$M_n(Q^2) \equiv \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$
$$= \sum_i \widetilde{C}_n^i(Q^2)\; \mathcal{A}_n^i(Q^2/\mu^2)$$

where $\widetilde{C}_n(Q^2)$ is Fourier transform of $C_n(z^2)$

- Reconstruct structure function from moments
  via inverse Mellin transform

- **Parton model:** $F_2(x,Q^2) = x \sum_q e_q^2\; q(x,Q^2)$

probability to find quark type "q" in nucleon,
carrying (light-cone) momentum fraction $x = \dfrac{p_q^+}{p_N^+} = \dfrac{p_q^0 + p_q^z}{p_N^0 + p_N^z}$

- Fourier transform of $J_\mu(z) J_\nu(0)$

$\longrightarrow$ series in $\left(\frac{1}{Q^2}\right)^{d-n-2}$, where $\tau \equiv d - n$

"twist"

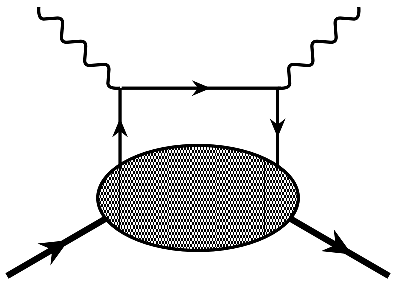
- Twist expansion of moments

$$M_n(Q^2) \;=\; A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

leading twist $(\tau = 2)$

e.g. $\bar{\psi}\, \gamma_\mu\, \psi$

$\longrightarrow$ *free quark scattering*



- Fourier transform of $J_\mu(z)J_\nu(0)$

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"twist"

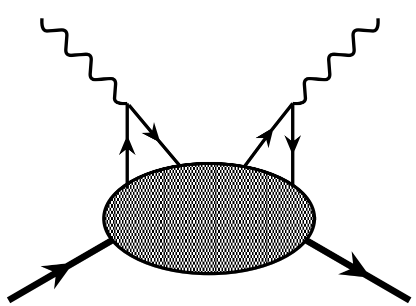
- Twist expansion of moments

$$M_n(Q^2) \;=\; A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

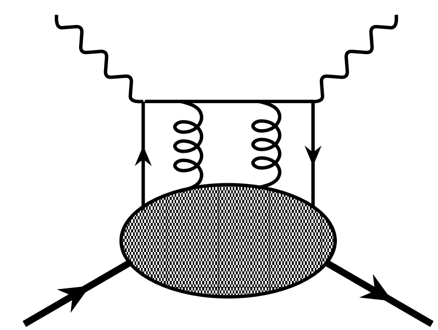
higher twists

e.g. $\bar\psi\ \gamma_\mu\ \psi\ \bar\psi\ \gamma_\nu\ \psi$

or $\bar\psi\ \widetilde{G}_{\mu\nu}\gamma^\nu\ \psi$



$\longrightarrow$ *multi-quark or quark-gluon correlations*



2.
Quark distributions

Parton distributions functions (PDFs)
(*leading twist*)

- PDFs provide basic information on structure of bound states in QCD

  → *momentum, flavour, spin ... distributions of quarks and gluons in hadrons*

- integrals of PDFs (*"moments"*) test fundamental sum rules (*Adler, Bjorken ...*)

  → *relate high-energy observables to low-energy hadron properties*

Parton distributions functions (PDFs)

(*leading twist*)

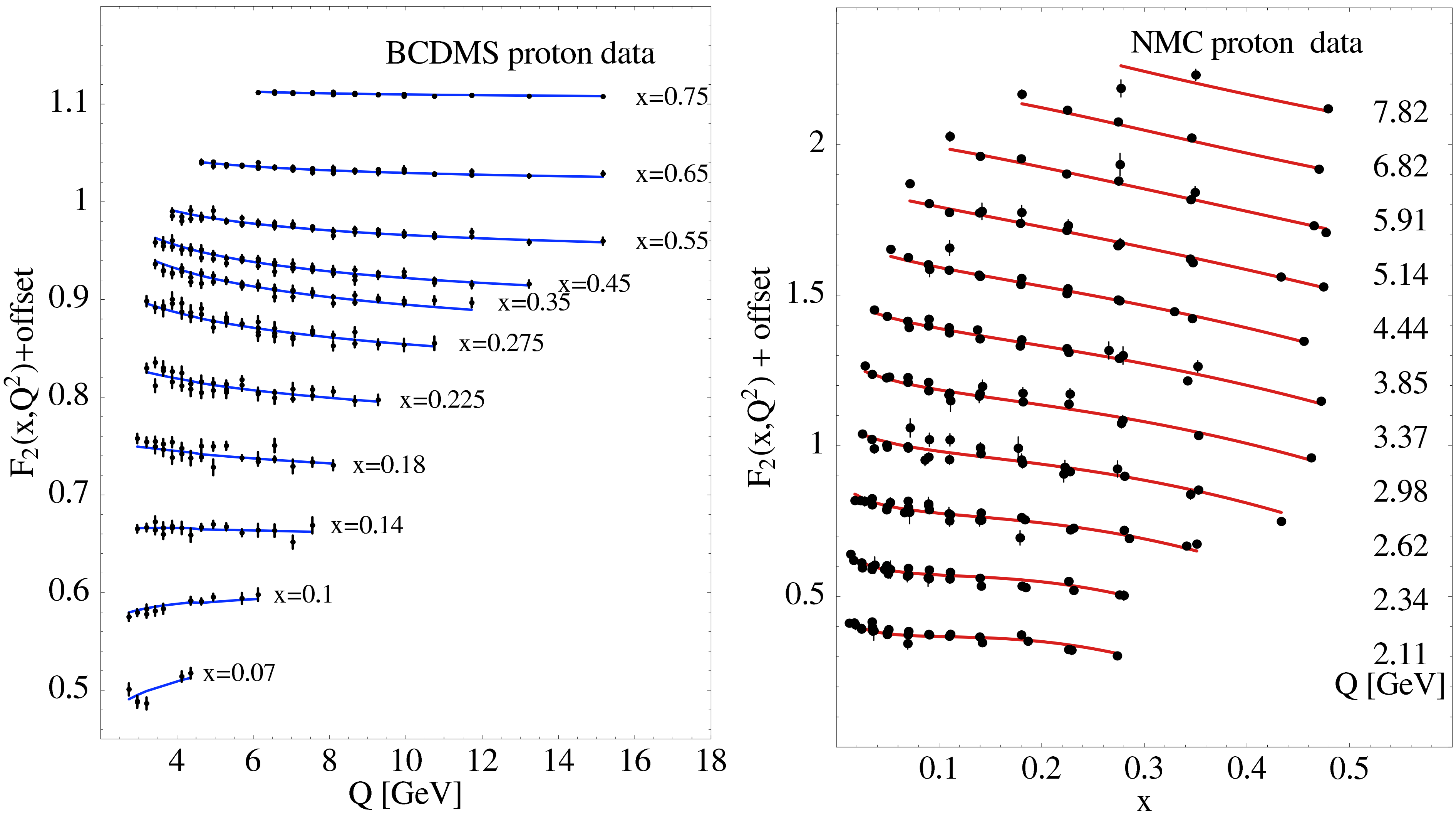
- provide input into nuclear physics and astrophysics calculations

  $\longrightarrow$ *e.g. relativistic heavy ion collisions*

- needed to understand backgrounds in searches for "new physics" beyond the Standard Model in high-energy colliders

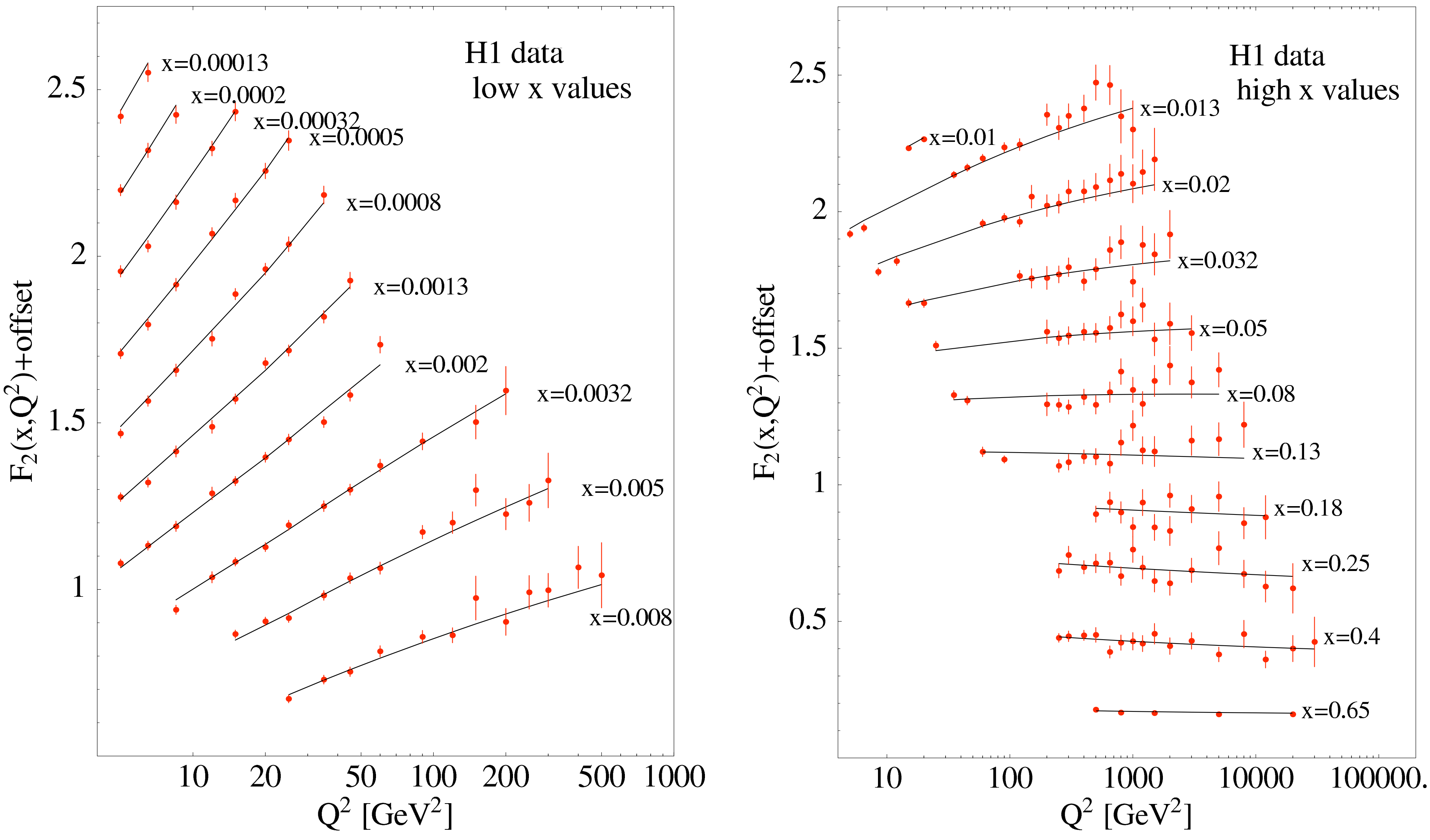
  $\longrightarrow$ *e.g. neutrino oscillations*

Structure function data



Lai et al., Eur. Phys. J. C12 (2000) 375

Structure function data



Lai et al., Eur. Phys. J. C12 (2000) 375

Parton distributions functions (PDFs)
(*leading twist*)

- PDFs extracted in global analyses of structure function data from electron, muon & neutrino scattering
  (also from Drell-Yan & W-boson production in hadronic collisions)

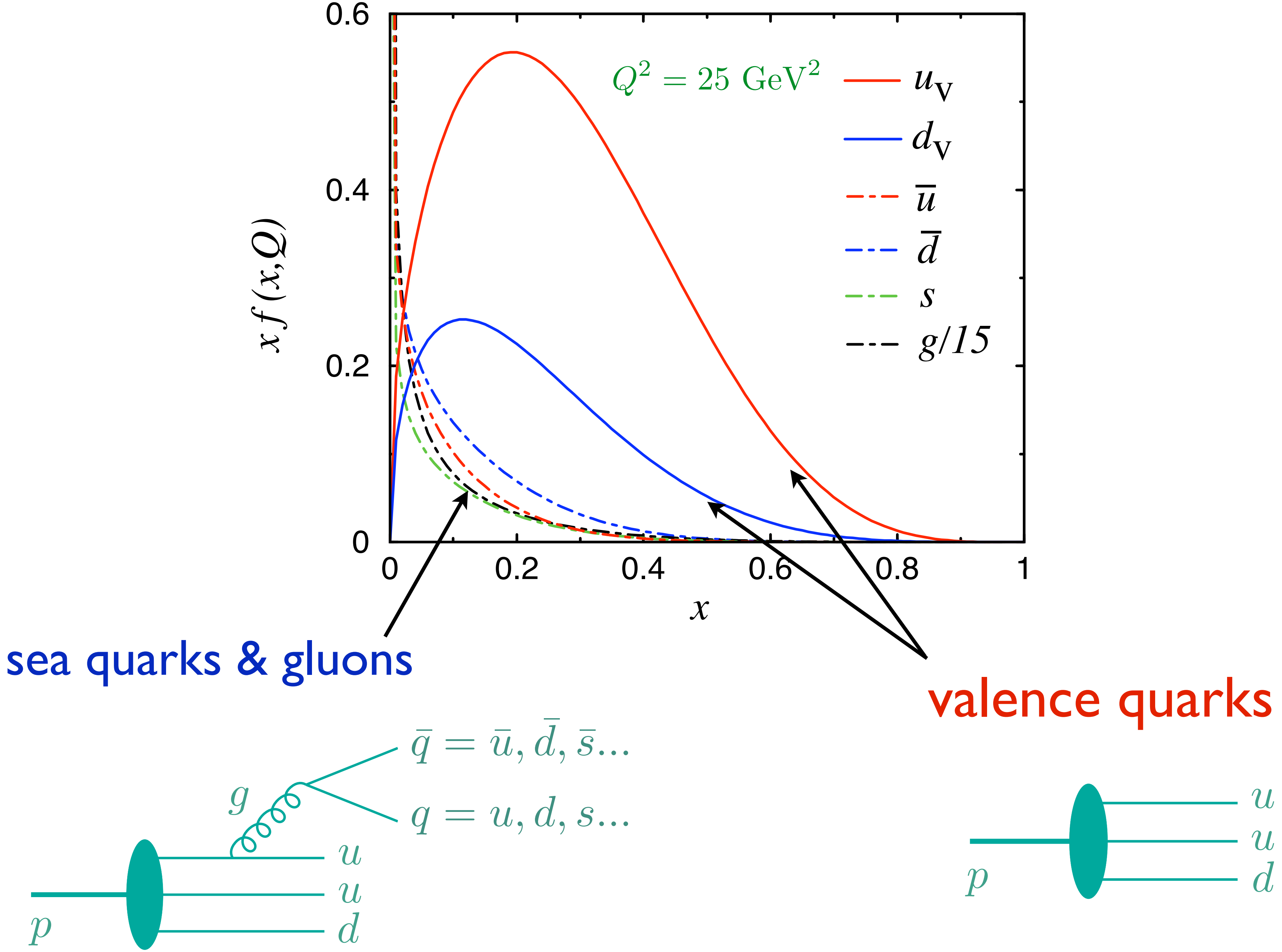
- parameterized using some functional form, e.g.

$$xq(x,Q^2) = A_0 \; x^{A_1}(1-x)^{A_2} \; e^{A_3 x}(1+e^{A_4}x)^{A_5}$$

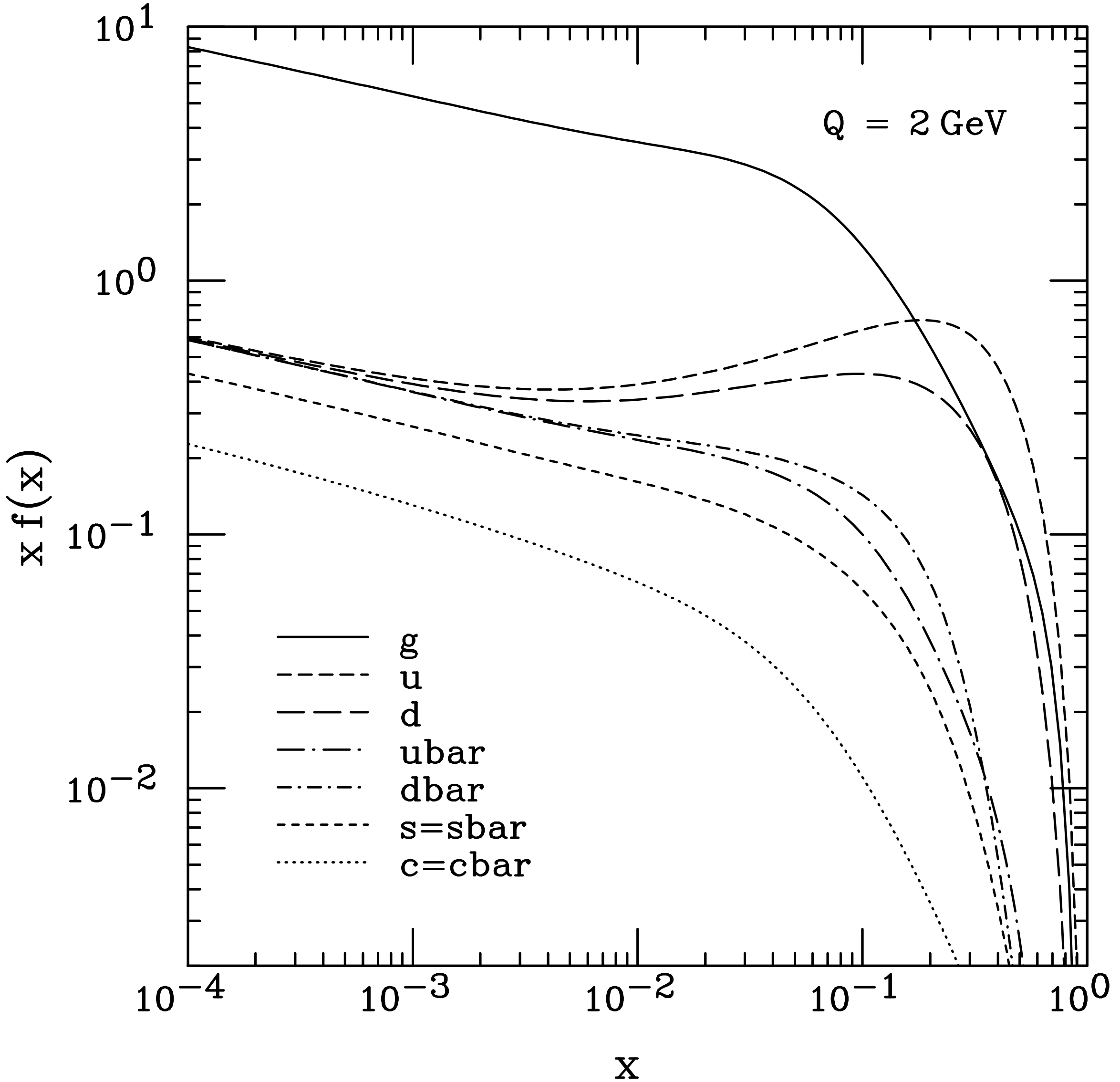
- determined over several orders of magnitude in x and Q^2

$$10^{-6} < x < 1$$

$$1 < Q^2 < 10^8 \; \mathrm{GeV}^2$$

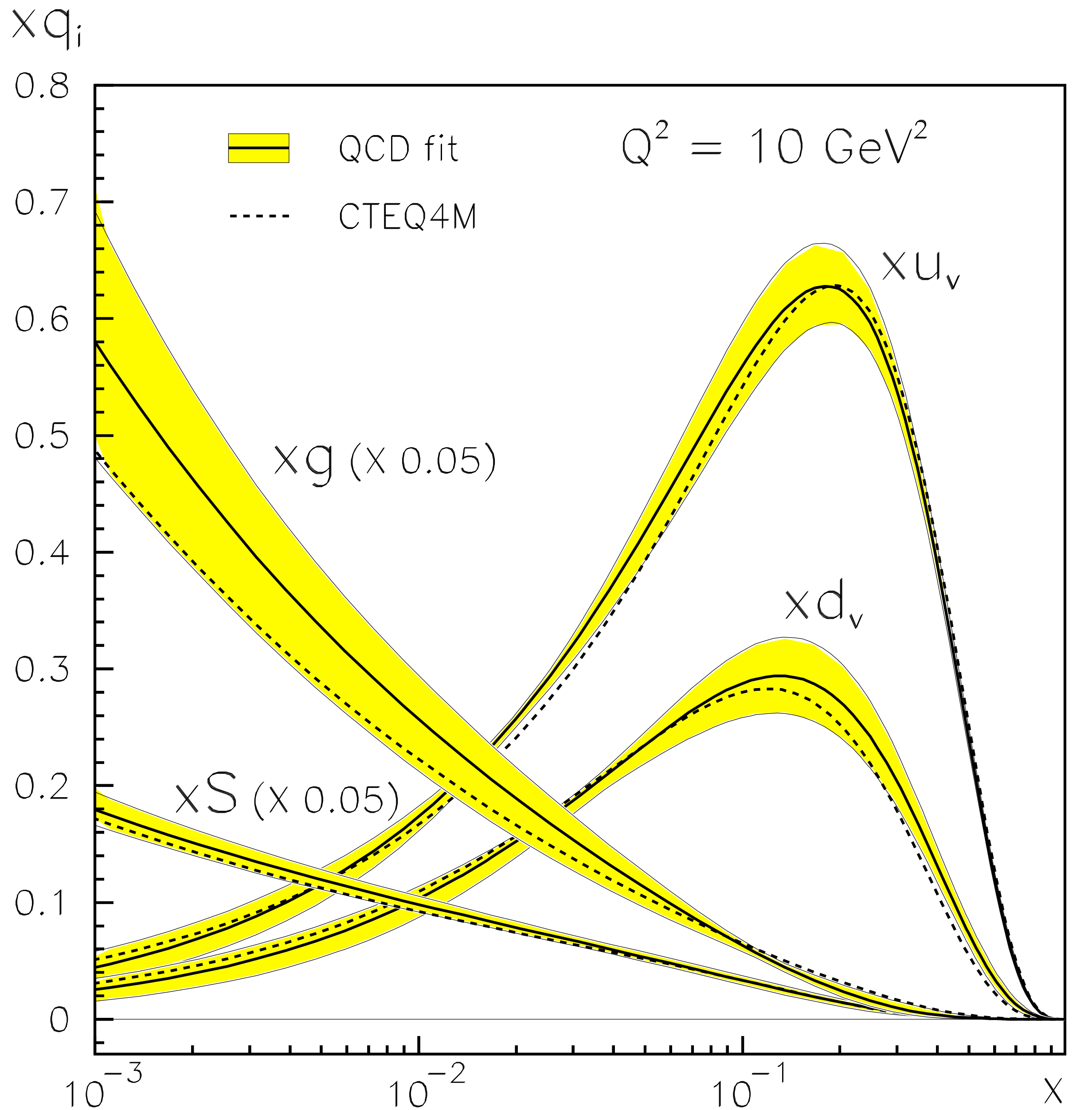


$Q^2 = 25\ \mathrm{GeV}^2$
u_V
d_V
$\bar{u}$
$\bar{d}$
s
$g/15$
$x\,f(x,Q)$
x
0
0.2
0.4
0.6
0.8
1
sea quarks & gluons
valence quarks
$\bar{q} = \bar{u}, \bar{d}, \bar{s}...$
$q = u, d, s...$
g
p
u
u
d
p
u
u
d



Virtual sea of $q\bar{q}$ pairs and gluons dominate small-x region

Lai et al., Eur. Phys. J. C12 (2000) 375



Botje, Eur. Phys. J. C 14 (2000) 285

2.
Quark distributions

- sea quarks

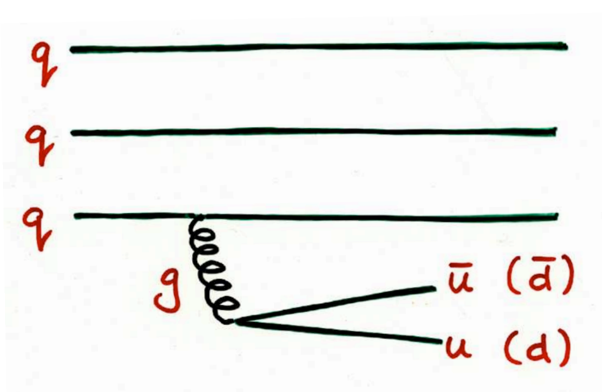
Sea quarks

- Because sea quarks & antiquarks are produced *"radiatively"* (by $g \to q\bar{q}$ radiation)

  $\longrightarrow$ expect flavour-symmetric sea
  IF quark masses are the same

  $\longrightarrow$ *e.g.* since $m_s \gg m_d \implies \bar{d}(x) > \bar{s}(x)$

- BUT since $m_u \approx m_d \implies$ expect $\bar{d}(x) \approx \bar{u}(x)$

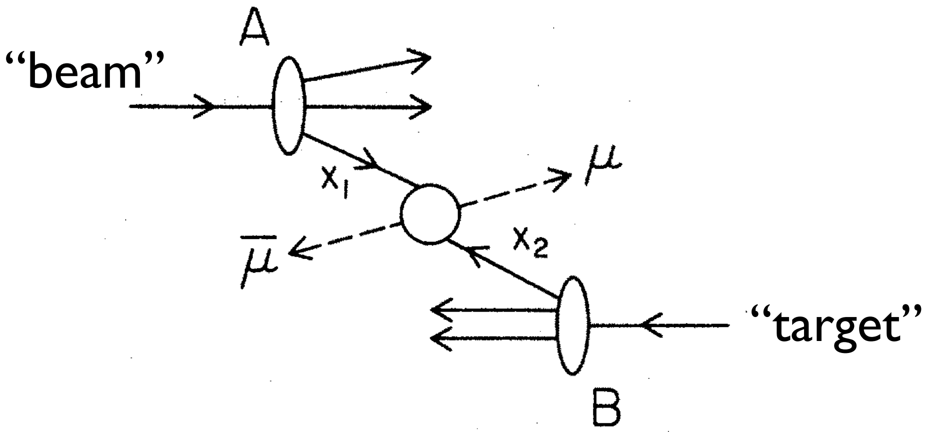


$\bar{d} = \bar{u}$

Fermilab E866 Drell-Yan experiment

$q\bar{q}$ annihilation in hadron-hadron collisions

$$q\bar{q} \rightarrow \gamma^* \rightarrow \mu^+\mu^-$$



Drell, Yan, Phys. Rev. Lett. 25 (1970) 316

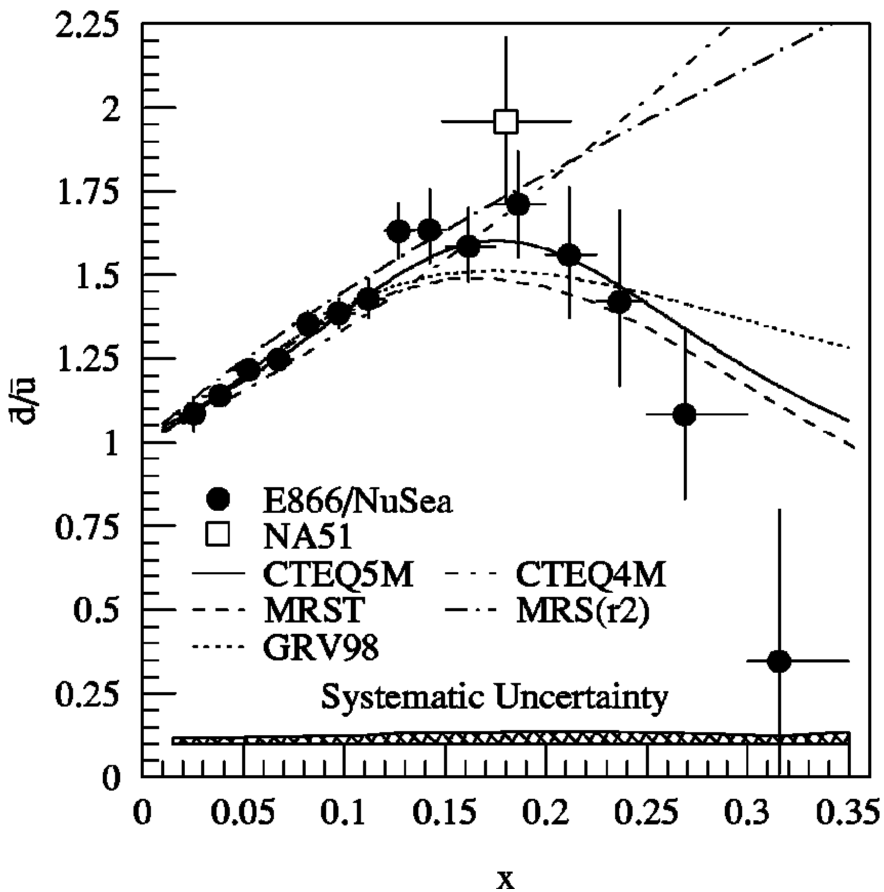
$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2 \left(q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t)\right)$$

For $x_b \gg x_t$

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)}\right)$$

Sea quarks

- Large $\bar{d} - \bar{u}$ asymmetry in proton observed in DIS (NMC) and Drell-Yan (CERN NA51 and FNAL E866) experiments



$$\int_0^1 dx\ (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

Towell et al., Phys. Rev. D 64 (2001) 052002

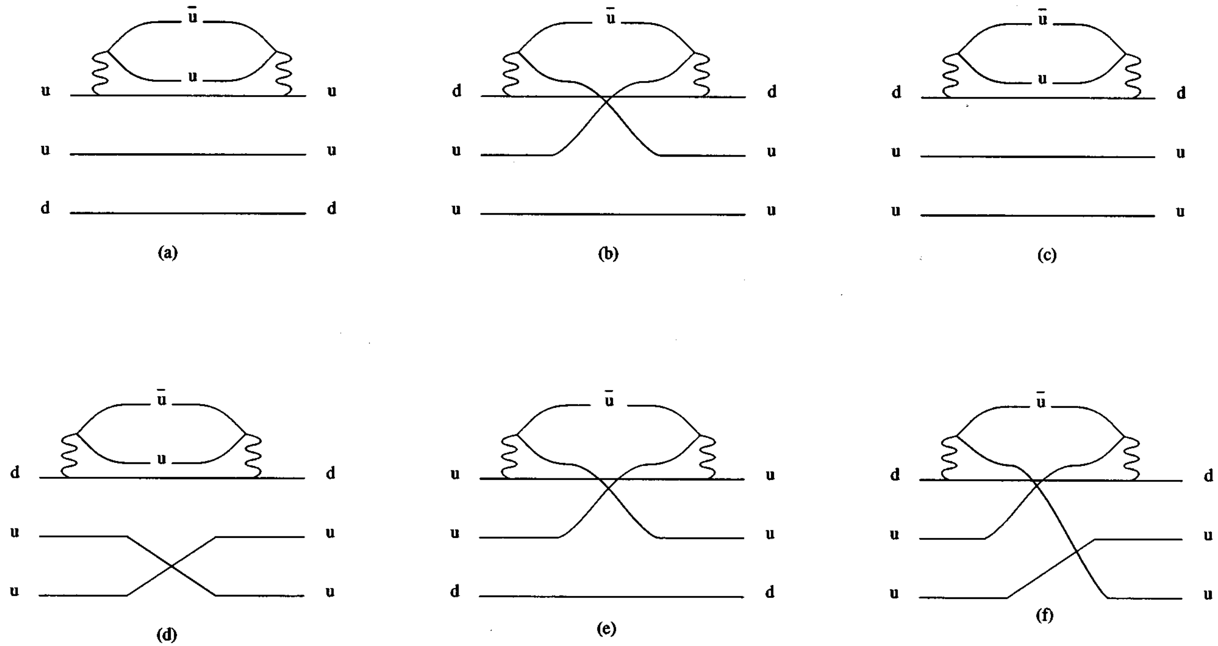
$\Longrightarrow$ why is $\bar{d} \gg \bar{u}$?

Sea quarks

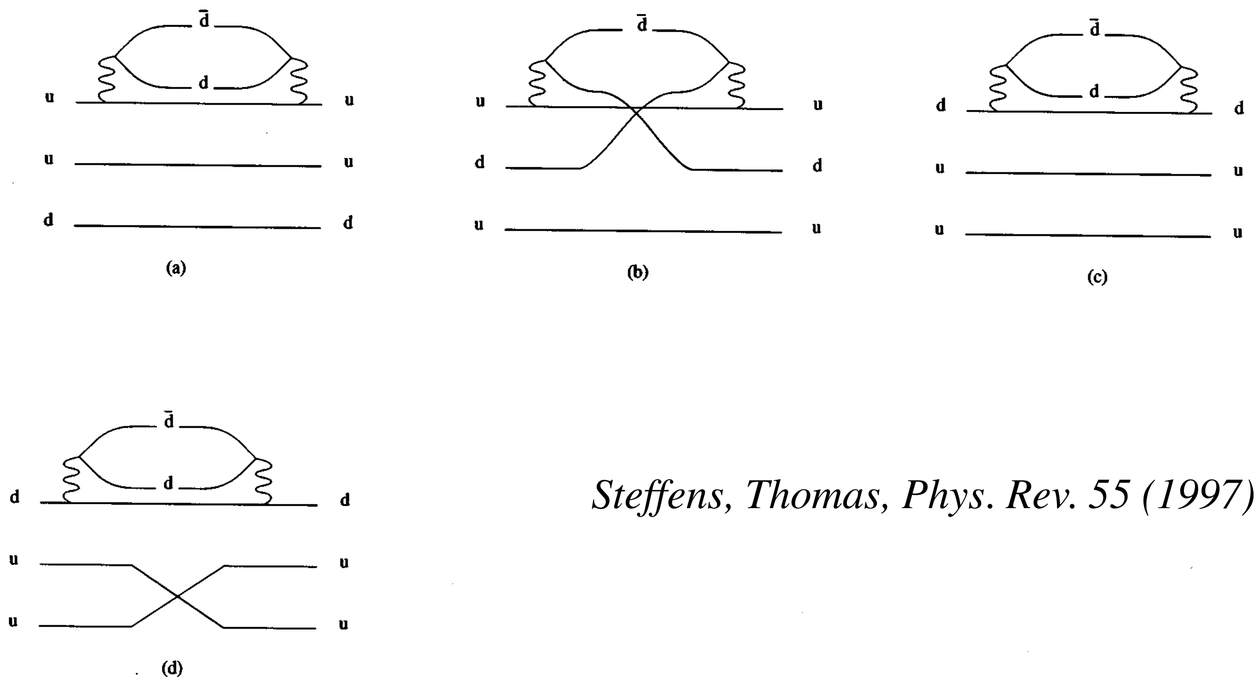
- Pauli Exclusion Principle

BARTENDER WITH PH.D. IN PHYSICS

$u\bar{u}$



$d\bar{d}$



Steffens, Thomas, Phys. Rev. 55 (1997) 900

Sea quarks

- Pauli Exclusion Principle

  ➡ since proton has more valence u than d
  $\longrightarrow$ easier to create $d\bar{d}$ than $u\bar{u}$

  Field, Feynman, Phys. Rev. D15 (1977) 2590

  ➡ explicit calculations of antisymmetrization effects in $g \to u\bar{u}$ and $g \to d\bar{d}$

  ➡ $\bar{u} > \bar{d}$

  asymmetry tiny

  Ross, Sachrajda, Nucl. Phys. B149 (1979) 497
  Steffens, Thomas, Phys. Rev. 55 (1997) 900

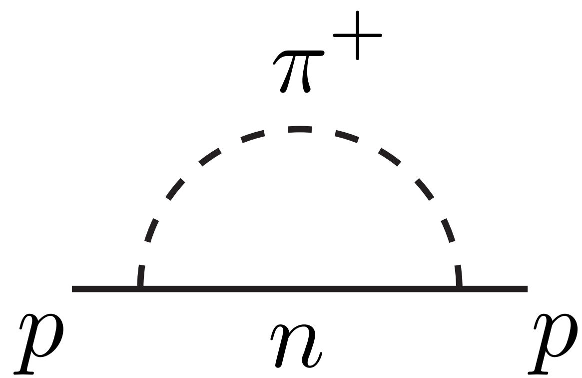
"BUT, HEISENBERG — YOU MUST BE CERTAIN ABOUT SOMETHING!"

Sea quarks

- Pion cloud

  ➡ some of the time the proton looks like a neutron & π^+

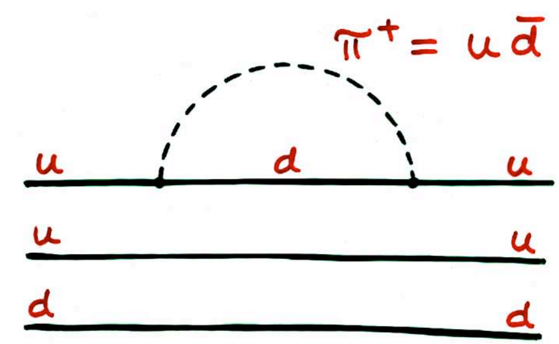
  (Heisenberg Uncertainty Principle)



$$p \to \pi^+ \; n \to p$$

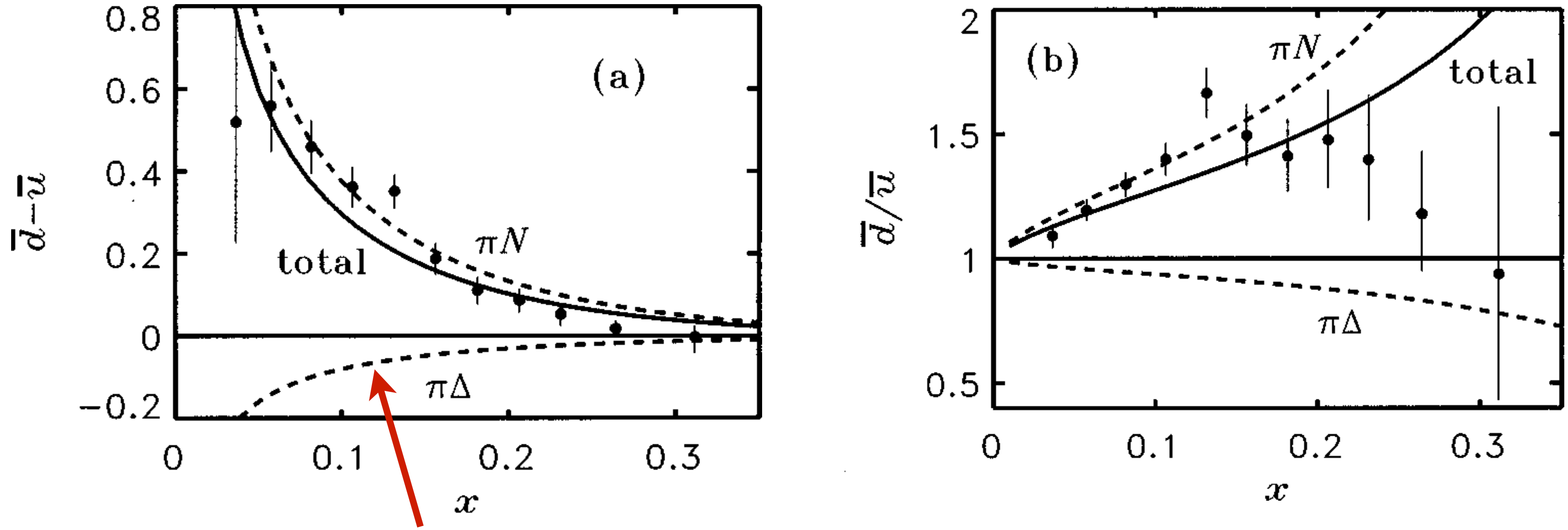
  ➡ at the quark level

$$uud \to (udd)(\bar{d}u) \to uud$$



  ➡ $\bar{d} > \bar{u}$!

Thomas, Phys. Lett. 126B (1983) 97

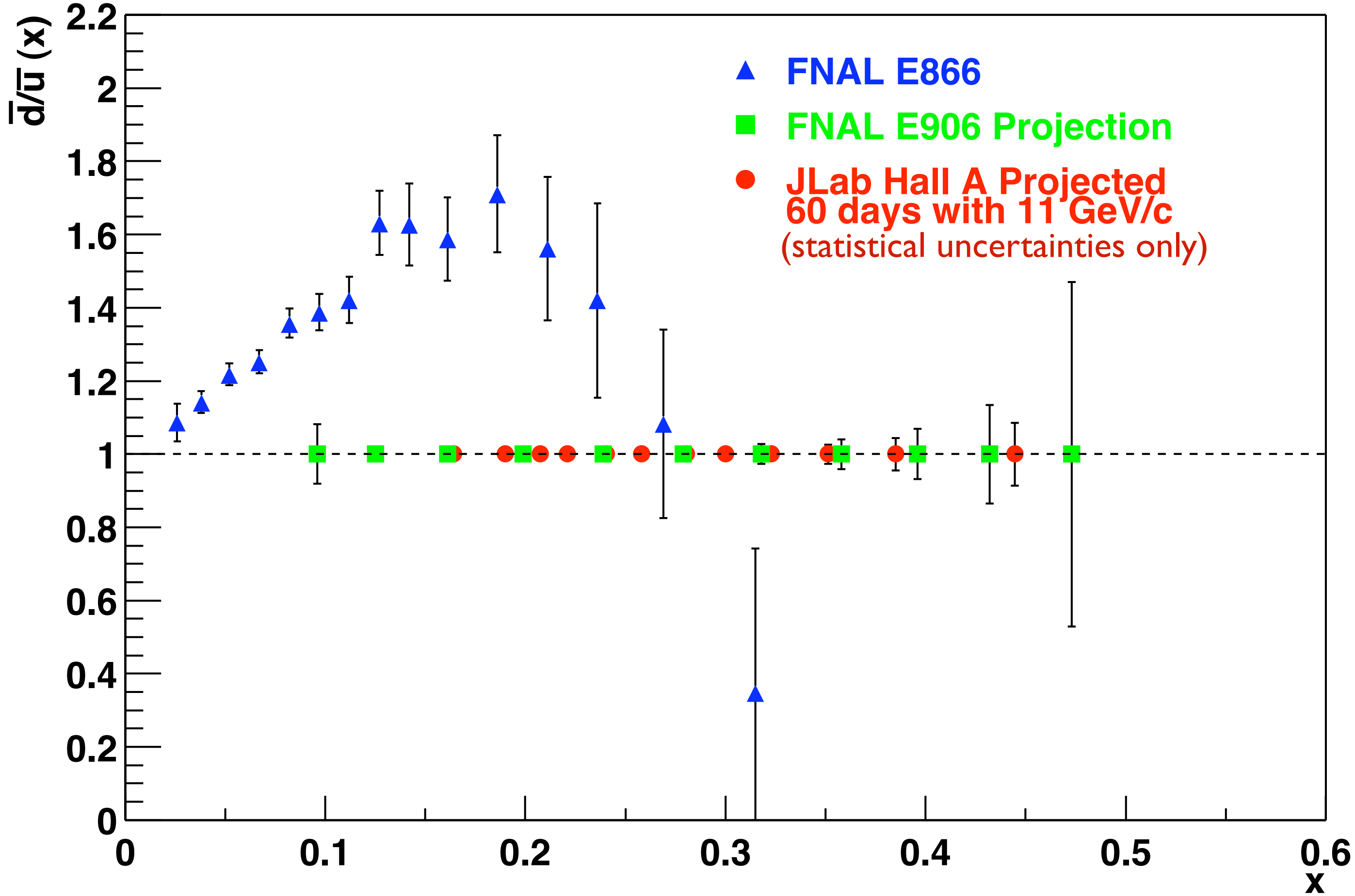


$$p(uud) \to \pi^-(d\bar{u}) + \Delta^{++}(uuu)$$
$$\Rightarrow \bar{u} > \bar{d}$$

WM, Speth, Thomas
Phys. Rev. D59 (1998) 014033

➡ difficult to understand quantitatively large x behavior

➡ JLab can significantly improve uncertainties at large x



$\bar{d}/\bar{u}$ (x)
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
x
FNAL E866
FNAL E906 Projection
JLab Hall A Projected
60 days with 11 GeV/c
(statistical uncertainties only)

Polarization asymmetry of proton sea
(aside...)

Neither gluon radiation nor pion cloud contribute to $\Delta\bar{d} - \Delta\bar{u}$

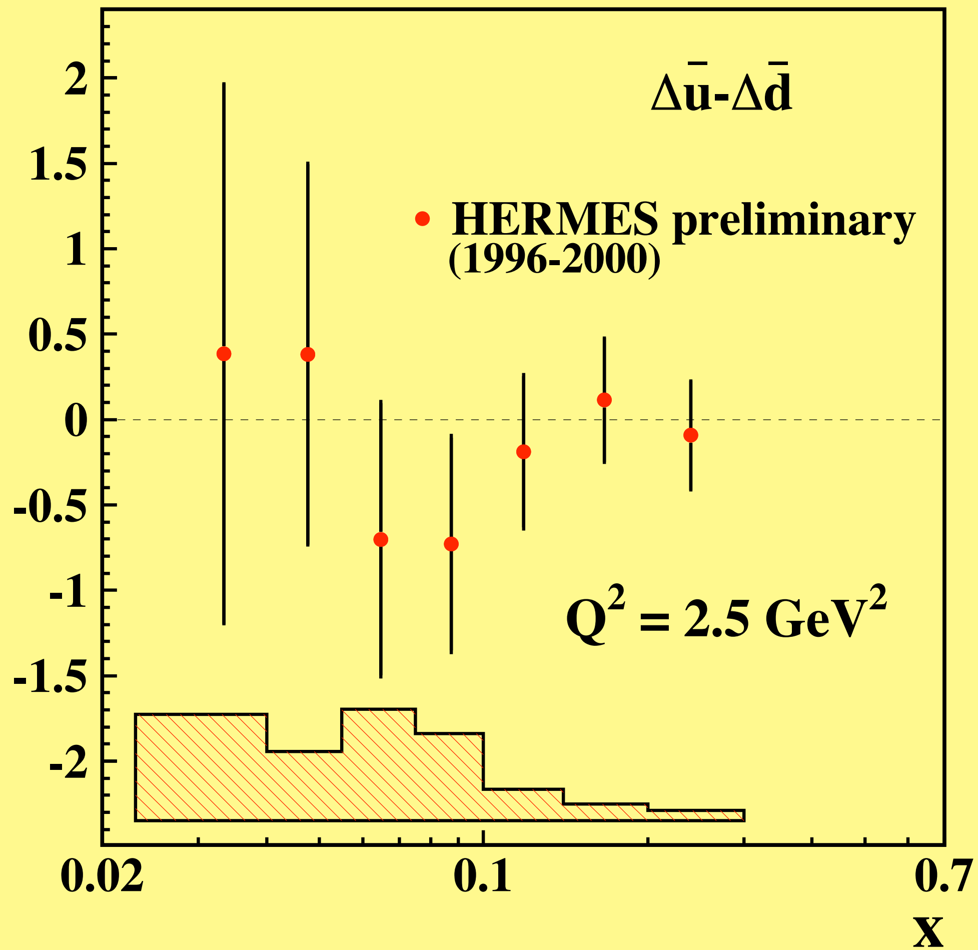
Pauli Exclusion Principle (antisymmetrization)

$$\Longrightarrow \quad \Delta\bar{u} - \Delta\bar{d} \;\approx\; \frac{5}{3}(\bar{d} - \bar{u})$$

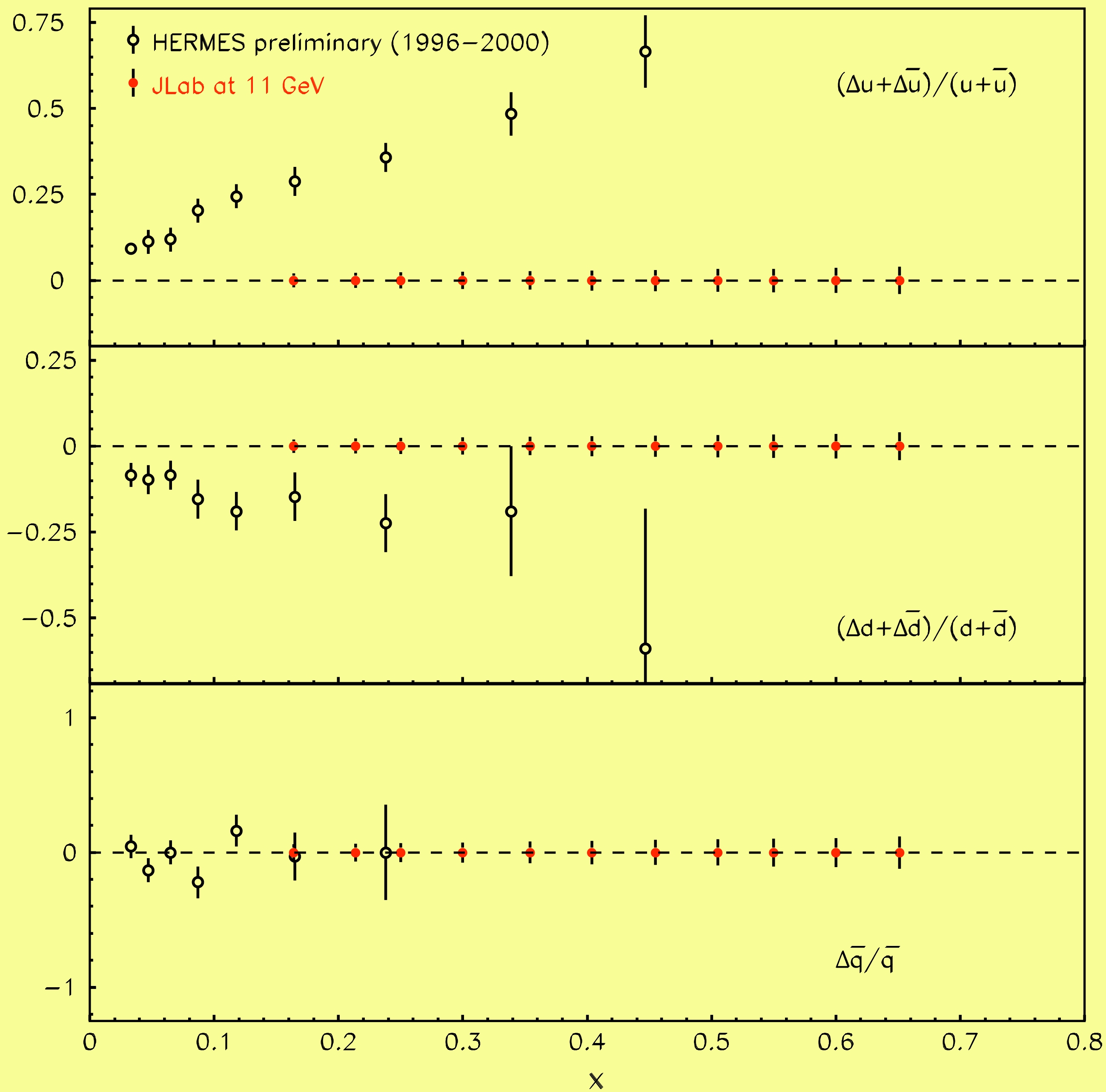
$\Longrightarrow$ also contributes to $\bar{d} - \bar{u}$

Disentangle origin of unpolarized and polarized asymmetries in sea via semi-inclusive DIS

Polarization asymmetry of proton sea
(aside...)



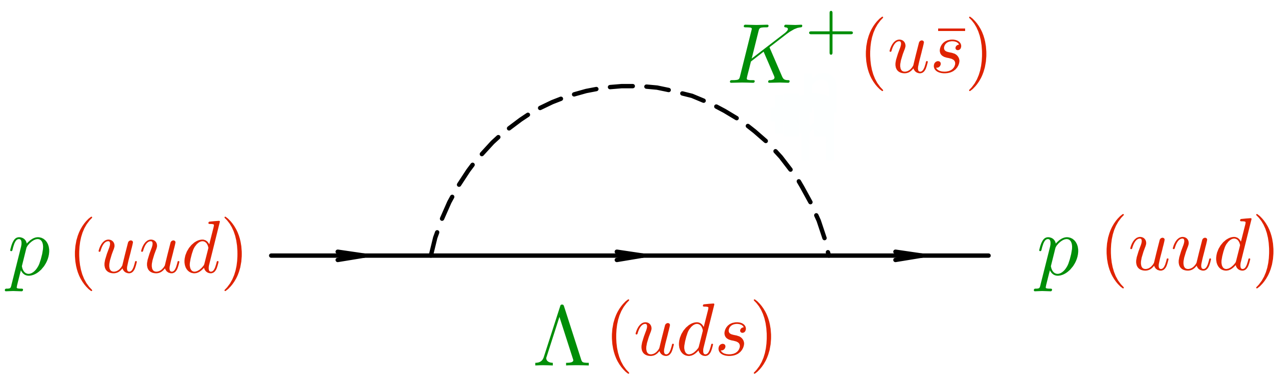
Polarization asymmetry of proton sea
(aside...)



Sea quarks

- Strange asymmetry

$s \neq \bar{s}$ can similarly be generated by nonperturbative *kaon cloud*



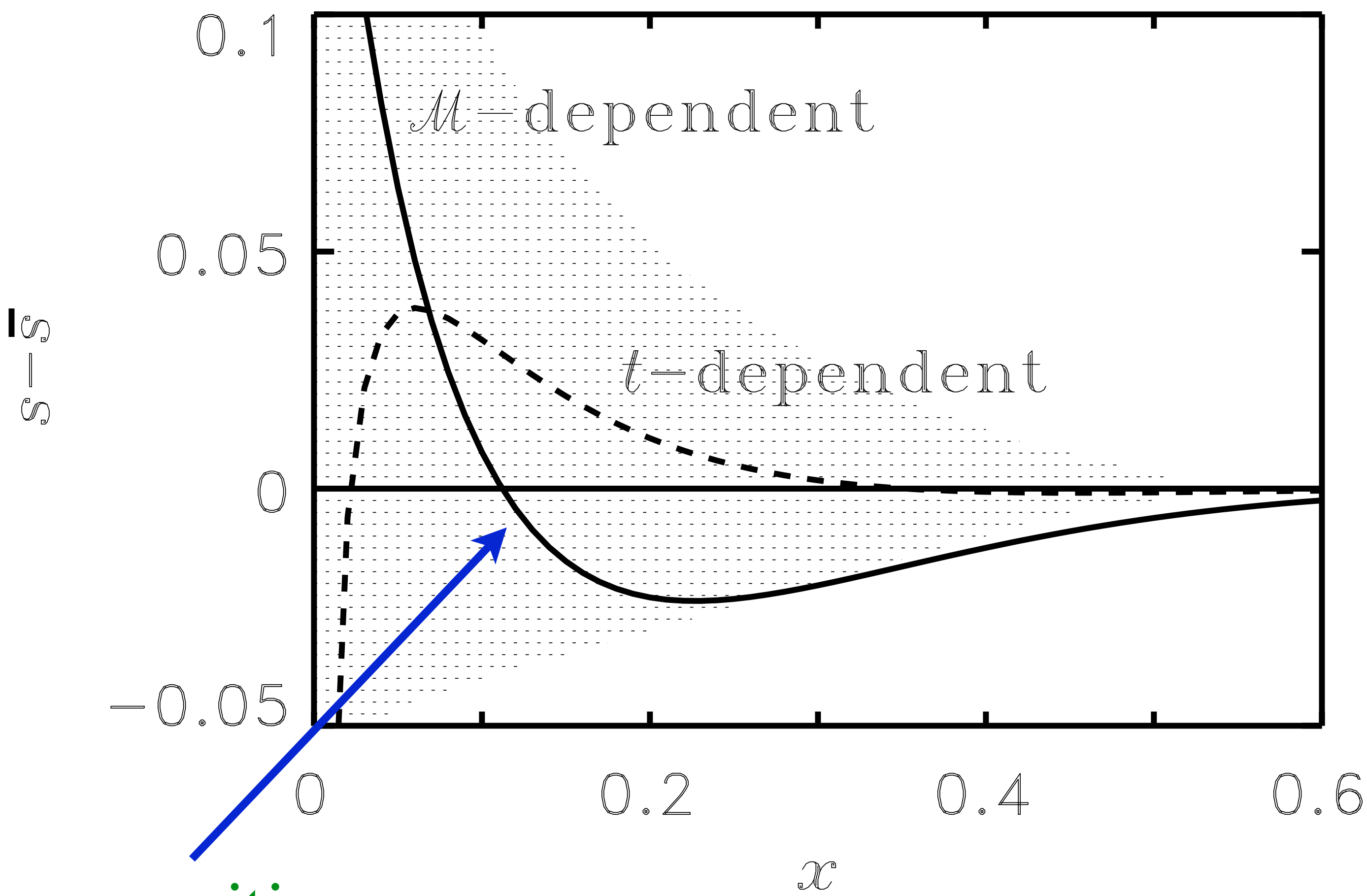
Signal, Thomas, Phys. Lett. B 191, 205 (1987)

⟹ net number of strange quarks must be zero

$$\int_0^1 dx\ (s - \bar{s}) = 0$$

Sea quarks

- Strange asymmetry



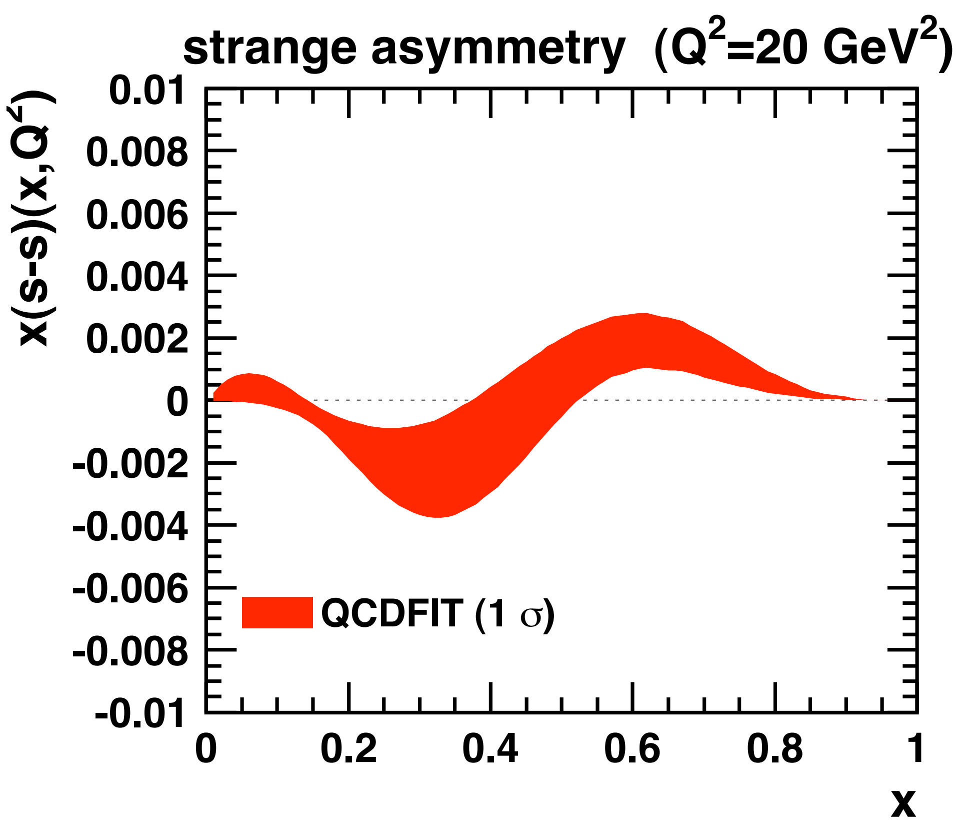
shape very sensitive to details of $Kp\Lambda$ interaction

WM, Malheiro, Phys. Lett. B451, 229 (1999)

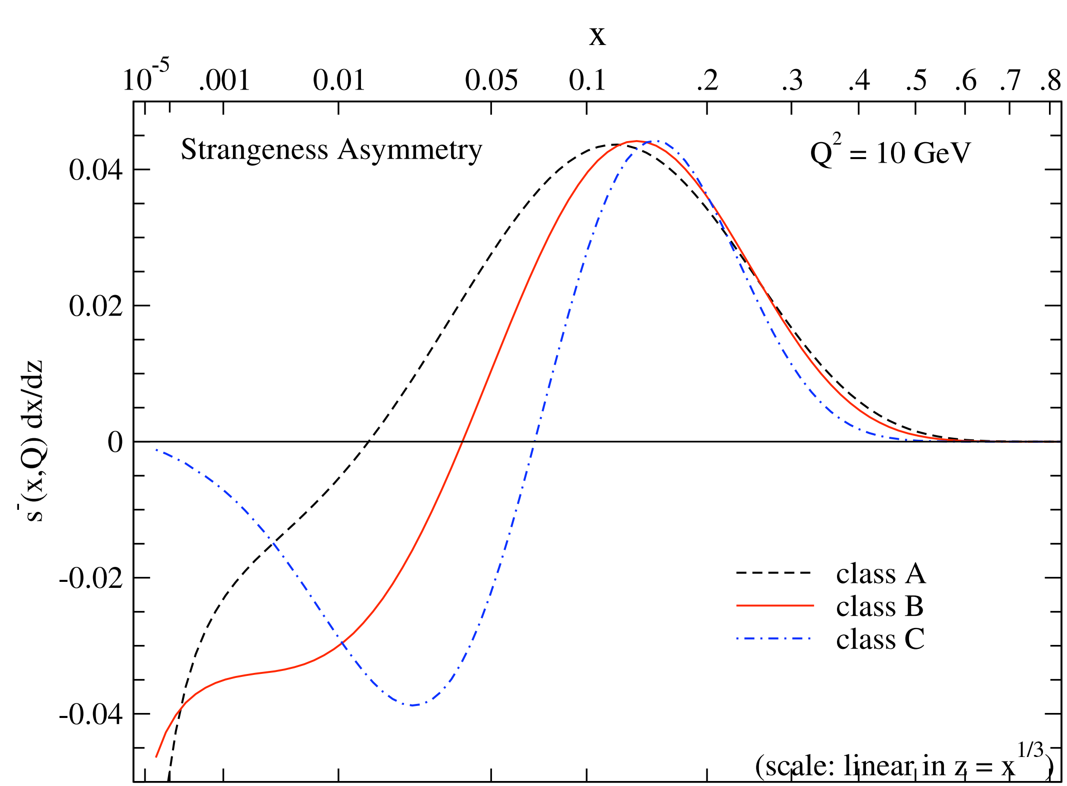
Sea quarks

- Strange asymmetry

  ⟹ shape from global fits also not well constrained



B. Portheault, hep-ph/0406226



S. Kretzer, hep-ph/0408287

Sea quarks

- Strange asymmetry

➡ can also be generated perturbatively by higher-order (3-loop) gluon radiation

Catani et al., hep-ph/0404240

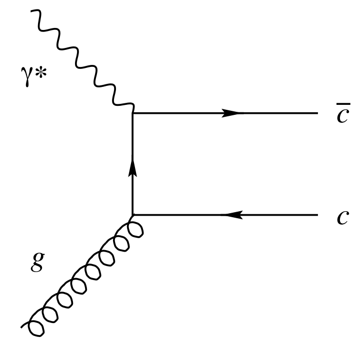
... though cannot predict shape

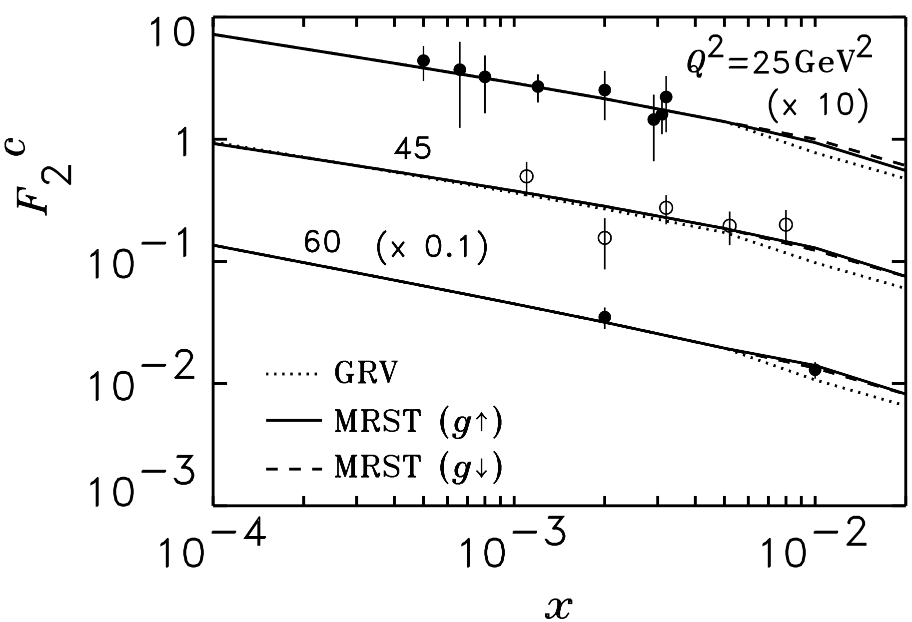
➡ $s \neq \bar{s}$ can have significant impact on extraction of $\sin^2\theta_W$ from $\nu,\ \bar{\nu}$ data

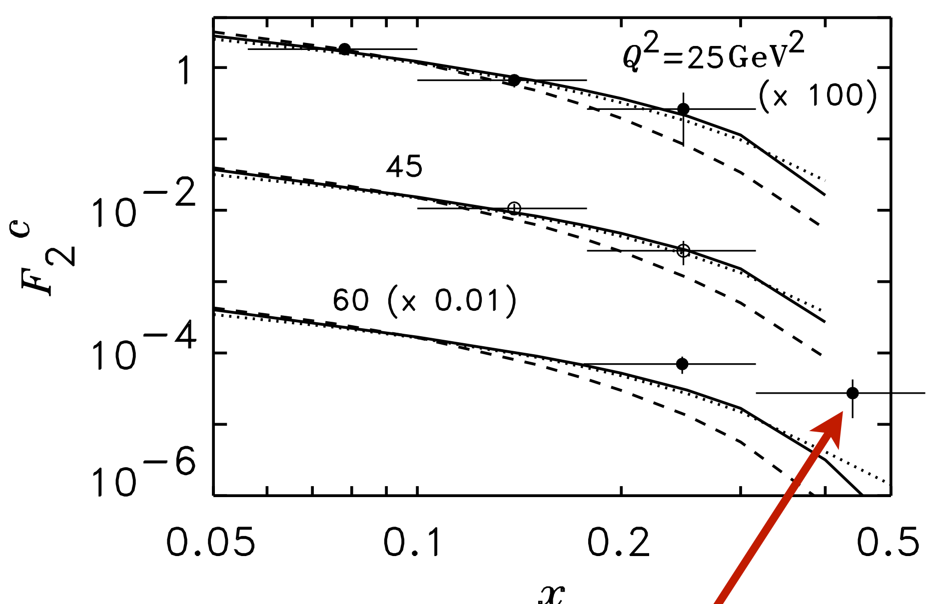
Sea quarks

- Charm structure function

photon-gluon fusion







disagreement with perturbative charm??

Sea quarks

- Intrinsic (nonperturbative) charm

➡ $|p\rangle = c_0|uud\rangle + c_1|uudc\bar{c}\rangle$ 1% normalisation

$$c^{\mathrm{IC1}}(x) = 6x^2\left((1-x)(1+10x+x^2) - 6x(1+x)\log 1/x\right)$$

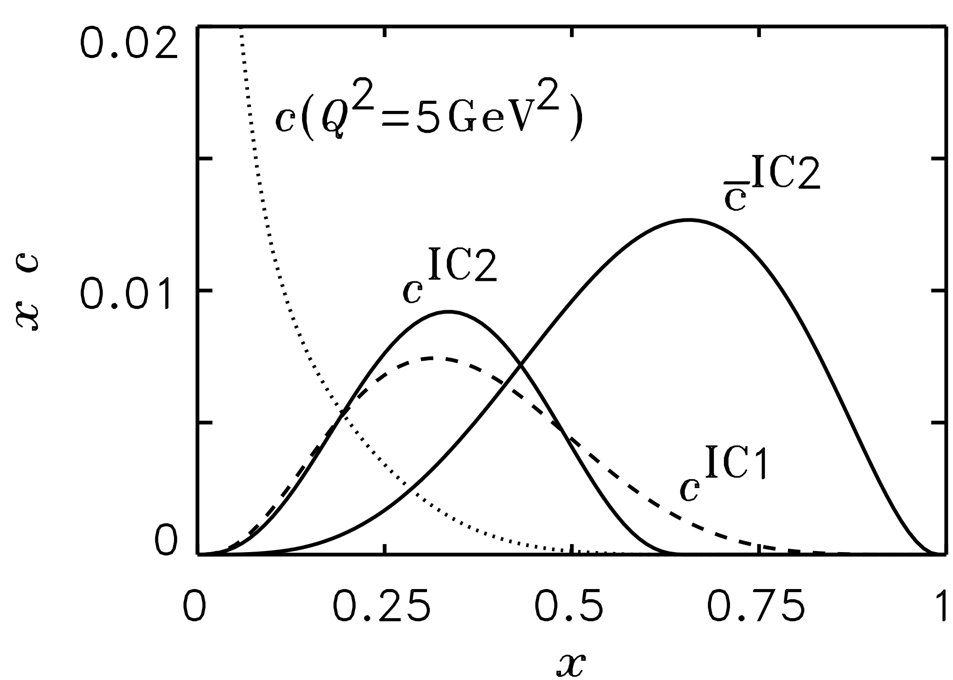
➡ meson cloud model

$$c^{\mathrm{IC2}}(x) = \int_x^1 \frac{dz}{z} f_{\Lambda_c/N}(z)\; c^{\Lambda_c}(x/z)$$

$$\approx \frac{3}{2} f_{\Lambda_c/N}(3x/2)$$

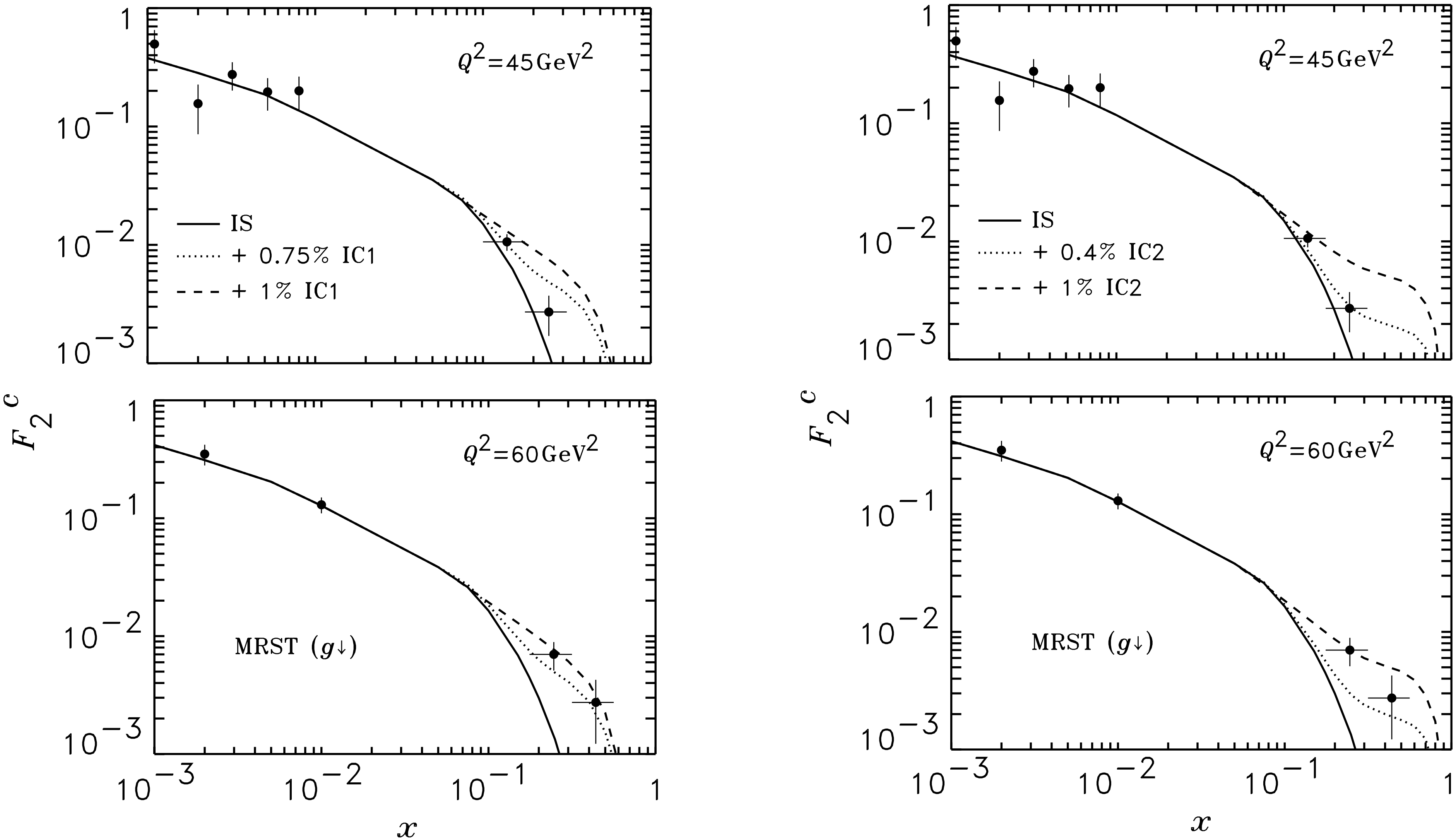
$$\bar{c}^{\mathrm{IC2}}(x) = \int_x^1 \frac{dz}{z} f_{\bar{D}/N}(z)\; \bar{c}^{D^-}(x/z)$$

$$\approx f_{\bar{D}/N}(x)$$



Sea quarks

- Perturbative + intrinsic charm



→ need more data at large x!

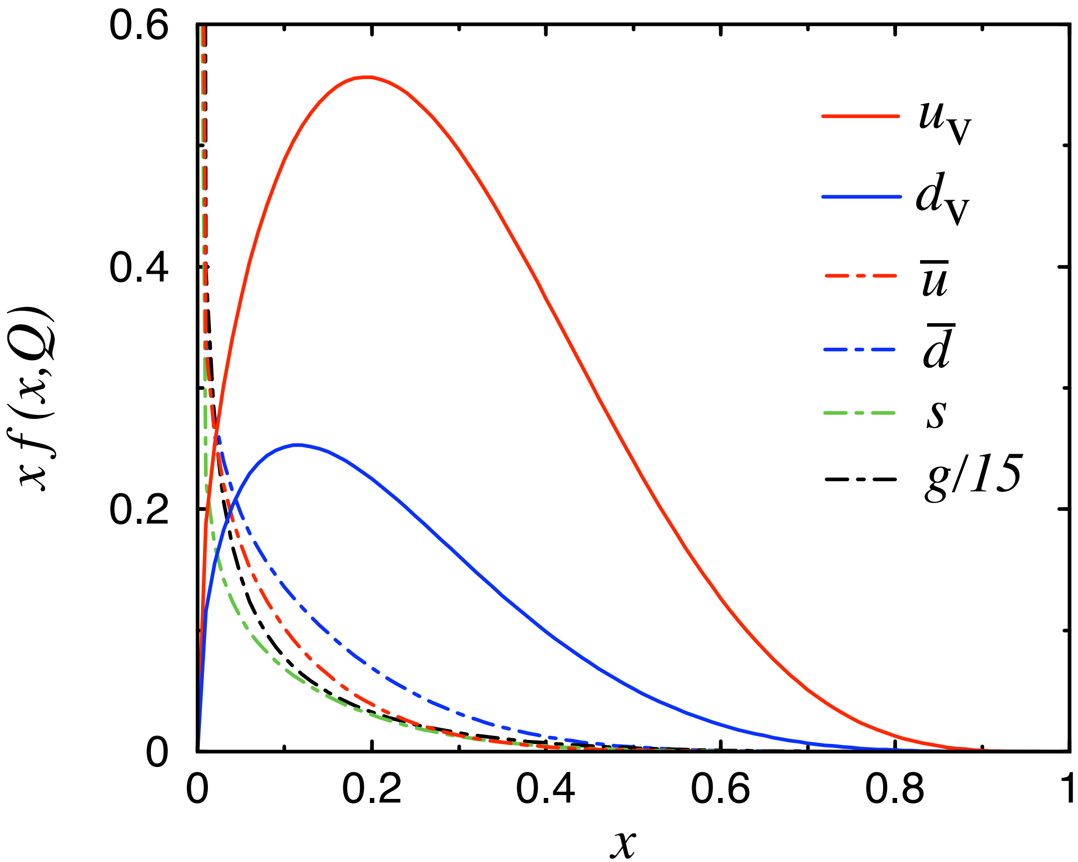
2.
Quark distributions

- valence quarks

Valence quarks

- Most direct connection between quark distributions and models of the nucleon is through *valence* quarks

- Nucleon structure at intermediate & large x dominated by valence quarks



Valence quarks

- At large x, valence u and d distributions extracted from p and n structure functions

$$F_2^p \approx \frac{4}{9} u_v + \frac{1}{9} d_v$$

$$F_2^n \approx \frac{4}{9} d_v + \frac{1}{9} u_v$$

- u quark distribution well determined from p

- d quark distribution requires n structure function

$$\Longrightarrow \quad \frac{d}{u} \approx \frac{4 - F_2^n/F_2^p}{4F_2^n/F_2^p - 1}$$

Valence quarks

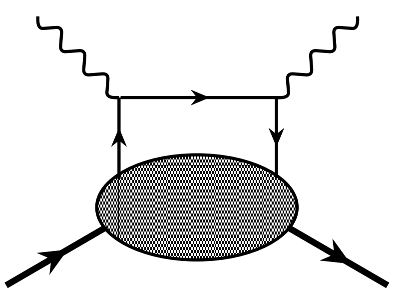
- Ratio of d to u quark distributions particularly sensitive to quark dynamics in nucleon

- <u>SU(6) spin-flavour symmetry</u>

proton wave function

$$p^{\uparrow} = -\frac{1}{3}d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3}d^{\downarrow}(uu)_1$$

$$+ \frac{\sqrt{2}}{6}u^{\uparrow}(ud)_1 - \frac{1}{3}u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}}u^{\uparrow}(ud)_0$$



interacting quark

spectator diquark

diquark spin

Valence quarks

- Ratio of d to u quark distributions particularly sensitive to quark dynamics in nucleon

- SU(6) spin-flavour symmetry

proton wave function

$$p^{\uparrow} = -\frac{1}{3}d^{\uparrow}(uu)_1 - \frac{\sqrt{2}}{3}d^{\downarrow}(uu)_1 + \frac{\sqrt{2}}{6}u^{\uparrow}(ud)_1 - \frac{1}{3}u^{\downarrow}(ud)_1 + \frac{1}{\sqrt{2}}u^{\uparrow}(ud)_0$$

$\longrightarrow$ $u(x) = 2\ d(x)$ for all x

$\longrightarrow$ $\dfrac{F_2^n}{F_2^p} = \dfrac{2}{3}$

Valence quarks

- scalar diquark dominance

$$M_\Delta > M_N \quad \Longrightarrow \quad (qq)_1 \text{ has larger energy than } (qq)_0$$

$$\Longrightarrow \quad \underline{\text{scalar diquark dominant in } x \to 1 \text{ limit}}$$

since only u quarks couple to scalar diquarks

$$\longrightarrow \quad \frac{d}{u} \to 0$$

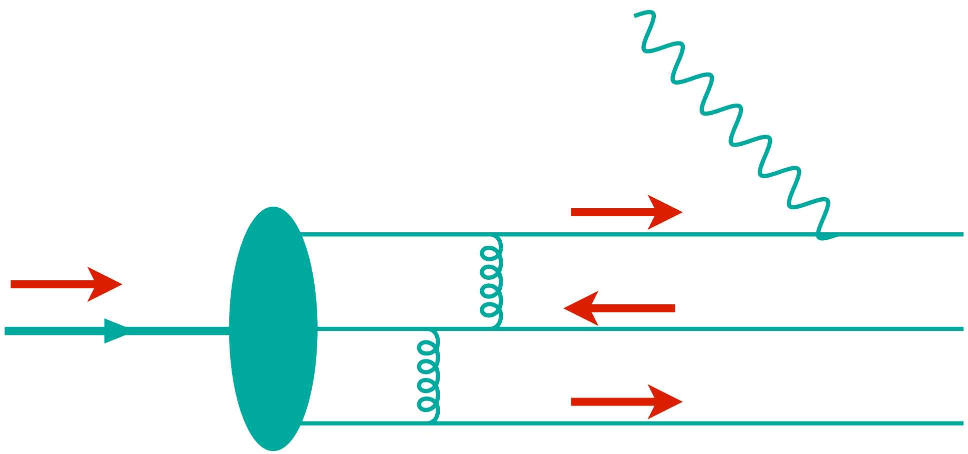
$$\longrightarrow \quad \frac{F_2^n}{F_2^p} \to \frac{1}{4}$$

Feynman 1972, Close 1973, Close/Thomas 1988

Valence quarks

- <u>hard gluon exchange</u>

at large x, helicity of struck quark = helicity of hadron



$$q^{\uparrow} \gg q^{\downarrow}$$

$\Longrightarrow$ <u>helicity-zero diquark dominant in $x \to 1$ limit</u>

$\longrightarrow$ $$\frac{d}{u} \to \frac{1}{5}$$

$\longrightarrow$ $$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$$

Farrar, Jackson 1975

Valence quarks

- BUT no free neutron targets!
  (neutron half-life ~ 12 mins)

  ⟹ use deuteron as "effective neutron target"

- However: deuteron is a nucleus, and $F_2^d \neq F_2^p + F_2^n$

  ⟹ nuclear effects (nuclear binding, Fermi motion, shadowing) *obscure neutron structure* information

  ⟹ "nuclear EMC effect"

2.
Quark distributions

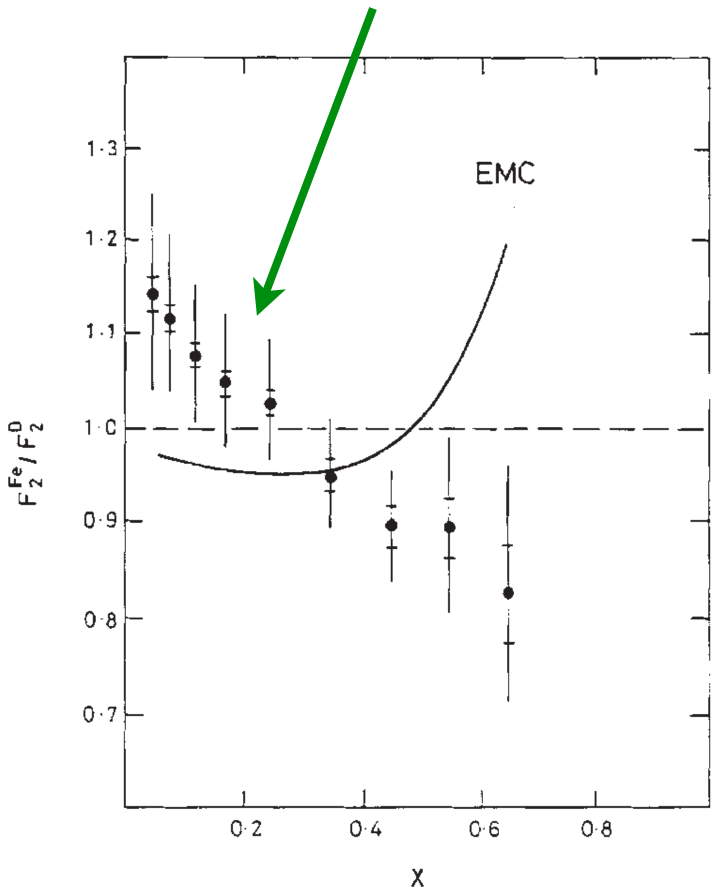
- nuclear effects

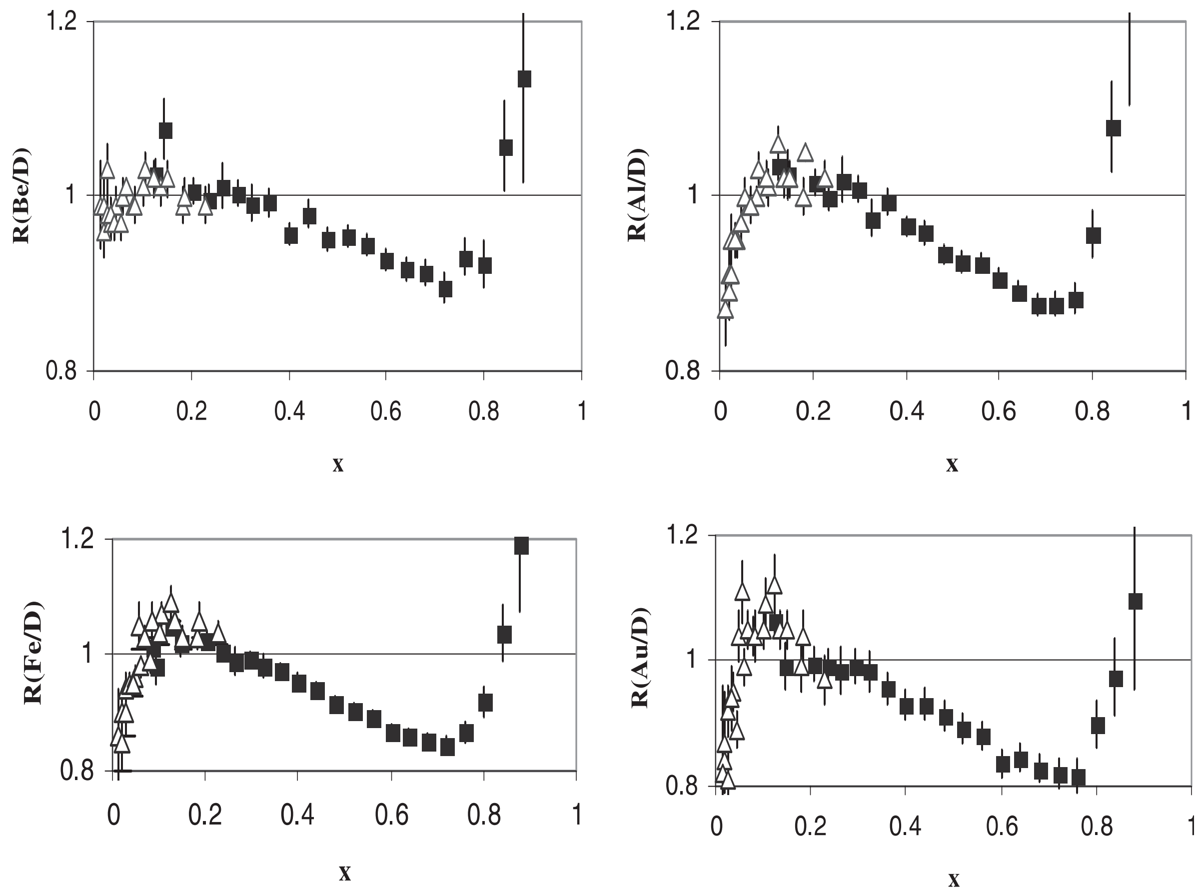
Nuclear "EMC effect"

$$F_2^A(x,Q^2) \neq AF_2^N(x,Q^2)$$

Original EMC data

Later SLAC data

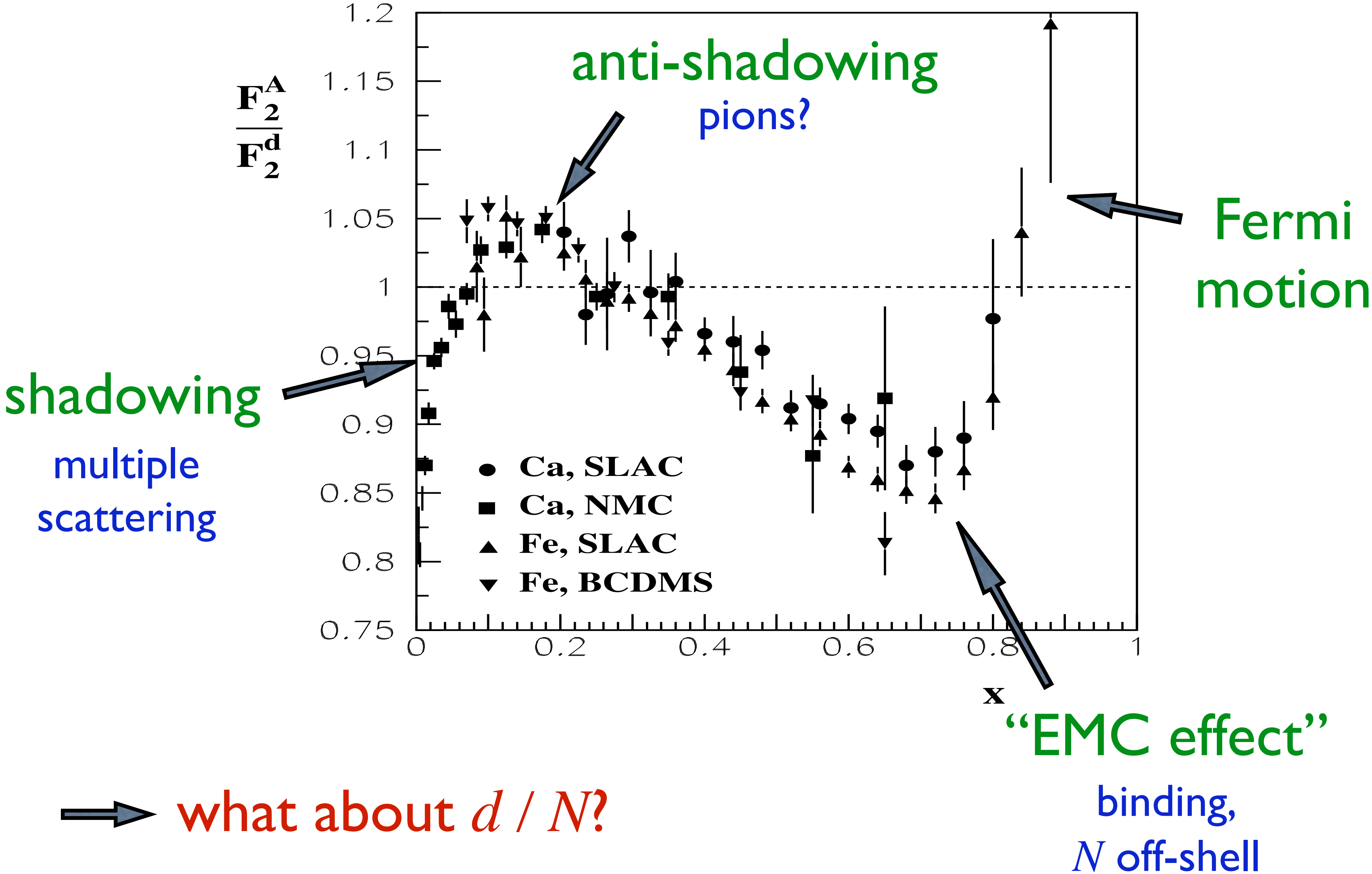




Aubert et al., Phys. Lett. B 123, 123 (1983)

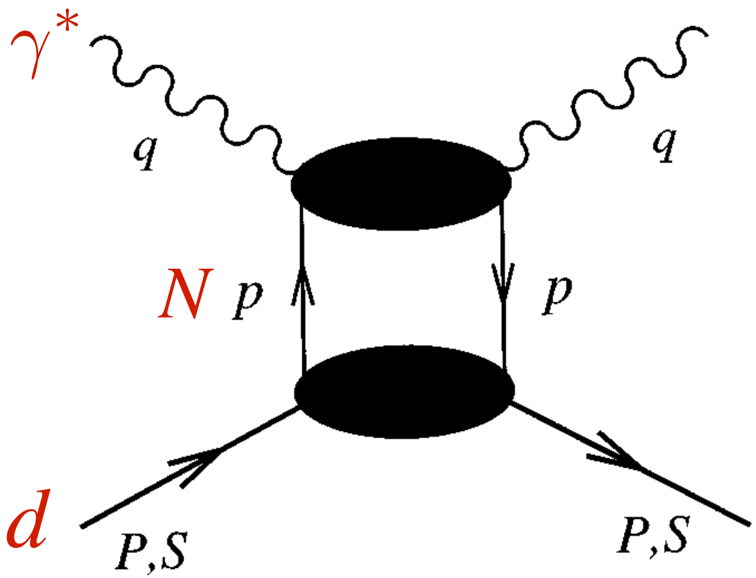
Gomez et al., Phys. Rev. D 49, 4348 (1994)

Nuclear "EMC effect"



→ what about d / N?

EMC effect in deuteron



Nuclear "impulse approximation"

$\Rightarrow$ incoherent scattering from individual nucleons in deuteron

$$F_2^d(x) = \int dy\ f_{N/d}(y)\ F_2^N(x/y)\ +\ \delta^{(\mathrm{off})} F_2^d(x)$$

nucleon momentum distribution

off-shell correction

EMC effect in deuteron

Nucleon momentum distribution in deuteron

➡ relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\mathrm{max}}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

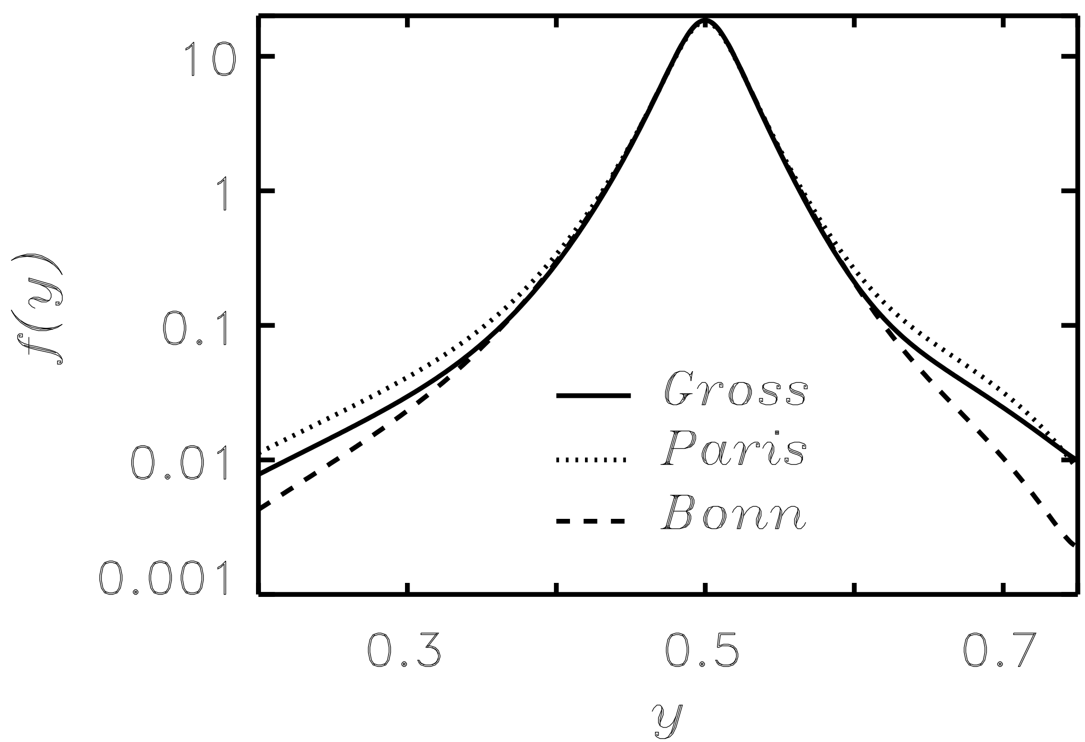
momentum fraction of deuteron
carried by nucleon

EMC effect in deuteron

Nucleon momentum distribution in deuteron

relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$



EMC effect in deuteron

Nucleon momentum distribution in deuteron

➡ relativistic dNN vertex function

$$f_{N/d}(y) = \frac{1}{4} M_d \; y \int_{-\infty}^{p^2_{\rm max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

Wave function dependence only at large $|y-1/2|$

➡ sensitive to large p components of wave function

➡ not very well known

EMC effect in deuteron

Nucleon momentum distribution in deuteron

→ relativistic dNN vertex function

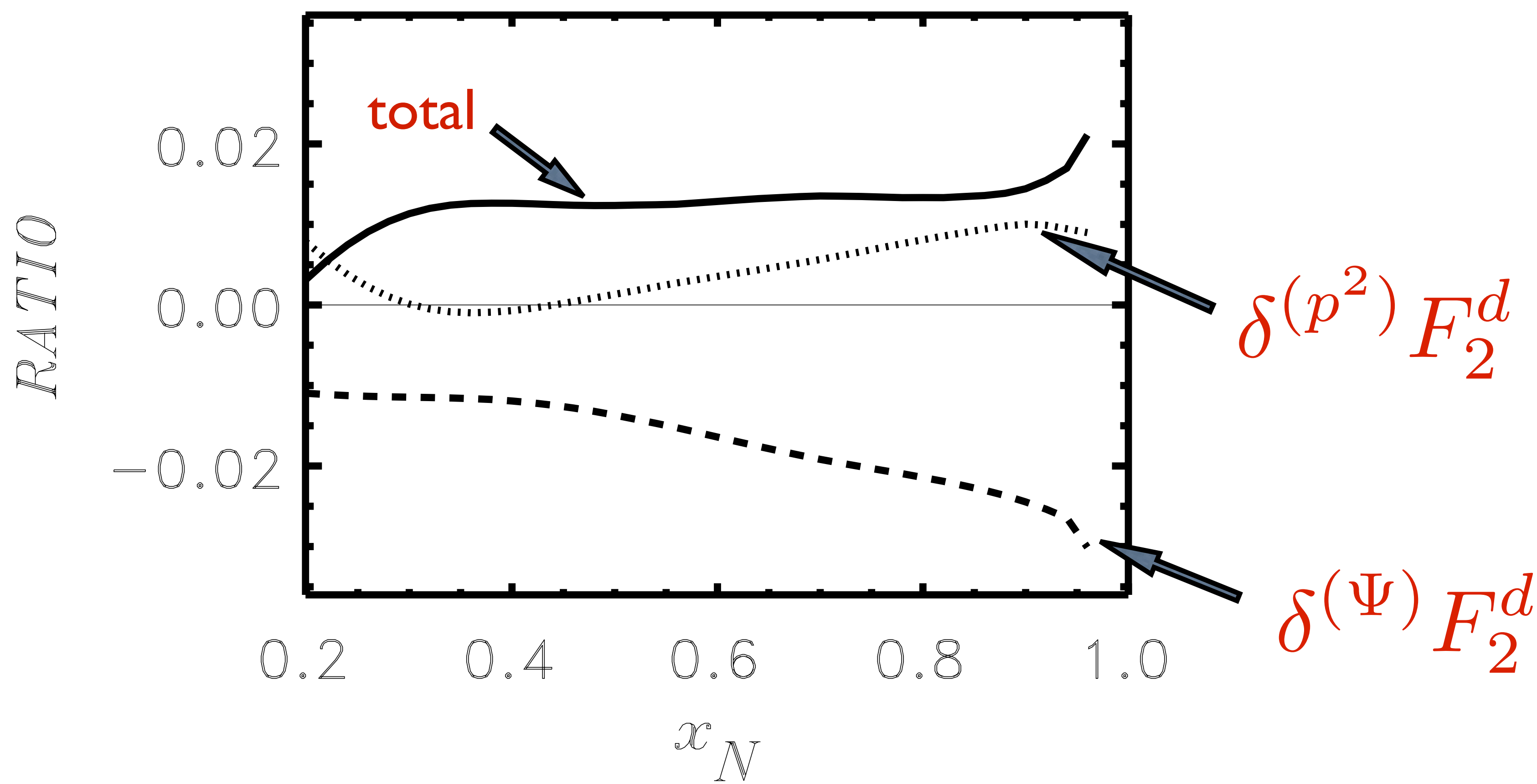
$$f_{N/d}(y) = \frac{1}{4} M_d \, y \int_{-\infty}^{p^2_{\max}} dp^2 \, \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^{\,2}) \right|^2$$

Nucleon off-shell correction

$\delta^{(\mathrm{off})} F_2^d \longrightarrow \delta^{(\Psi)} F_2^d$ negative energy components of d wave function

$\longrightarrow \delta^{(p^2)} F_2^d$ off-shell N structure function

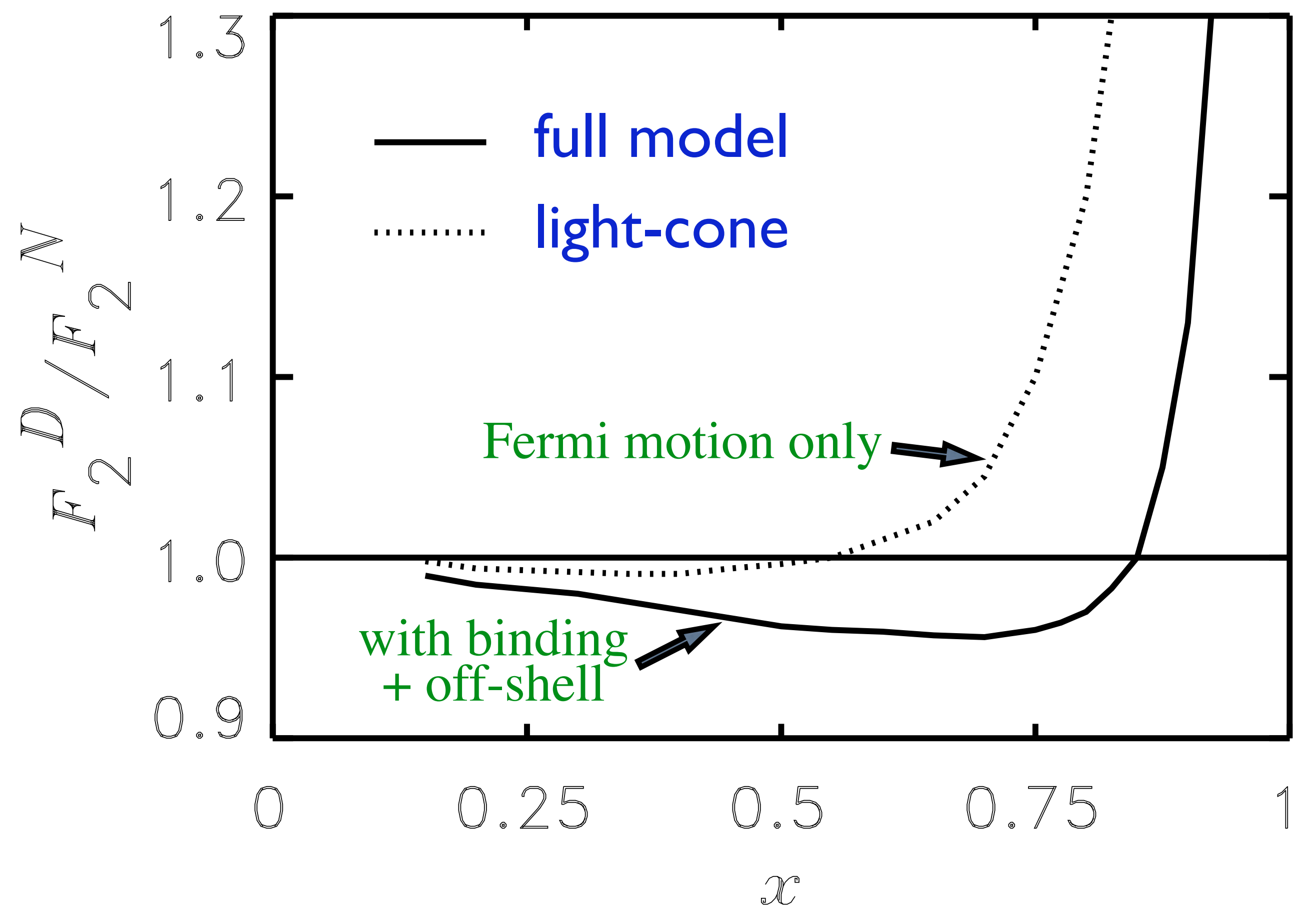
Off-shell correction



$\Longrightarrow$ $\leq 1 - 2\ \%$ **effect**

WM, Schreiber, Thomas, Phys. Lett. B 335 (1994) 11

EMC effect in deuteron



Larger EMC effect (smaller d/N ratio)

→ F_2^n underestimated at large x

Unsmearing

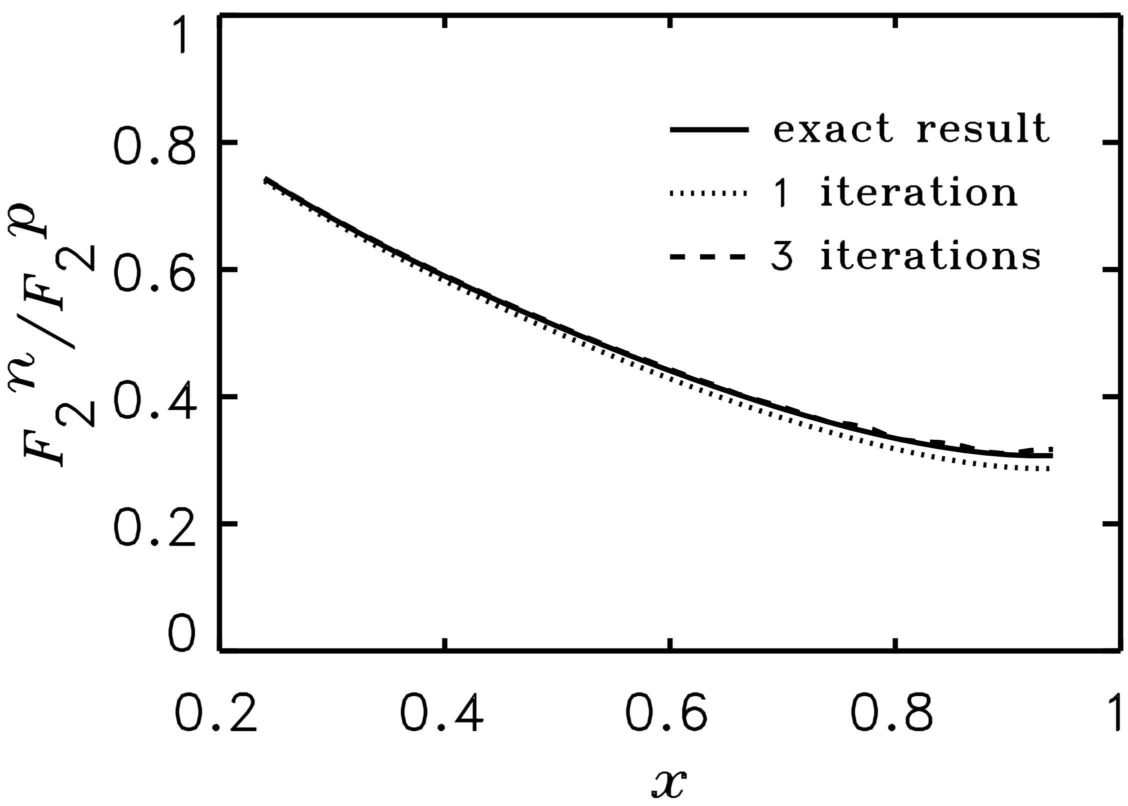
Note: calculated d/N ratio depends on input F_2^n

⟹ extracted n depends on input n ... cyclic argument

Solution: <u>iteration procedure</u>

0. subtract $\delta^{(\mathrm{off})} F_2^d$ from d data: $F_2^d \to F_2^d - \delta^{(\mathrm{off})} F_2^d$
1. smear F_2^p with $f_{N/d}$: $f_{N/d} \otimes F_2^p \equiv S_p\, F_2^p$
2. extract neutron via $F_2^n = S_n(F_2^d - F_2^p / S_p)$
   starting with $e.g.$ $S_n = S_p$
3. smear F_2^n with $f_{N/d}$ to get new S_n
4. repeat 2-3 until convergence

Unsmearing

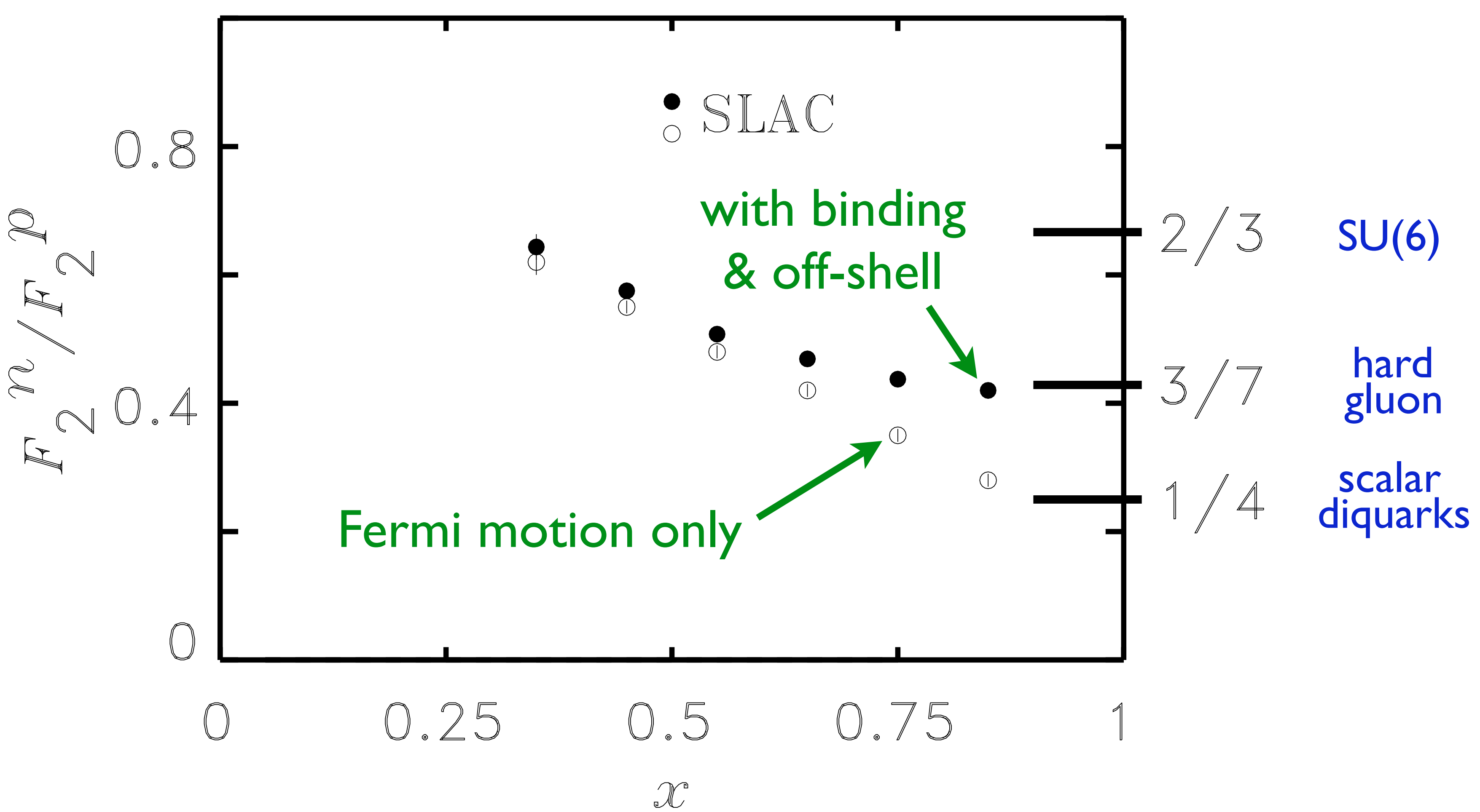


Afnan, Bissey, Gomez, Liuti, WM, Thomas et al., Phys. Rev. C68 (2003) 035201

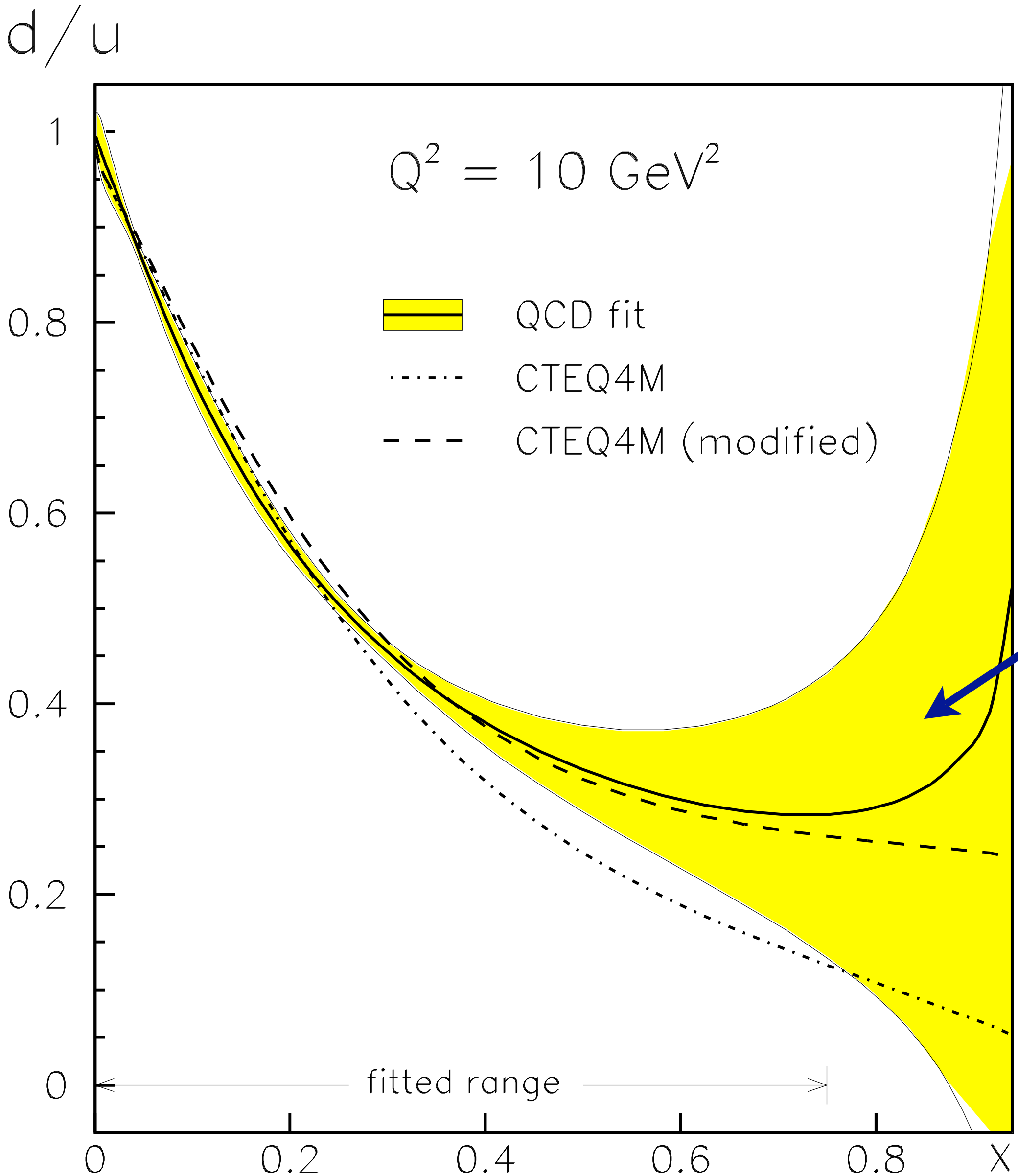
- good convergence after several iterations
- resulting F_2^n <u>independent</u> of starting assumptions

depends only on smearing function $f_{N/d}$

Effect on *n*/*p* ratio



WM, Thomas, Phys. Lett. B 377 (1996) 11



uncertainty due to nuclear effects in neutron
(full range of nuclear models)

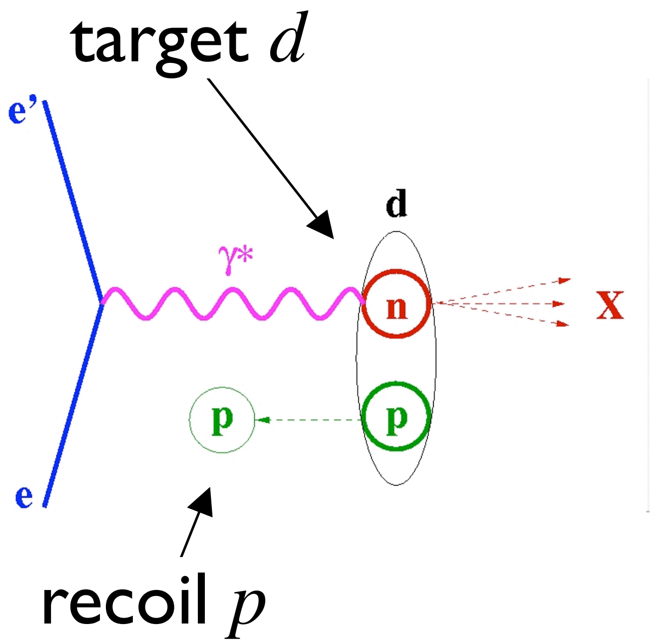
$\Rightarrow$ d distribution poorly known beyond $x \sim 0.5$

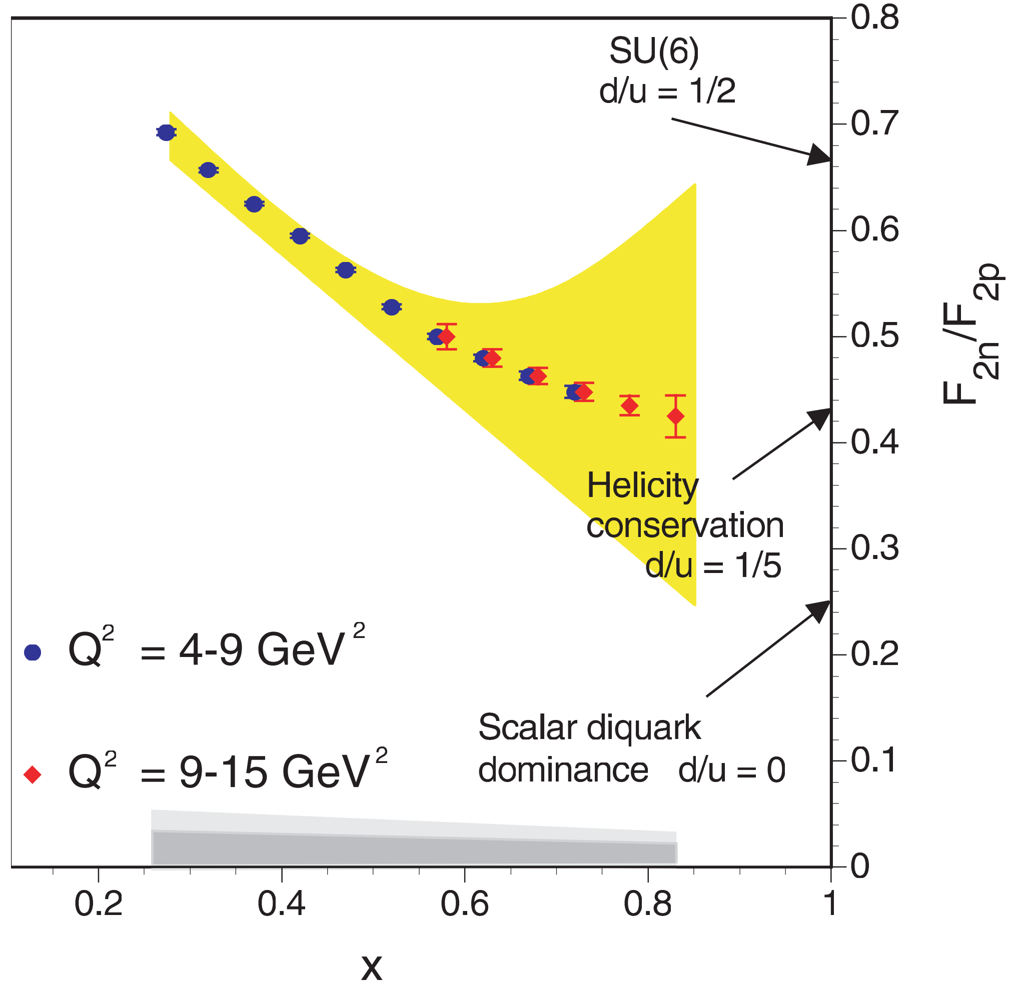
“Cleaner” methods of determining d/u

$e^{\mp}\ p \to \nu(\bar{\nu})X$	need high luminosity
$\nu(\bar{\nu})\ p \to l^{\mp}\ X$	low statistics
$p\ p(\bar{p}) \to W^{\pm}X$	need large lepton rapidity
$\vec{e}_L(\vec{e}_R)\ p \to e\ X$	low count rate
$e\ p \to e\ \pi^{\pm}\ X$	need $z \sim 1$, factorization
$e\ {}^3\mathrm{He}({}^3\mathrm{H}) \to e\ X$	tritium target

"Cleaner" methods of determining d/u

$$e\ d \rightarrow e\ p\ X$$





slow backward p

→ neutron nearly on-shell

→ minimize rescattering

JLab Hall B experiment ("BoNuS")
completed run Dec. 2005

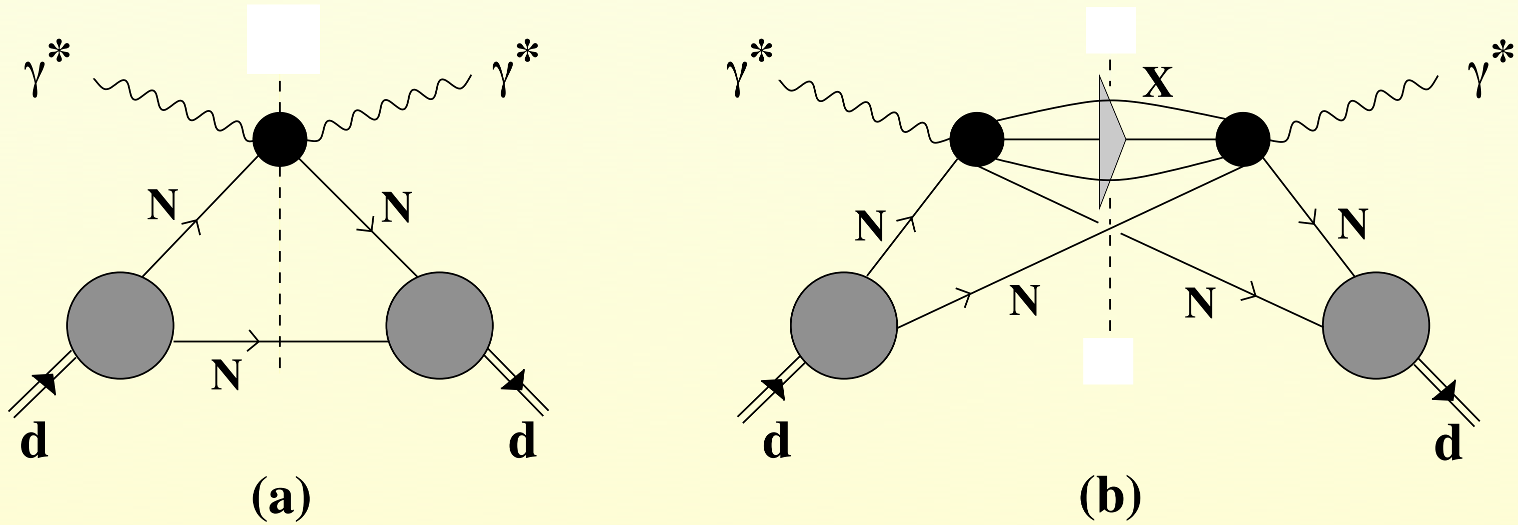
2.
Quark distributions

- nuclear shadowing

Nuclear shadowing

Interference of multiple scattering amplitudes

For deuteron:



Nuclear impulse approximation

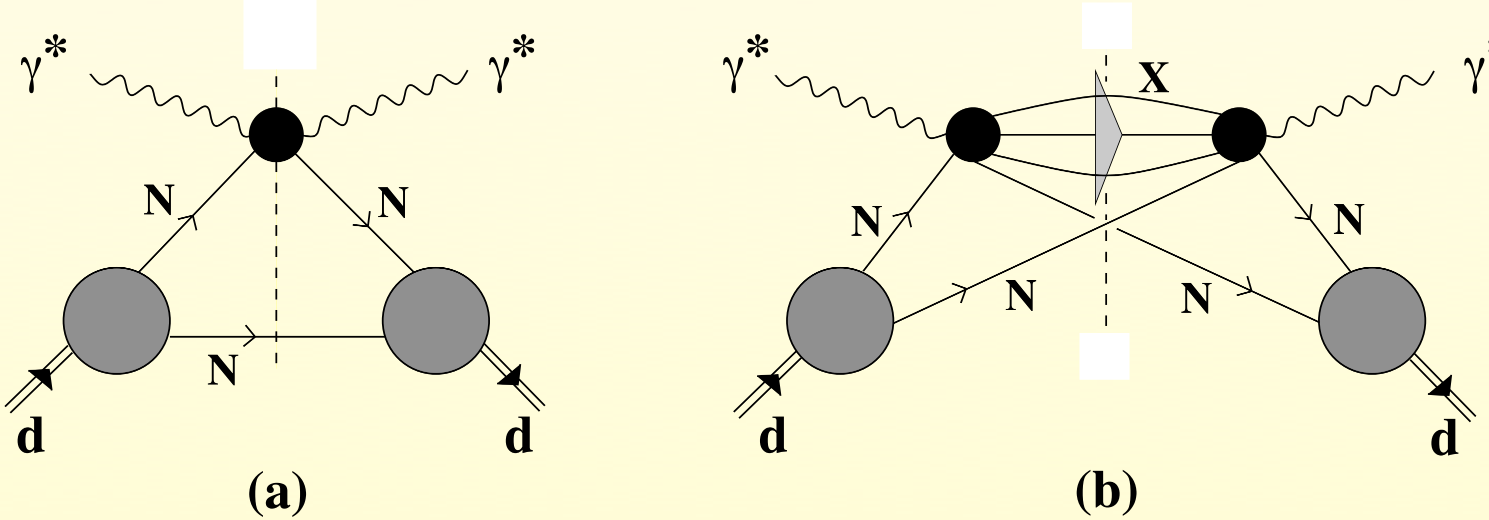
Double scattering

e.g. Piller, Weise, Phys. Rep. 330 (2000) 1

Nuclear shadowing

Interference of multiple scattering amplitudes

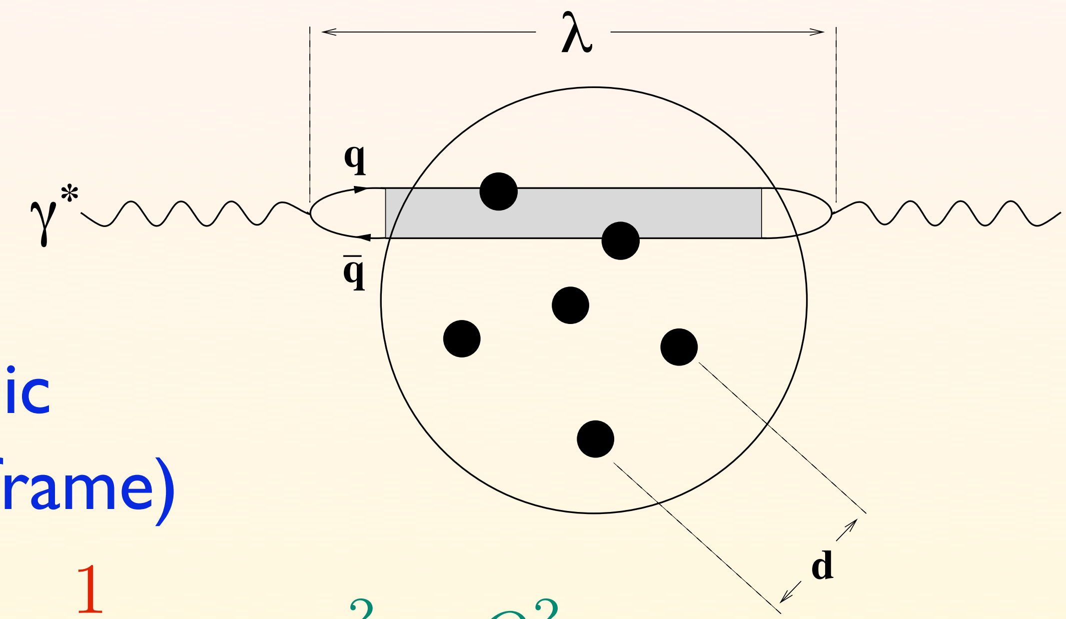
For deuteron:



$$F_2^d = F_2^p + F_2^n + \delta^{(\text{shad})} F_2^d$$

e.g. Piller, Weise, Phys. Rep. 330 (2000) 1

Space-time view of shadowing



propagation length of hadronic fluctuation of mass μ (in lab frame)

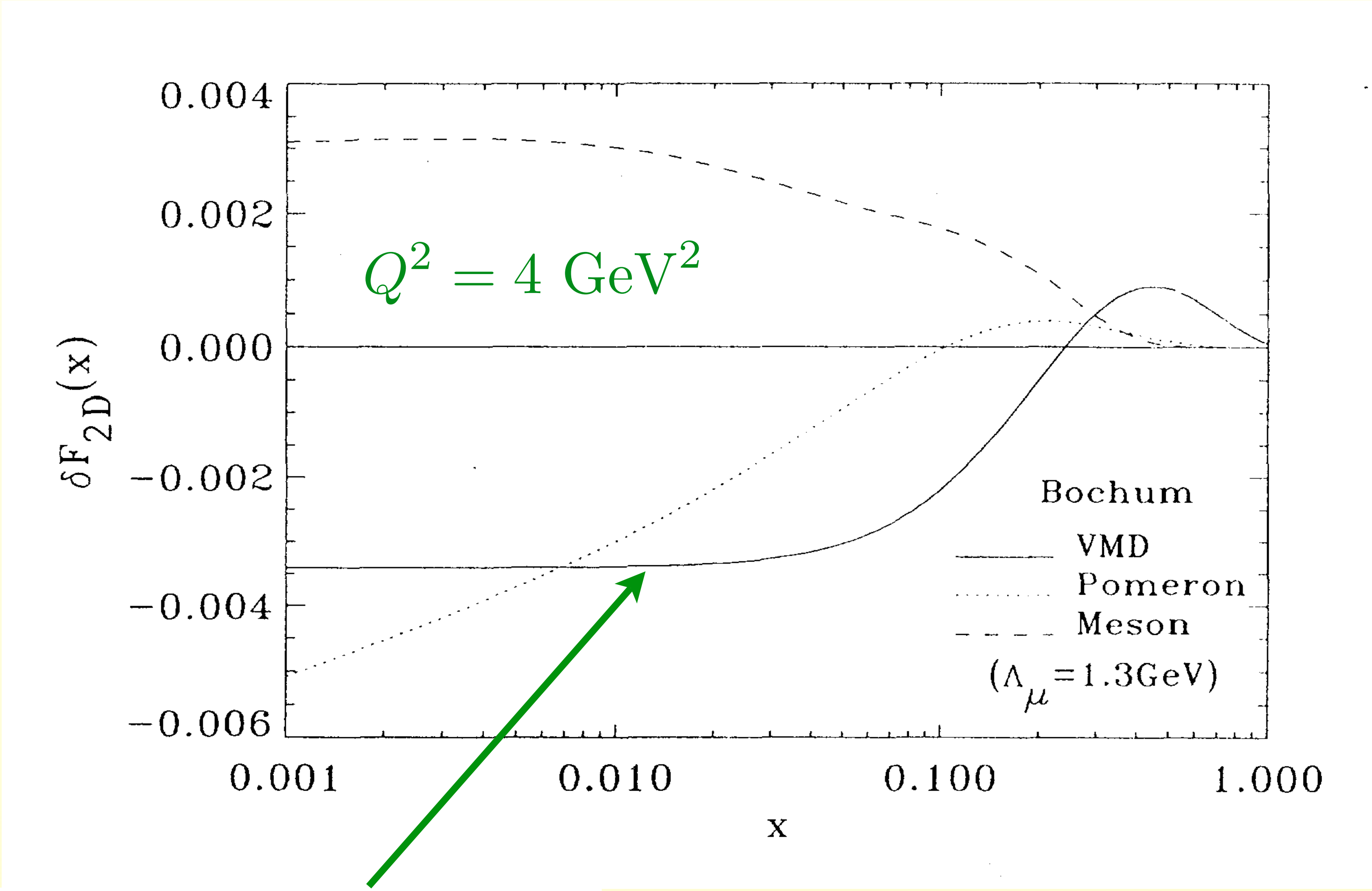
$$\lambda \sim \frac{1}{\Delta E} = \frac{2\nu}{\mu^2 + Q^2} \to \frac{1}{2xM} \,, \quad \mu^2 \sim Q^2$$

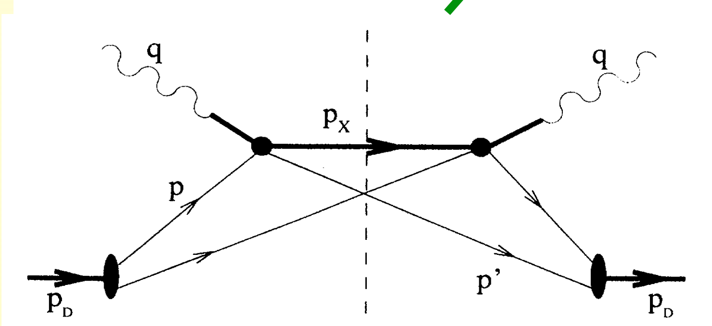
if propagation length exceeds average distance between nucleons $\lambda > d \approx 2 \text{ fm}$

⇨ coherent multiple scattering can occur

$$x < 0.05$$

Shadowing in deuterium

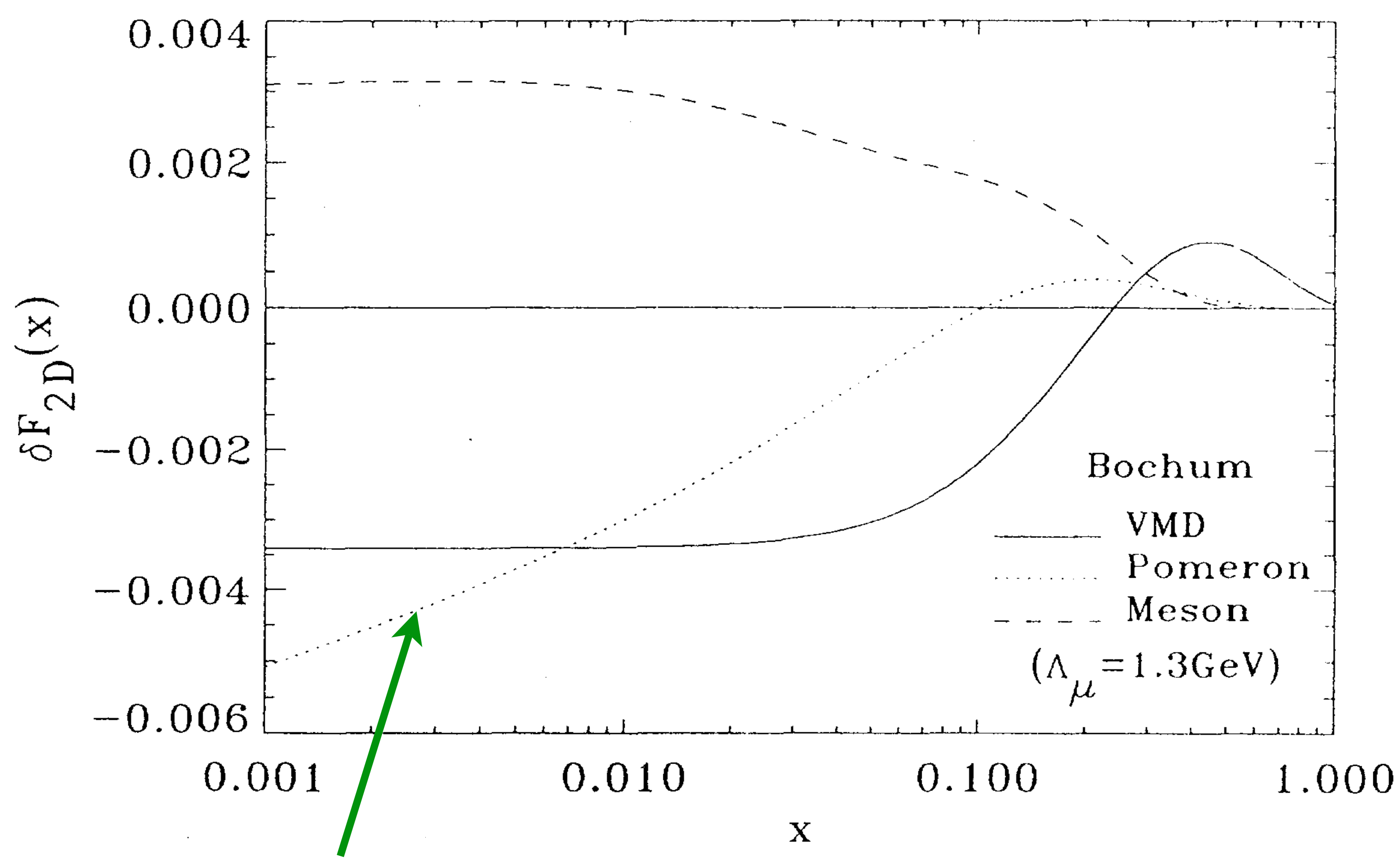


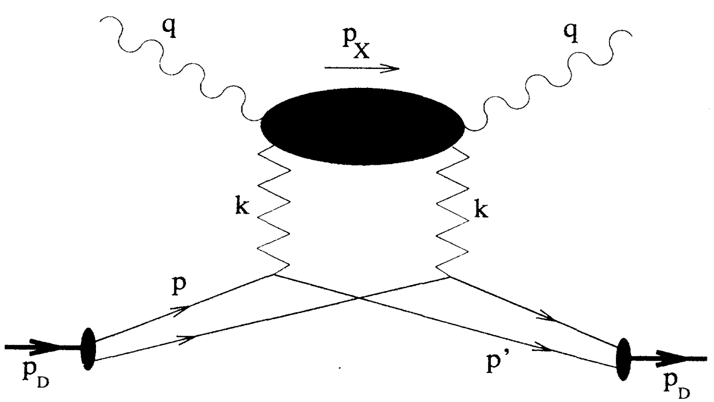


vector meson dominance

(higher twist)

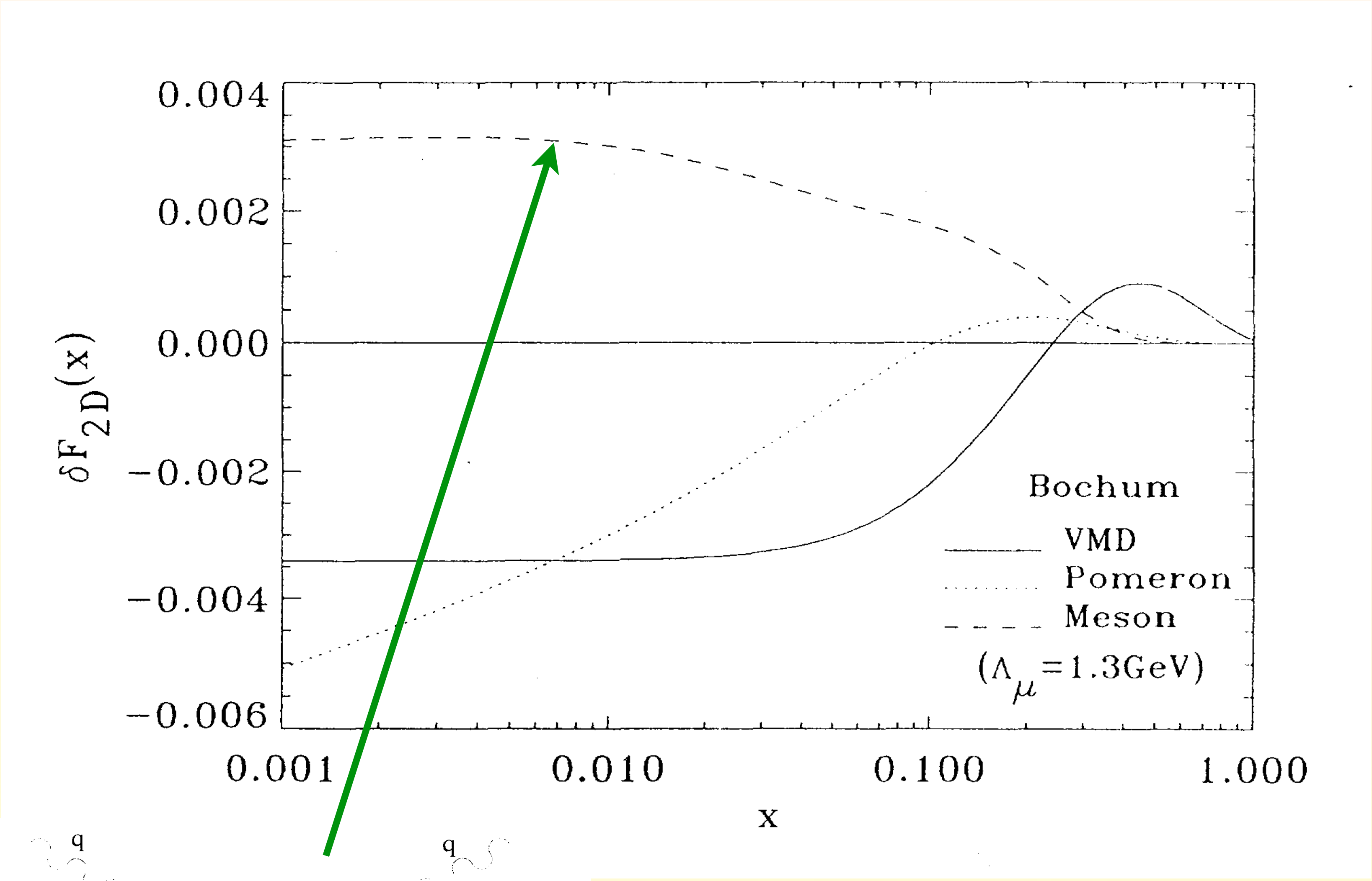
Shadowing in deuterium

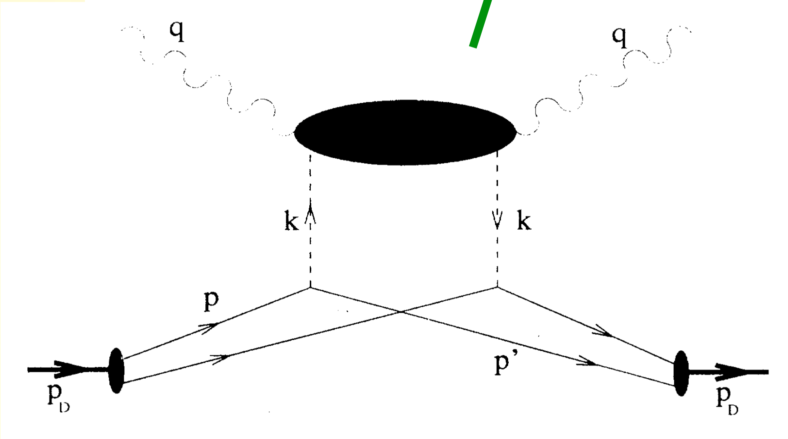




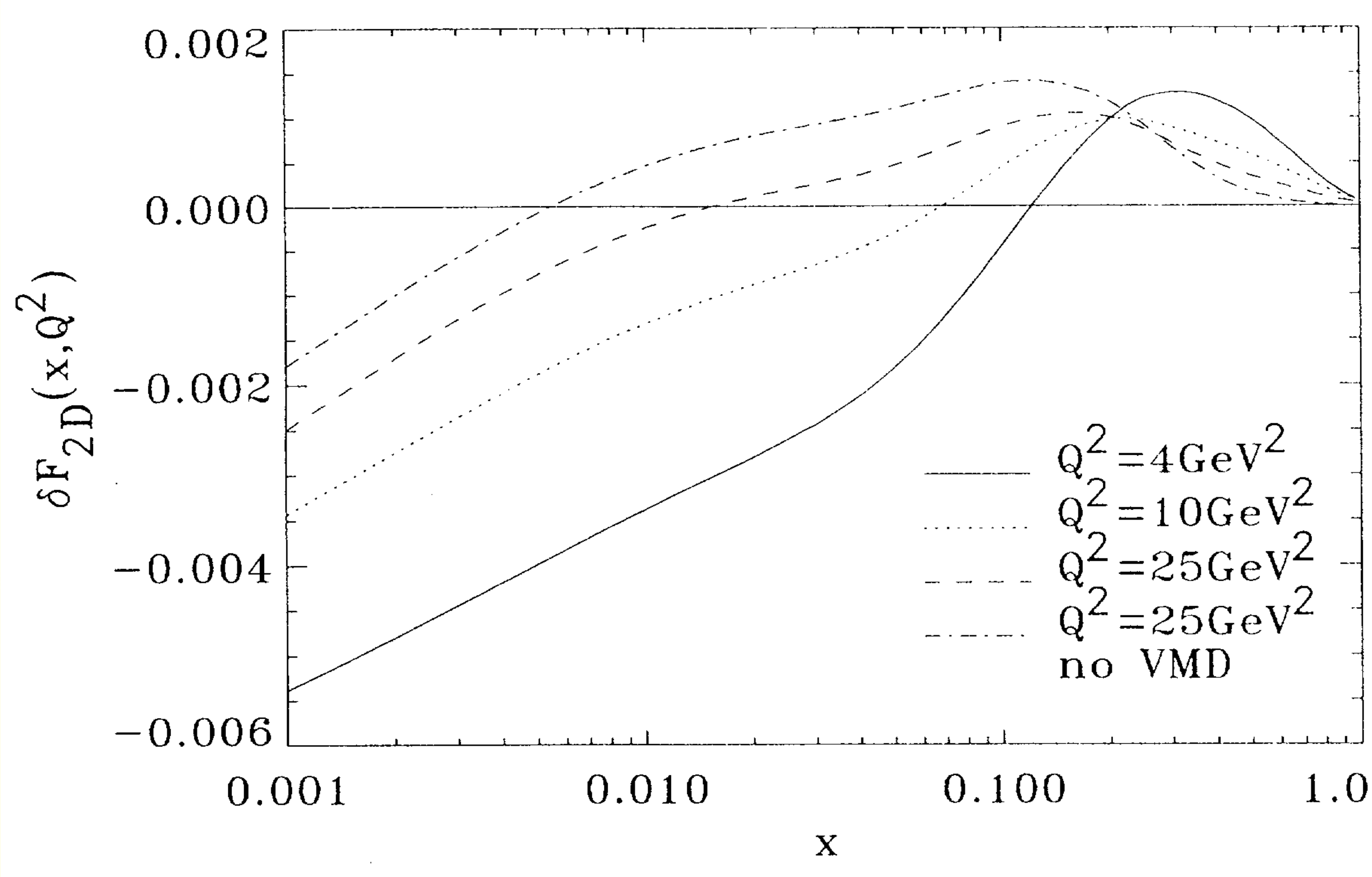
Pomeron exchange

Anti-shadowing in deuterium





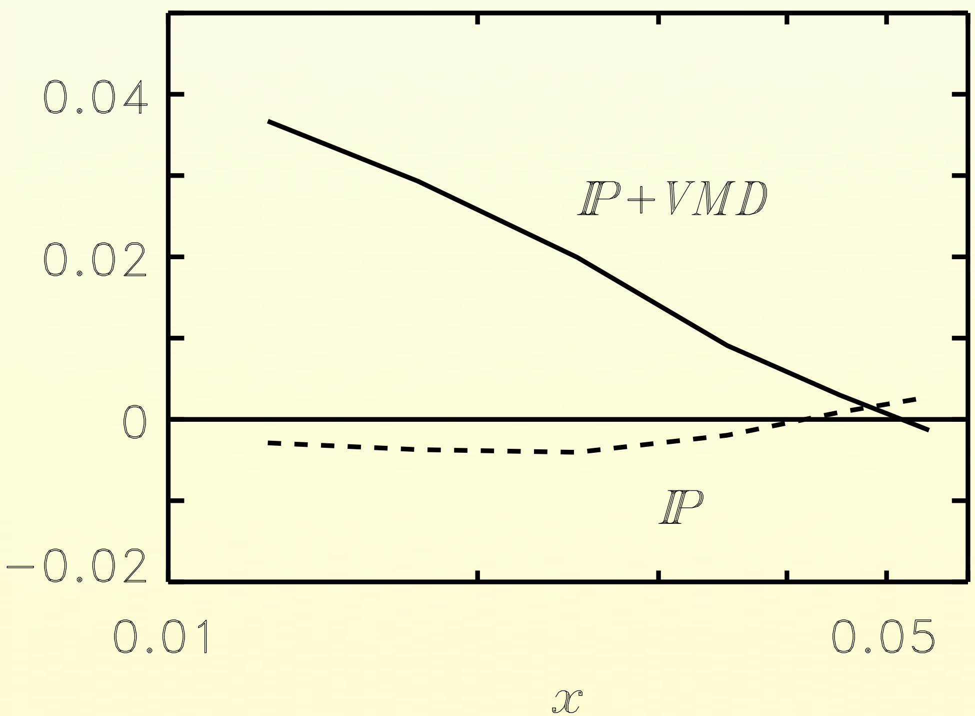
meson (pion) exchange



VMD important even at moderate Q^2

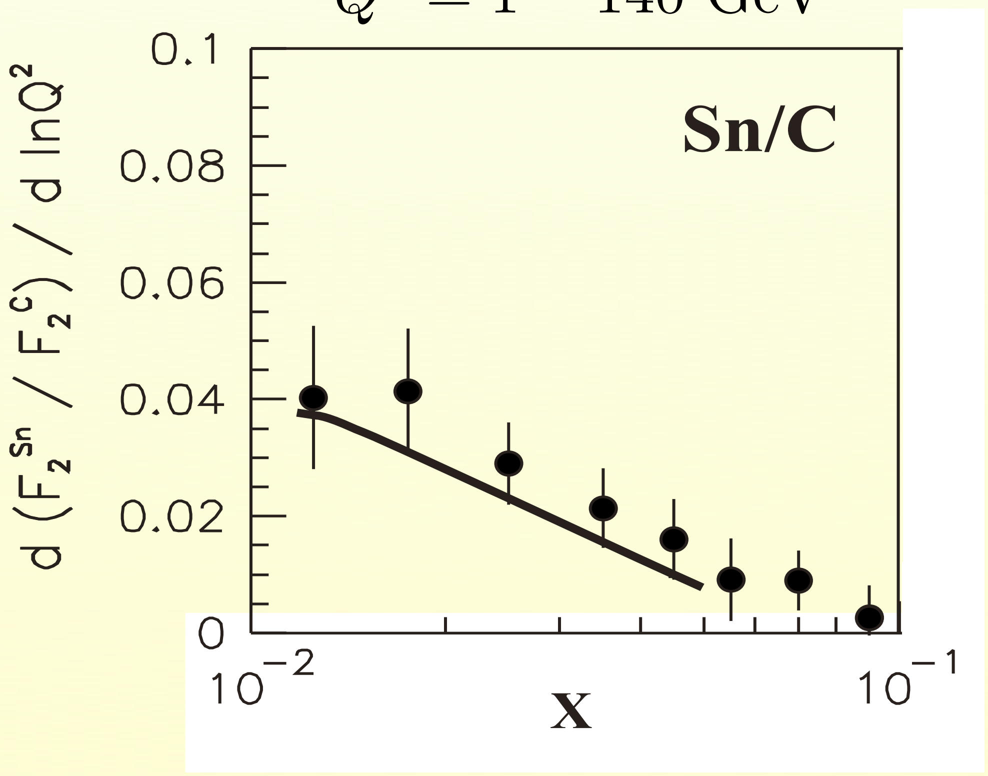
Shadowing in nuclei

Perturbative or nonperturbative origin of Q^2 dependence?



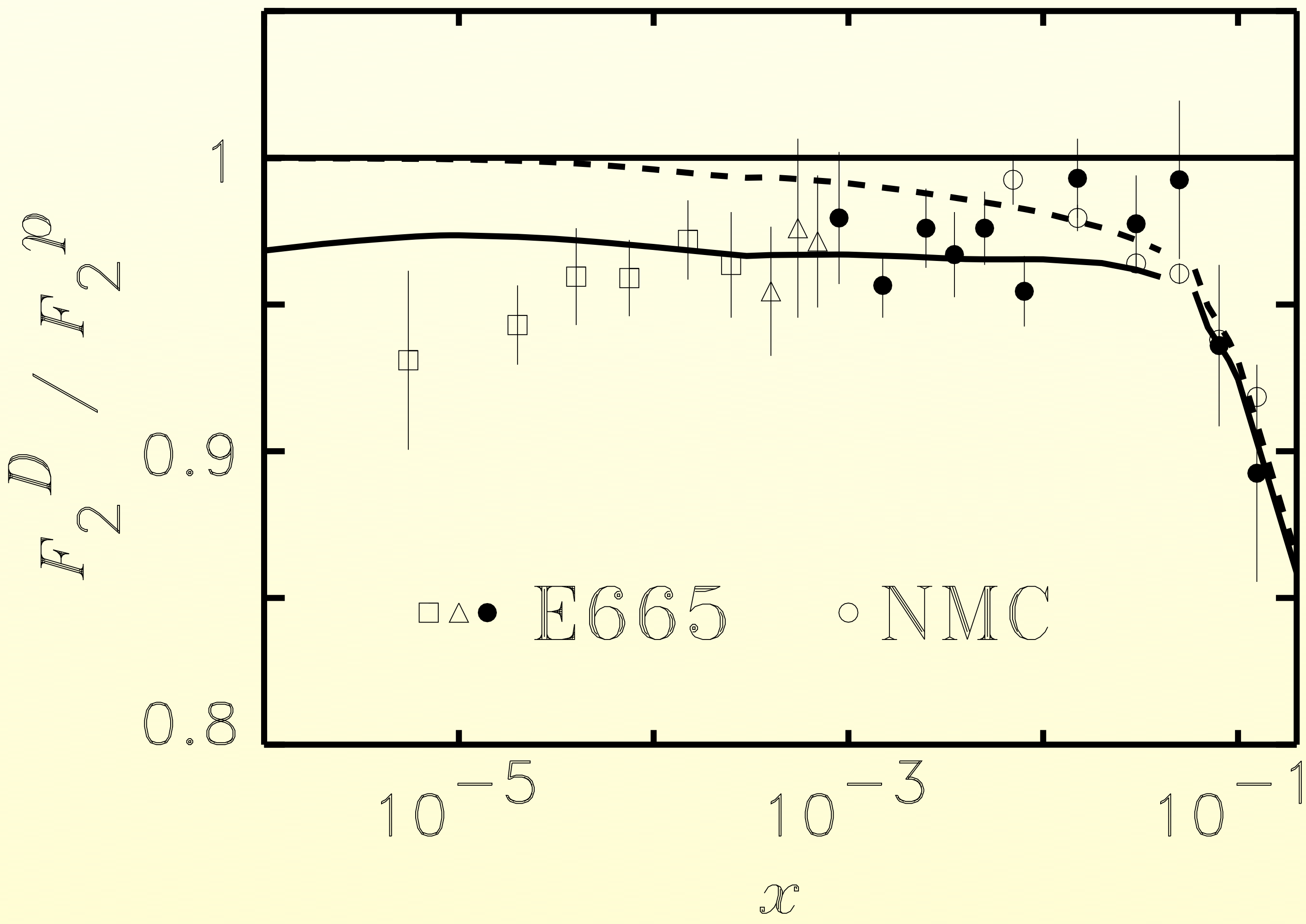
WM, Thomas, Phys. Rev. C52 (1995) 3373

$Q^2 = 1 - 140\ \mathrm{GeV}^2$



NMC, Nucl. Phys. B481 (1996) 23

Comparison with data



WM, Thomas, Phys. Rev. C 52 (1995) 3373
- see also Badelek, Kwiecinski (1992),
Nikolaev, Zoller (1992)

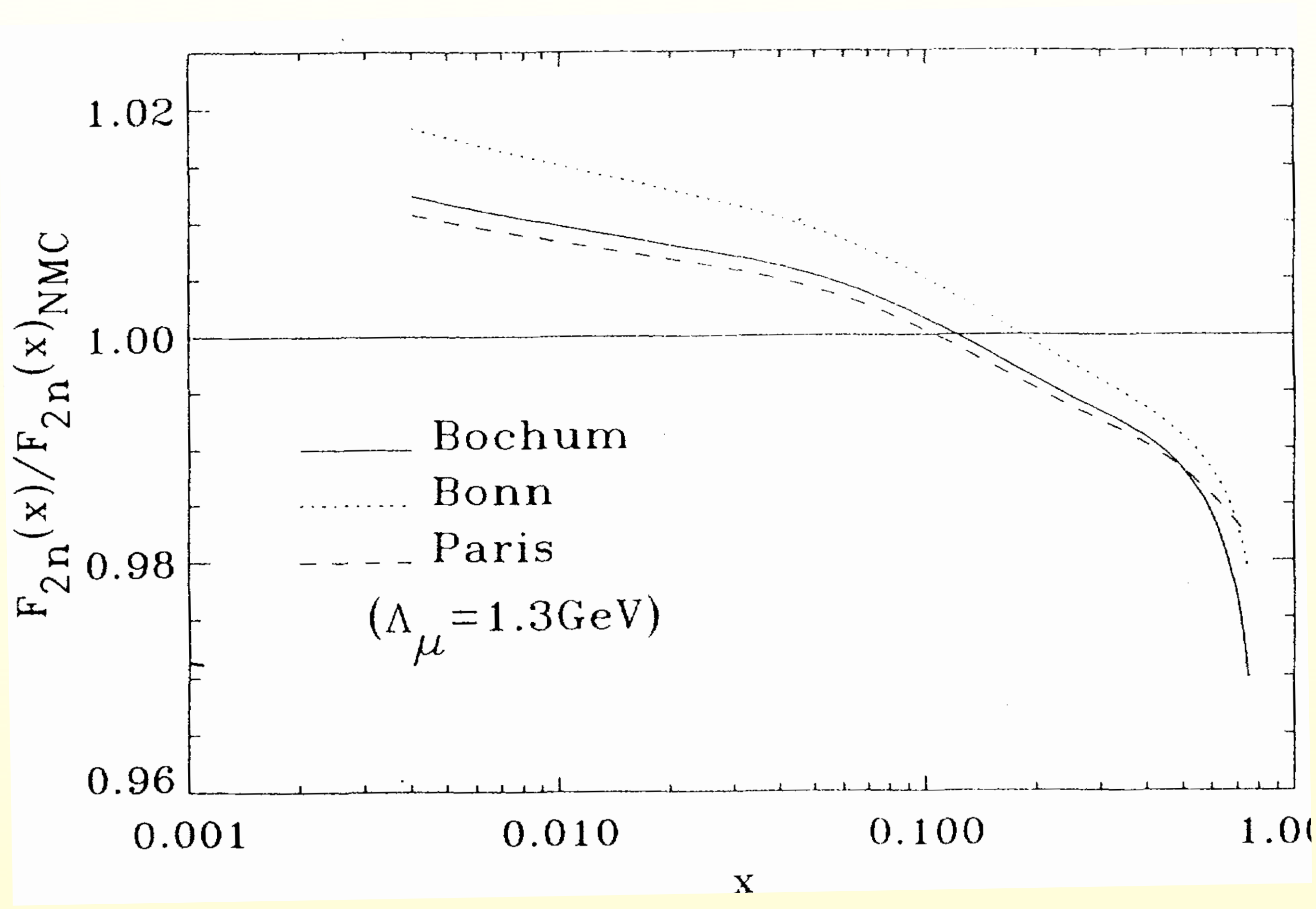
Effect on neutron structure function at small x

$$\frac{F_2^n}{(F_2^n)_{\rm exp}} = 1 - \frac{\delta F_2^d}{F_2^d} \left(\frac{1 + (F_2^n/F_2^p)_{\rm exp}}{(F_2^n/F_2^p)_{\rm exp}} \right)$$

where "experimental" n/p ratio is defined as

$$\left. \frac{F_2^n}{F_2^p} \right|_{\rm exp} \equiv \frac{F_2^d}{F_2^p} - 1$$

Effect on neutron structure function at small x



1-2% enhancement at $x \sim 0.01$

Gottfried sum rule

Integrated difference of p and n structure functions

$$S_G = \int_0^1 dx \, \frac{F_2^p(x) - F_2^n(x)}{x}$$

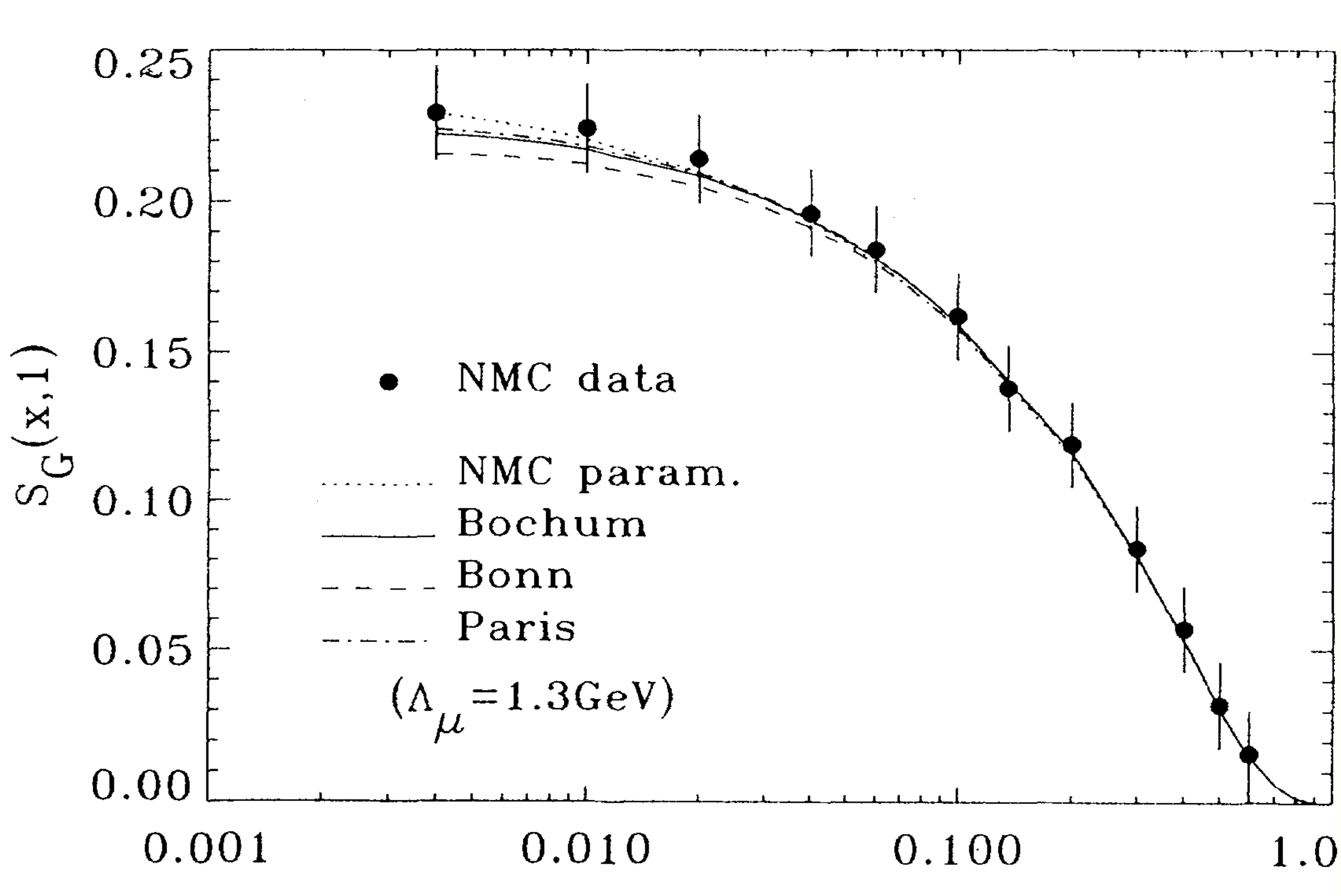
$$= \frac{1}{3} + \frac{2}{3} \int_0^1 dx \, \left(\bar{u}(x) - \bar{d}(x)\right)$$

Experiment: $S_G = 0.235 \pm 0.026$

NMC, Phys. Rev. D 50 (1994) 1

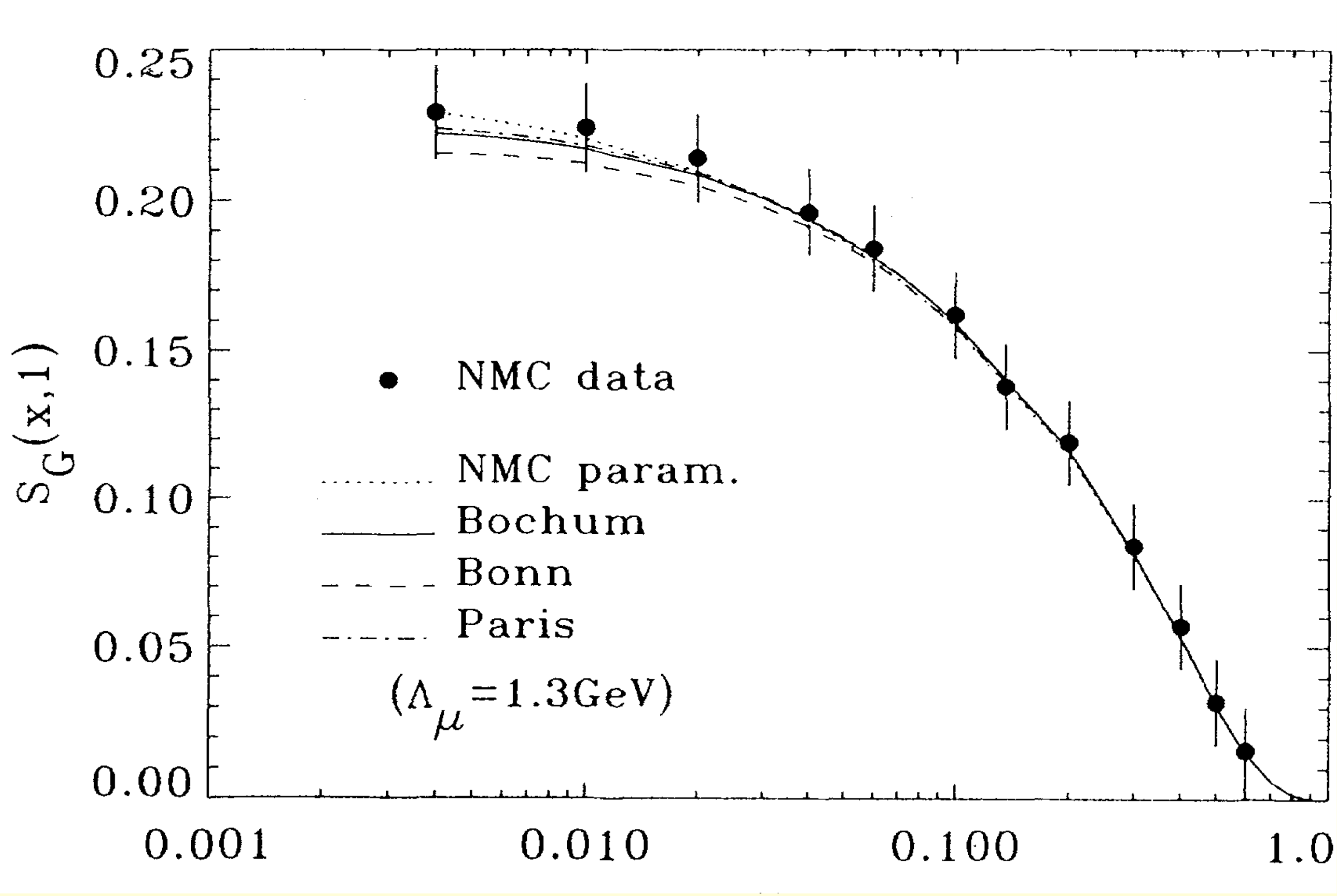
$\Rightarrow$ $\bar{d}(x) \neq \bar{u}(x)$ flavor asymmetric sea

Saturation of Gottfried sum rule



$$S_G(x,1) = \int_x^1 dx' \; \frac{F_2^p(x') - F_2^n(x')}{x'}$$

Saturation of Gottfried sum rule



correction to $S_G(0,1) \approx -0.02$

~ 10% decrease due to shadowing

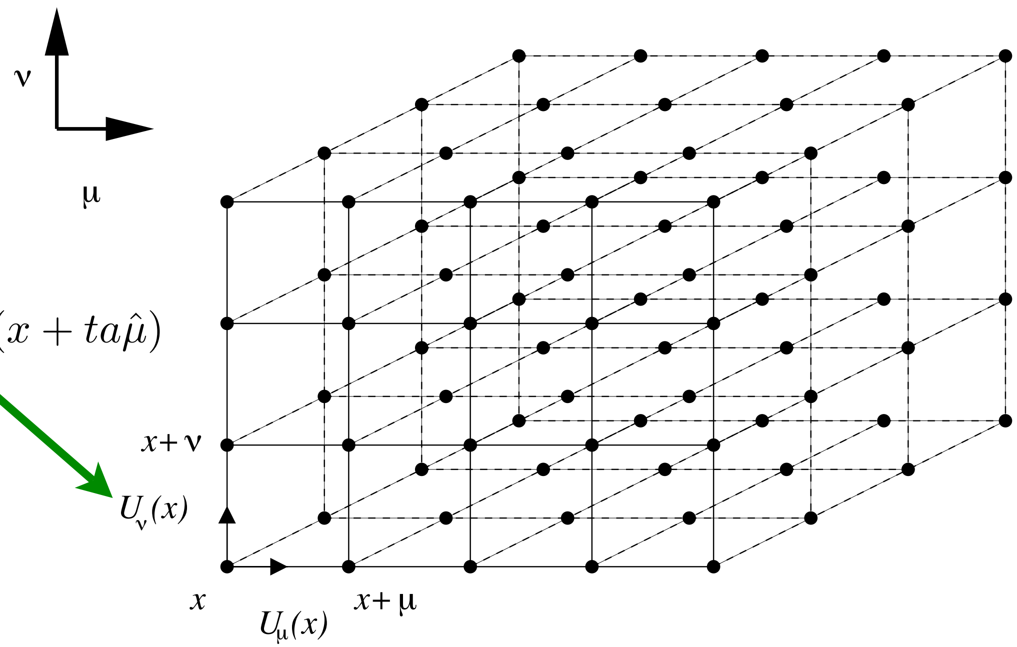
2.
Quark distributions

- from lattice QCD

Lattice QCD

Solve QCD equations of motion *numerically* on discretized space-time grid

$$U_\mu(x) = \exp i g\, a \int_0^1 dt\, A_\mu(x + t a\hat{\mu})$$



Wilson (1974)

- quarks on lattice nodes
- gluons as links between nodes

Observables calculated from path integrals in Euclidean space

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U\, O(U) e^{-S_G(U)}$$

generating functional

$$Z = \int \mathcal{D}U \; \det M(U)\, e^{-S_G(U)}$$

Fermion mass matrix

$$M(x, y, U) = m\, \delta_{x,y} + \frac{1}{2} \sum_{\mu} \gamma_\mu \left(U_\mu(x) \delta_{y,x+\hat{\mu}} - U_\mu^\dagger(x - \hat{\mu}) \delta_{y,x-\hat{\mu}} \right)$$

Approximations

- finite lattice spacing a $(\to 0)$
- finite lattice volume V $(\to \infty)$
- large quark mass m_q $(\to m_q^{\text{phys}})$ $\Longleftarrow$ $\text{cost} \propto m_q^{-4}$
- "quenching" - suppression of background $q\bar{q}$ loops $\Longleftarrow$ $\det M \to 1$

PDFs from Lattice QCD

Cannot calculate x-distribution on lattice

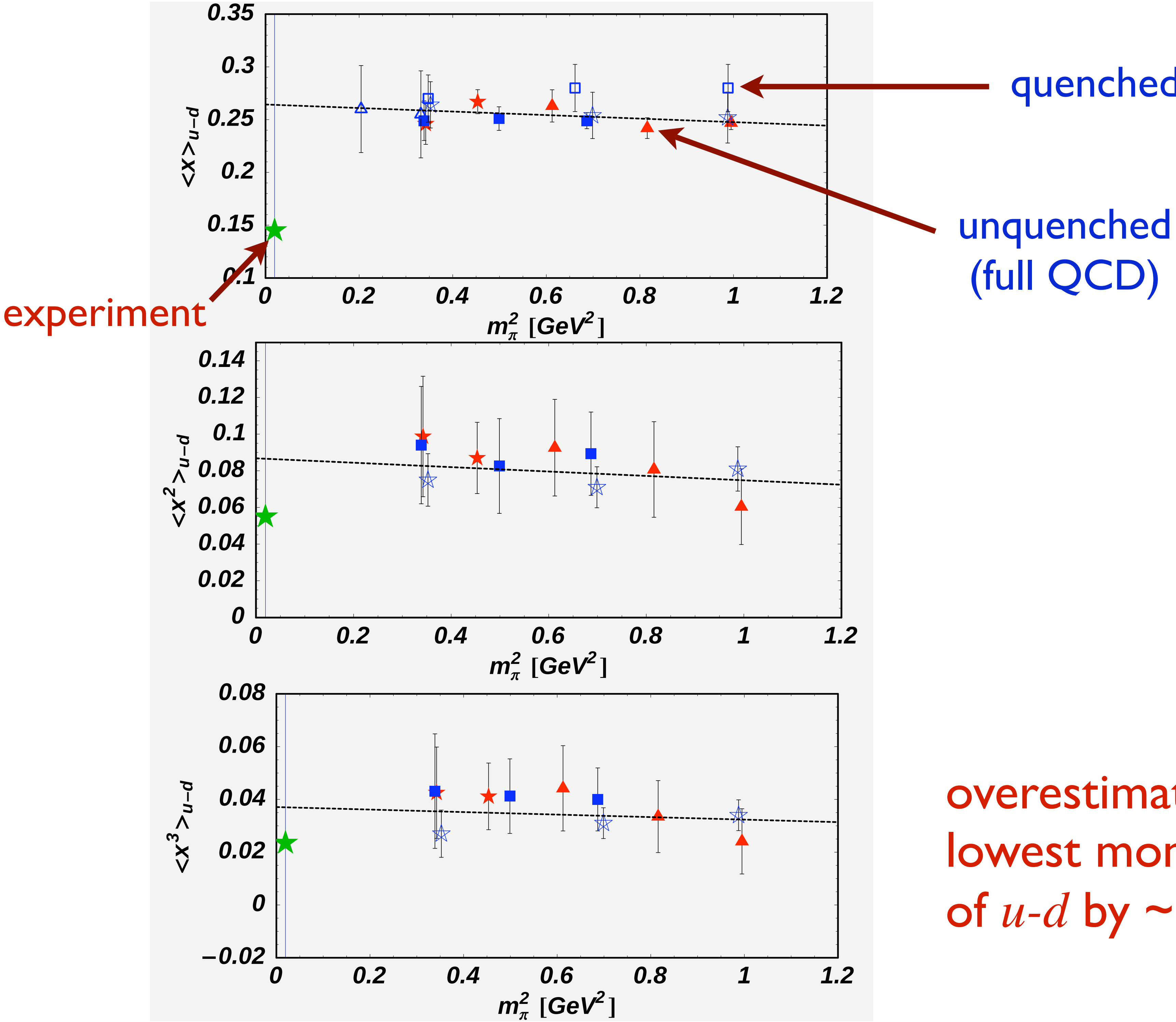
(no light-cone in Euclidean space) - *only moments*

$$\langle x^n \rangle_q = \int_0^1 dx\ x^n\ \Big(q(x) + (-1)^{n+1} \bar{q}(x) \Big)$$

⟹ use OPE to relate moments of PDFs to matrix elements of local operators

$$\langle x^n \rangle\ p_{\mu_1} \cdots p_{\mu_{n+1}} = \langle N |\ \mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} | N \rangle$$

twist-2 operators

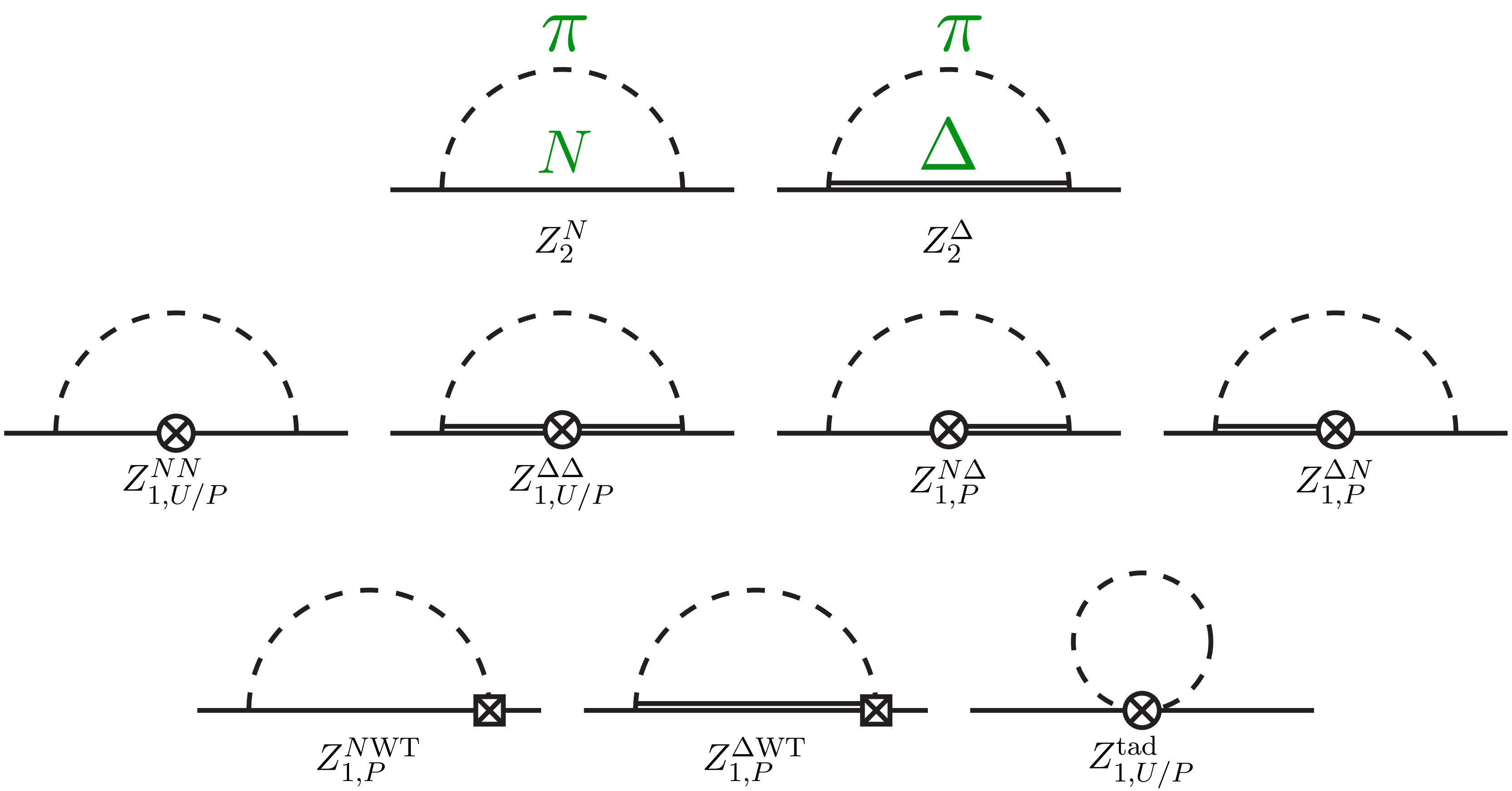


overestimates
lowest moment
of u-d by ~50% !

Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies

➡ their moments have chiral expansion



Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies
⇒ their moments have chiral expansion

$$\langle x^n \rangle_{u-d} \;=\; a_n \left(1 + c_{\mathrm{LNA}} m_\pi^2 \log \frac{m_\pi^2}{m_\pi^2 + \mu^2}\right) \;+\; b_n \frac{m_\pi^2}{m_\pi^2 + m_{b,n}^2}$$

Detmold et al., Phys. Rev. Lett. 87 (2001) 172001

Leading non-analytic coefficient (non-analytic in $m_q \sim m_\pi^2$)

$$c_{\mathrm{LNA}} = -(1 + 3g_A^2)/(4\pi f_\pi)^2$$

calculated from chiral perturbation theory

Arndt, Savage (2001)
Ji, Chen (2001)

PDF in heavy quark limit

$$u(x) - d(x) \xrightarrow{m_q \to \infty} \delta(x - \frac{1}{3})$$

Moment

$$\langle x^n \rangle_{u-d} \xrightarrow{m_q \to \infty} \frac{1}{3^n}$$

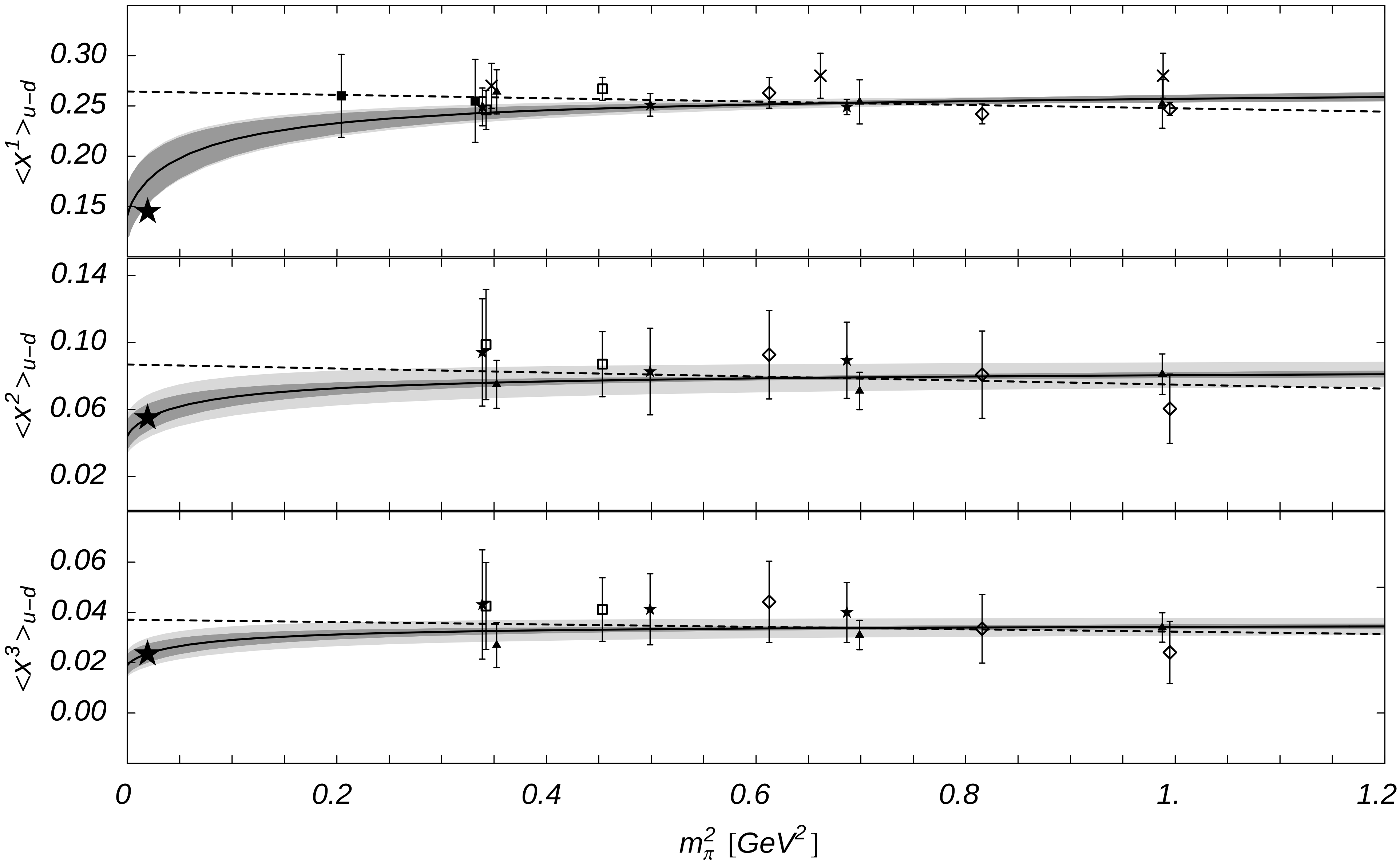
Coefficient ensures correct $m_\pi \to \infty$ behavior

$$b_n = \frac{1}{3^n} - a_n \left(1 - \mu^2 c_{\mathrm{LNA}}\right)$$

Parameter μ determines amount of curvature at low m_π^2

$(m_\pi^2 \propto m_q)$

Detmold, WM, Thomas, Mod. Phys. Lett. A18 (2003) 2681



Chiral physics *vital* for understanding lattice data

Odds and evens

For unpolarized parton distributions

- n even $\Longrightarrow$ total $q + \bar{q}$

- n odd $\Longrightarrow$ valence $q - \bar{q}$

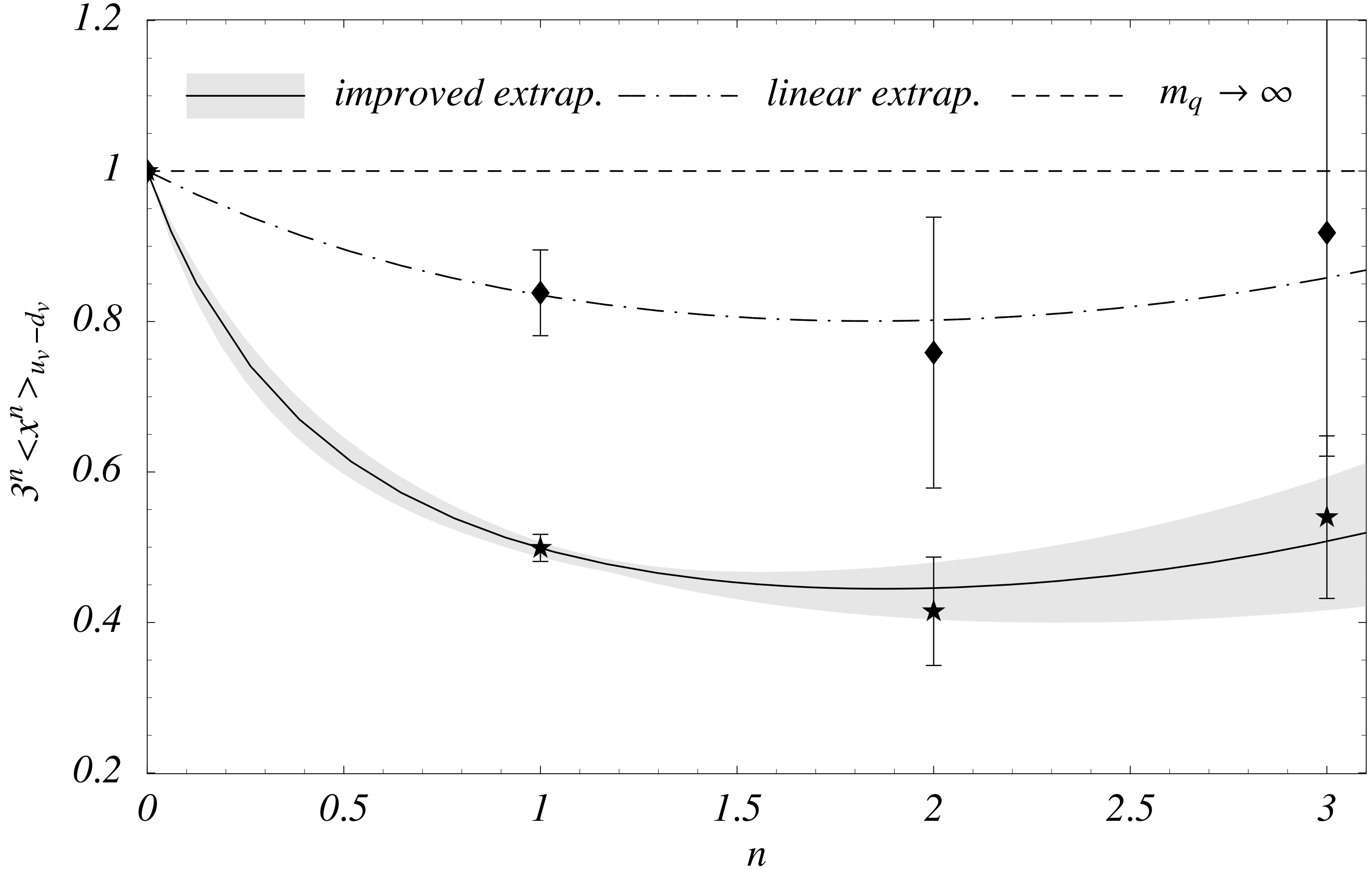
If have sufficient number of moments

- fit odd and even moments separately
  to obtain both valence and total

- subtract 2 x empirical sea from odd moments

$$q_v \equiv q - \bar{q} = q + \bar{q} - 2\bar{q}$$

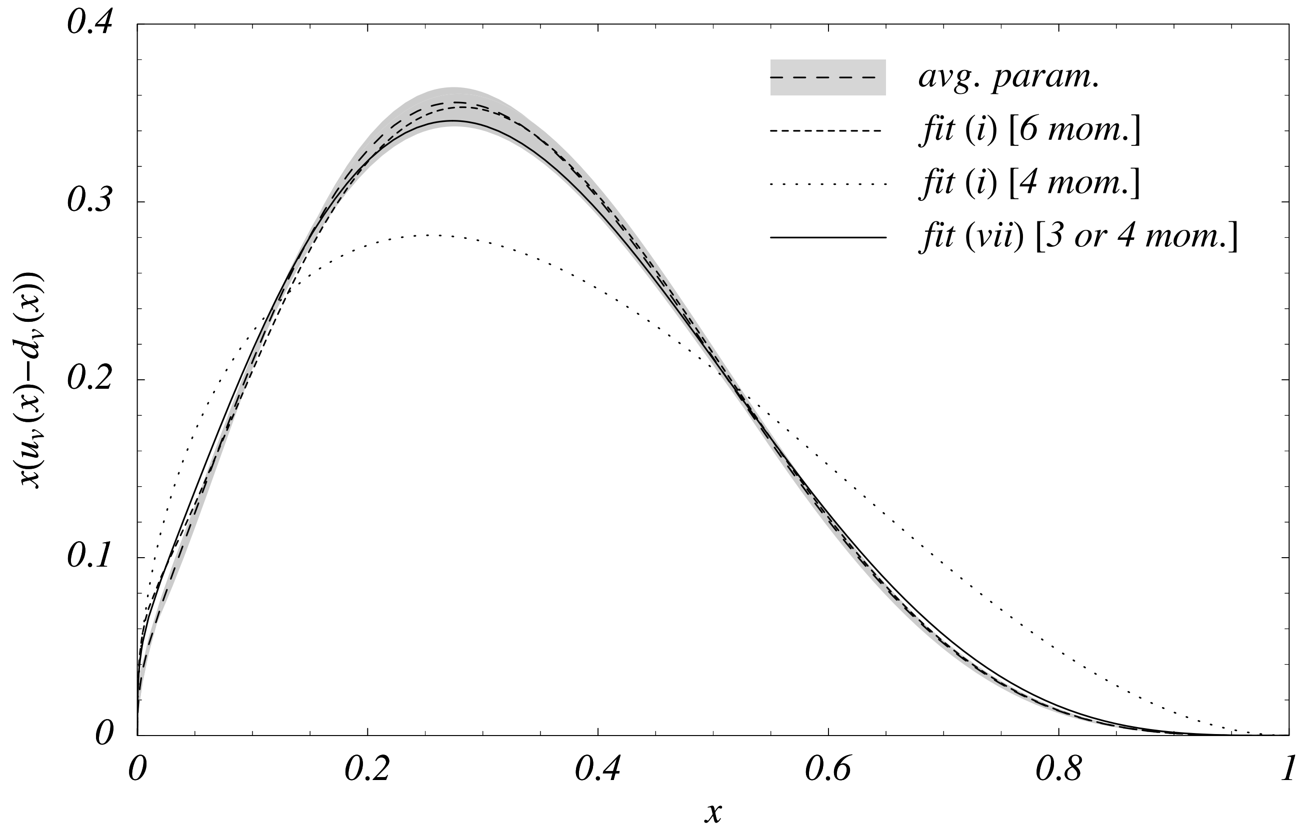
Chiral extrapolation of valence moments



Moments of $u_v - d_v$ (scaled by 3^n)

How well can one reconstruct PDFs from a few moments?

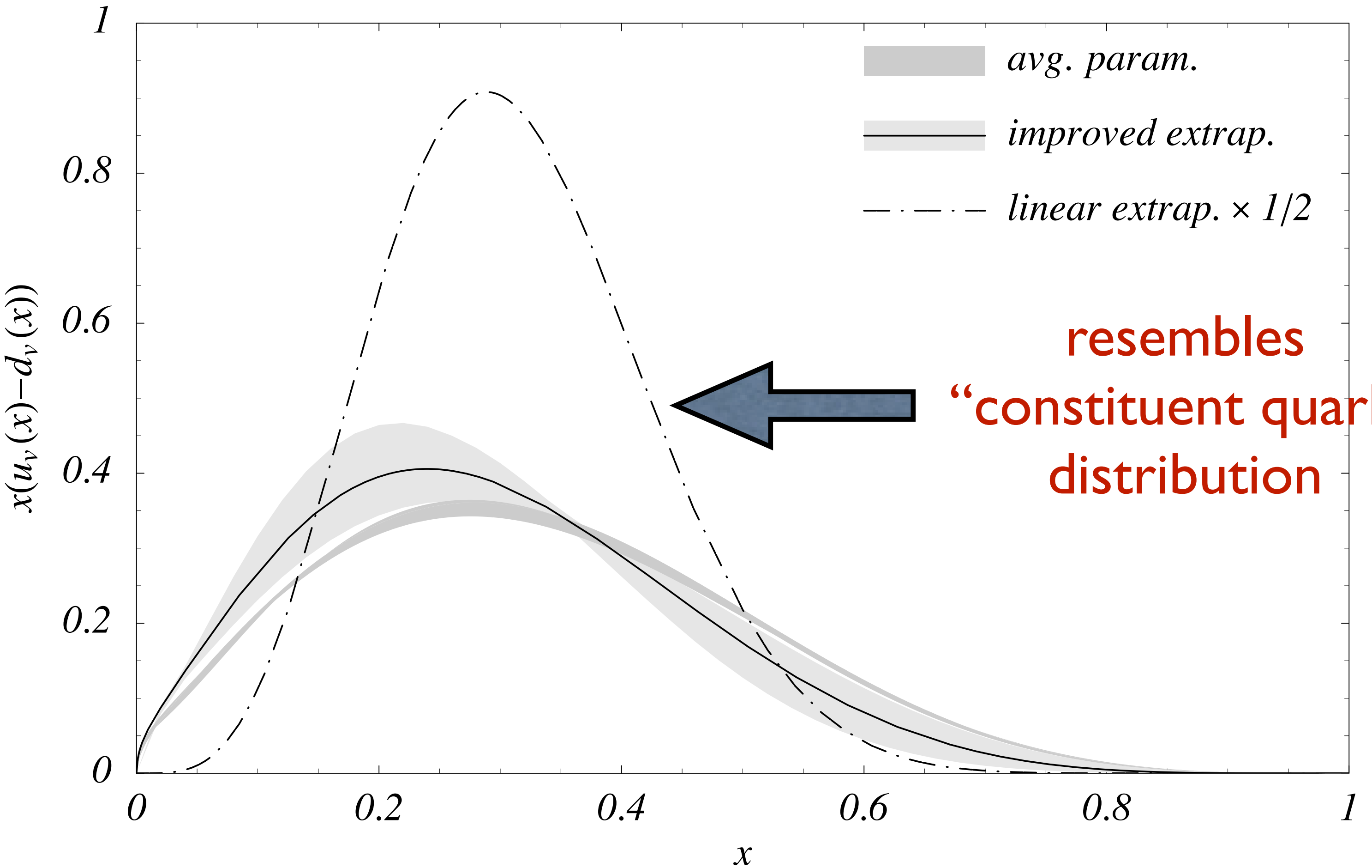
Test case:



$$xq(x) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$

- $fit(i)$: 4 unconstrained parameters (b, c, ϵ, γ)
- $fit(vii)$: 2 unconstrained parameters (b, c)

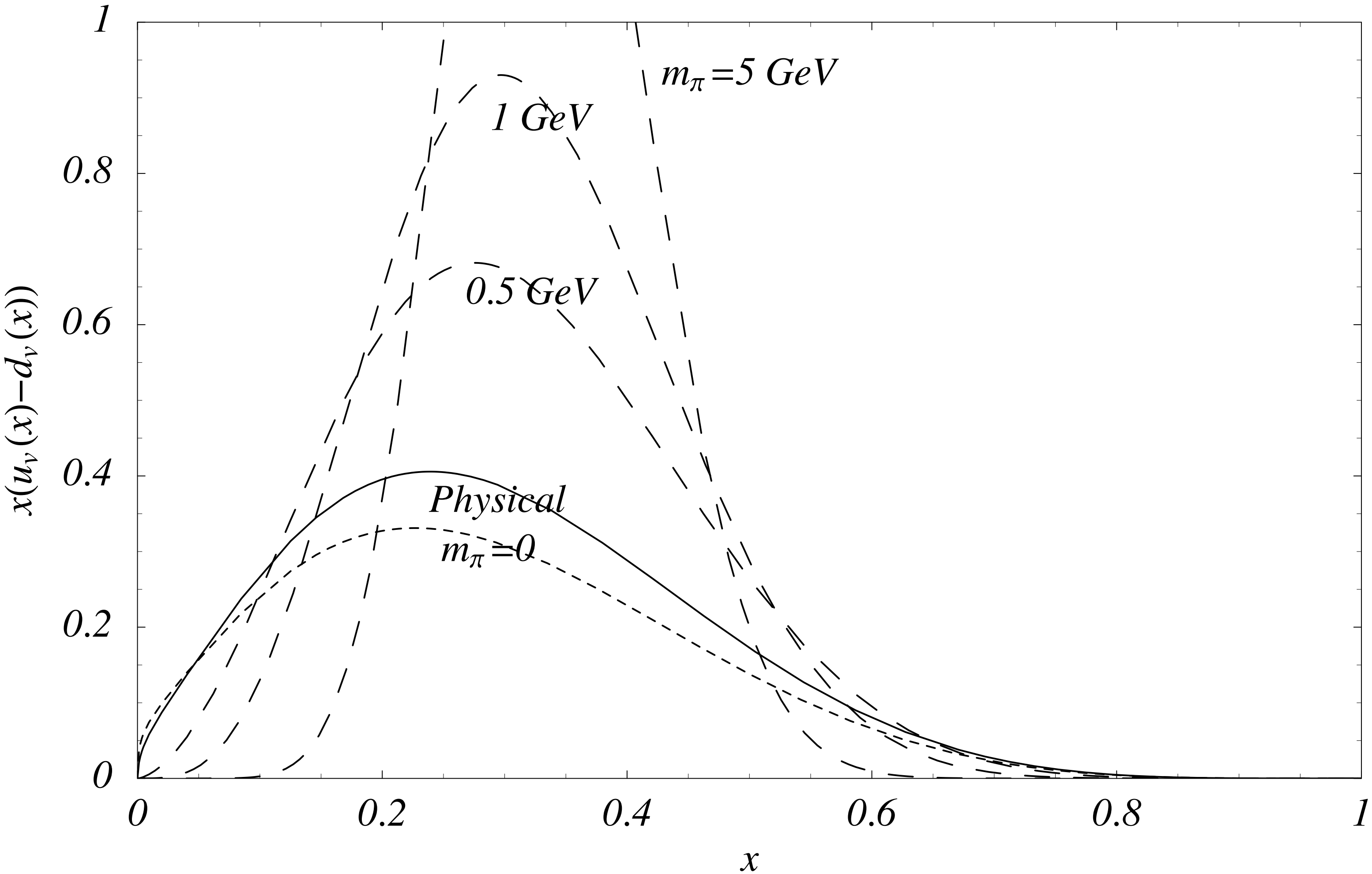
Reconstructed distribution



resembles "constituent quark" distribution

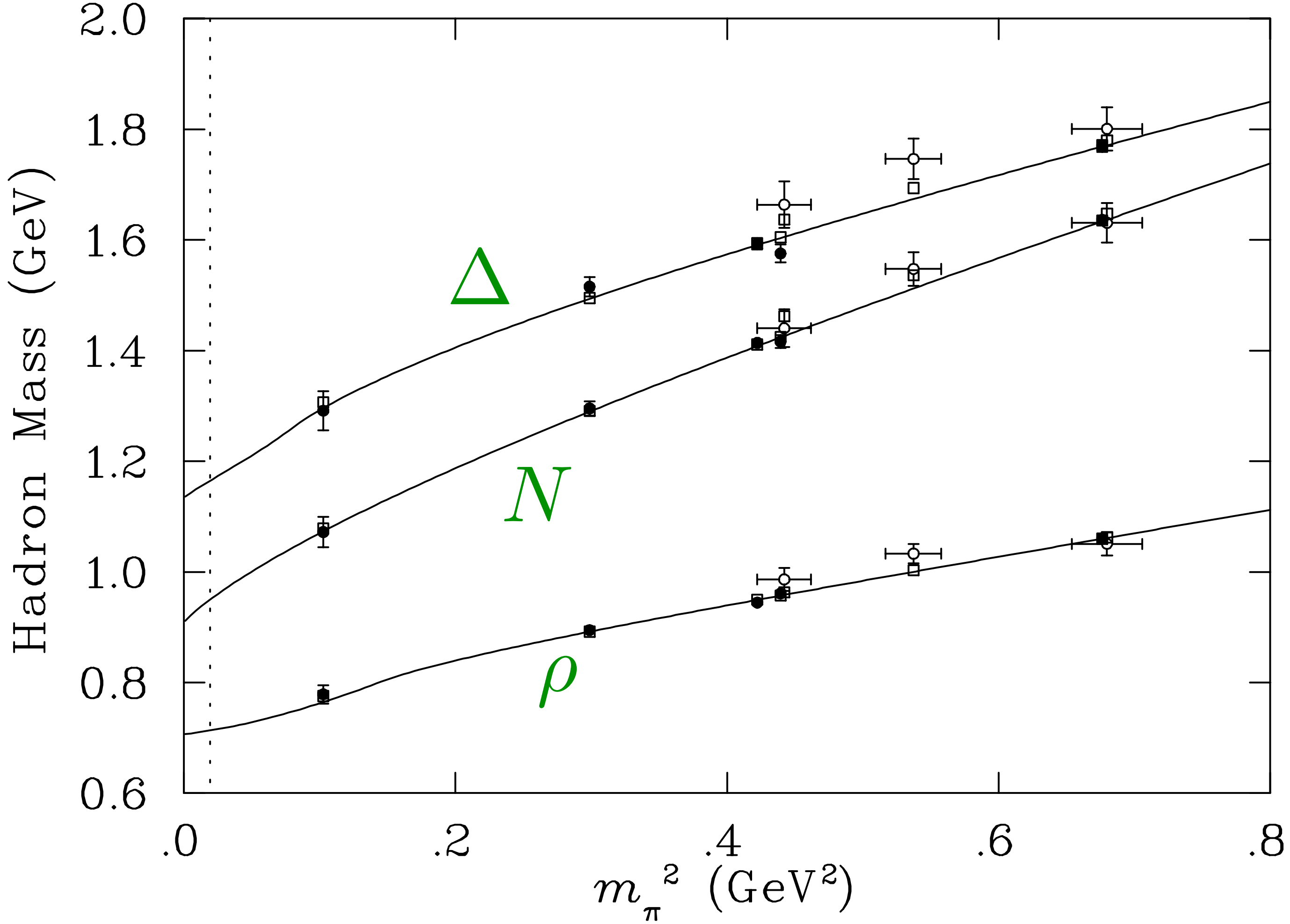
$$xq(x) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$

Quark mass dependence of PDFs



Looks like "constituent quark" distribution
in heavy quark limit !

Connecting models with lattice QCD



Young, Wright, Leinweber, Thomas et al.

Connecting models with lattice QCD

- At large quark masses, observables display "constituent quark" behavior

⟹ $$M_{\text{baryon}} \sim 3m_q$$
$$M_{\text{meson}} \sim 2m_q$$

⟹ suggests new approach to modeling QCD

- construct "constituent quark" model at large quark masses

- extrapolate to physical quark mass using known chiral behavior

Summary - quark distributions

- Sea quarks

  $\longrightarrow$ asymmetry $\bar{d} > \bar{u}$ arises from nonperturbative QCD effects such as pion cloud of the nucleon

  $\longrightarrow$ similarly, strong indications that $s \neq \bar{s}$

- Valence quarks

  $\longrightarrow$ d quark poorly known at large x

  $\longrightarrow$ n structure obscured by nuclear effects in deuteron (also nuclear shadowing at small x)

- Progress in extracting quark distributions from lattice QCD

  $\longrightarrow$ need to extrapolate lattice data to physical regime

3.
Quark-hadron duality

Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

Can use either set of complete basis states to describe all physical phenomena

Duality in Nature

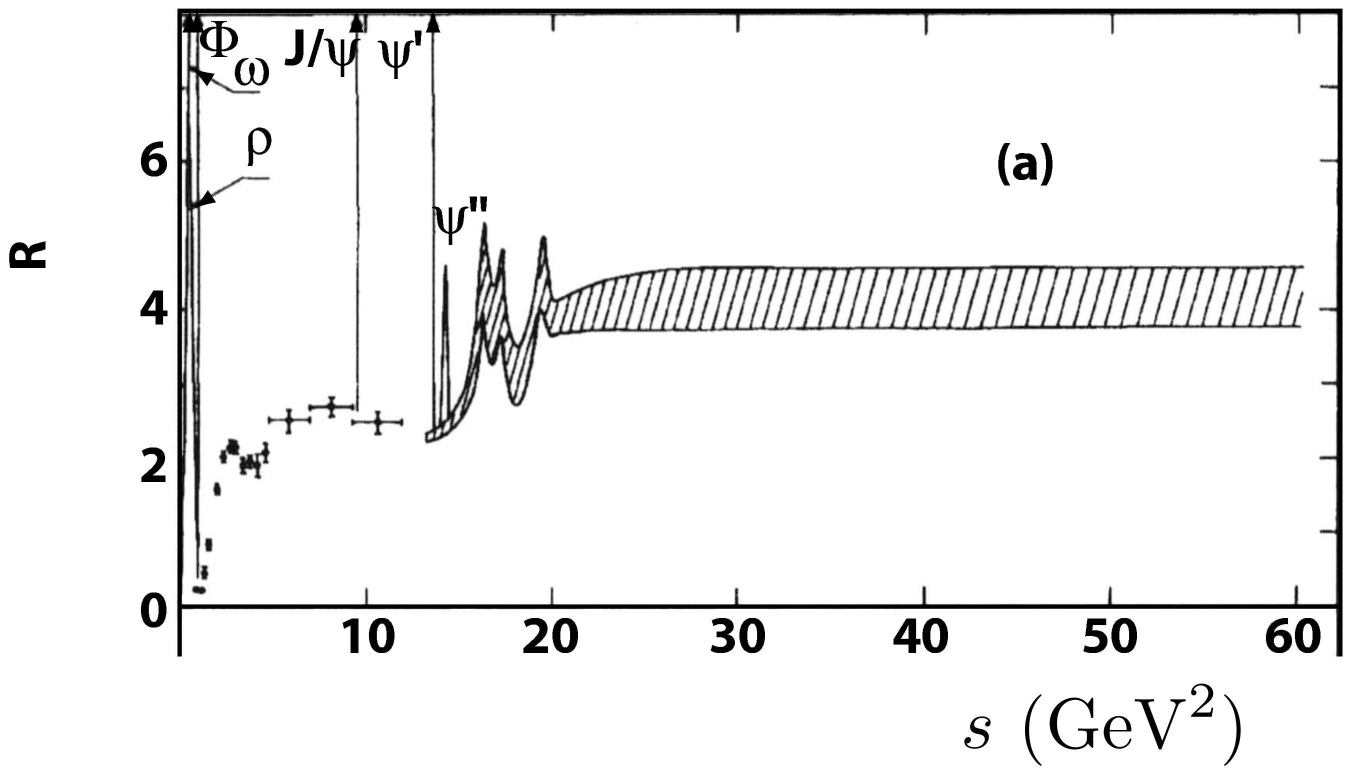
- Duality between quarks *(high energy)* and hadrons *(low energy)* manifests itself in many processes

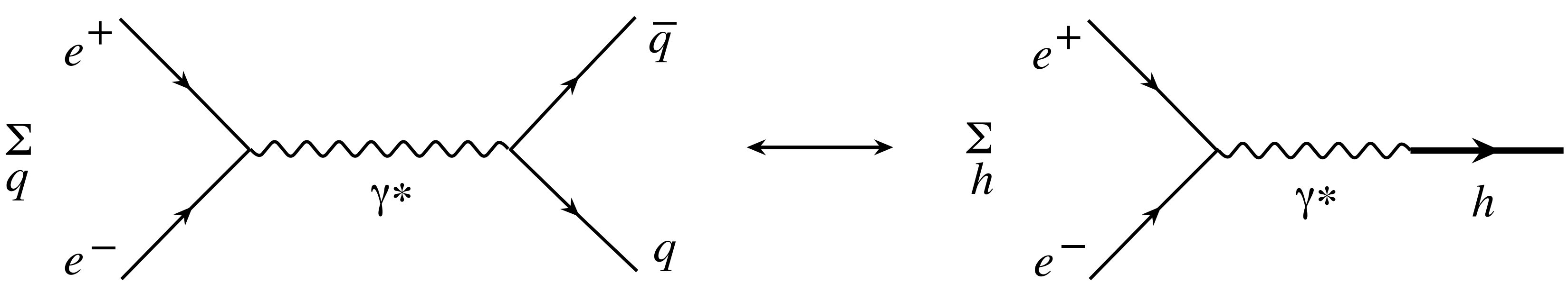
- *e*+ *e*- annihilation
  - *total hadronic cross section at high energy averages resonance cross section*

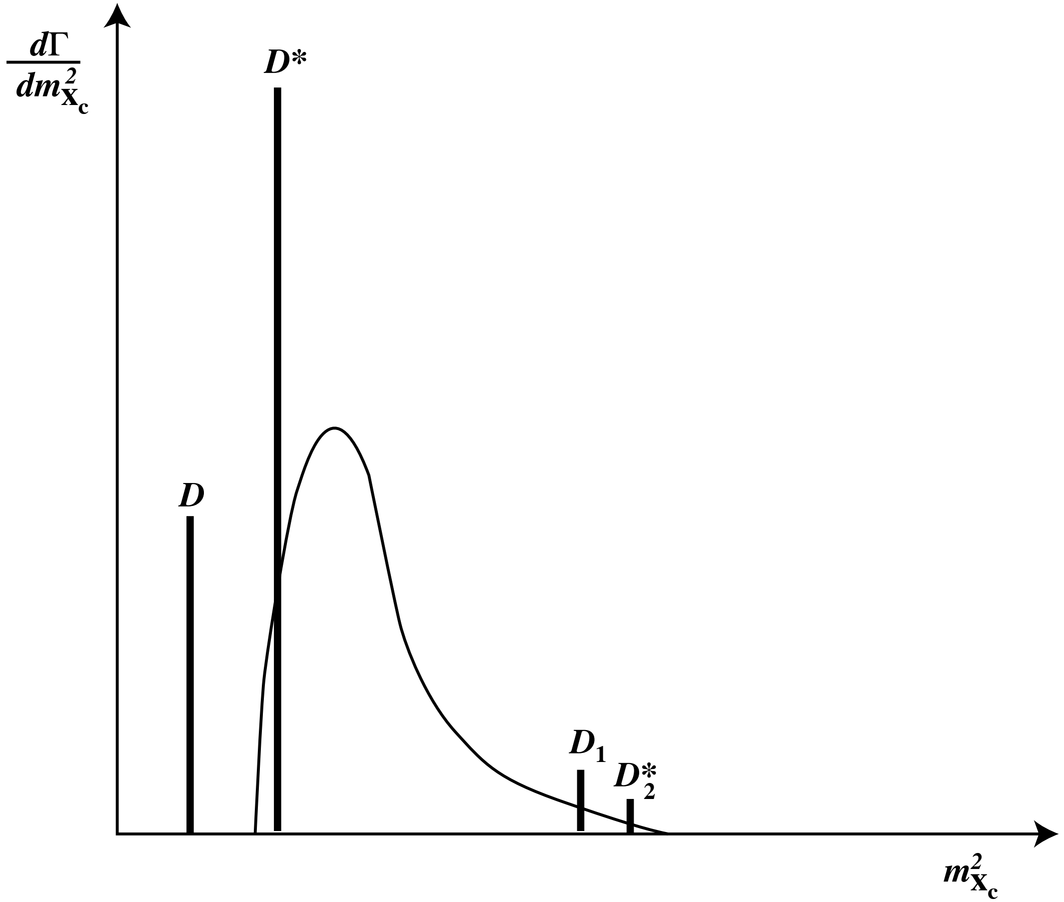
- Heavy meson decays
  - *duality between hadronic & quark descriptions of decays in* $m_Q \to \infty$ *limit*

- Duality between *s*-channel resonances and *t*-channel (Regge) poles in hadronic reactions

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



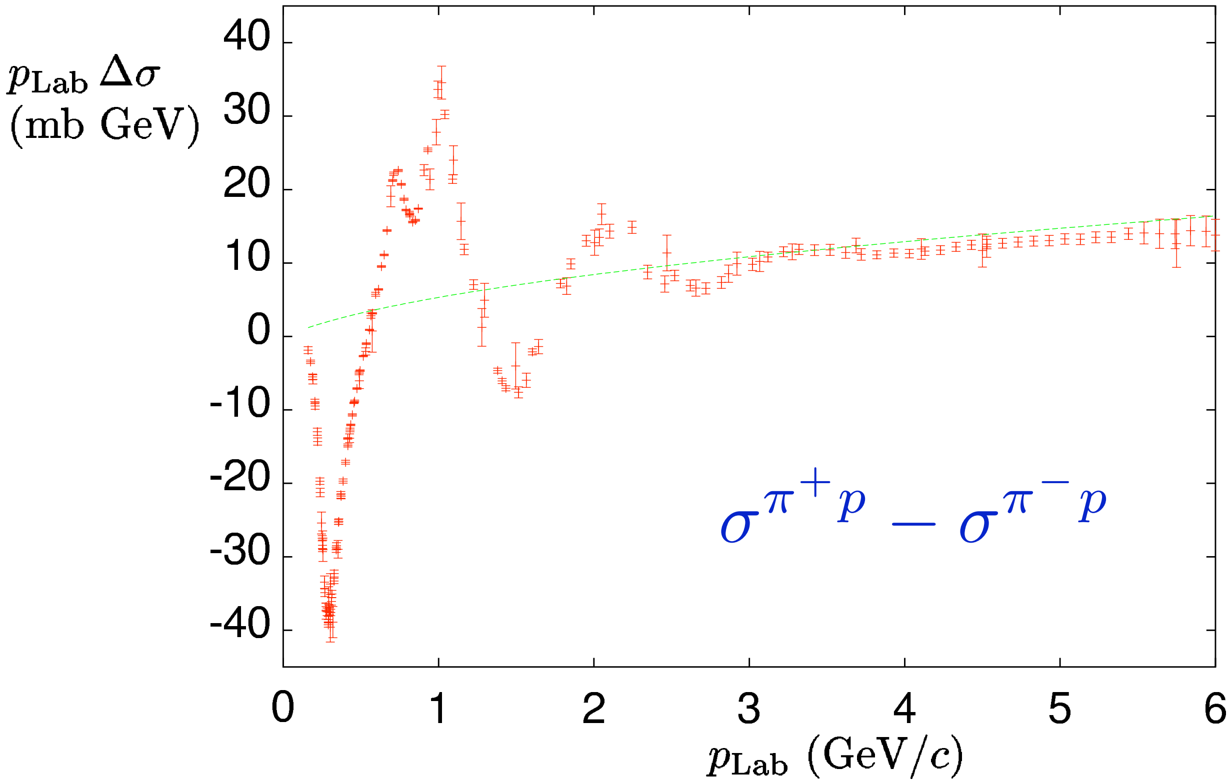


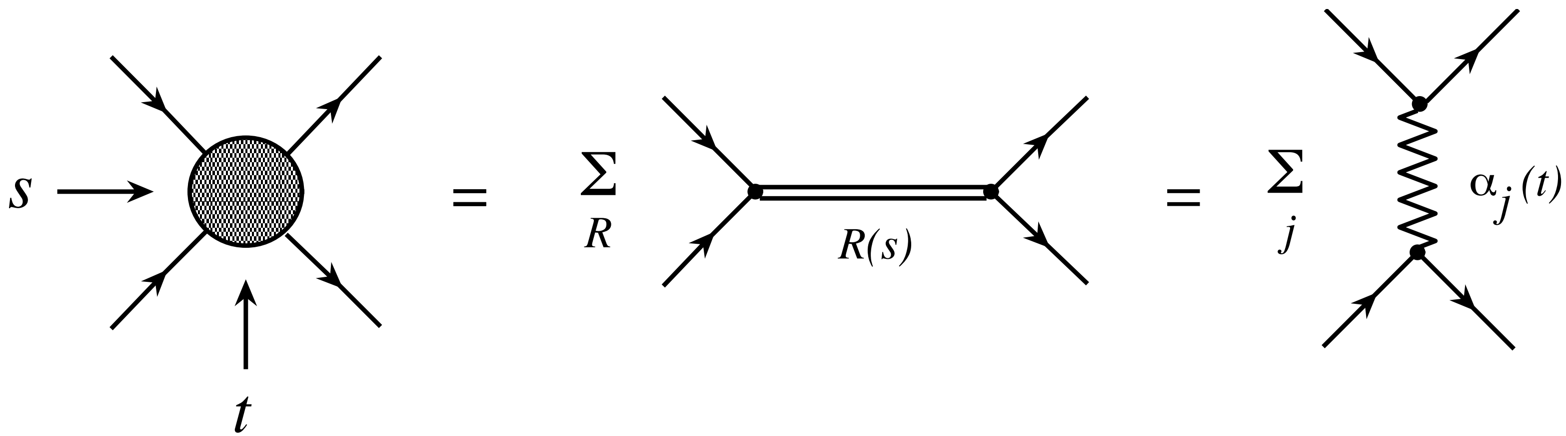


$$B \to D\, l\, \nu$$

$$b \to c\, l\, \nu$$

Voloshin, Shifman, Sov. J. Nucl. Phys. 41 (1985) 120
Isgur, Phys. Lett. B448 (1999) 111





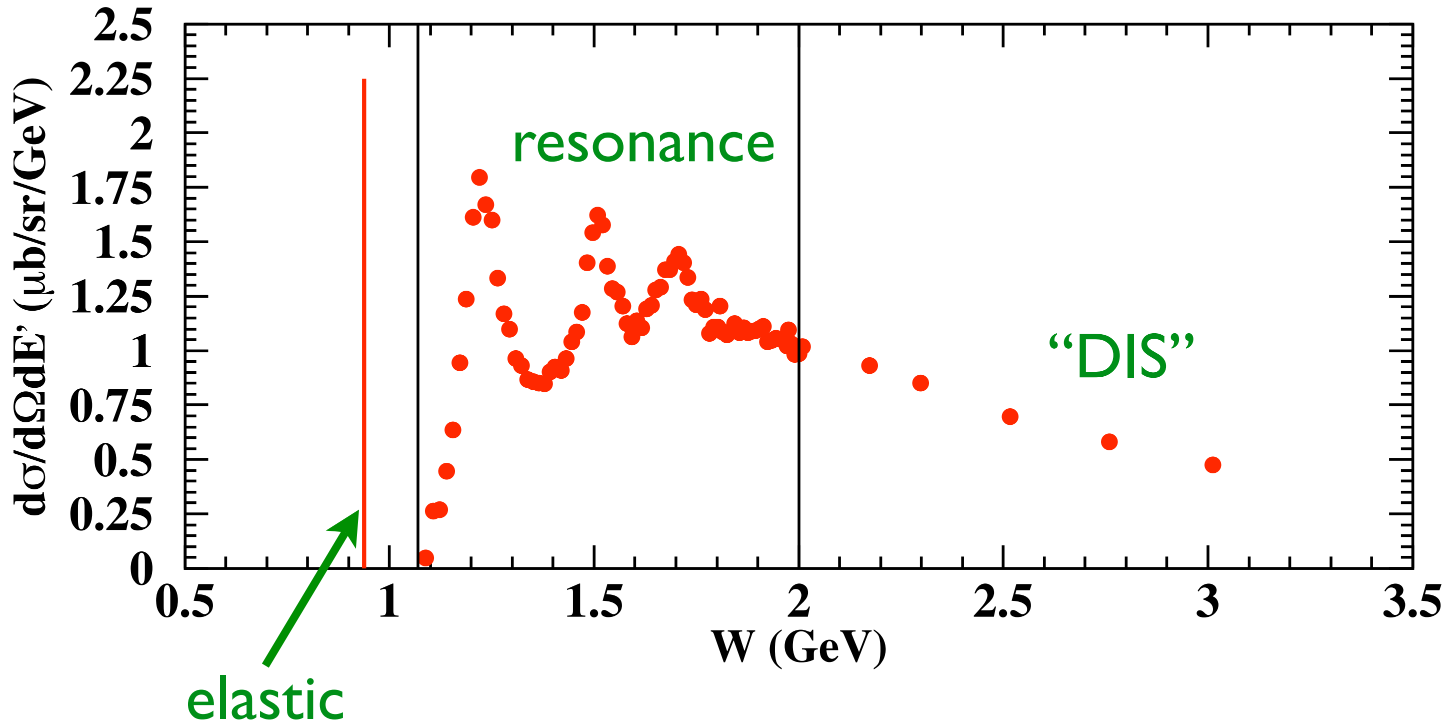
"Finite energy sum rules"

Igi (1962), Dolen, Horn, Schmidt (1968)

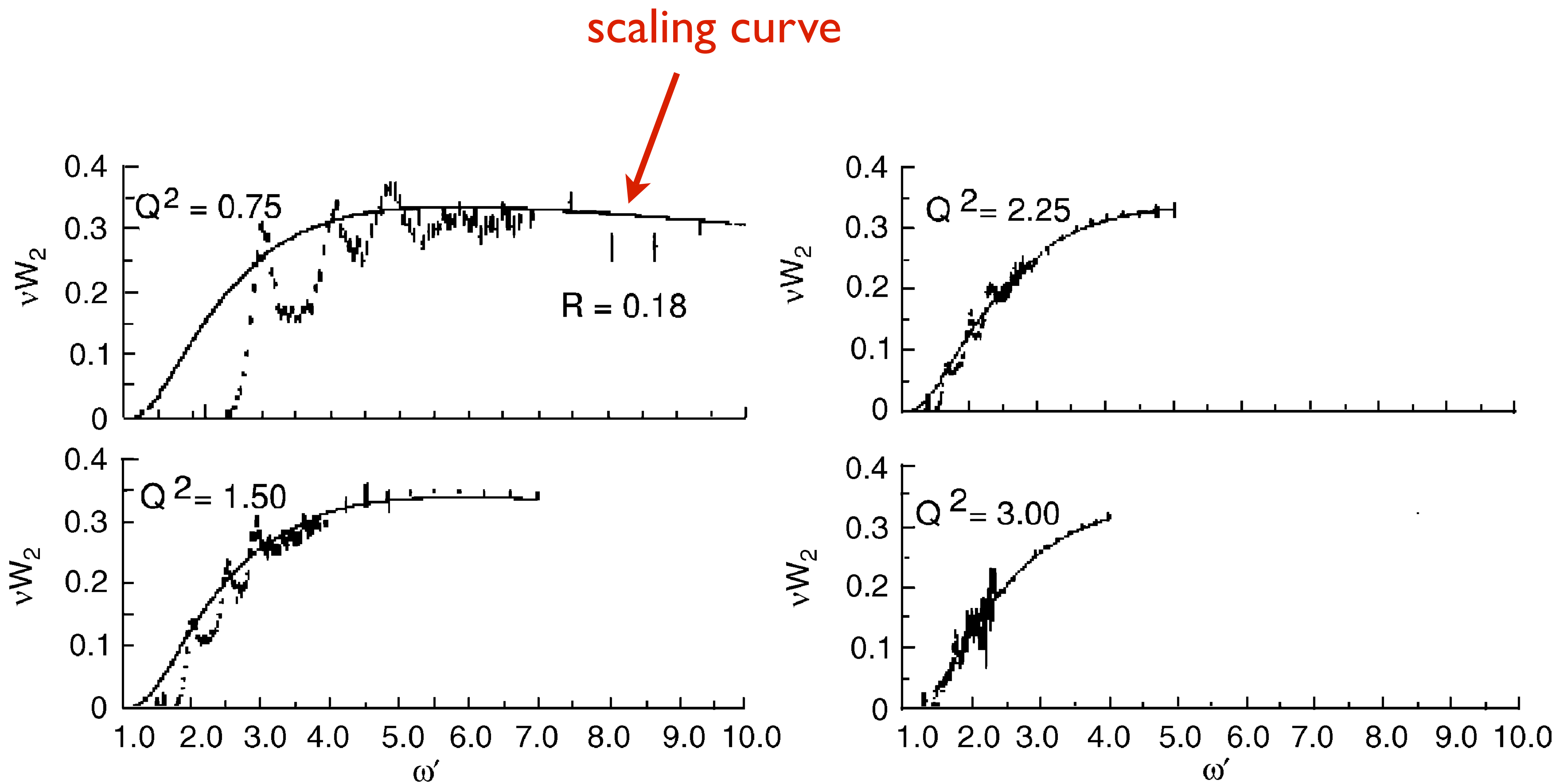
3.
Quark-hadron duality
- Bloom-Gilman duality

Resonances

As W decreases, DIS region gives way to region dominated by nucleon resonances



$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$



Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Quark-hadron duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

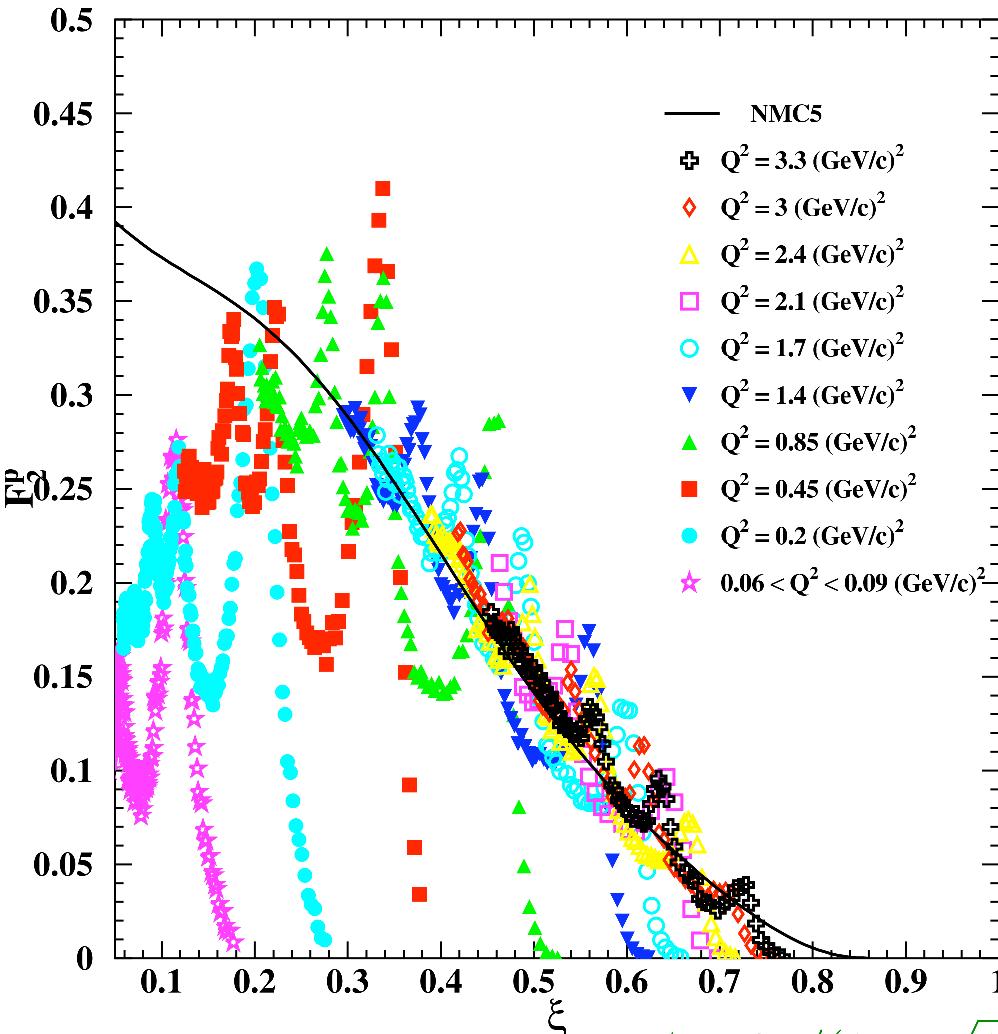
"Finite energy sum rule" for eN scattering

$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ F_2(\nu, Q^2) = \int_1^{\omega'} d\omega'\ F_2(\omega')$$

$$\omega' = 1/x + M^2/Q^2$$

Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

Bloom-Gilman duality



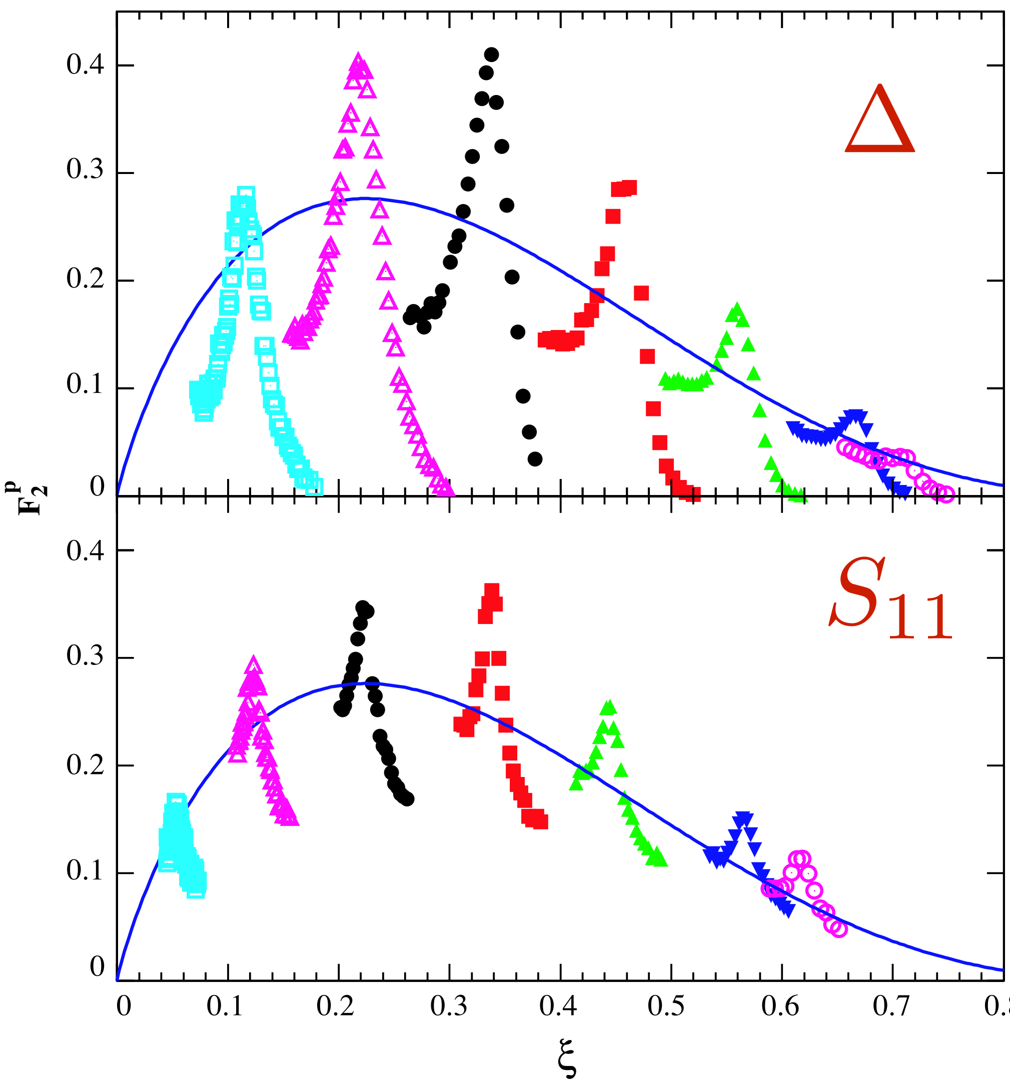
$\xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2})$

Average over
(strongly Q^2 dependent)
resonances
$\approx$ Q^2 independent
scaling function

Jefferson Lab (Hall C)

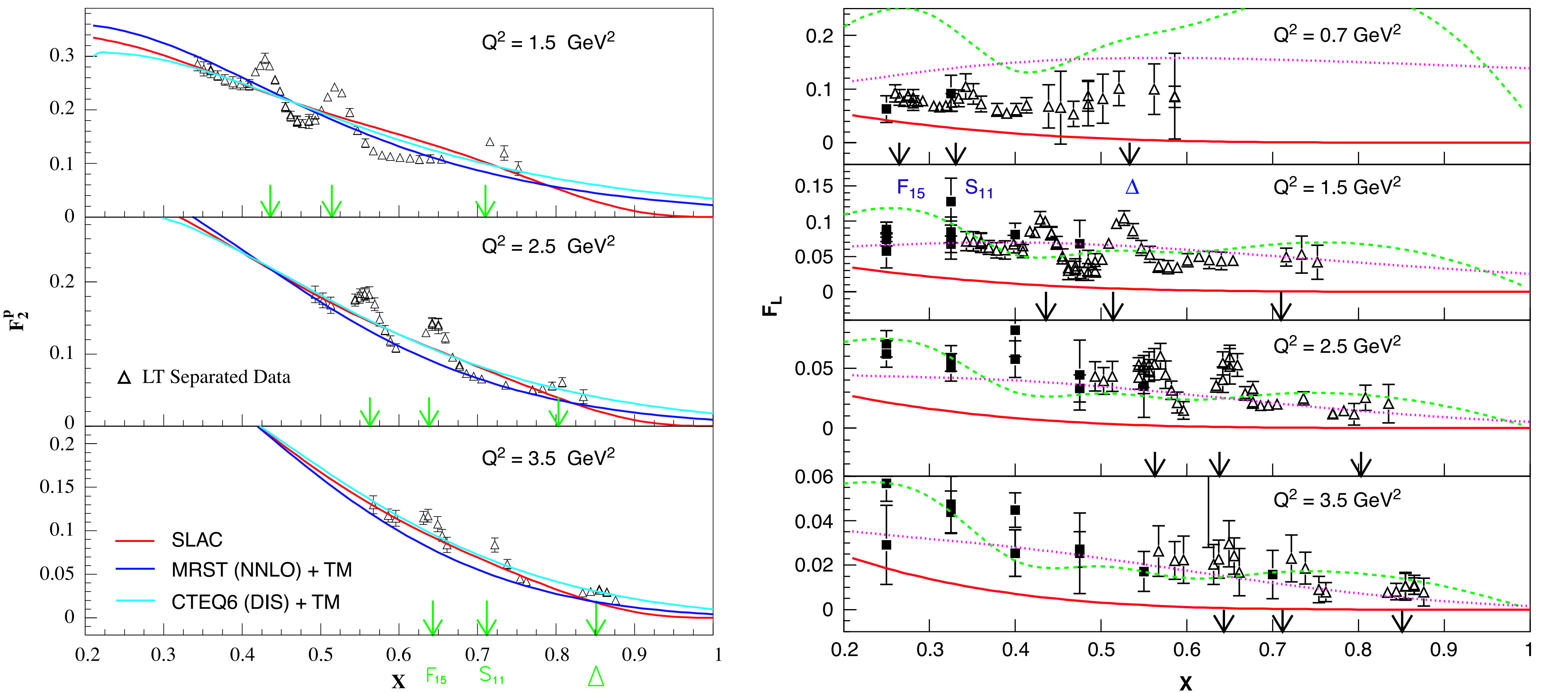
Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Local Bloom-Gilman duality



$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$ Nachtmann scaling variable

Local Bloom-Gilman duality



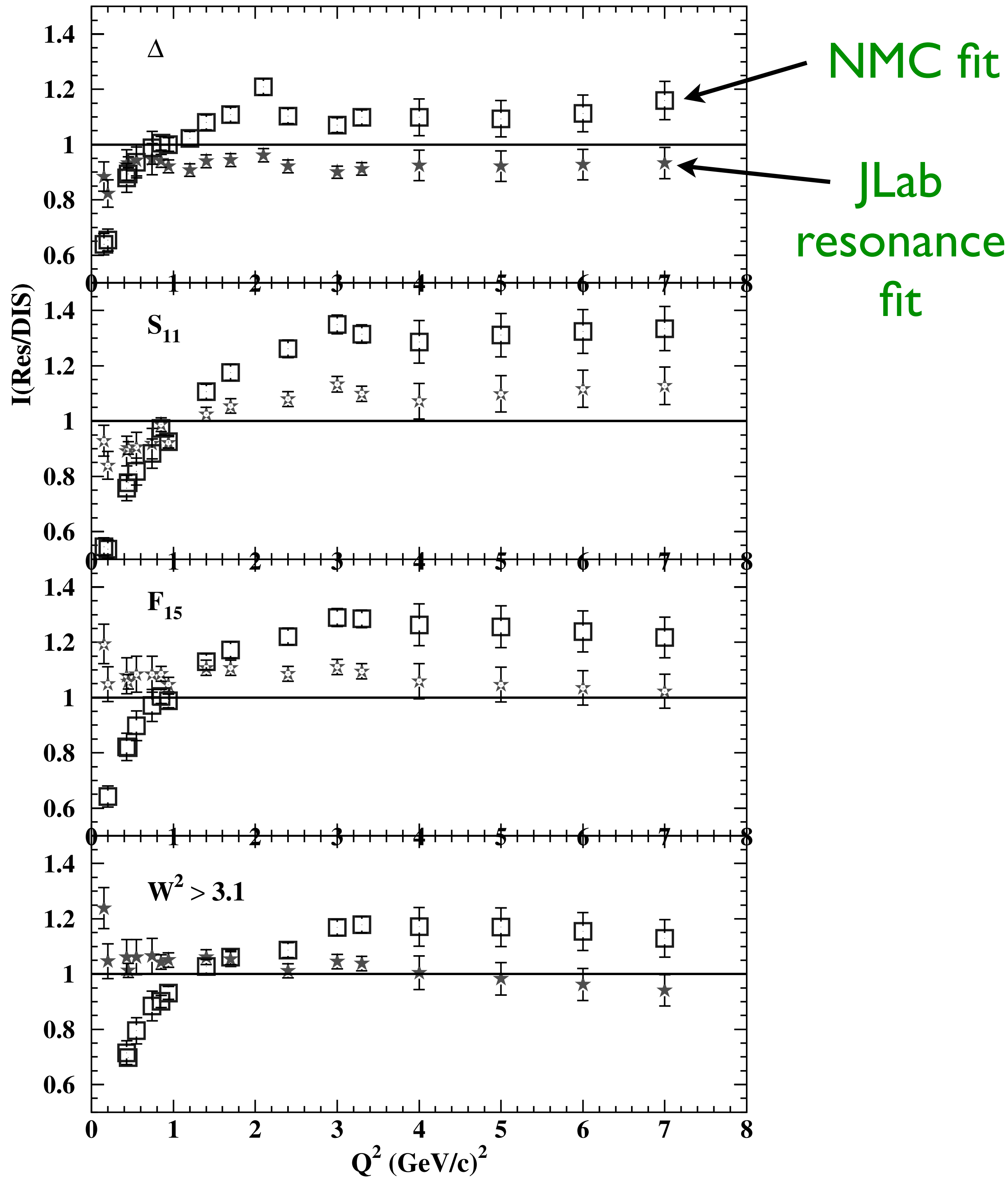
E. Christy et al. (2005)

duality in F_2 and F_L structure functions

(from longitudinal-transverse separation)

⟹ importance of target mass corrections

Integrated strength

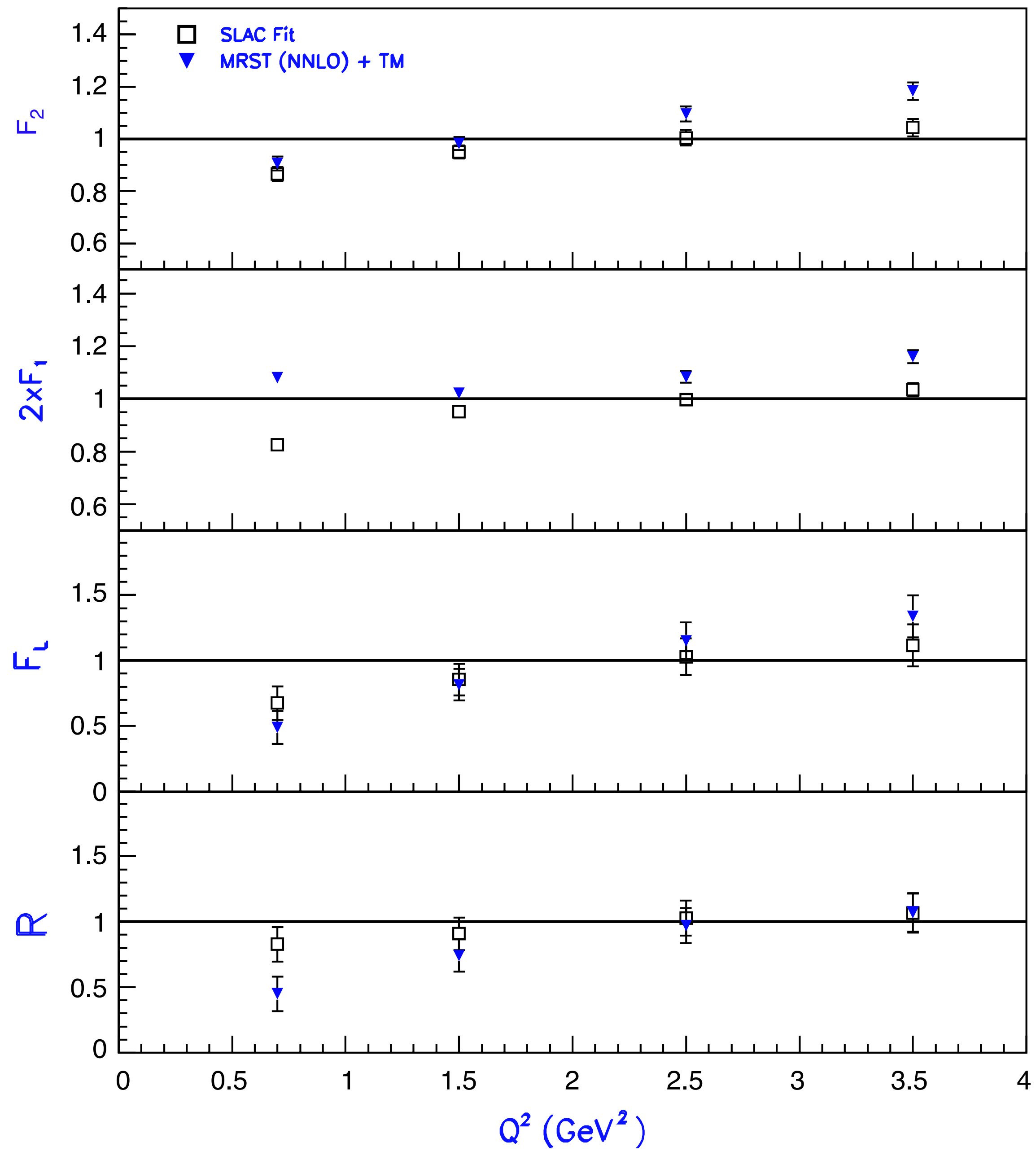


~10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al, Phys. Rev. Lett. 85 (2000) 1186

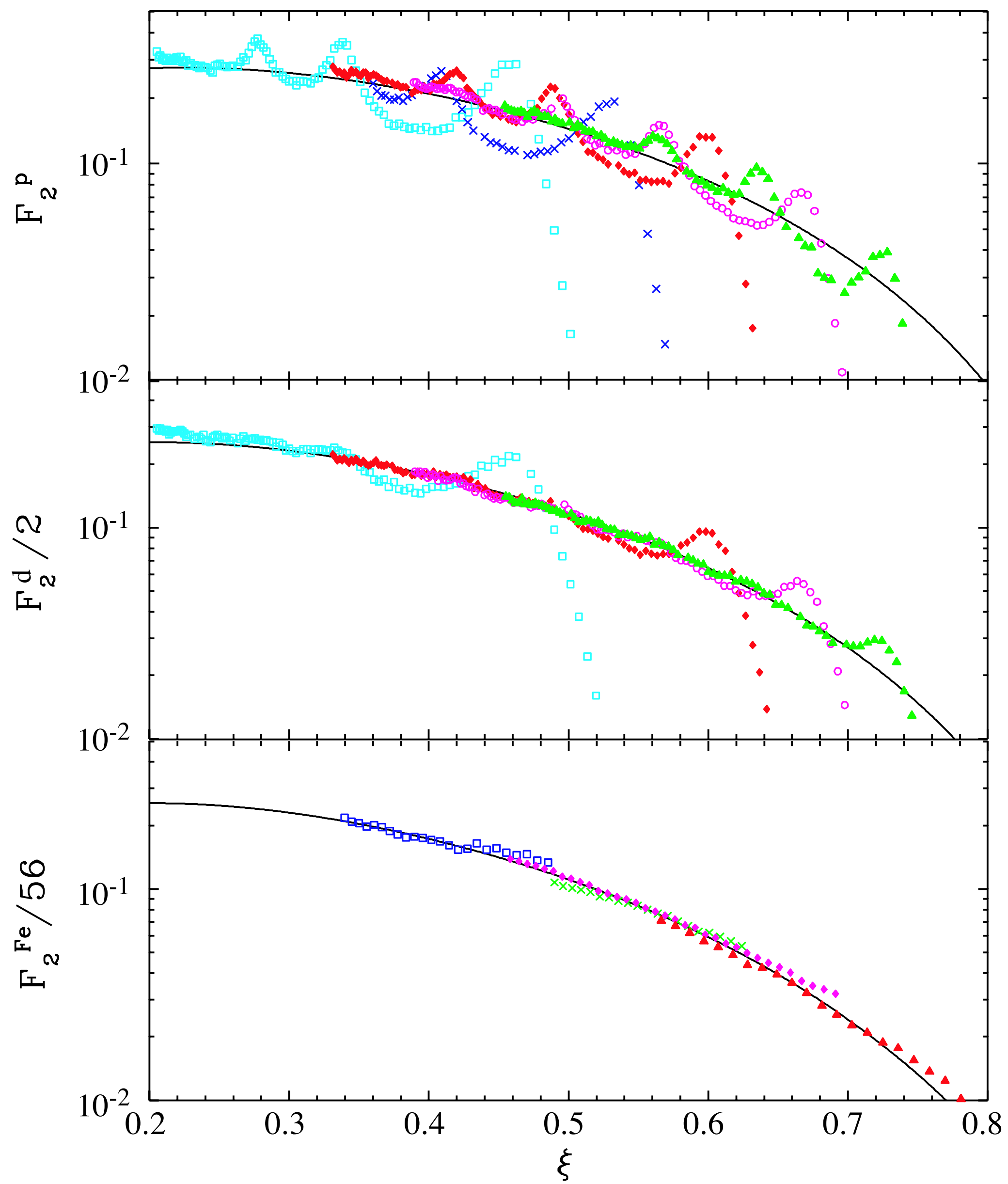
Moments

data from longitudinal-transverse separation !



Jefferson Lab (Hall C)

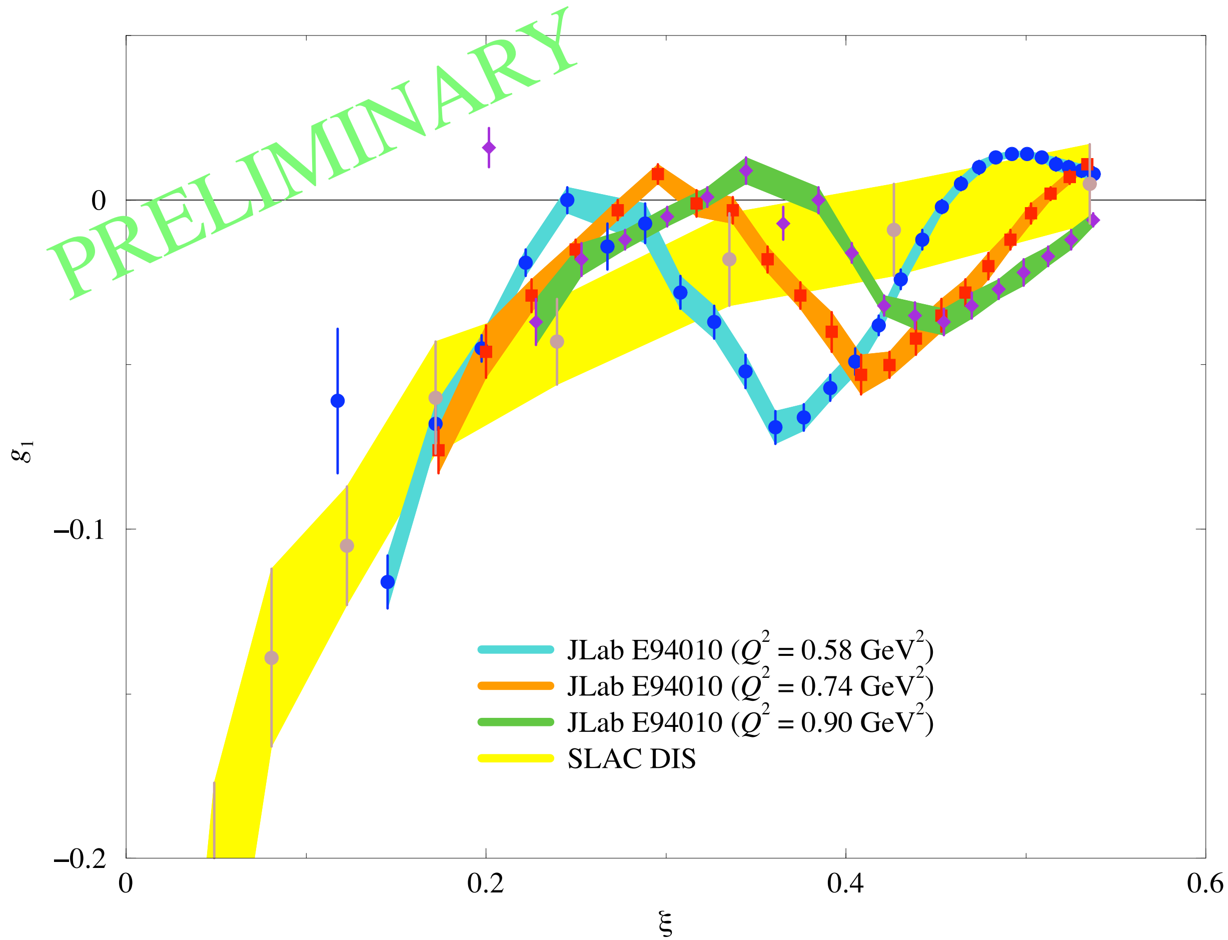
Nuclear structure functions



for larger nuclei, Fermi motion does resonance averaging automatically !

Jefferson Lab (Hall C)

Neutron (^{3}He) g_1 structure function



Liyanage et al. (JLab Hall A)

3.
Quark-hadron duality

- duality in QCD

Duality and QCD

Operator product expansion

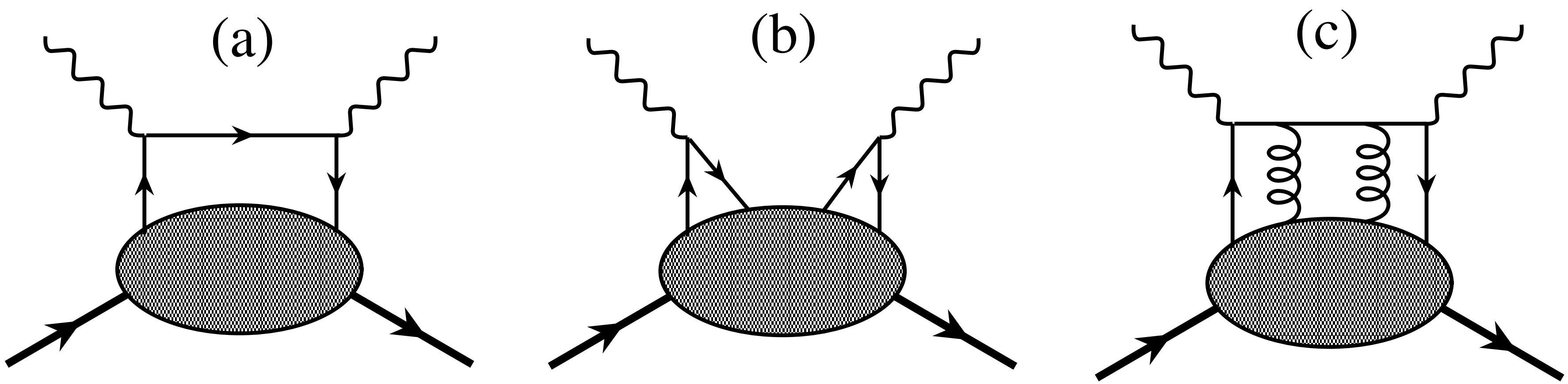
expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

Higher twists



$\tau = 2$

$\tau > 2$

single quark scattering

qq and qg correlations

Duality and QCD

Operator product expansion

➡ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

➡ higher twist terms $A_n^{(\tau>2)}$ small

Duality and QCD

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x, Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer, Ann. Phys. 103 (1975) 315

Applications of duality

If higher twists are small (duality "works")

⟹ can use single-parton approximation to describe structure functions

⟹ extract *leading twist* parton distributions

If duality is violated, and if violations are small

⟹ can use duality violations to extract *higher twist matrix elements*

⟹ learn about nonperturbative *qq* or *qg* correlations

Example: Lowest moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 dx\; g_1(x, Q^2)$$
$$= \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots$$

Twist 2

$$\mu_2^{p(n)} = \left(\pm\frac{1}{12} g_A + \frac{1}{36} a_8\right) C_{ns}(Q^2) + \frac{1}{9}\Delta\Sigma\; C_s(Q^2)$$

triplet *octet* *RGI singlet axial charge*

Higher twist terms

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4 d_2 + 4 f_2 \right)$$

target mass correction

quark-gluon correlations

Higher twist terms

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4 d_2 + 4 f_2 \right)$$

$$d_2 \;\rightarrow\; \langle N | \, \bar{\psi} \, \widetilde{G}^{\mu\{\nu} \gamma^{\alpha\}} \, \psi \, | N \rangle$$

twist 3

$$f_2 \;\rightarrow\; \langle N | \, \bar{\psi} \, \widetilde{G}^{\mu\nu} \gamma_\nu \, \psi \, | N \rangle$$

twist 4

Color polarizabilities

$1/Q^2$ correction to g_1 moment

$$\mu_4 = \frac{1}{9} M^2 \left(a_2 + 4d_2 + 4f_2\right)$$

color *electric* polarizability

$$\chi_E \;=\; \frac{1}{3}(4d_2 + 2f_2) \;\sim\; \langle \vec{j}_a \times \vec{E}_a \rangle_z$$

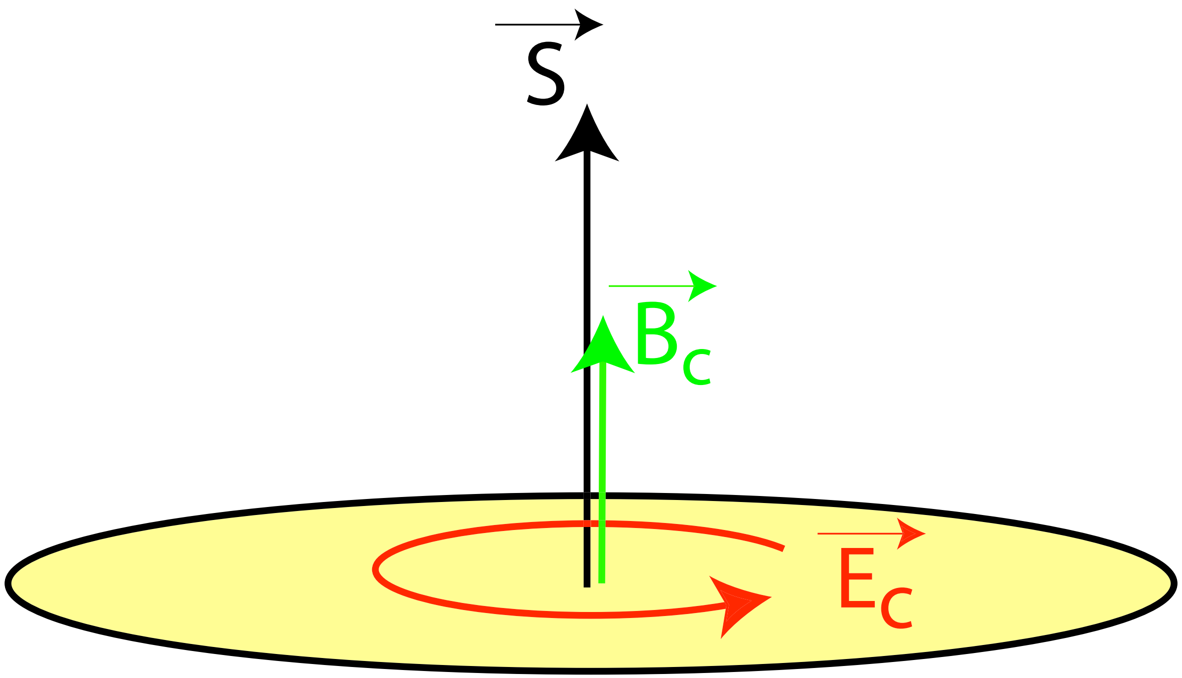
$$j_a^\mu = g_s \psi \gamma^\mu \mathbf{t}_a \psi$$

color *magnetic* polarizability

$$\chi_B \;=\; \frac{1}{3}(4d_2 - f_2) \;\sim\; \langle j_a^0 \; \vec{B}_a \rangle_z$$

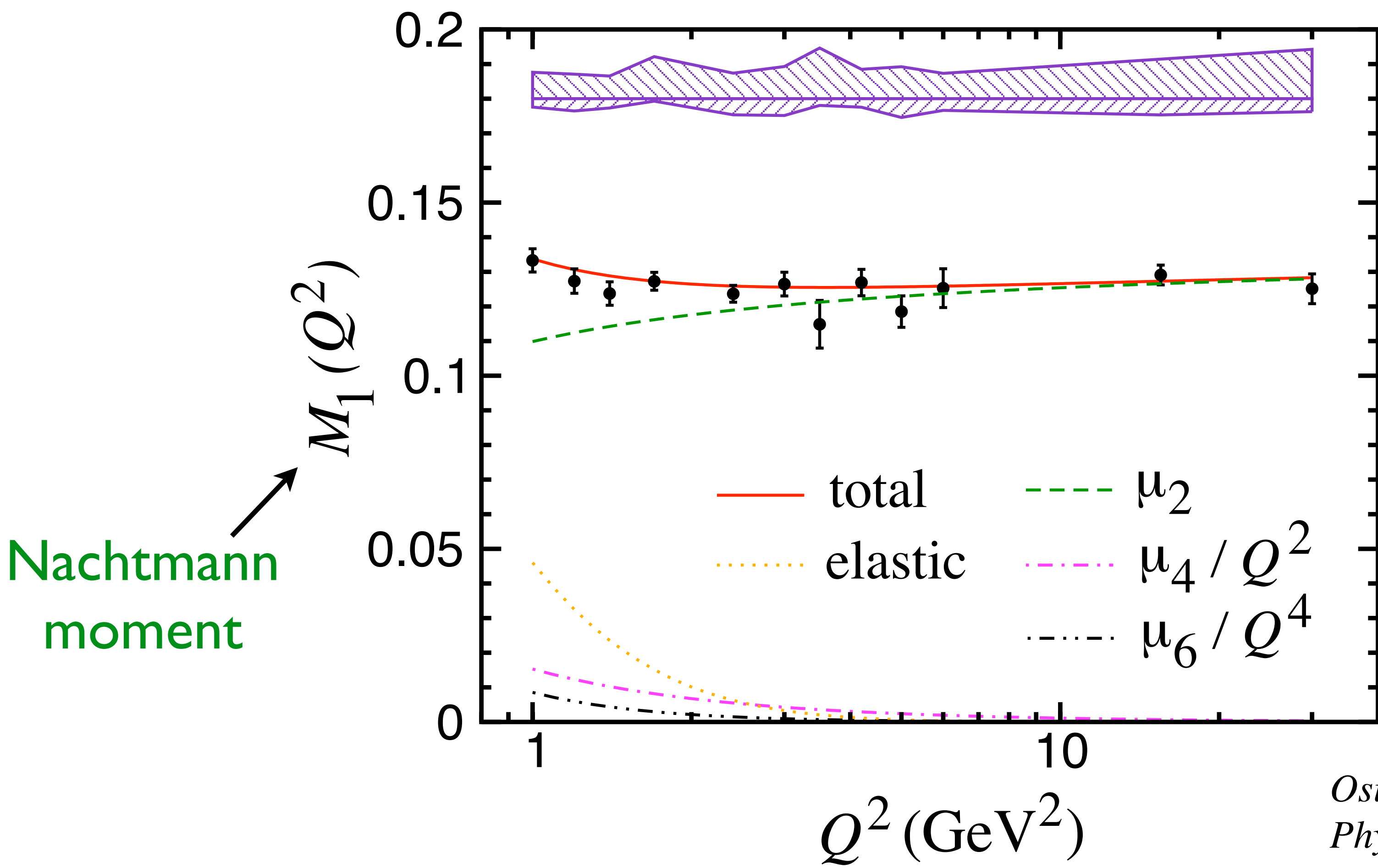
Ji (1995), Schafer, Mankiewicz, ... (1995)

Color polarizabilities



response of collective color electric and magnetic fields to spin of nucleon

Proton g_1 moment



Osipenko, WM et al,
Phys. Lett. B 609 (2005) 259

$$M_1 = \int_0^1 dx \frac{\xi^2}{x^2}\left[g_1\left(\frac{x}{\xi} - \frac{M^2 x\xi}{9Q^2}\right) - g_2 \frac{4M^2x^2}{3Q^2}\right] \;=\; \mu_2 + \frac{4M^2}{9Q^2} f_2 + \cdots$$

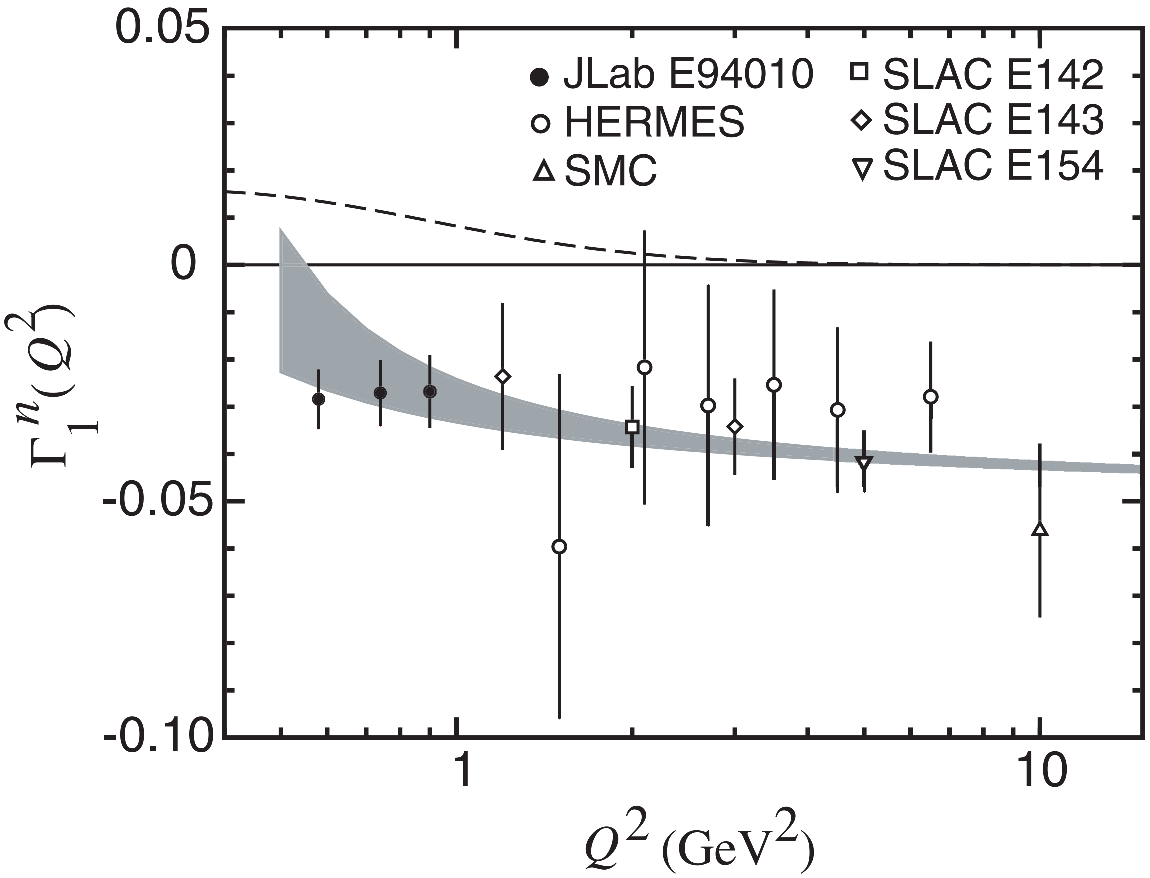
$$\chi_E^p = 0.026 \pm 0.015 \text{ (stat)} \pm 0.021 \text{ (sys)}$$

$$\chi_B^p = -0.013 \pm 0.007 \text{ (stat)} \pm 0.011 \text{ (sys)}$$

Compare with theoretical calculations:

	χ_E^p	χ_B^p
QCD sum rules	-0.04	0.01
MIT bag	0.05	0.02
Instanton	-0.03	0.02
Lattice	?	?

Neutron g_1 moment



Meziani, WM et al,
Phys. Lett. B613 (2005) 148

Γ_1^n extracted from $\Gamma_1^{^3\mathrm{He}}$ data

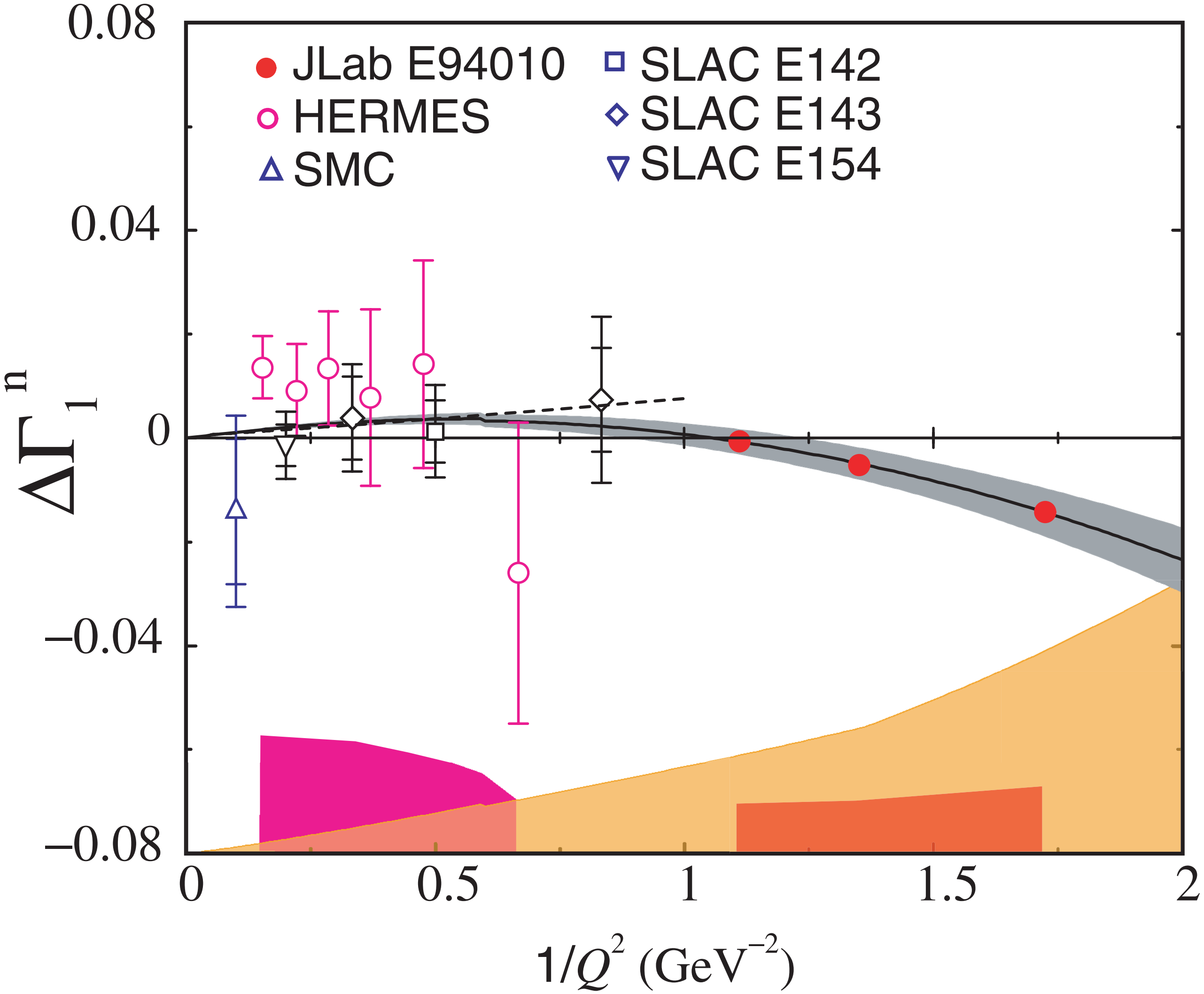
correcting for nuclear effects

$$\chi_E^n = +0.033 \pm 0.029$$

$$\chi_B^n = -0.001 \pm 0.016$$

Compare with theoretical calculations:

	χ_E^n	χ_B^n
QCD sum rules	-0.04	-0.02
MIT bag	0.00	0.00
Instanton	0.03	-0.01
Lattice	?	?



Higher twist contribution to neutron moment

Total higher twist $\sim$ *zero* at $Q^2 \sim 1-2\ \mathrm{GeV}^2$

⟹ nonperturbative interactions between quarks and gluons not dominant at these scales

⟹ suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

⟹ OPE does not tell us *why* higher twists are small !

Can we understand this behavior dynamically?

How do cancellations between *coherent* resonances produce *incoherent* scaling function?

3.
Quark-hadron duality

- local duality

Coherence vs. incoherence

Exclusive form factors

⟹ *coherent* scattering from quarks

$$d\sigma \sim \left(\sum_i e_i \right)^2$$

Inclusive structure functions

⟹ *incoherent* scattering from quarks

$$d\sigma \sim \sum_i e_i^2$$

⟹ How can the *square of a sum* become the *sum of squares*?

Pedagogical model

Two quarks bound in a harmonic oscillator potential

$\Longrightarrow$ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left| G_{0,n}(\mathbf{q}^2) \right|^2 \; \delta(E_n - E_0 - \nu)$$

Charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites

even partial waves with strength $\propto (e_1 + e_2)^2$

odd partial waves with strength $\propto (e_1 - e_2)^2$

Pedagogical model

Resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{ (e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1} \right\}$$

If states degenerate, cross terms $(\sim e_1 e_2)$ cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

$\Longrightarrow$ *at least one complete set of <u>even</u> and <u>odd</u> parity resonances must be summed over*

Close, Isgur, Phys. Lett. B509 (2001) 81

Quark model

Even and odd parity states generalize to $\mathbf{56}^{+}$ $(L=0)$ and $\mathbf{70}^{-}$ $(L=1)$ multiplets of spin-flavor SU(6)

$\Longrightarrow$ scaling occurs if contributions from $\mathbf{56}^{+}$ and $\mathbf{70}^{-}$ have equal overall strengths

Simplified case: magnetic coupling of γ^{*} to quark

$\Longrightarrow$ expect dominance over electric at large Q^{2}

Quark model

Even and odd parity states generalize to $\mathbf{56}^+$ ($L=0$) and $\mathbf{70}^-$ ($L=1$) multiplets of spin-flavor SU(6)

$\Longrightarrow$ scaling occurs if contributions from $\mathbf{56}^+$ and $\mathbf{70}^-$ have equal overall strengths

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$
g_1^p	$9\rho^2$	$-4\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2-3\lambda^2$
g_1^n	$(3\rho+\lambda)^2/4$	$-4\lambda^2$	$(3\rho-\lambda)^2/4$	$-2\lambda^2$	λ^2	$(9\rho^2-9\lambda^2)/2$

$\lambda\ (\rho) =$ (anti) symmetric component of ground state wfn.

Quark model

SU(6) limit ⟹ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},\mathbf{0}^+]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{56},\mathbf{0}^+]^{\mathbf{4}}\mathbf{10}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{2}}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{4}}\mathbf{8}$	$[\mathbf{70},\mathbf{1}^-]^{\mathbf{2}}\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

Summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

⟹ $R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$ $\quad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$ $\quad A_1^n = \frac{g_1^n}{F_1^n} = 0$

⟹ as in quark-parton model !

Quark model

SU(6) limit ⟹ $\lambda = \rho$

$SU(6):$	$[\mathbf{56},0^+]^2\mathbf{8}$	$[\mathbf{56},0^+]^4\mathbf{10}$	$[\mathbf{70},1^-]^2\mathbf{8}$	$[\mathbf{70},1^-]^4\mathbf{8}$	$[\mathbf{70},1^-]^2\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18
g_1^p	9	-4	9	0	1	15
g_1^n	4	-4	1	-2	1	0

⟹ expect duality to appear earlier for F_1^p than F_1^n

⟹ earlier onset for g_1^n than g_1^p

⟹ cancellations *within* multiplets for g_1^n

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

gives $\Delta u/u > 1$ $\Longrightarrow$ *inconsistent with duality*

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

$^4\mathbf{10}$ $[\mathbf{56}^+]$ and $^4\mathbf{8}$ $[\mathbf{70}^-]$
suppressed

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large x

→ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

helicity 3/2 suppression

$N \to N^*$ transitions for helicity-1/2 dominance

SU(6) representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
$F_1^p = g_1^p$	9	2	9	0	1	21
$F_1^n = g_1^n$	4	2	1	1	1	9

polarization asymmetries $A_1^N \to 1$

→ cf. pQCD "counting rules"

→ hard gluon exchange between quarks

neutron to proton ratio $F_2^n/F_2^p \to 3/7$

→ cf. "helicity retention" model

Farrar, Jackson, Phys. Rev. Lett. 35 (1975) 1416

Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

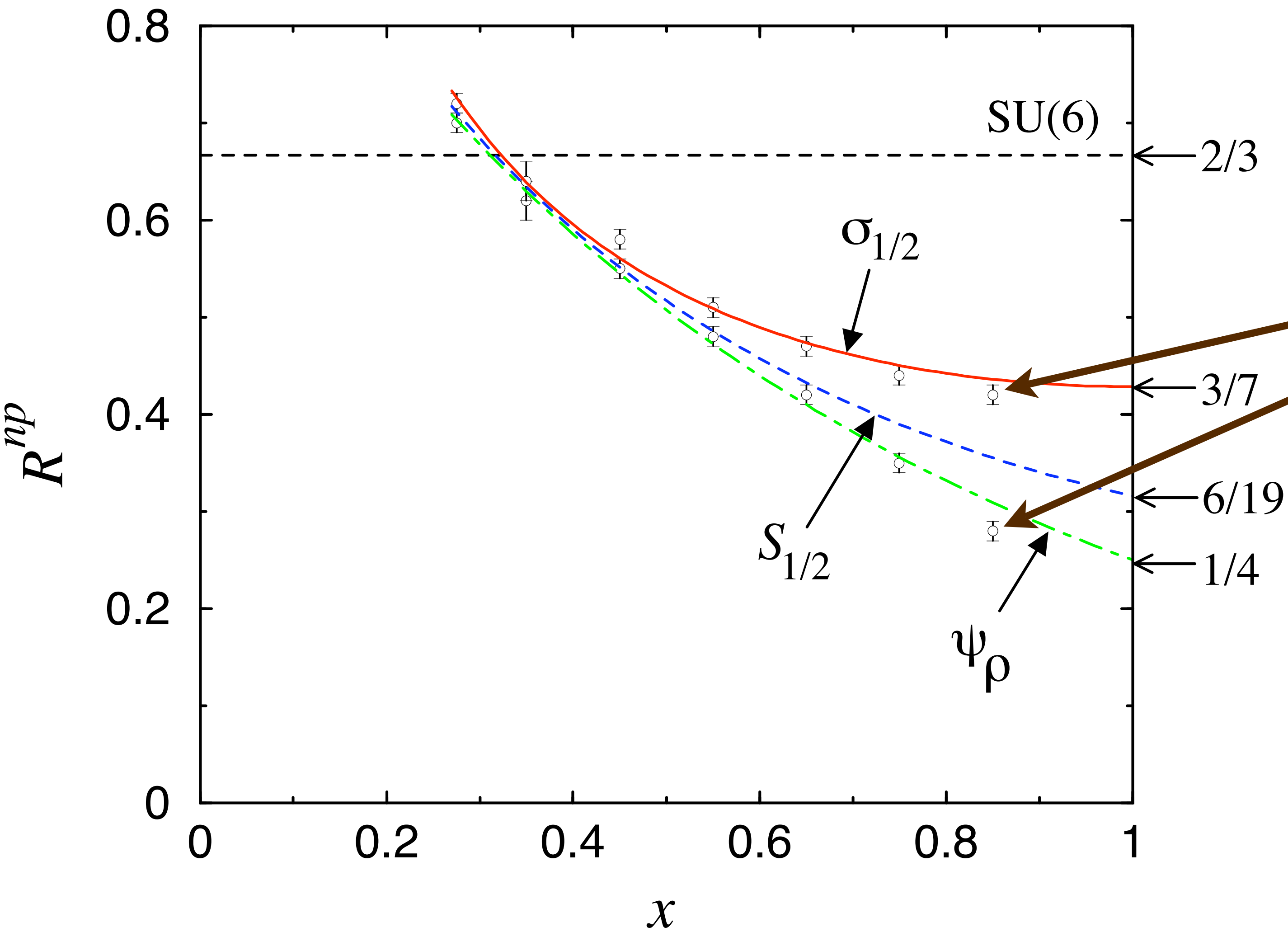
But significant deviations at large x

$\Longrightarrow$ which combinations of resonances reproduce behavior of structure functions at large x?

Model	SU(6)	No $^4\mathbf{10}$	No $^2\mathbf{10}$, $^4\mathbf{10}$	No $S_{3/2}$	No $\sigma_{3/2}$	No ψ_λ
R^{np}	2/3	10/19	1/2	6/19	3/7	1/4
A_1^p	5/9	1	1	1	1	1
A_1^n	0	2/5	1/3	1	1	1

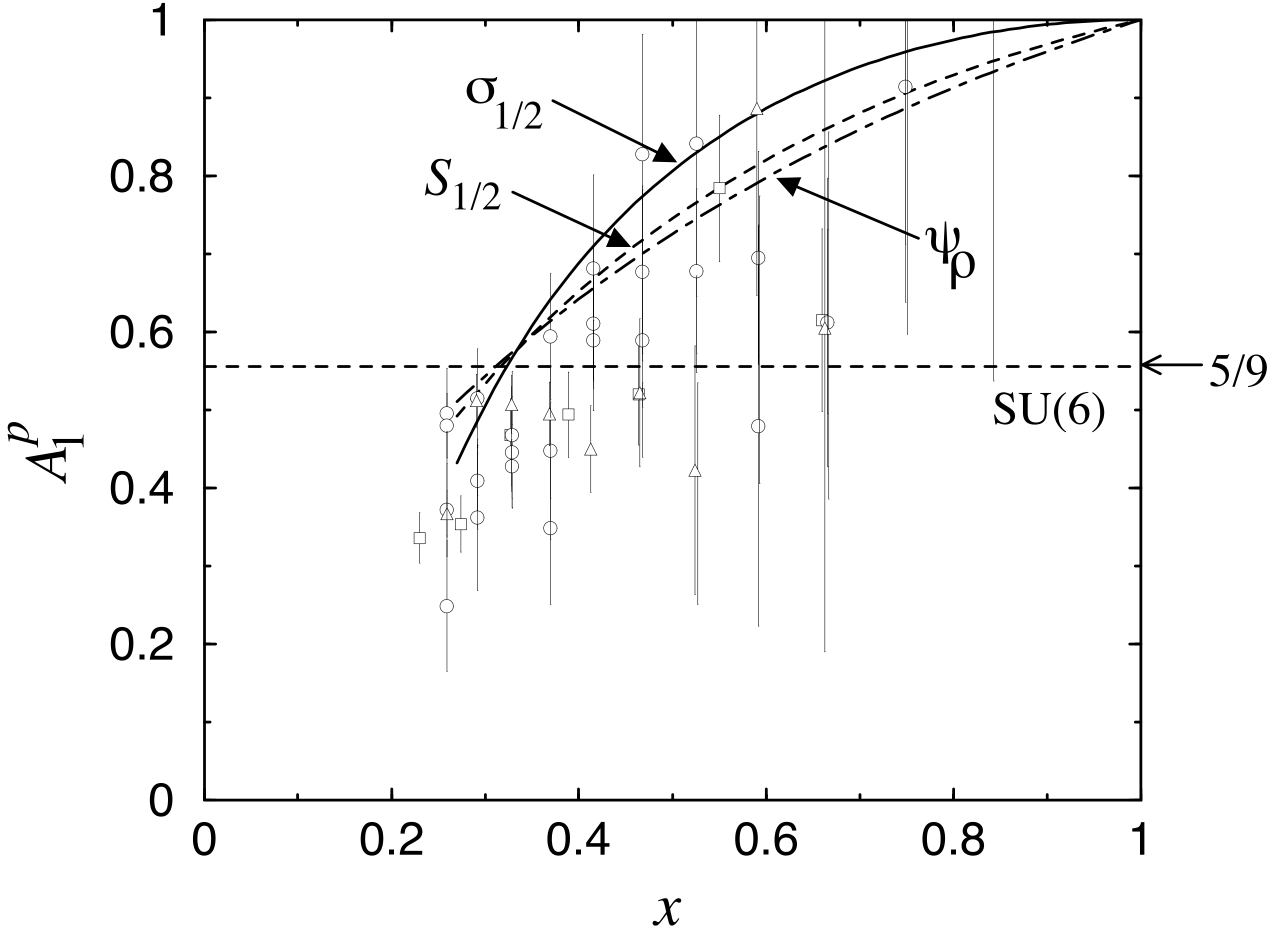
e.g. through $\vec{S}_i \cdot \vec{S}_j$ interaction between quarks $\Longleftarrow$ suppression of symmetric part of spin-flavor wfn.

Fit to $\left\{ \begin{array}{l} \text{SU(6) symmetry at } x \sim 1/3 \\ \text{SU(6) breaking at } x \sim 1 \end{array} \right.$

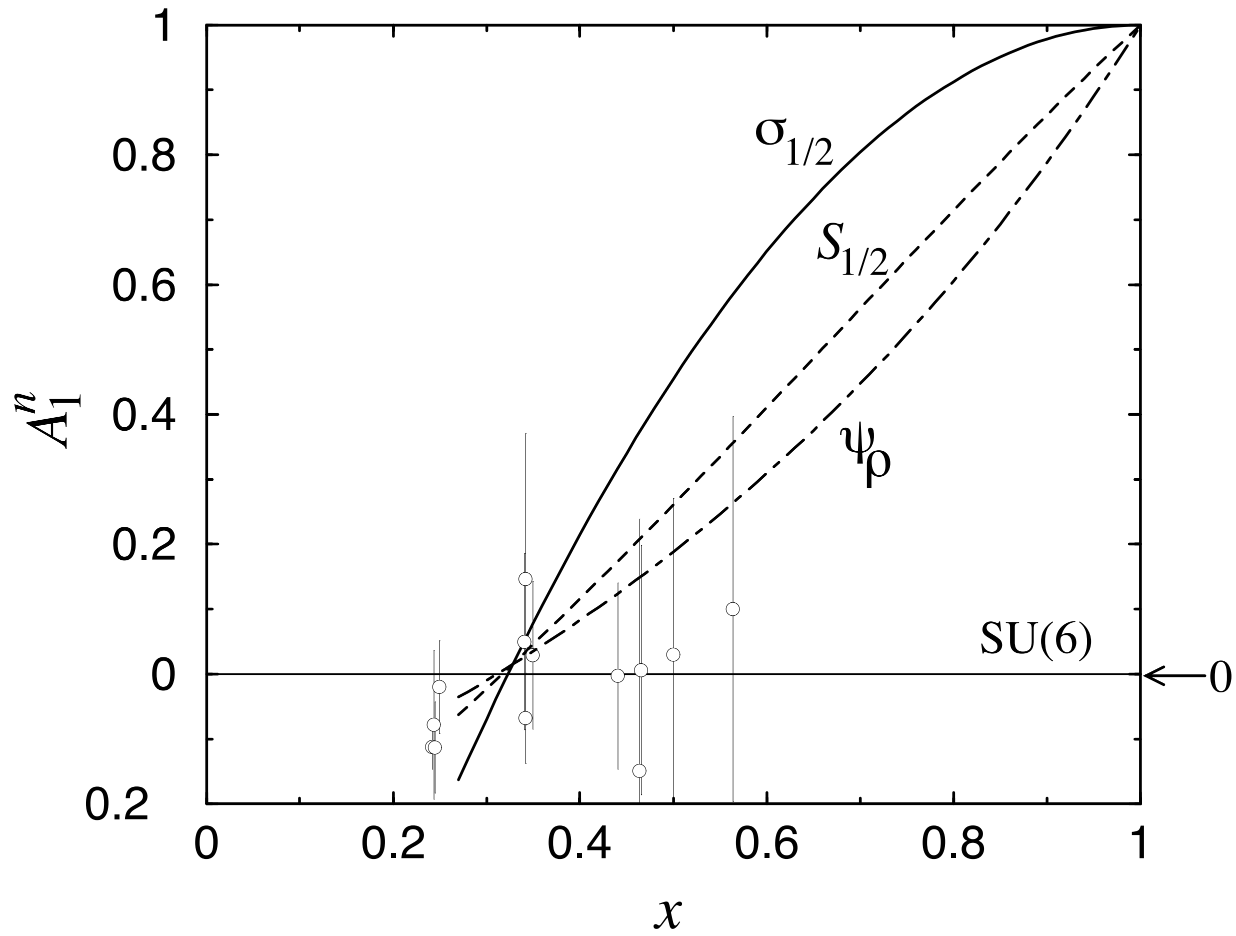


uncertainty in F_2^n due to nuclear effects in deuteron

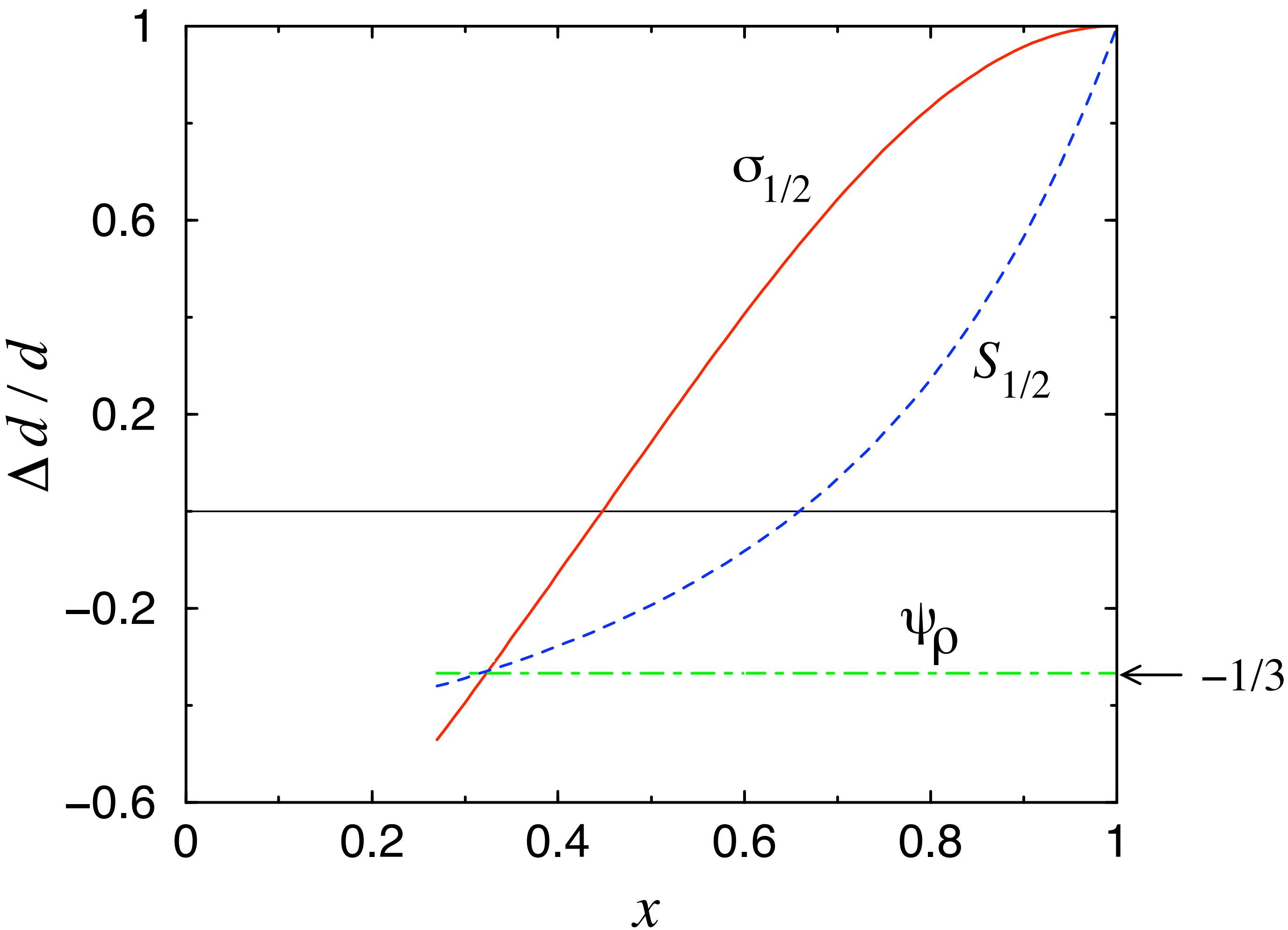
Polarization asymmetry A_1^p



Polarization asymmetry A_1^n



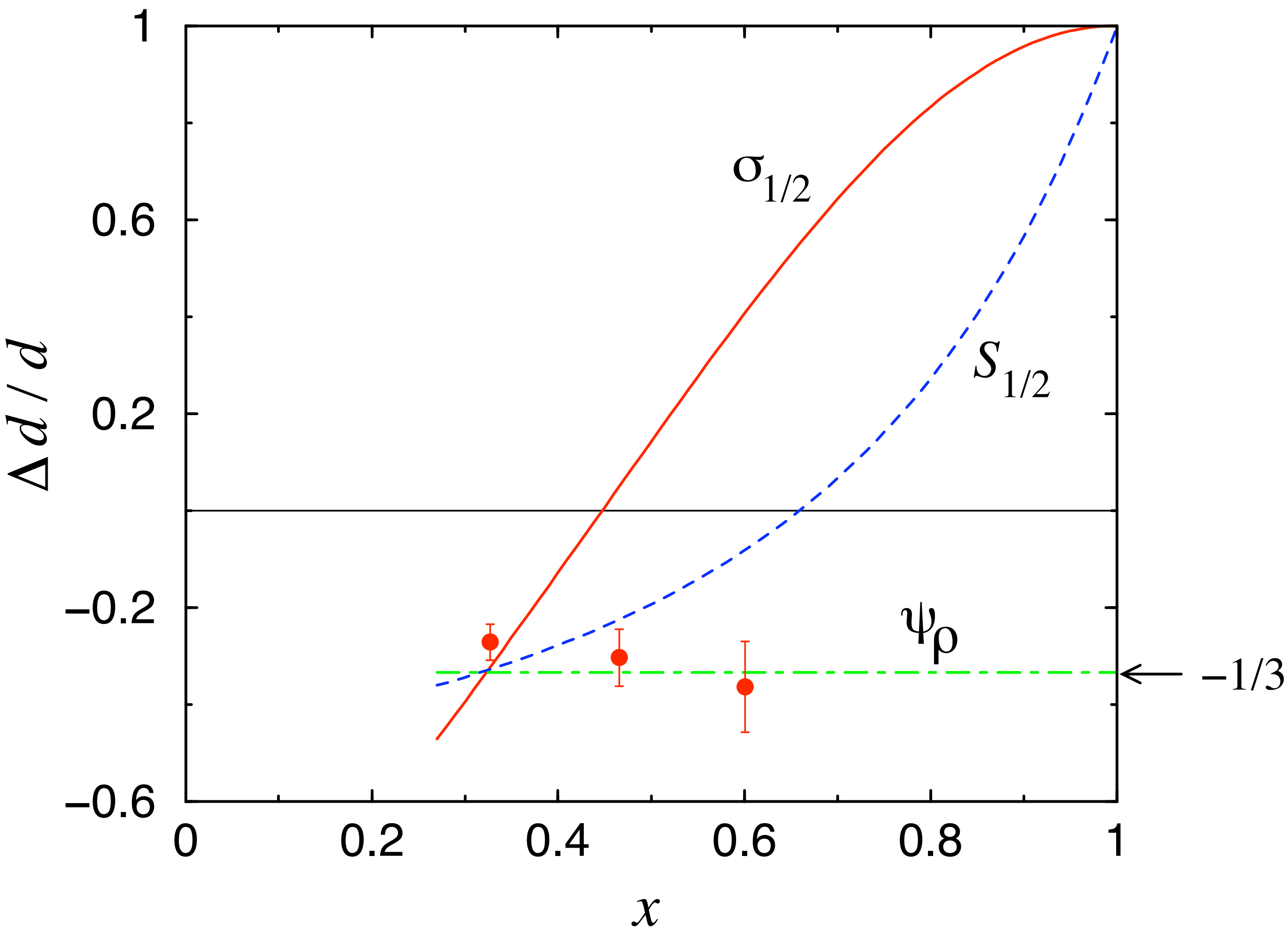
$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Close, WM, Phys. Rev. C68 (2003) 035210

$$\frac{\Delta d}{d} = \frac{4}{15} A_1^n \left(4 + \frac{u}{d}\right) - \frac{1}{15} A_1^p \left(1 + 4\frac{u}{d}\right)$$



$$\frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1}$$

Zheng et al. [JLab Hall A], Phys. Rev. Lett. (2004) 012004

Summary - quark-hadron duality

- Remarkable confirmation of quark-hadron duality in structure functions
  $\rightarrow$ higher twists "small" down to low $Q^2 (\sim 1\ \mathrm{GeV}^2)$

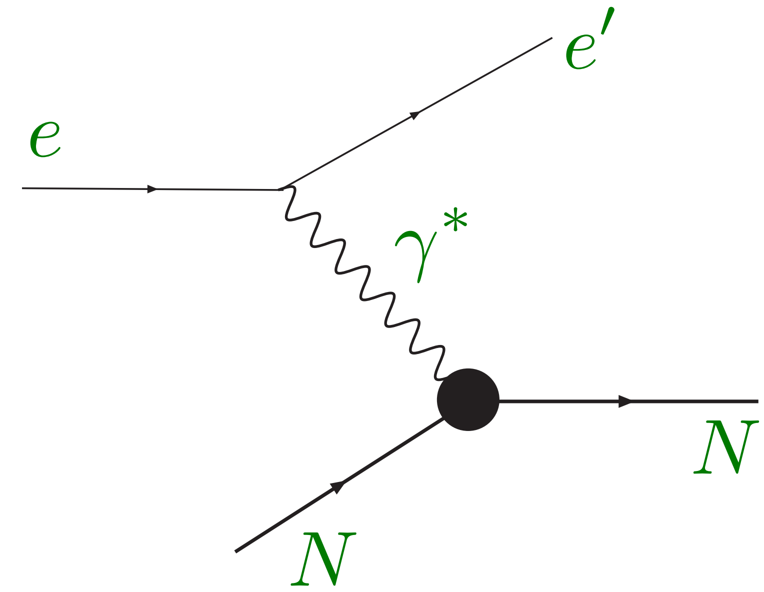
- Use duality violations to extract higher twist matrix elements $\longrightarrow$ color polarizabilities

- Quark models provide clues to origin of resonance cancellations $\longrightarrow$ local duality

- Practical applications
  $\rightarrow$ broaden kinematic region for studying
  (leading and higher twist) quark-gluon structure
  of nucleon

4.
Form factors

Elastic eN scattering



Elastic cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \; \frac{\tau}{\varepsilon \; (1+\tau)} \; \sigma_R$$

$$\tau = Q^2/4M^2$$

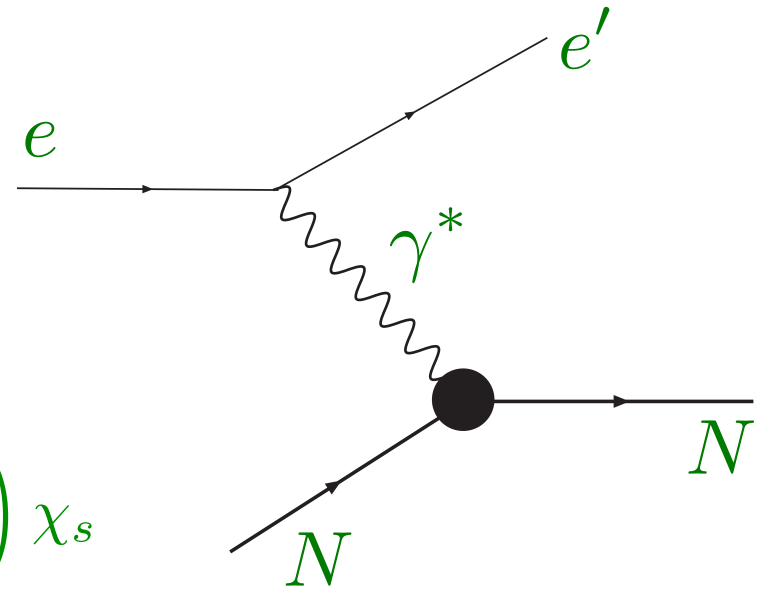
$$\varepsilon = \left(1 + 2(1+\tau)\tan^2(\theta/2)\right)^{-1}$$

$$\sigma_{\text{Mott}} = \frac{\alpha^2 E' \cos^2\frac{\theta}{2}}{4E^3 \sin^4\frac{\theta}{2}} \longleftarrow$$ cross section for scattering from point particle

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \longleftarrow$$ reduced cross section

G_E , G_M Sachs electric and magnetic form factors

Elastic eN scattering



In Breit frame

$$\nu = 0 \ , \quad Q^2 = \vec{q}^{\,2}$$

electromagnetic current is

$$\bar{u}(p', s')\Gamma^\mu u(p, s) = \chi^\dagger_{s'} \left(G_E + \frac{i\vec{\sigma} \times \vec{q}}{2M} G_M \right) \chi_s$$

cf. classical (<u>N</u>on-<u>R</u>elativistic) current density

$$J^{\rm NR} = \left(e\ \rho_E^{\rm NR} \ , \ \mu\ \vec{\sigma} \times \vec{\nabla} \rho_M^{\rm NR} \right)$$

$\Longrightarrow$ $$\rho_E^{\rm NR}(r) = \frac{2}{\pi} \int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_E(\vec{q}^{\,2})$$ *<u>charge density</u>*

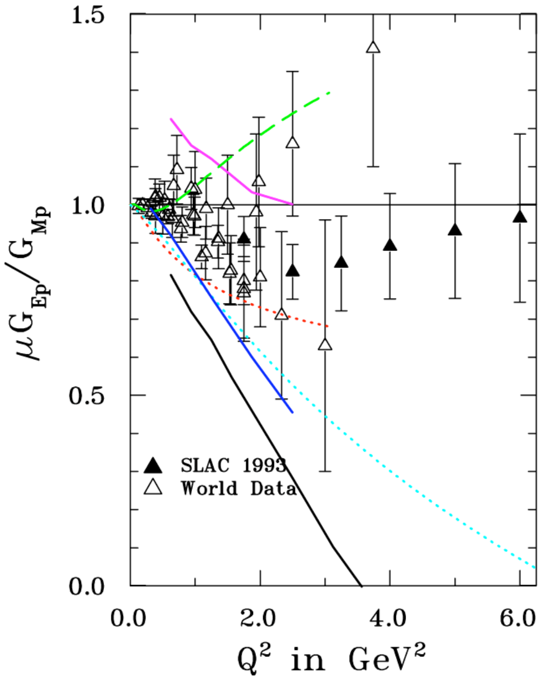
$$\mu\ \rho_M^{\rm NR}(r) = \frac{2}{\pi} \int_0^\infty dq\ \vec{q}^{\,2}\ j_0(qr)\ G_M(\vec{q}^{\,2})$$ *<u>magnetisation density</u>*

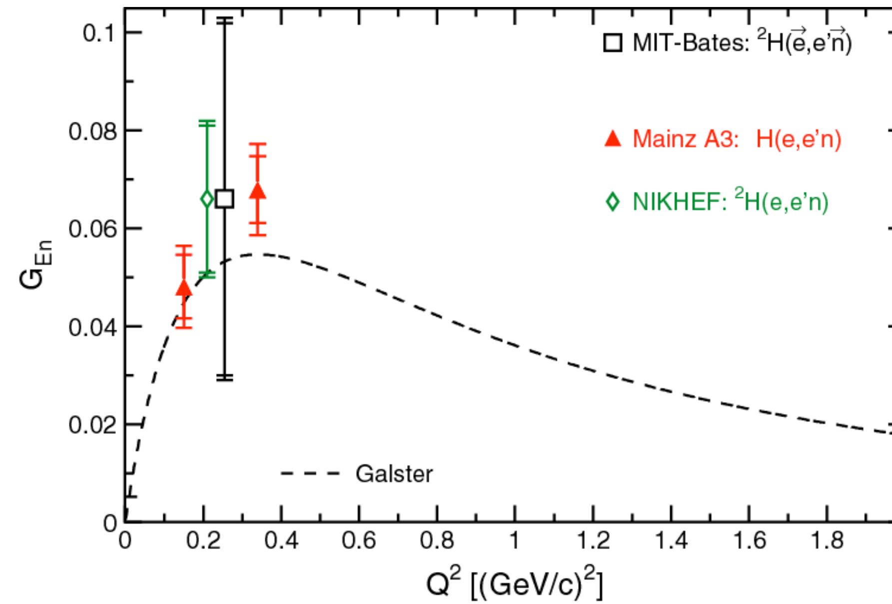
Until recently...

proton

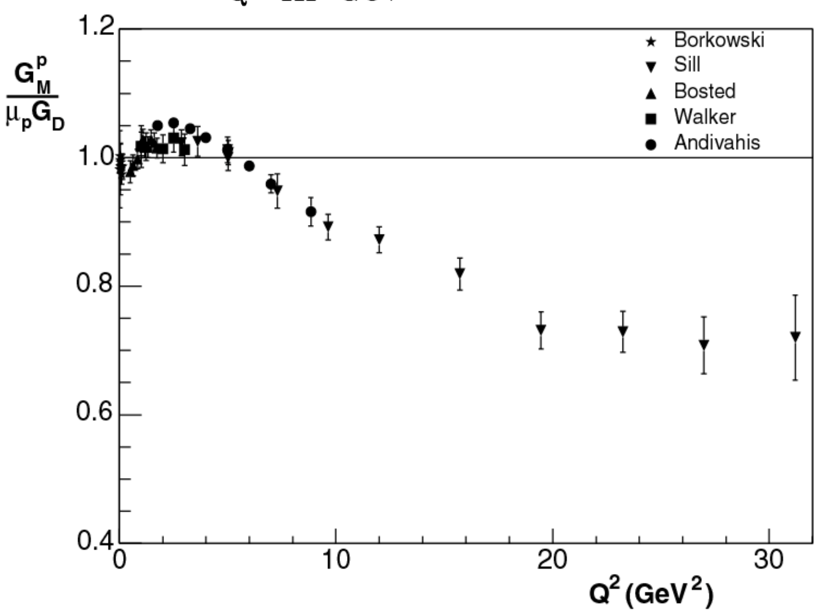
neutron

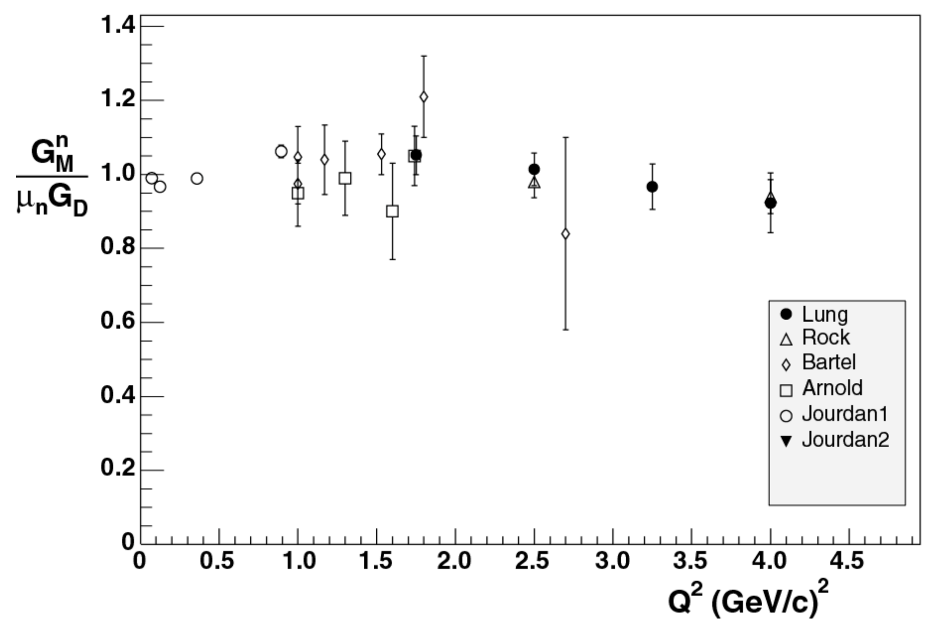
Electric





Magnetic



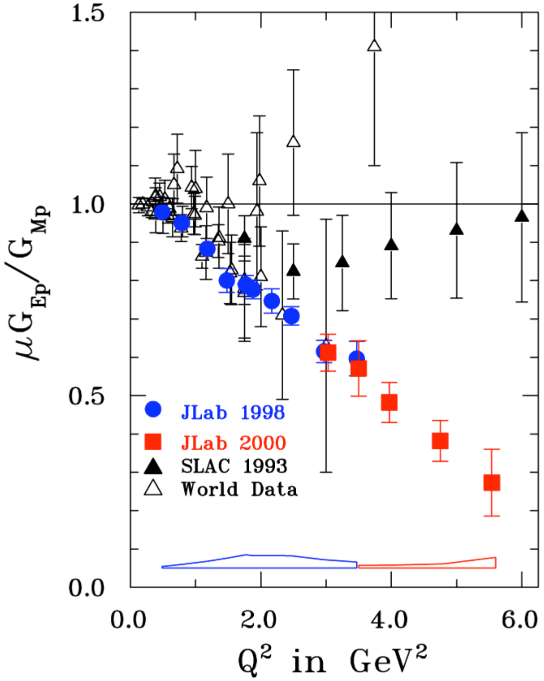


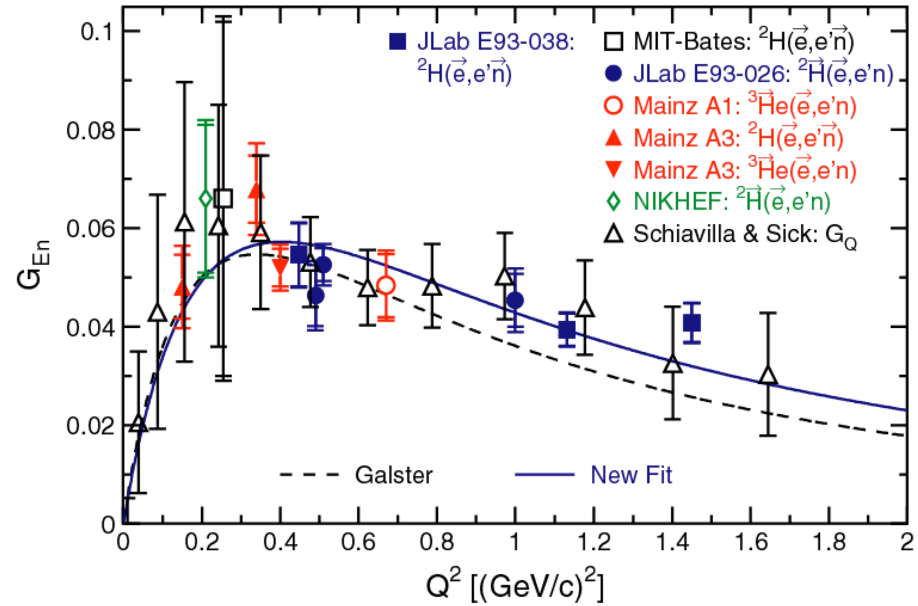
Latest data...

proton

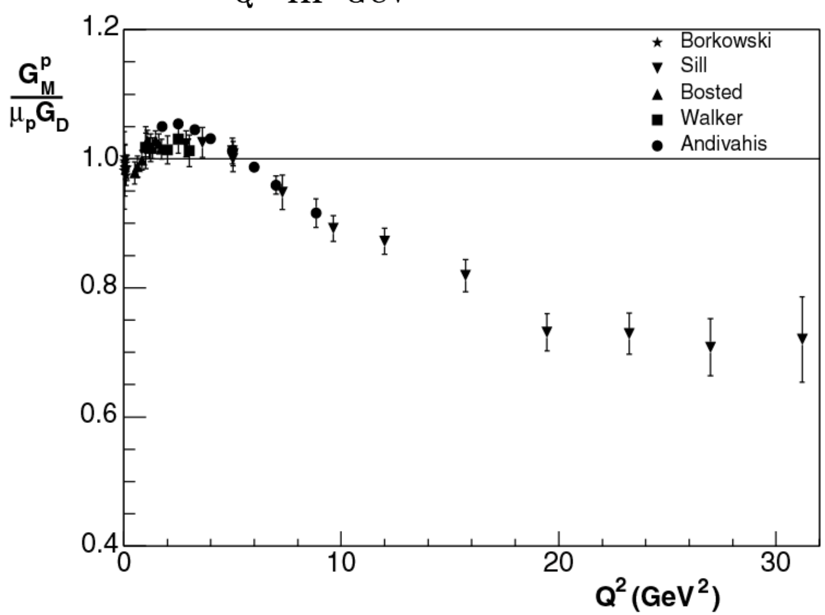
neutron

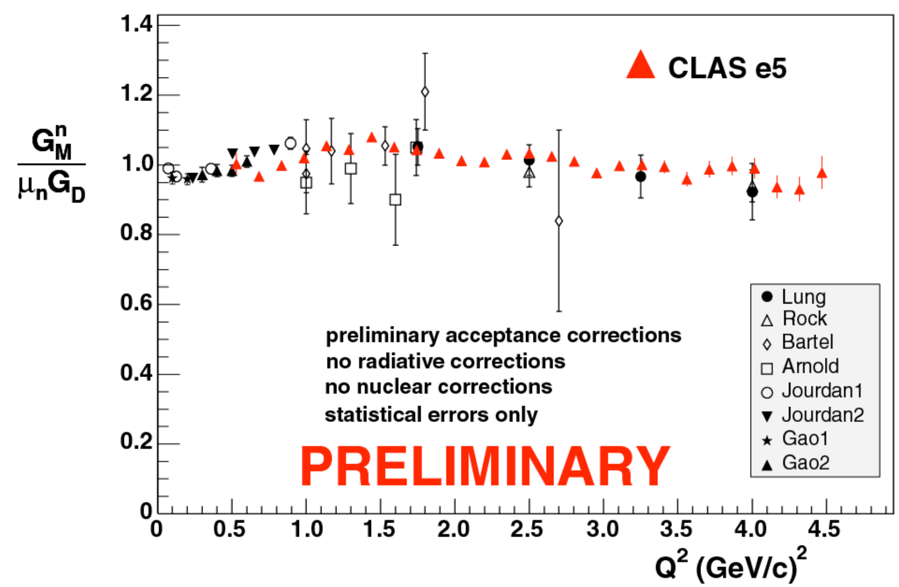
Electric

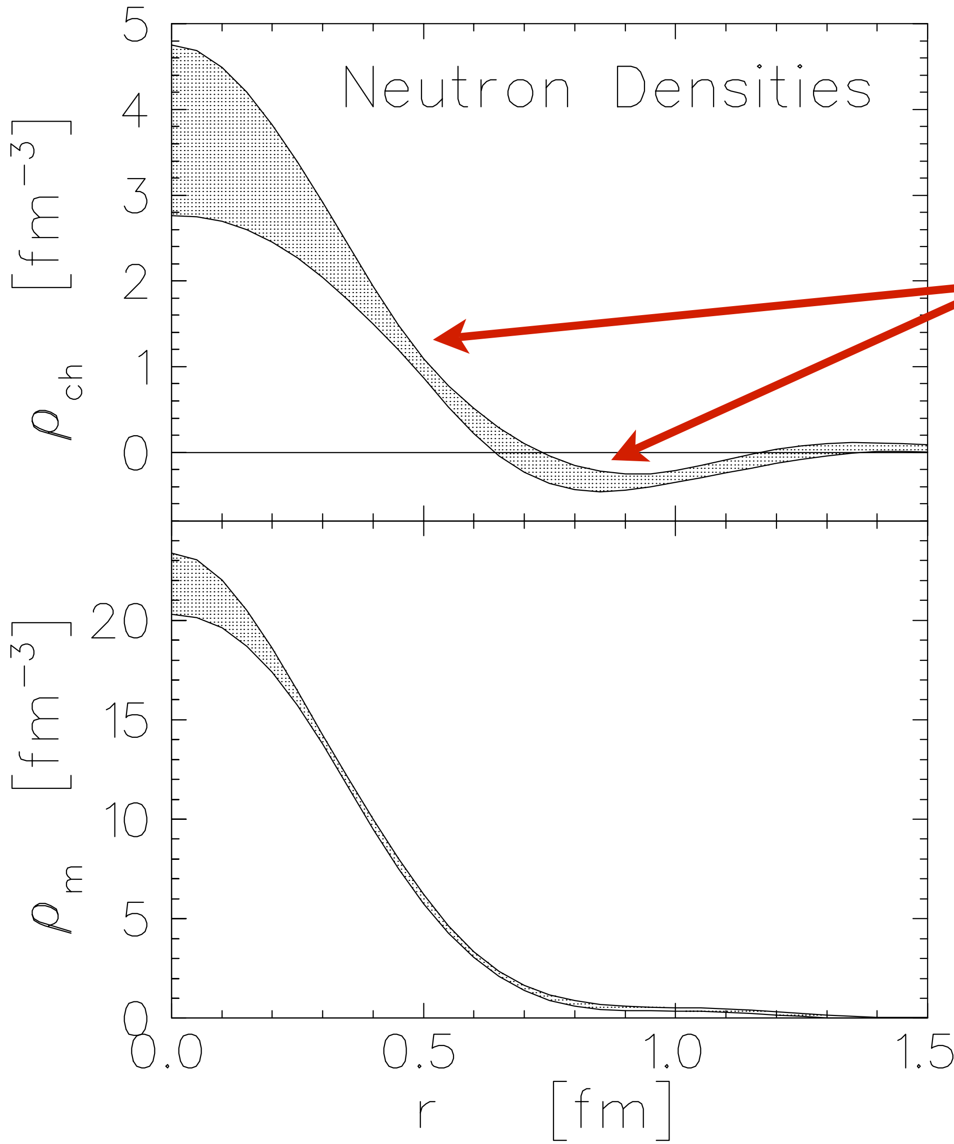




Magnetic





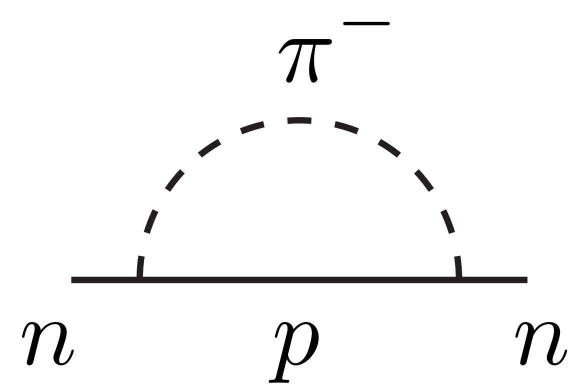


J.Kelly, Phys. Rev. C 66 (2002) 065203

note neutron $\rho_E > 0$ at small r, but < 0 at larger r

same physics which gives $\bar{d} > \bar{u}$ also gives shape of neutron ρ_E

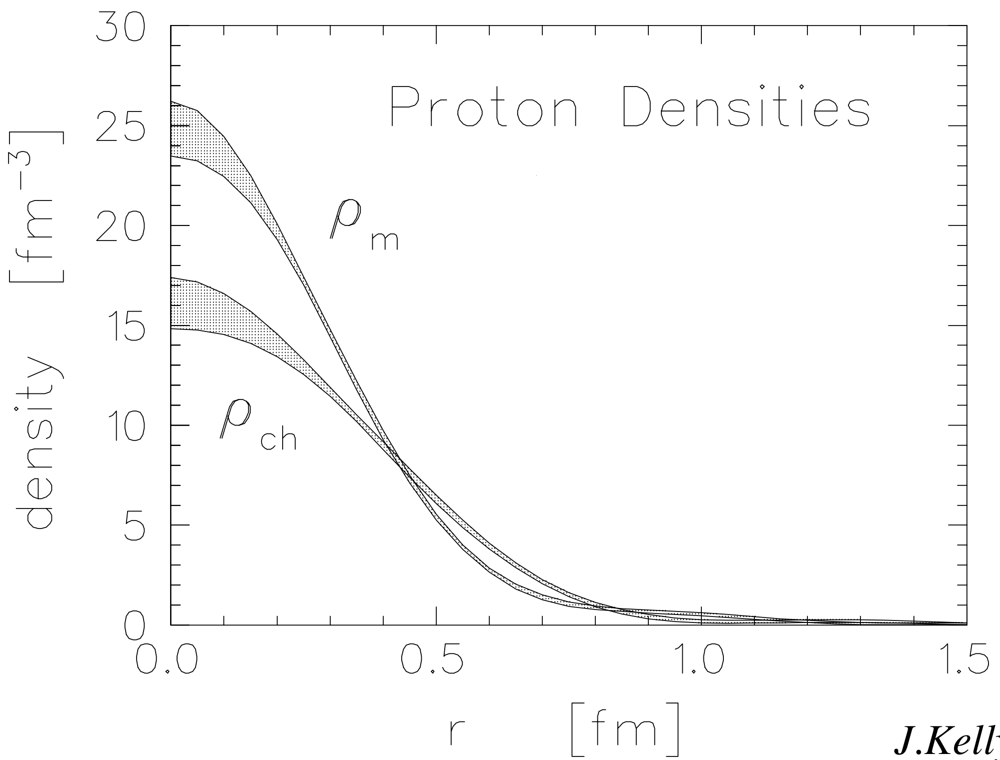
$\Longrightarrow$ pion cloud



Surprising result for G_E^p/G_M^p

$\longrightarrow$ expect $G_E^p/G_M^p \to$ constant at high Q^2

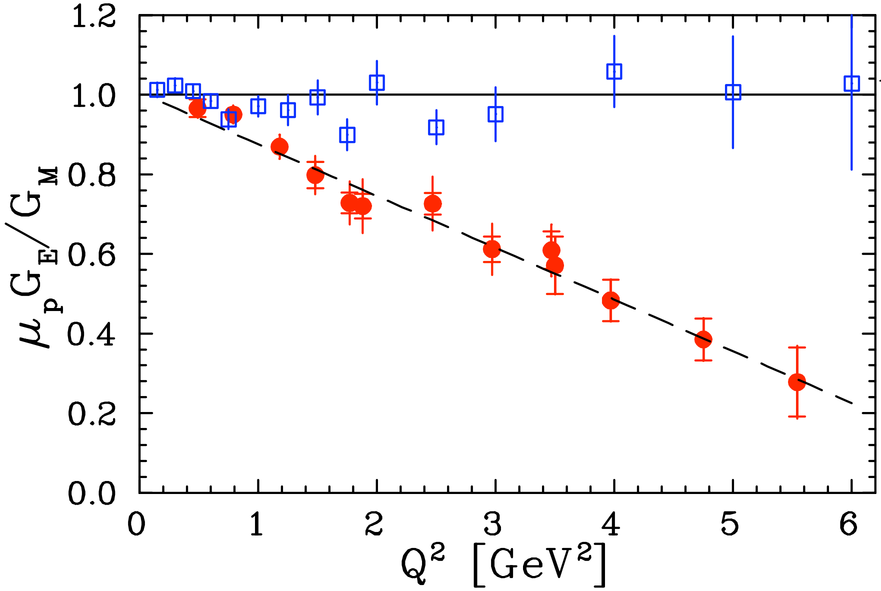
$\longrightarrow$ implies very different proton charge and magnetization densities at small r



J.Kelly, Phys. Rev. C 66 (2002) 065203

Are the G_E^p/G_M^p data consistent ?

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

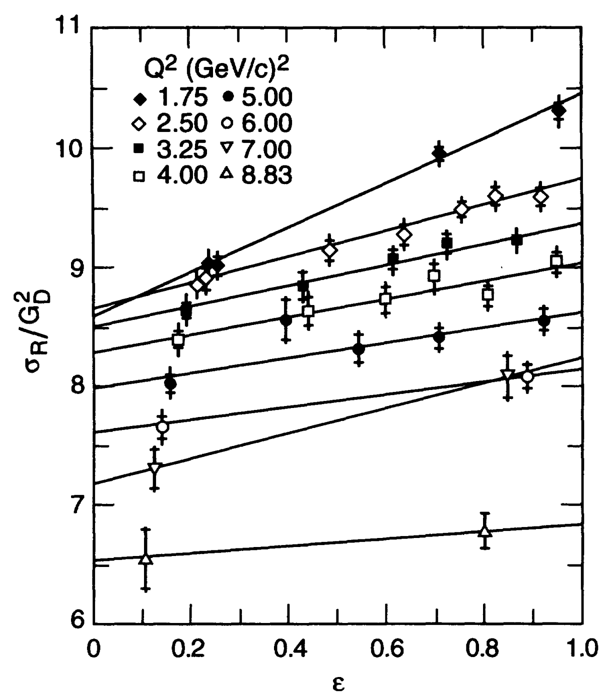
LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

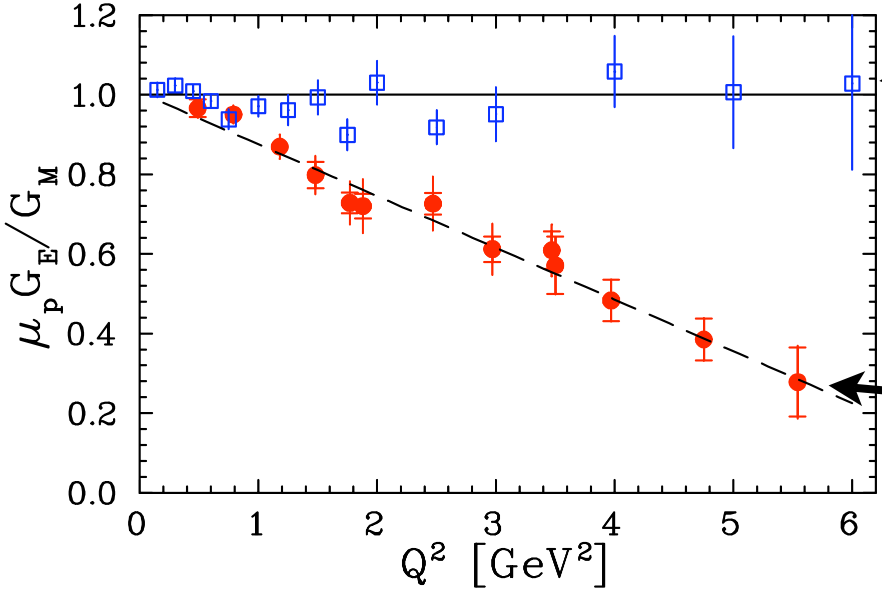
$$\tau = Q^2/4M^2$$

$$\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$$

G_E/G_M from slope in ε plot



Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = Q^2/4M^2$$
$$\varepsilon = \left[1 + 2(1+\tau)\tan^2\theta/2\right]^{-1}$$

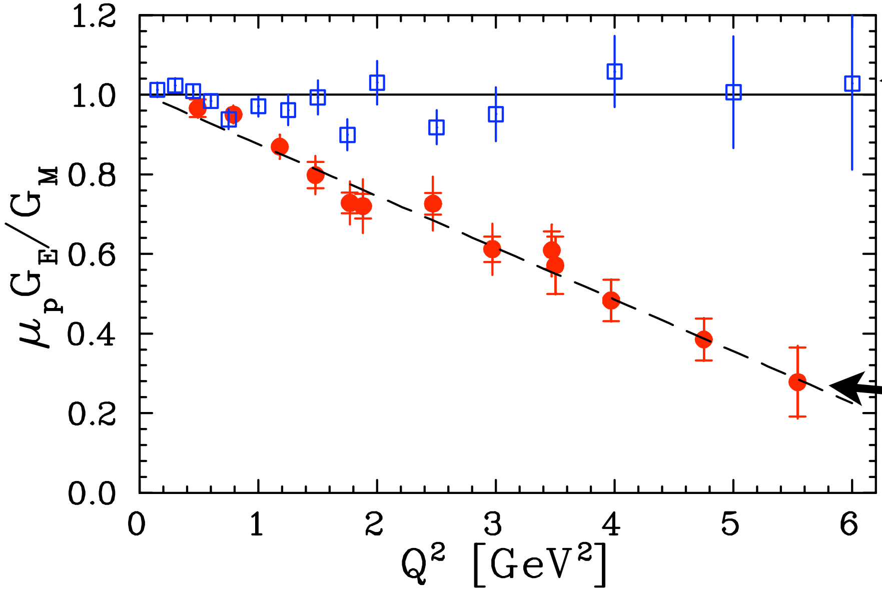
G_E/G_M from slope in ε plot

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_T}{P_L}$$

$P_{T,L}$ polarization of recoil proton

Proton G_E/G_M Ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

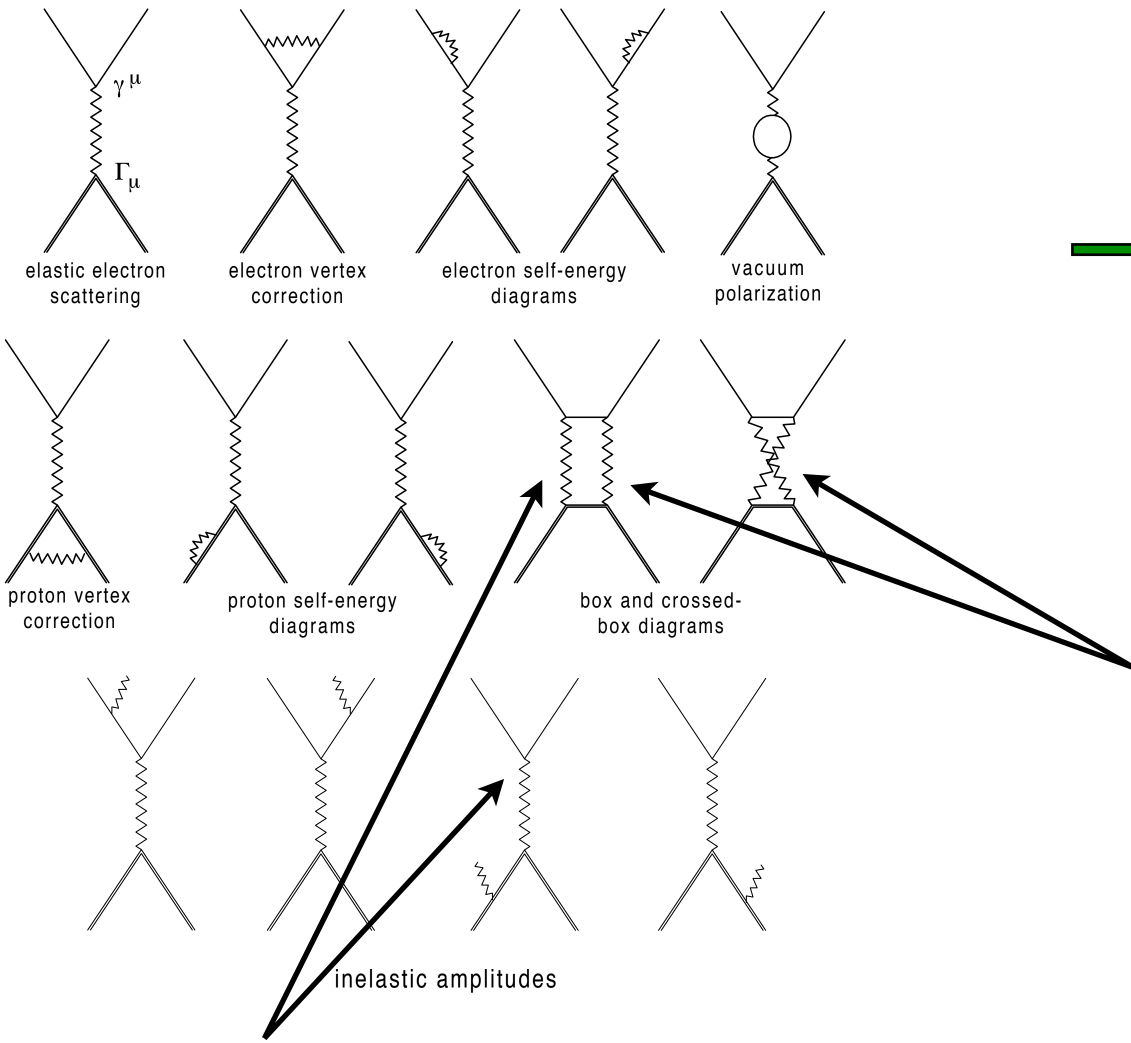
PT method

Why is there a discrepancy between the two methods?

4.
Form factors
- two photon exchange

QED Radiative Corrections

cross section modified by 1γ loop effects



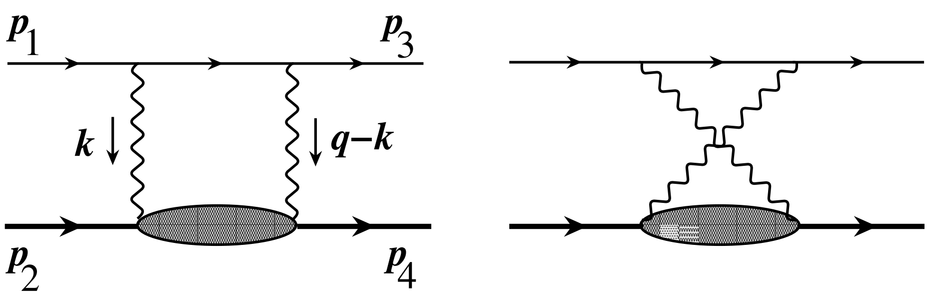
$$d\sigma = d\sigma_0\ (1 + \delta)$$

δ contains additional ε dependence

mostly from box (and crossed box) diagram

infrared divergences cancel

Box diagram



$\longrightarrow$ nucleon elastic contribution

$$\mathcal{M}_{\gamma\gamma} \;=\; e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)}$$

where

$$\begin{aligned} N(k) \;&=\; \bar{u}(p_3)\,\gamma_\mu(\not{p}_1 - \not{k} + m_e)\gamma_\nu\; u(p_1) \\ &\times\; \bar{u}(p_4)\;\Gamma^\mu(q-k)\;(\not{p}_2 + \not{k} + M)\;\Gamma^\nu(k)\;u(p_2) \end{aligned}$$

and

$$\begin{aligned} D(k) \;&=\; (k^2 - \lambda^2)\;((k-q)^2 - \lambda^2) \\ &\times\; ((p_1 - k)^2 - m^2)\;((p_2 + k)^2 - M^2) \end{aligned}$$

with λ an IR regulator, and e.m. current is

$$\Gamma^\mu(q) \;=\; \gamma^\mu\; F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}\; F_2(q^2)$$

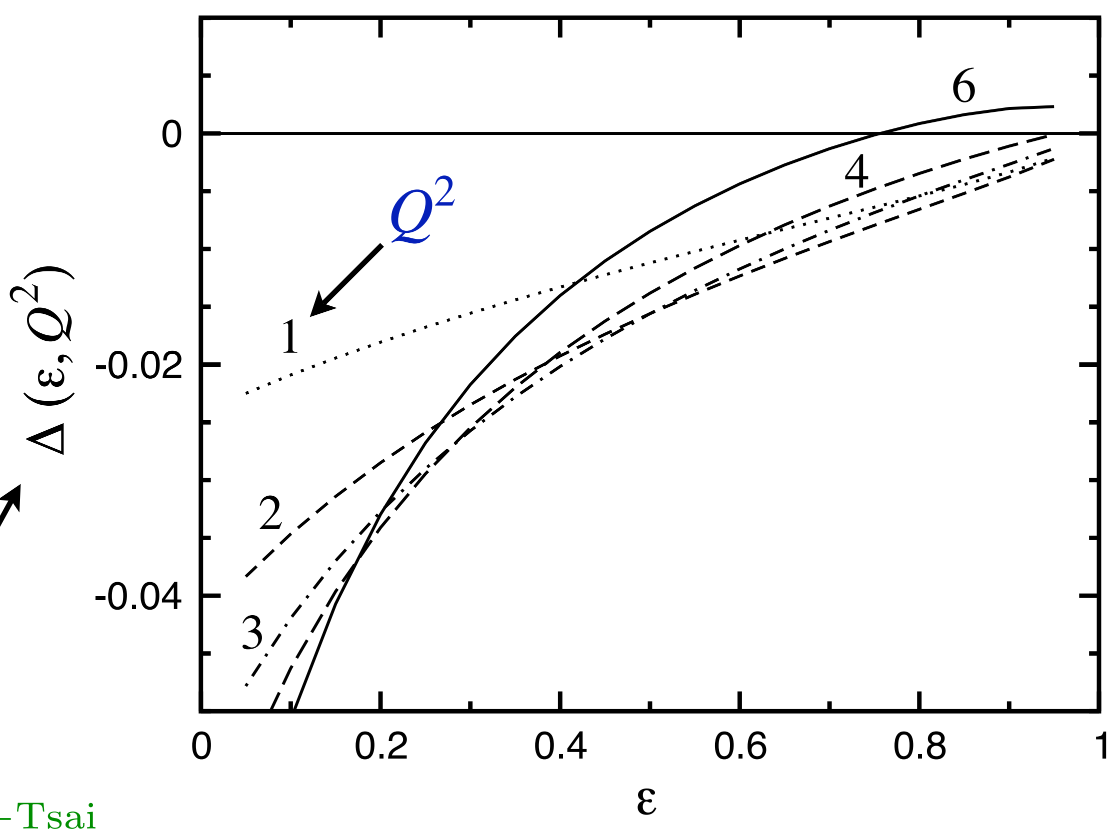
Various approximations to $\mathcal{M}_{\gamma\gamma}$ used

- Mo-Tsai: soft γ approximation
  $\longrightarrow$ integrand most singular when $k = 0$
  and $k = q$
  $\longrightarrow$ replace γ propagator which is
  not at pole by $1/q^2$
  $\longrightarrow$ approximate numerator $N(k) \approx N(0)$
  $\longrightarrow$ neglect all structure effects

- Maximon-Tjon: improved loop calculation
  $\longrightarrow$ exact treatment of propagators
  $\longrightarrow$ still evaluate $N(k)$ at $k = 0$
  $\longrightarrow$ first study of form factor effects
  $\longrightarrow$ additional ε dependence

- Blunden-WM-Tjon: exact loop calculation
  $\longrightarrow$ no approximation in $N(k)$ or $D(k)$
  $\longrightarrow$ include form factors

Two-photon correction



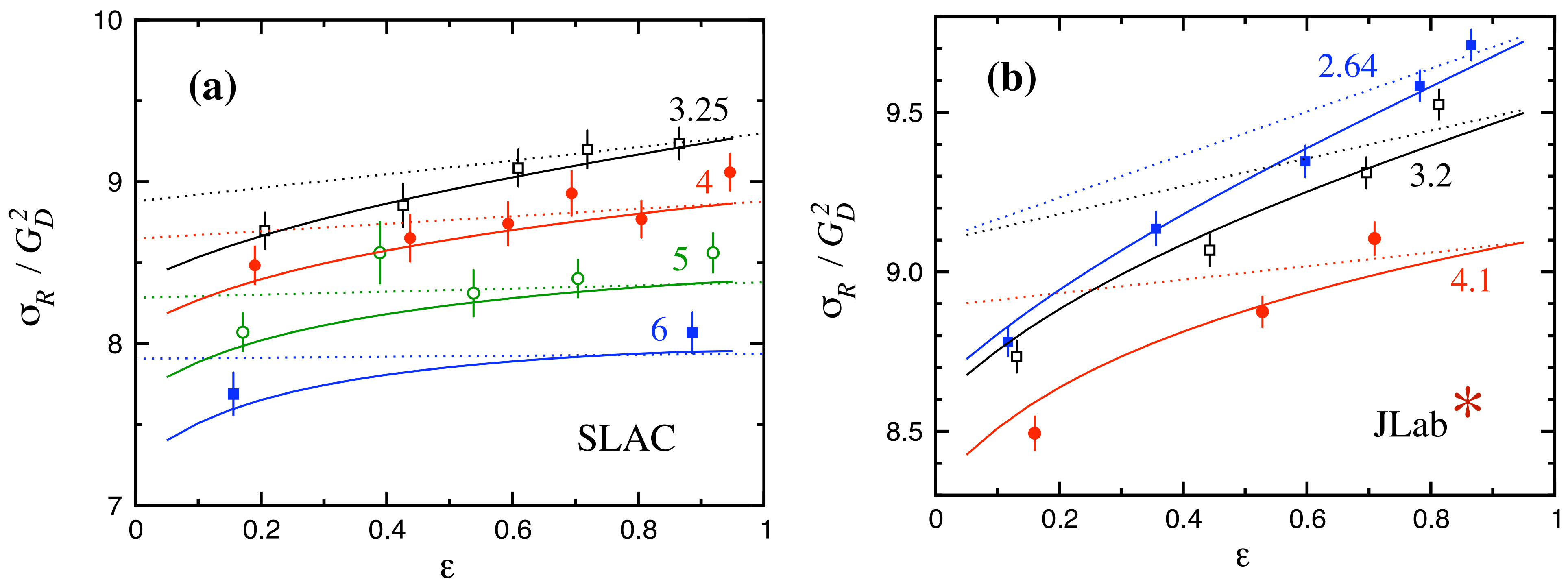
$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

$$\delta^{(2\gamma)} \rightarrow \frac{2\mathcal{R}e\{\mathcal{M}_0^\dagger \mathcal{M}_{\gamma\gamma}\}}{|\mathcal{M}_0|^2}$$

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC72 (2005) 034612

- few % magnitude
- positive slope
- non-linearity in ε

Effect on cross section



Born cross section with PT form factors

including TPE effects

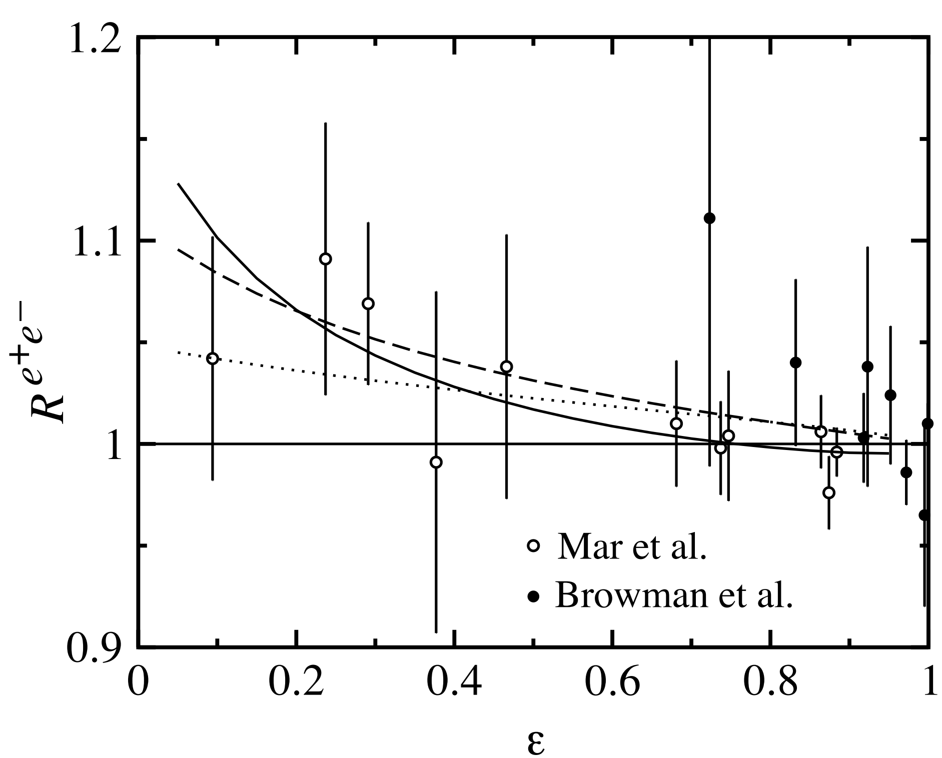
* Super-Rosenbluth

Qattan et al.,

PRL 94, 142301 (2005)

e^+/e^- comparison

- 1γ exchange changes sign under $e^+ \leftrightarrow e^-$
- 2γ exchange invariant under $e^+ \leftrightarrow e^-$
- ratio of $e^+p\,/\,e^-p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$



$$R^{e^+e^-} = \frac{d\sigma^{e^+}}{d\sigma^{e^-}}$$

$$\approx 1 - 2\,\Delta$$

$Q^2 = 1\ \mathrm{GeV}^2$ (dotted)
$Q^2 = 3\ \mathrm{GeV}^2$ (dashed)
$Q^2 = 6\ \mathrm{GeV}^2$ (solid)

⇨ simultaneous e^-p/e^+p measurement using <u>tertiary</u> e^+/e^- beam planned in Hall B (to $Q^2 \sim 1\ \mathrm{GeV}^2$)

G_E^p / G_M^p ratio

- estimate effect of TPE on ε dependence

- approximate correction by linear function of ε

$$1+\Delta \approx a + b\varepsilon$$

→ reduced cross section is then

$$\sigma_R \approx a\, G_M^2 \left[1 + \frac{\varepsilon}{\mu^2\,\tau} \left(R^2 (1 + \varepsilon\, b/a) + \mu^2\, \tau\, b/a \right) \right]$$

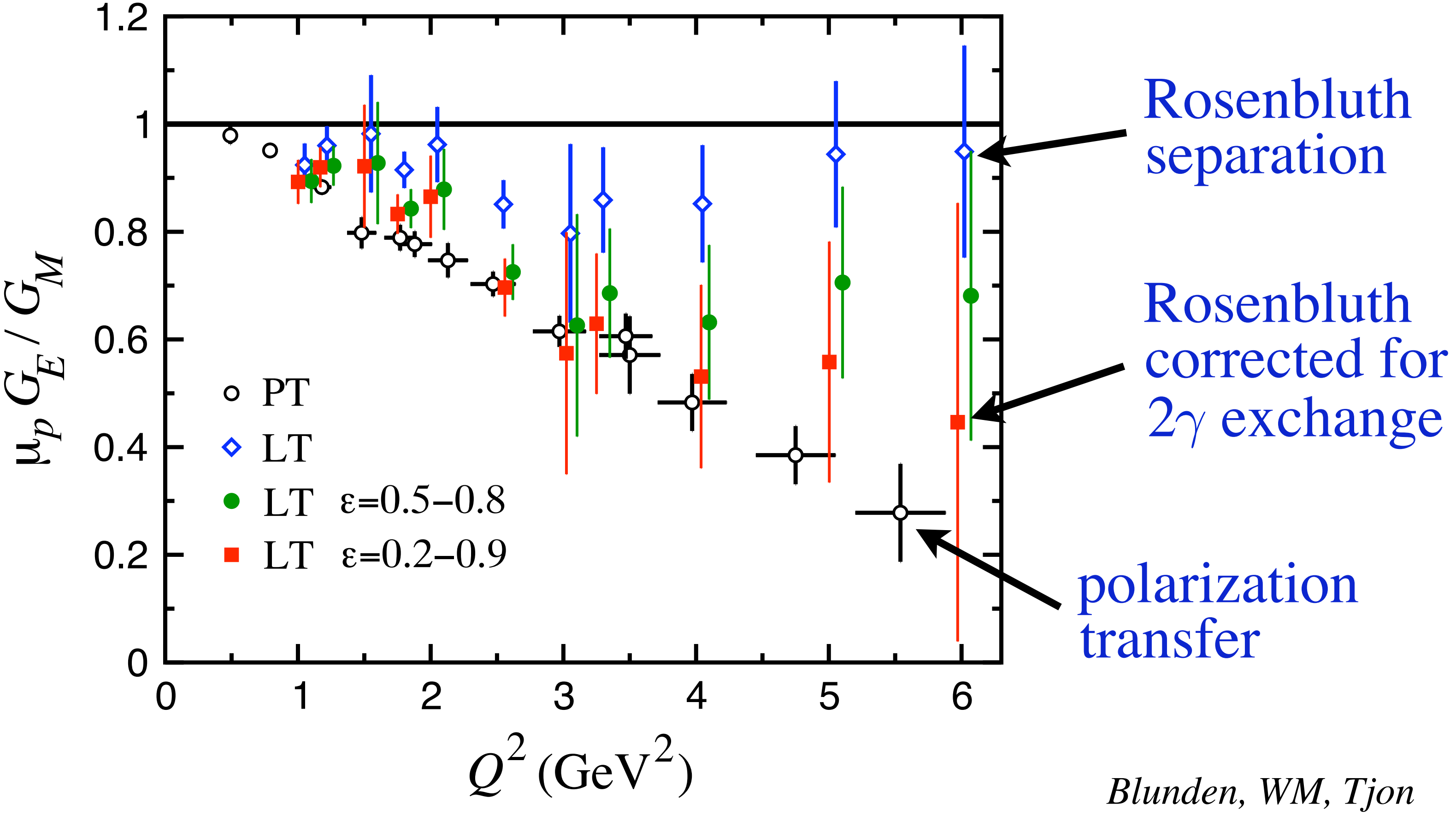
where "true" ratio is

$$R^2 = \frac{\widetilde{R}^2 - \mu^2\,\tau\, b/a}{1 + \bar{\varepsilon}\, b/a}$$

"effective" ratio
contaminated by TPE

average value of ε
over range fitted

$G_E^p \,/\, G_M^p$ ratio



Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

⟹ resolves much of the form factor discrepancy

- how does TPE affect polarization transfer ratio?

$$\widetilde{R} = R \left(\frac{1+\Delta_T}{1+\Delta_L} \right)$$

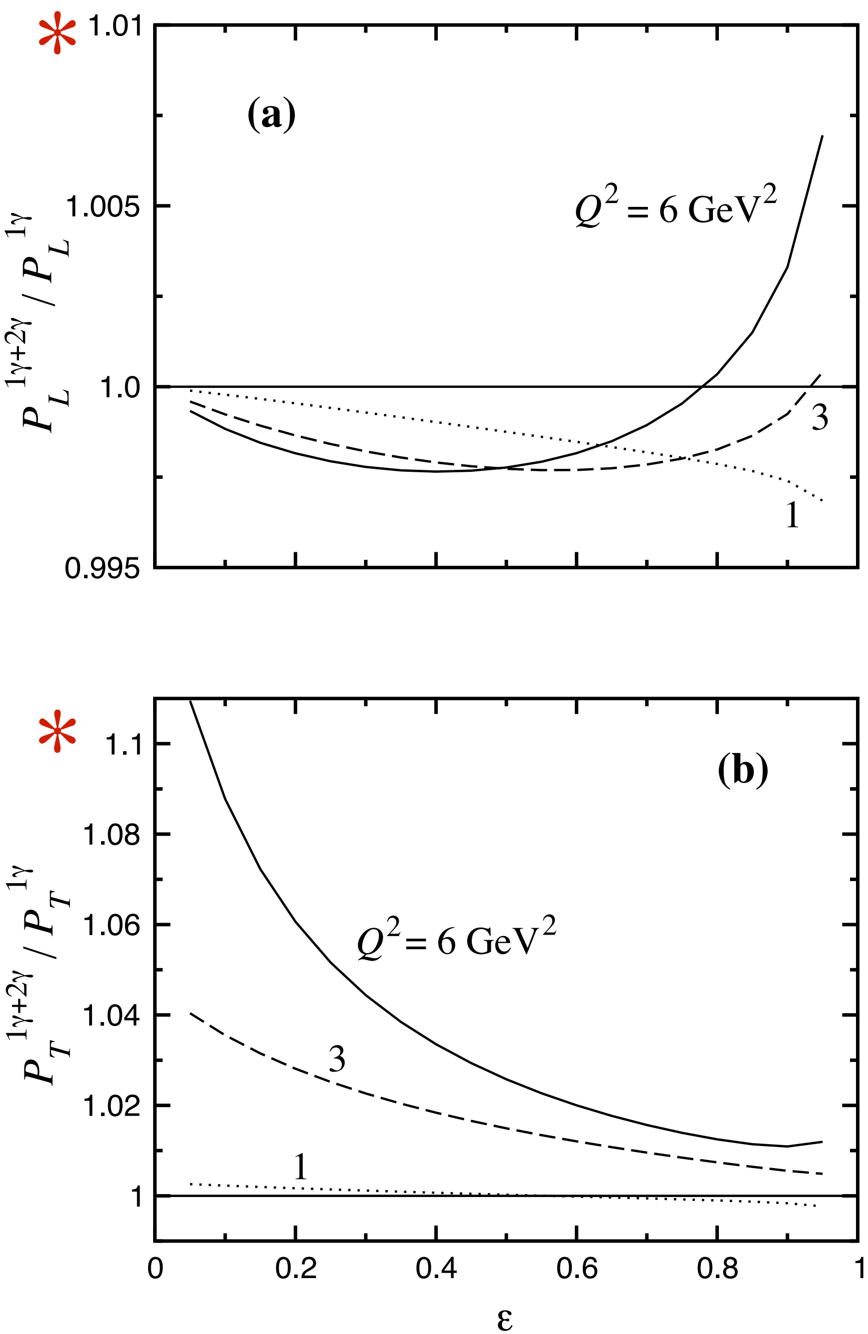
where $\Delta_{L,T} = \delta_{L,T}^{\rm full} - \delta_{\rm IR}^{\rm Mo-Tsai}$ is finite part of 2γ contribution relative to IR part of Mo-Tsai

- experimentally measure ratio of polarized to unpolarized cross sections

$$\frac{P_{L,T}^{1\gamma+2\gamma}}{P_{L,T}^{1\gamma}} = \frac{1+\Delta_{L,T}}{1+\Delta}$$

Longitudinal & transverse polarizations

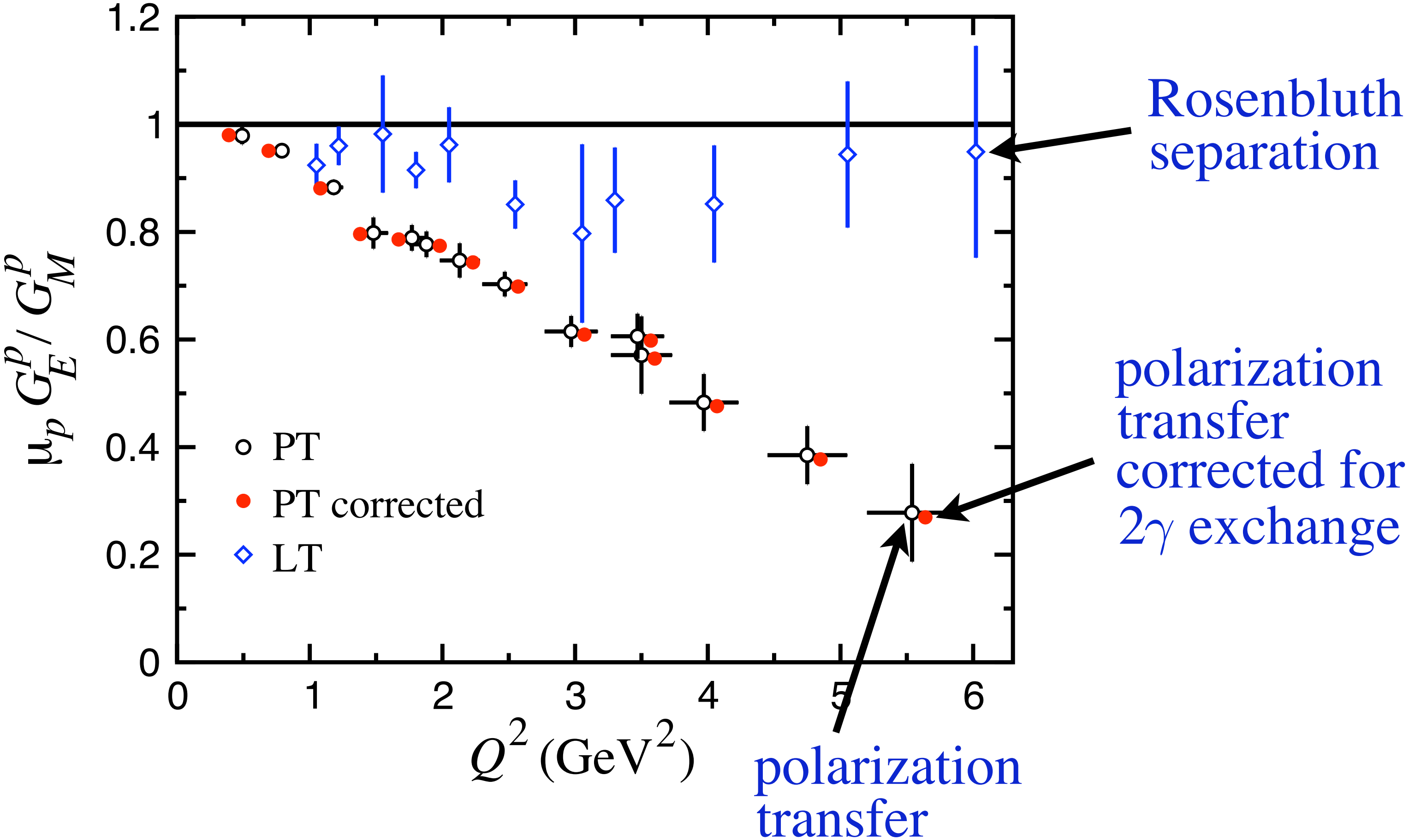
* Note scales!



➡ small effect on P_L

➡ large effect on P_T

$G_E^p \,/\, G_M^p$ ratio



- large Q^2 data typically at large ε
- $< 3\%$ suppression at large Q^2

4.
Form factors
- excited intermediate states

Lowest mass excitation is P_{33} Δ resonance

➡ relativistic $\gamma^* N \Delta$ vertex

$$\Gamma^{\nu\alpha}_{\gamma\Delta\to N}(p,q) \equiv i V^{\nu\alpha}_{\Delta in}(p,q) = i\frac{eF_\Delta(q^2)}{2M_\Delta^2}\Big\{ g_1 \left[g^{\nu\alpha} \not{p}\not{q} - p^\nu \gamma^\alpha \not{q} - \gamma^\nu \gamma^\alpha p\cdot q + \gamma^\nu \not{p} q^\alpha \right]$$
$$+ g_2 \left[p^\nu q^\alpha - g^{\nu\alpha} p\cdot q \right] + (g_3/M_\Delta)\left[q^2 (p^\nu \gamma^\alpha - g^{\nu\alpha}\not{p}) + q^\nu (q^\alpha \not{p} - \gamma^\alpha p\cdot q) \right] \Big\} \gamma_5 T_3$$

form factor $\dfrac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$

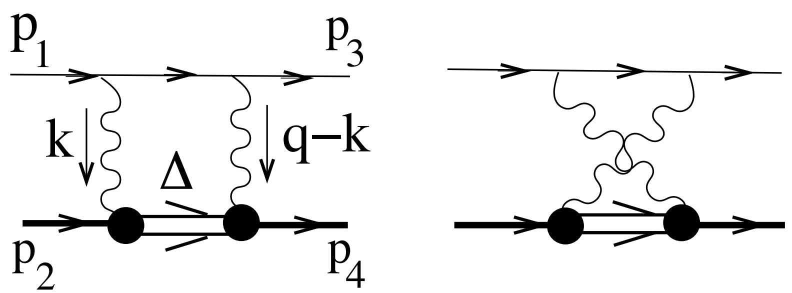
➡ coupling constants

g_1 magnetic ➡ 7

$g_2 - g_1$ electric ➡ 9

g_3 Coulomb ➡ -2 ... 0

Two-photon exchange amplitude with Δ intermediate state



$$\mathcal{M}_{\Delta}^{\gamma\gamma} = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{box}^{\Delta}(k)}{D_{box}^{\Delta}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N_{x-box}^{\Delta}(k)}{D_{x-box}^{\Delta}(k)}$$

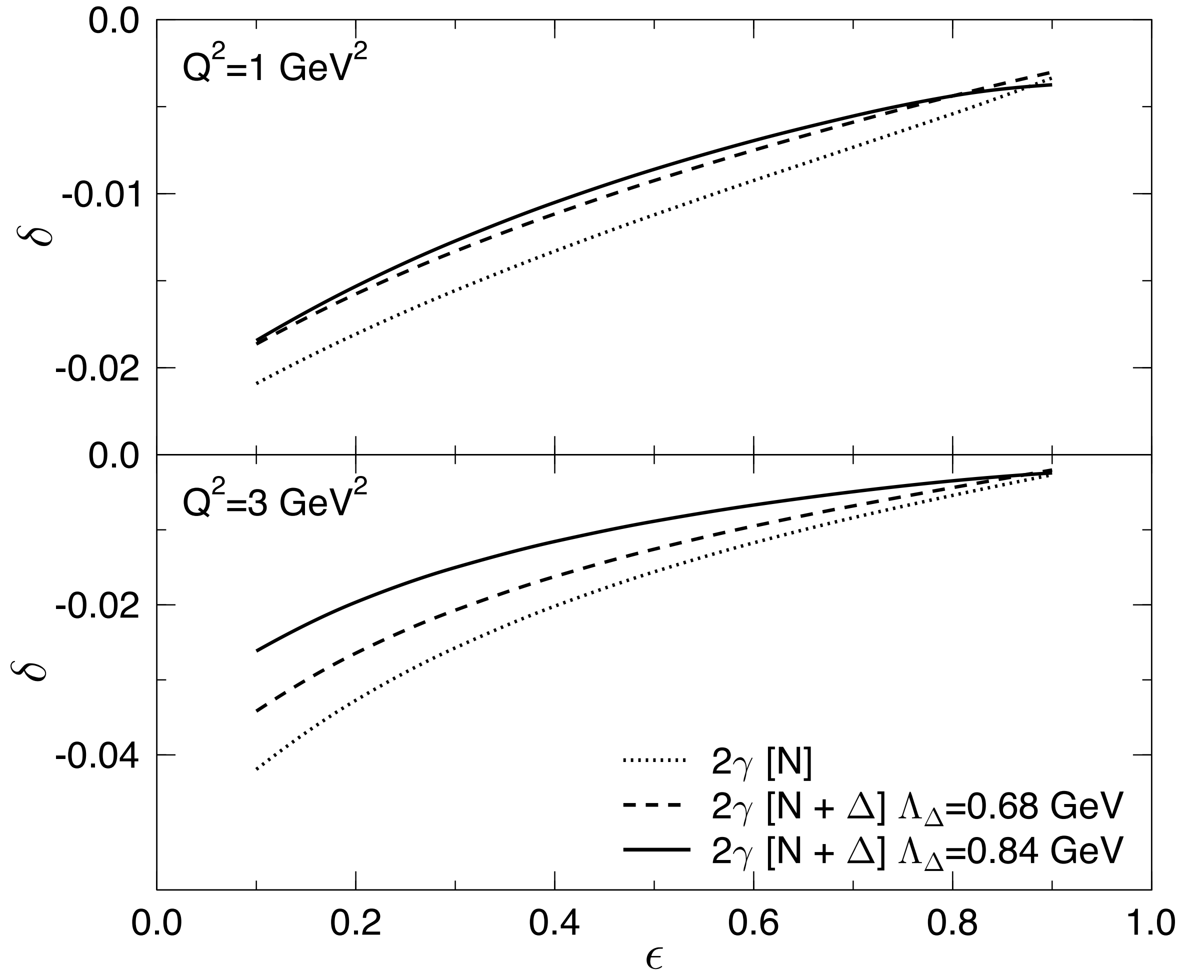
numerators

$$\begin{aligned} N_{box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_{\Delta}\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_{\mu} \left[\not{p}_1 - \not{k} + m_e\right] \gamma_{\nu} u(p_1) \end{aligned}$$

$$\begin{aligned} N_{x-box}^{\Delta}(k) &= \overline{U}(p_4) V_{\Delta in}^{\mu\alpha}(p_2+k, q-k) \left[\not{p}_2 + \not{k} + M_{\Delta}\right] \mathcal{P}_{\alpha\beta}^{3/2}(p_2+k) V_{\Delta out}^{\beta\nu}(p_2+k, k) U(p_2) \\ &\times \; \overline{u}(p_3)\gamma_{\nu} \left[\not{p}_3 + \not{k} + m_e\right] \gamma_{\mu} u(p_1) \end{aligned}$$

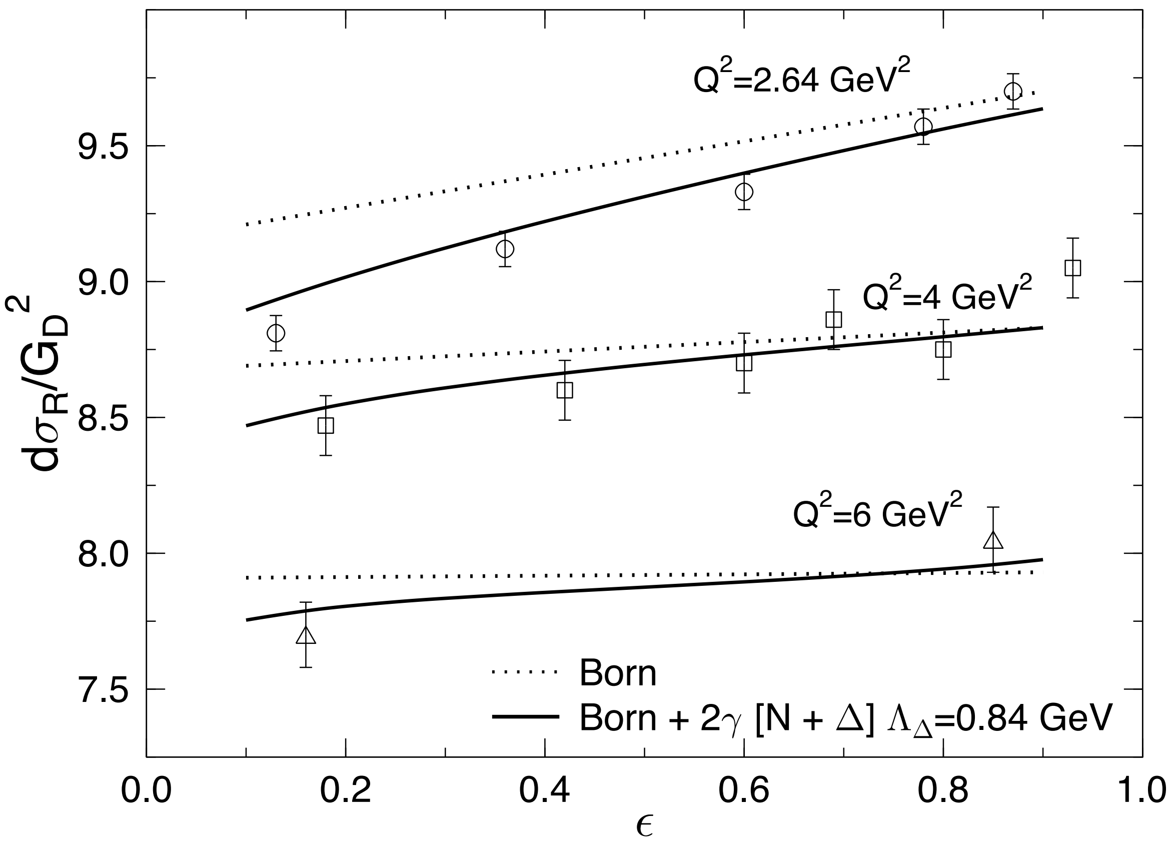
spin-3/2 projection operator

$$\mathcal{P}_{\alpha\beta}^{3/2}(p) = g_{\alpha\beta} - \frac{1}{3}\gamma_{\alpha}\gamma_{\beta} - \frac{1}{3p^2}\left(\not{p}\gamma_{\alpha}p_{\beta} + p_{\alpha}\gamma_{\beta}\not{p}\right)$$



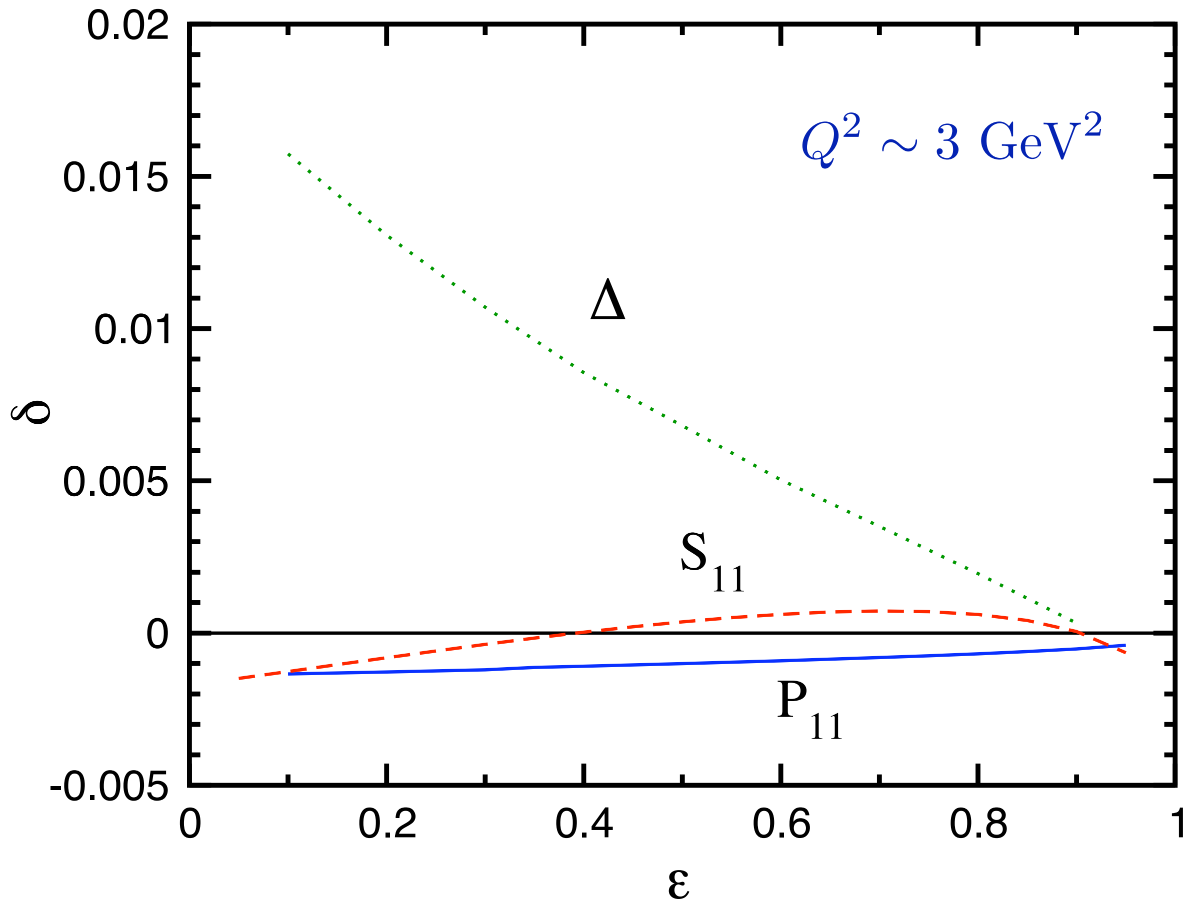
Kondratyuk, Blunden, WM, Tjon
Phys. Rev. Lett. 2006

- ➡ Δ has <u>opposite</u> slope to N
- ➡ cancels some of TPE correction from N



- weaker ε dependence than with N alone
- better fit to JLab data!

$J^P = \frac{1}{2}^+, \quad \frac{1}{2}^-$ excited N^* states



Tjon, WM, et al. (2005)

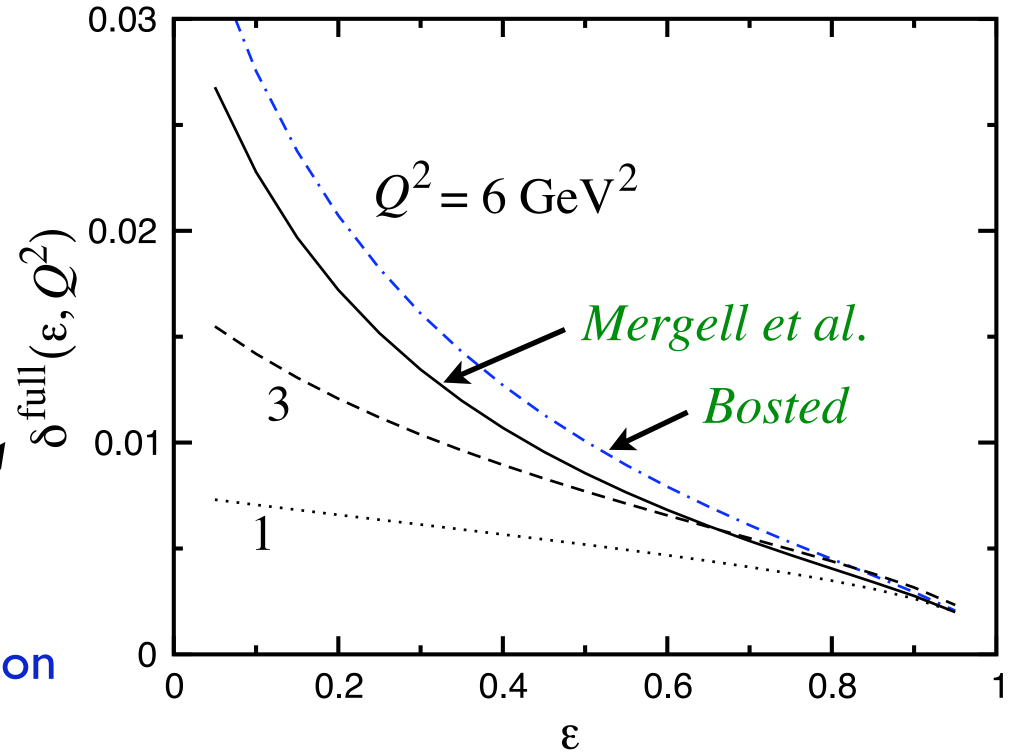
- higher mass resonance contributions small
- enhance nucleon elastic contribution

4.
Form factors

- effect on neutron

Neutron correction

➡ since G_E^n is small, effect may be relatively large

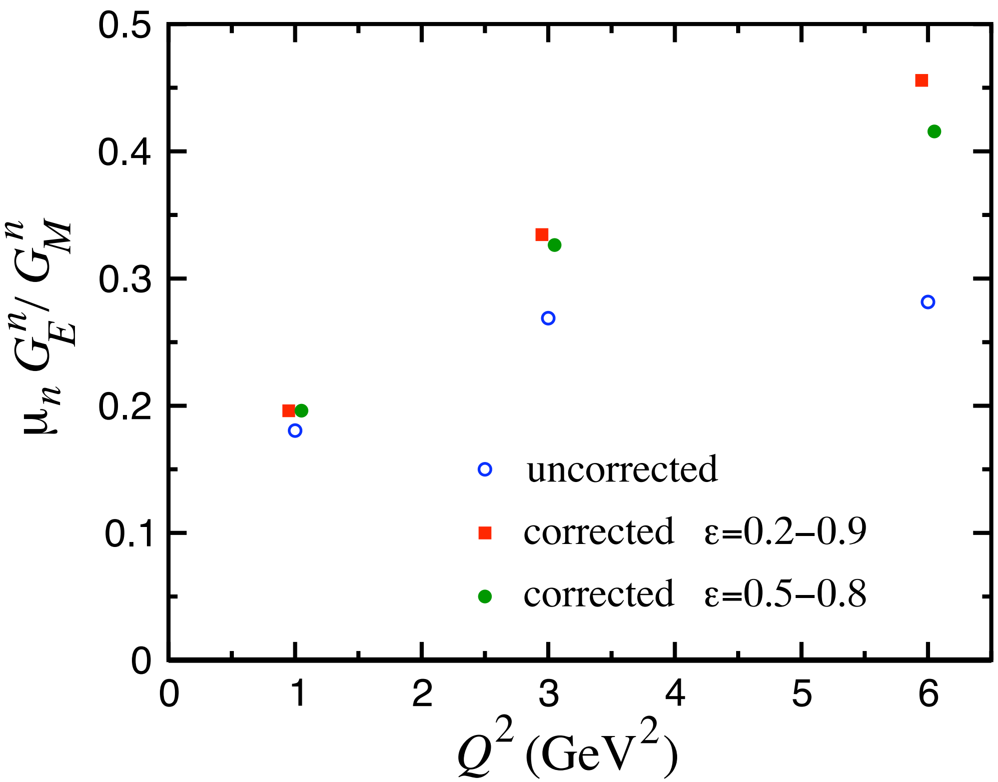


full correction
- no IR contribution

Blunden, WM, Tjon
Phys. Rev. C72 (2005) 034612

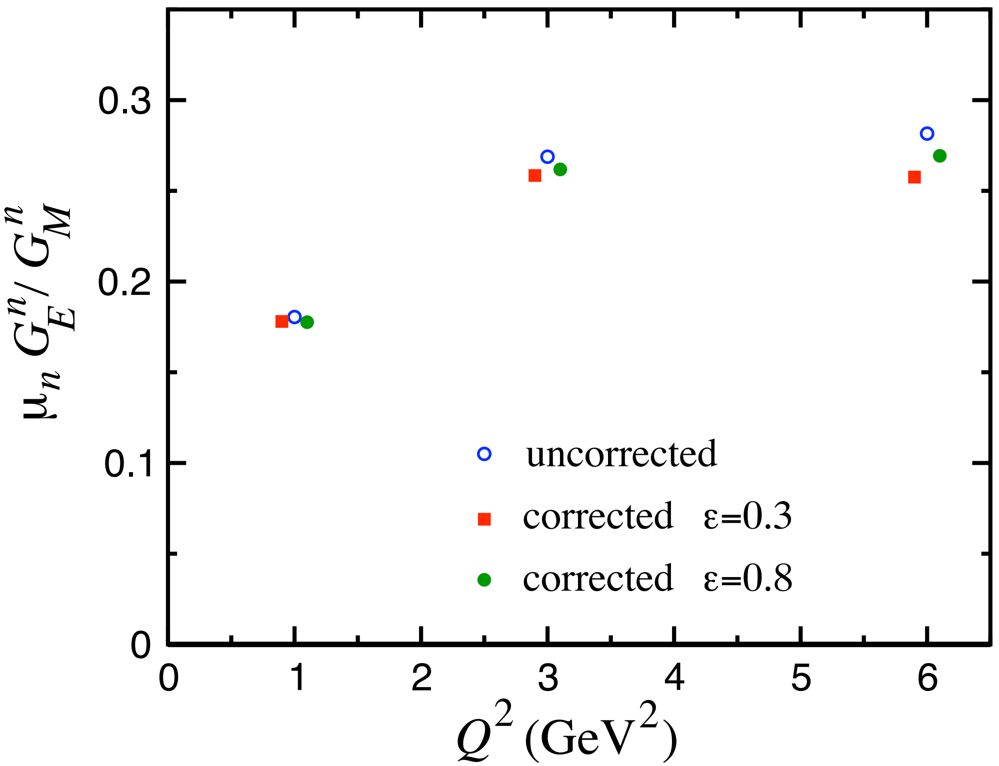
➡ sign opposite to proton (since $\kappa_n < 0$)

Effect on neutron LT form factors



- large effect at high Q^2 for LT-separation method
- LT method unreliable for neutron

Effect on neutron PT form factors



➡ small correction for PT

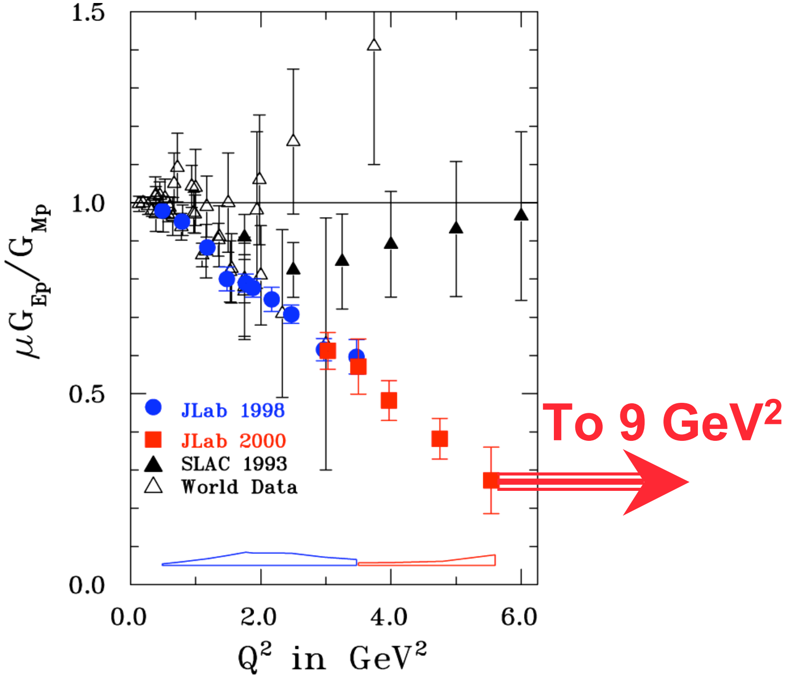
➡ 4% (3%) suppression at $\varepsilon = 0.3\ (0.8)$ for $Q^2 = 3\ \mathrm{GeV}^2$
10% (5%) suppression at $\varepsilon = 0.3\ (0.8)$ for $Q^2 = 6\ \mathrm{GeV}^2$

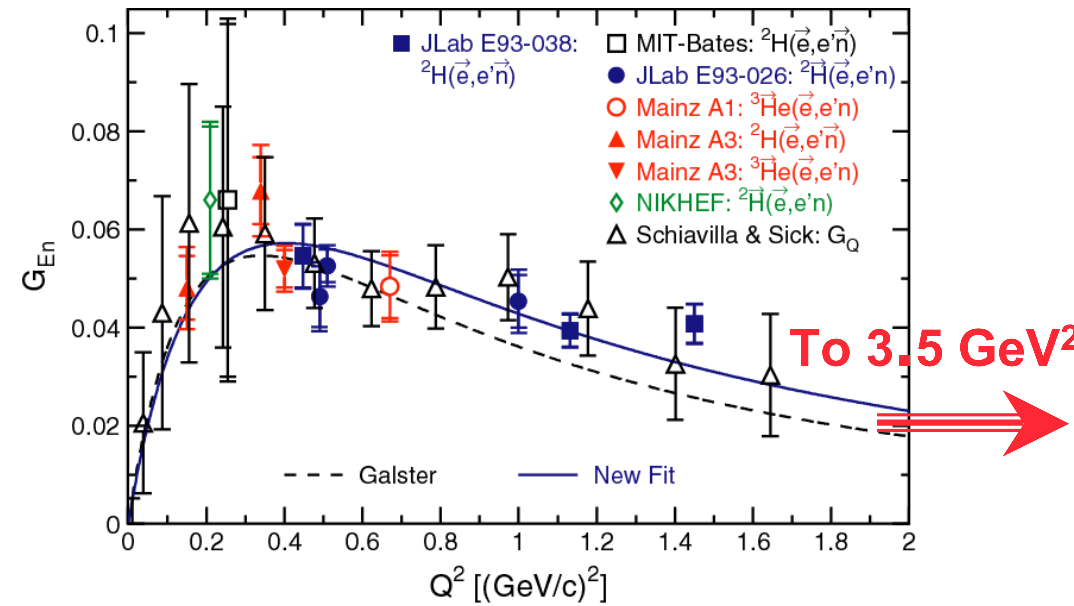
Next 5 years

proton

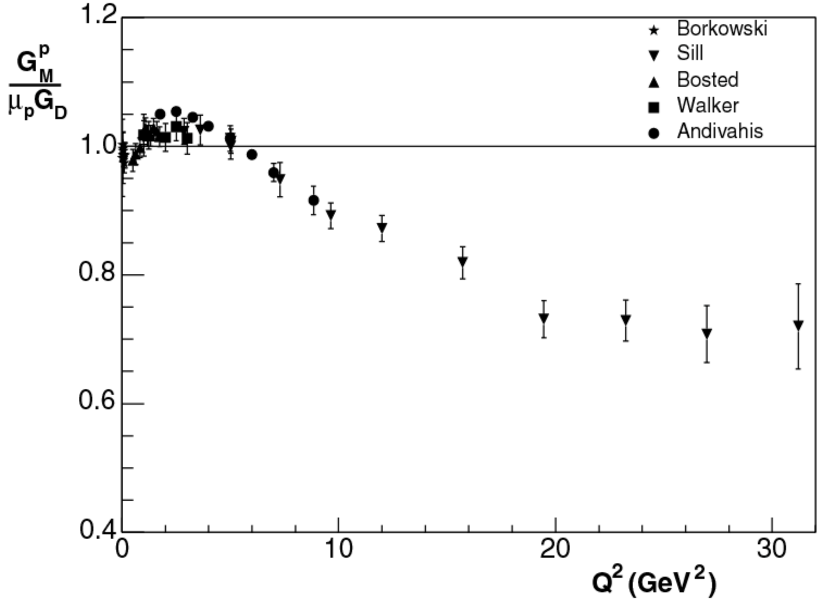
neutron

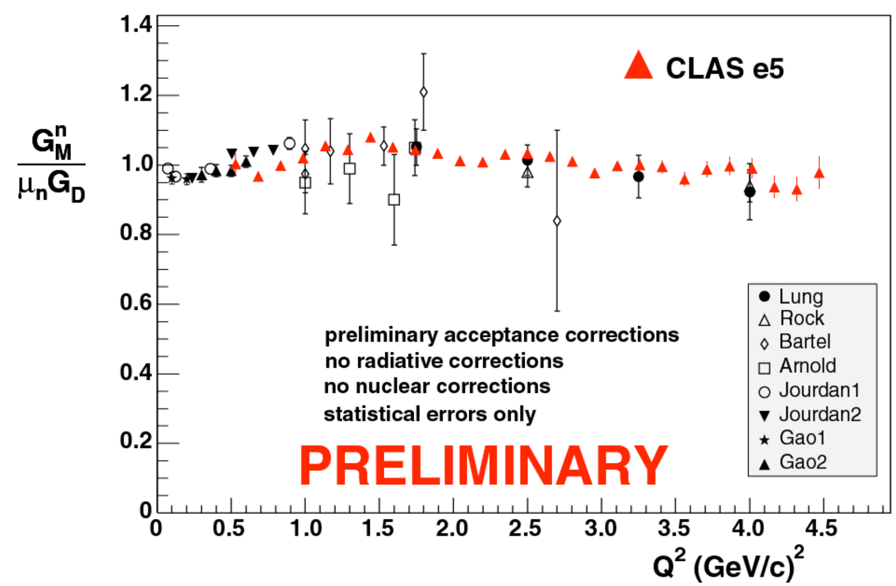
Electric





Magnetic





4.
Form factors
- strangeness in the nucleon

Strangeness Widely Believed to Play a Major Role – Does It?

- As much as 100 to 300 MeV of proton mass:

$$M_N = \langle N(P)| -\frac{9\,\alpha_s}{4\,\pi}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d + m_s\bar{\psi}_s\psi_s|N(P)\rangle$$

$$\Delta M_N^{\mathrm{strange}} = \frac{y\ m_s}{m_u + m_d}\ \sigma_N$$

$$y = 0.2 \pm 0.2 \qquad \sigma_N = 45 \pm 8\ \mathrm{MeV}$$

$$\Rightarrow\quad \Delta M_N^{\mathrm{strange}} \sim 110 \pm 110\ \mathrm{MeV}$$

- Through proton spin crisis:
  As much as 10% of the spin of the proton

- HOW MUCH OF THE MAGNETIC FORM FACTOR?

Strangeness in the Nucleon

Proton and neutron electromagnetic form factors give two combinations of 3 unknowns

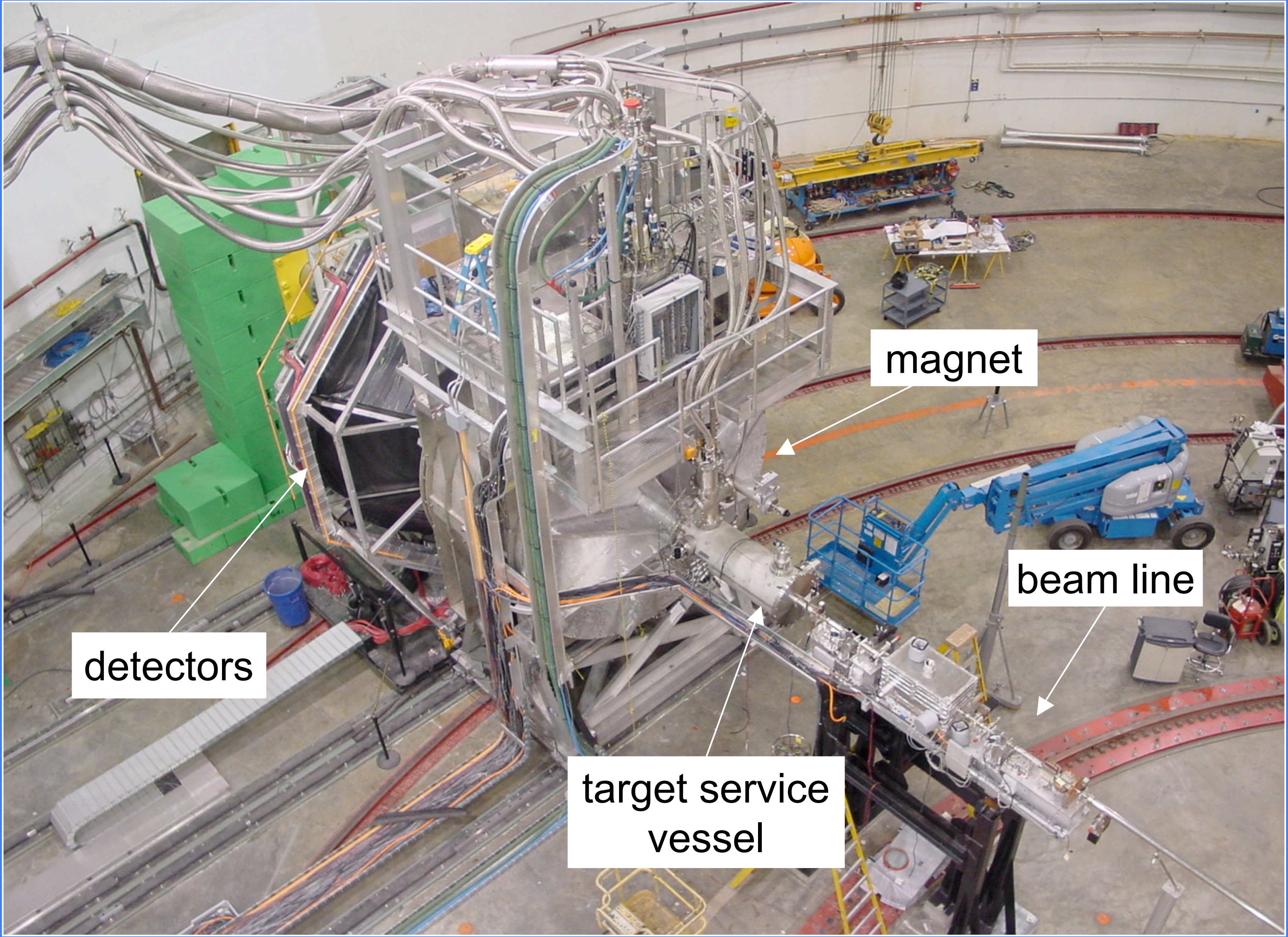
$$G^p_{E,M} = \frac{2}{3} G^u_{E,M} - \frac{1}{3} G^d_{E,M} - \frac{1}{3} G^s_{E,M}$$

$$G^n_{E,M} = \frac{2}{3} G^d_{E,M} - \frac{1}{3} G^u_{E,M} - \frac{1}{3} G^s_{E,M}$$

⟹ need 3rd observable to extract $G^s_{E,M}$

⟹ parity-violating e scattering (interference of γ and Z^0 exchange)

G0 Experiment at Jefferson Lab

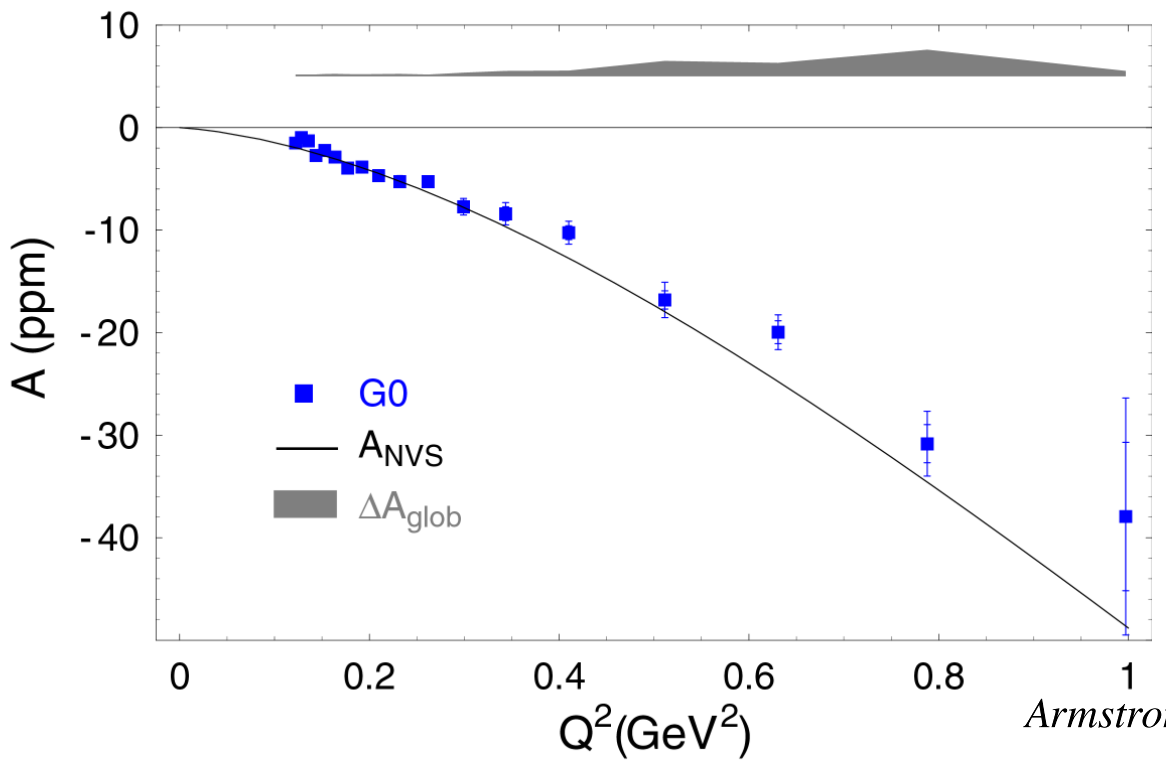


Strangeness in the Nucleon

Parity-violating e scattering

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{p\gamma} G_E^{pZ} + \tau G_M^{p\gamma} G_M^{pZ} - \frac{1}{2}(1-4\sin^2\theta_W)\varepsilon' G_M^{p\gamma} G_A^{pZ}}{\varepsilon (G_E^{p\gamma})^2 + \tau (G_M^{p\gamma})^2}$$

$$G_{E,M}^{pZ} = \frac{1}{4}(G_{E,M}^{p\gamma} - G_{E,M}^{n\gamma}) - \sin^2\theta_W G_{E,M}^{p\gamma} - \frac{1}{4} G_{E,M}^{s}$$

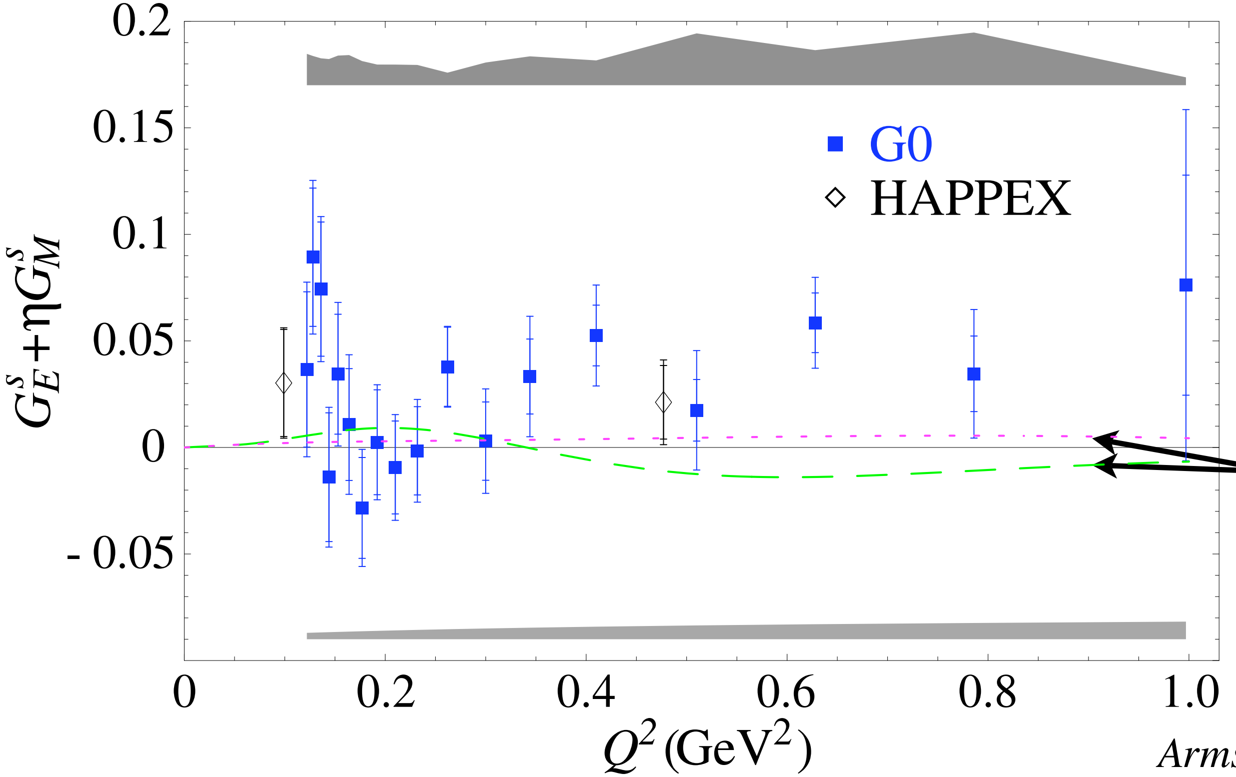


Armstrong et al. [G0 Collaboration]
nucl-ex/0506021

http://www.npl.uiuc.edu/exp/G0Forward

Strangeness in the Nucleon

Parity-violating e scattering



$\eta = \tau G_M / \varepsilon G_E$

$\sim \ 0.94\ Q^2$

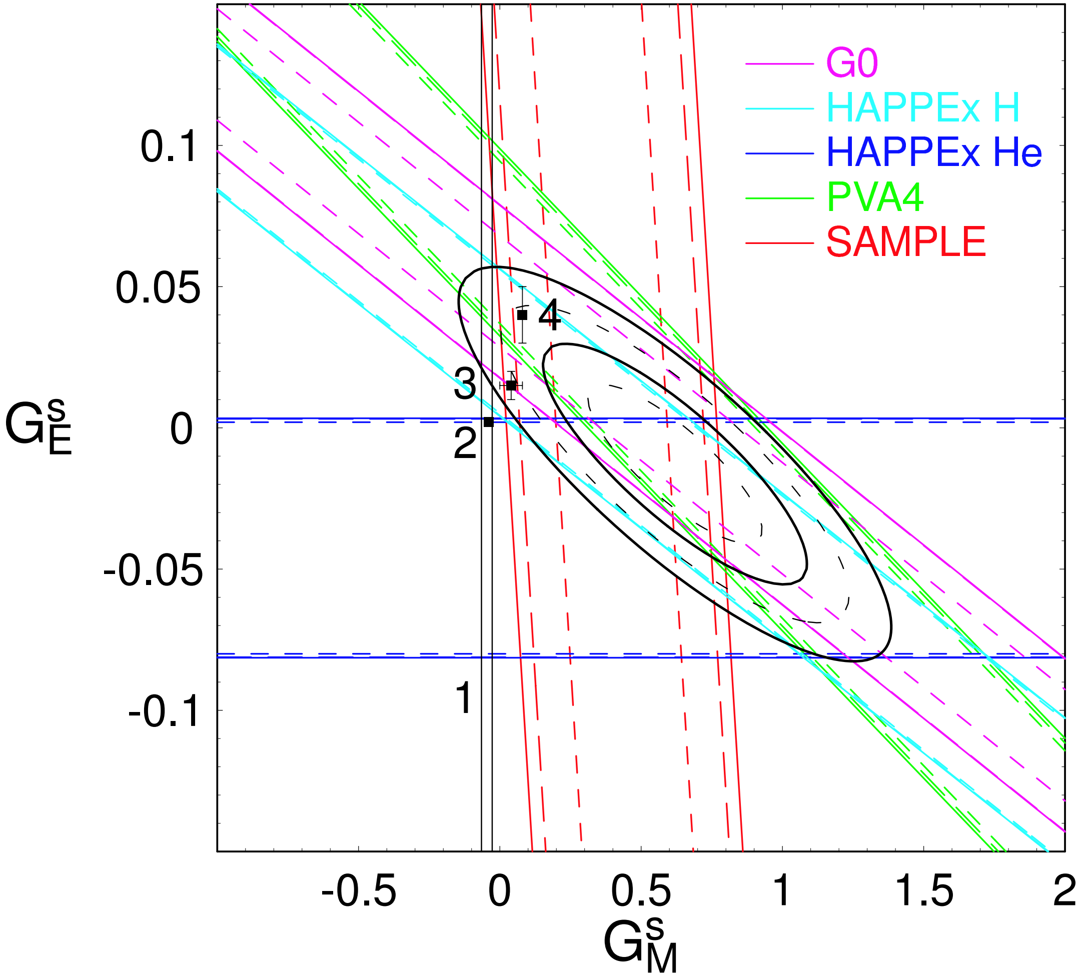
dependence of "zero-point" on e.m. form factors

Armstrong et al. [G0 Collaboration] nucl-ex/0506021

- intriguing Q^2 dependence !
- trend to positive values at larger Q^2

Strangeness in the Nucleon

combined world data at $Q^2 = 0.1\ \mathrm{GeV}^2$



$G_E^s = -0.013 \pm 0.028$

$G_M^s = +0.62 \pm 0.31$

$(\pm\ 0.62\ 2\sigma)$

<u>Theories</u>

1. Leinweber, et al. PRL **94** (05) 212001 — *lattice*
2. Lyubovitskij, et al. PRC **66** (02) 055204 — *chiral quark model*
3. Lewis, et al. PRD **67** (03) 013003 — *chiral EFT*
4. Silva, et al. PRD **65** (01) 014016 — *quark soliton model*

⇒ huge effect!

⇒ can theory explain result?

Lattice Results

Dong *et al.* PRD(1998) $G_M^s = -0.36 \pm 0.20$

Mathur & Dong NPB(2001) $G_M^s = -0.27 \pm 0.10$

Lewis *et al.* PRD(2003) $G_M^s(0.1\,\mathrm{GeV}^2) = +0.05 \pm 0.06$

Leinweber *et al.* PRL(2005) $G_M^s = -0.046 \pm 0.019$

Charge Symmetry Constraint

$$p = \frac{2}{3}u^p - \frac{1}{3}u^n + O_N$$
$$n = -\frac{1}{3}u^p + \frac{2}{3}u^n + O_N$$

$$3O_N = 2p + n - u^p$$
$$3O_N = p + 2n - u^n$$

Lattice QCD

$$\Sigma^+ = \frac{2}{3}u^\Sigma - \frac{1}{3}s^\Sigma + O_\Sigma$$
$$\Sigma^- = -\frac{1}{3}u^\Sigma - \frac{1}{3}s^\Sigma + O_\Sigma$$

$$\Sigma^+ - \Sigma^- = u^\Sigma$$

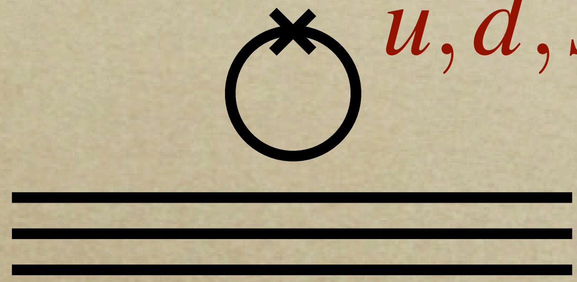
$$3O_N = 2p + n - \frac{u^p}{u^\Sigma}(\Sigma^+ - \Sigma^-)$$
$$3O_N = p + 2n - \frac{u^n}{u^\Xi}(\Xi^0 - \Xi^-)$$

Ross Young et al.
(JLab/CSSM)

Disconnected Loops

"l" loop contribution

$O_N =$



$$= \frac{2}{3}{}^{l}G_M^u - \frac{1}{3}{}^{l}G_M^d - \frac{1}{3}{}^{l}G_M^s$$

$${}^{l}G_M^u = {}^{l}G_M^d$$

QCD equality for $m_u = m_d$

$$O_N = -\frac{1}{3}\left({}^{l}G_M^d + {}^{l}G_M^s\right)$$

$$= \frac{{}^{l}G_M^s}{3}\left(\frac{1 - {}^{l}R_d^s}{{}^{l}R_d^s}\right)$$

$${}^{l}R_d^s = {}^{l}G_M^s / {}^{l}G_M^d = 0.139 \pm 0.042$$

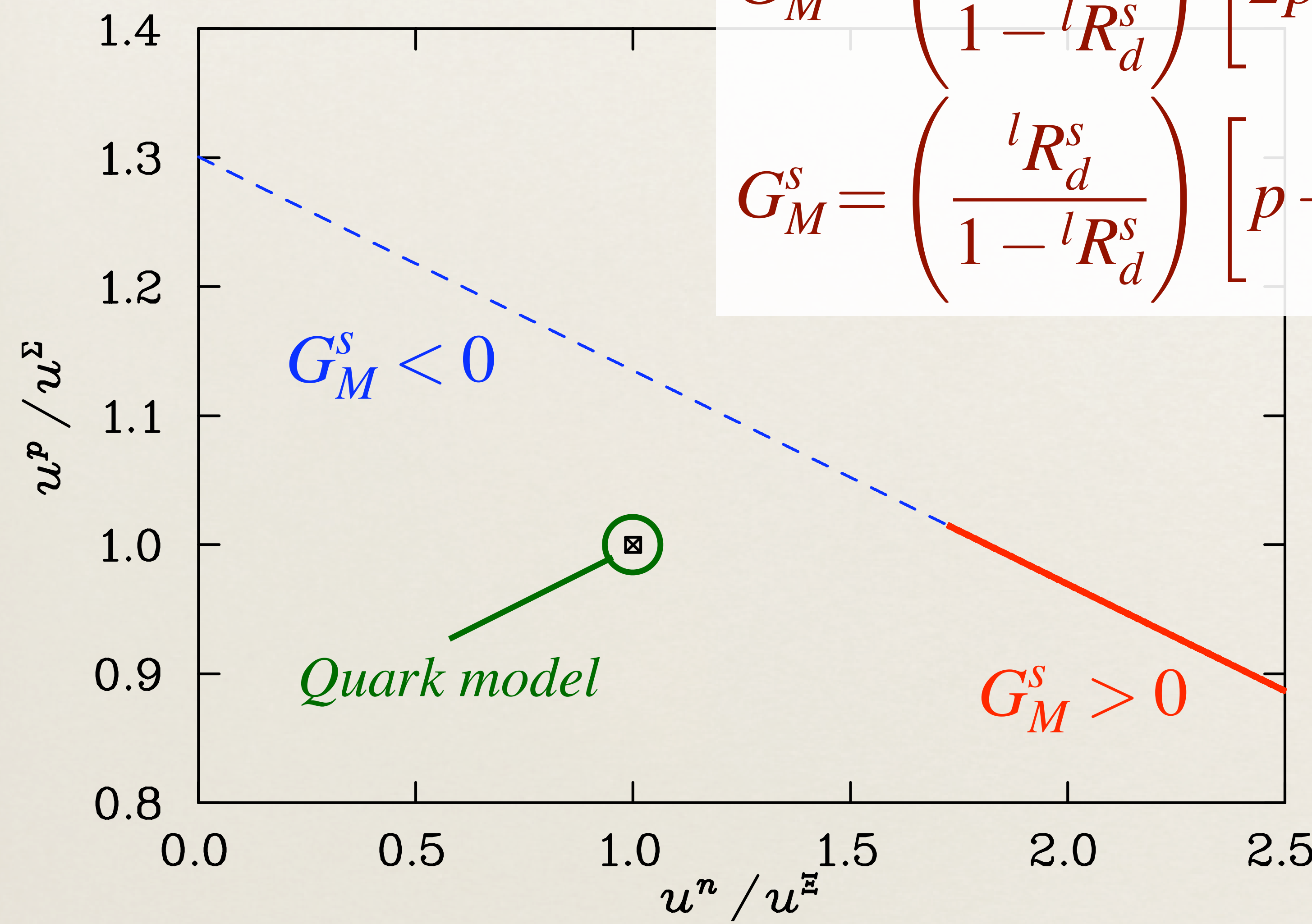
chiral phenomenology

Constraint on GMs

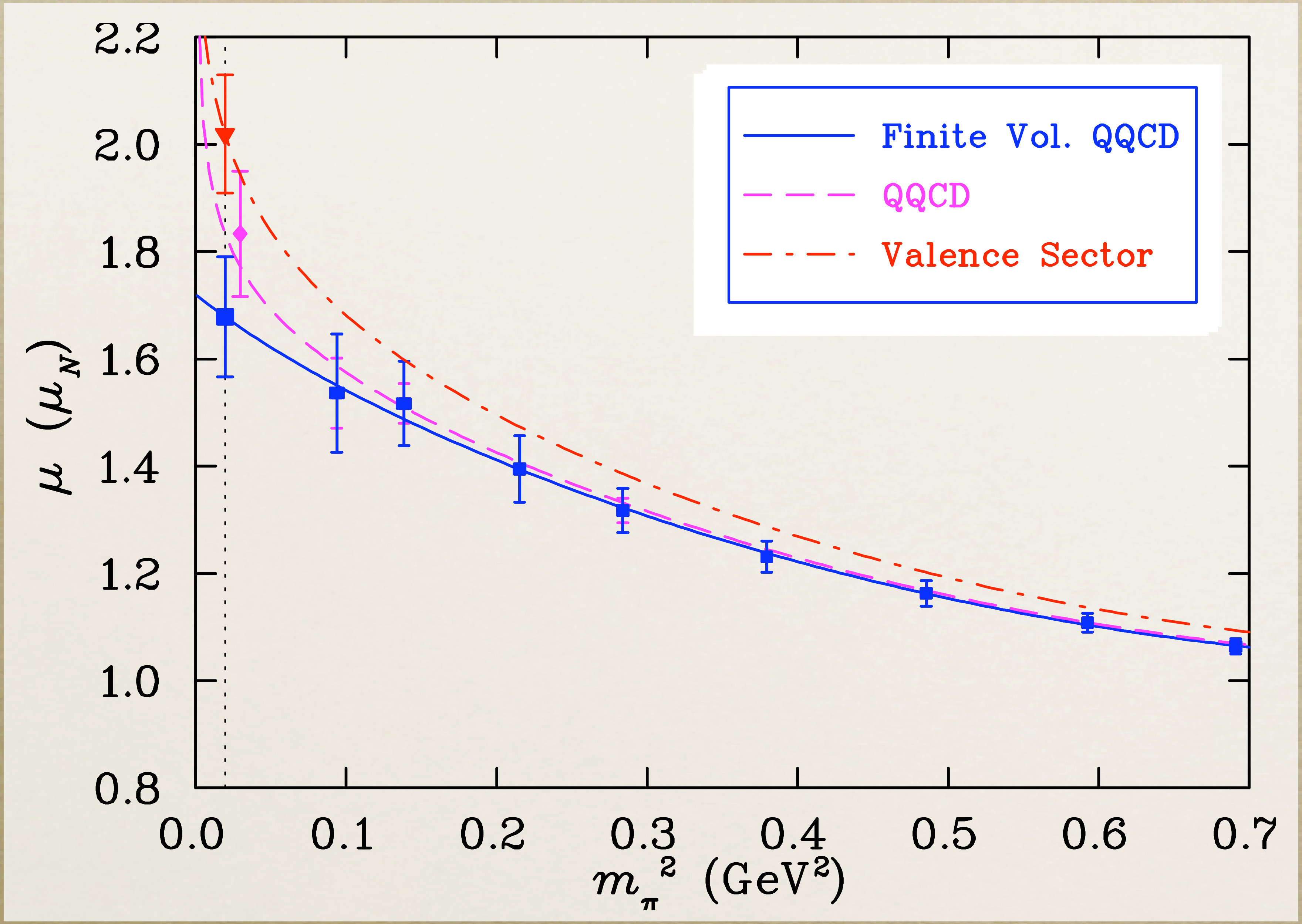
$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[2p + n - \frac{u^p}{u^\Sigma}(\Sigma^+ - \Sigma^-)\right]$$

$$G_M^s = \left(\frac{{}^lR_d^s}{1 - {}^lR_d^s}\right)\left[p + 2n - \frac{u^n}{u^\Xi}(\Xi^0 - \Xi^-)\right]$$

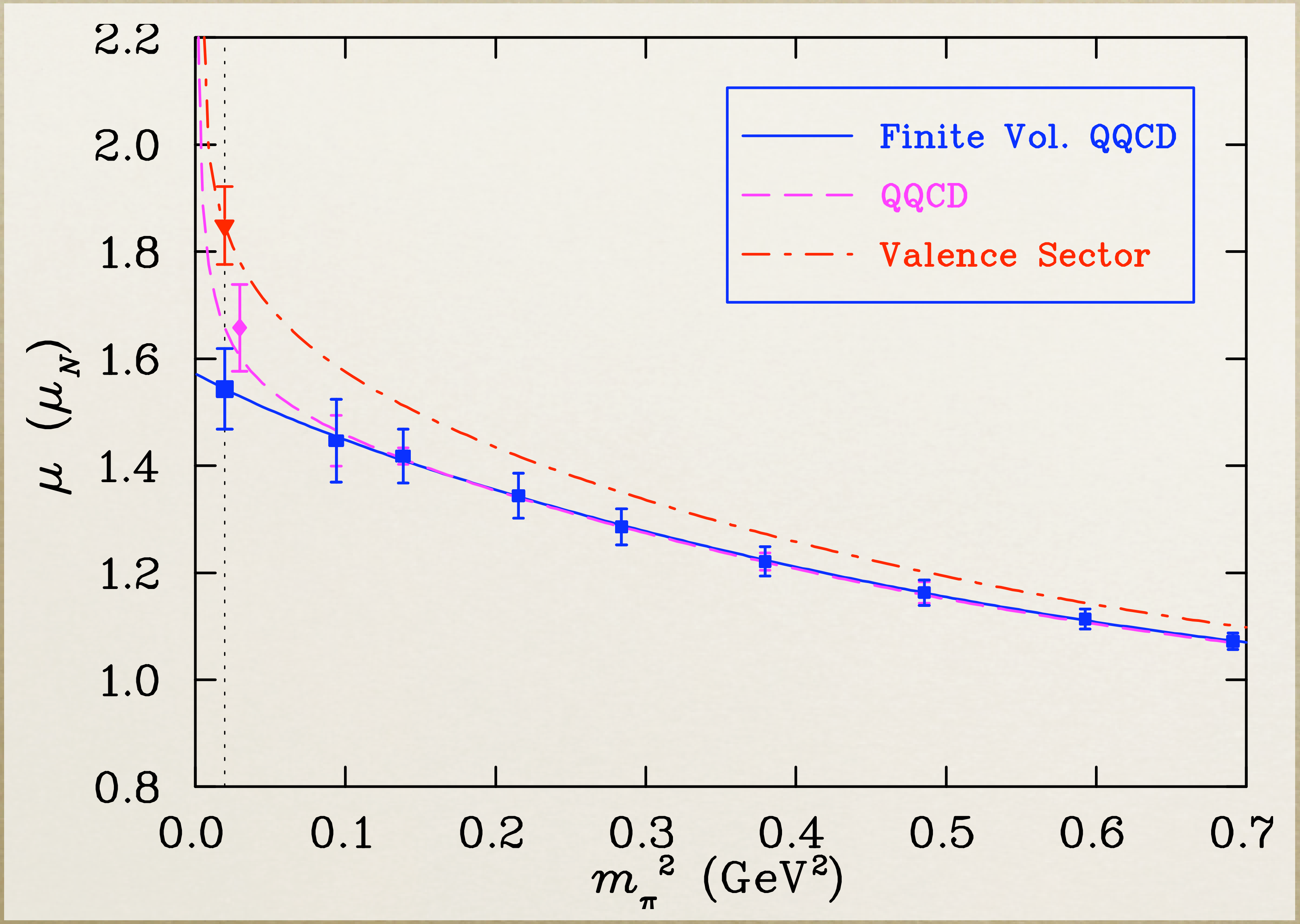
Lattice QCD



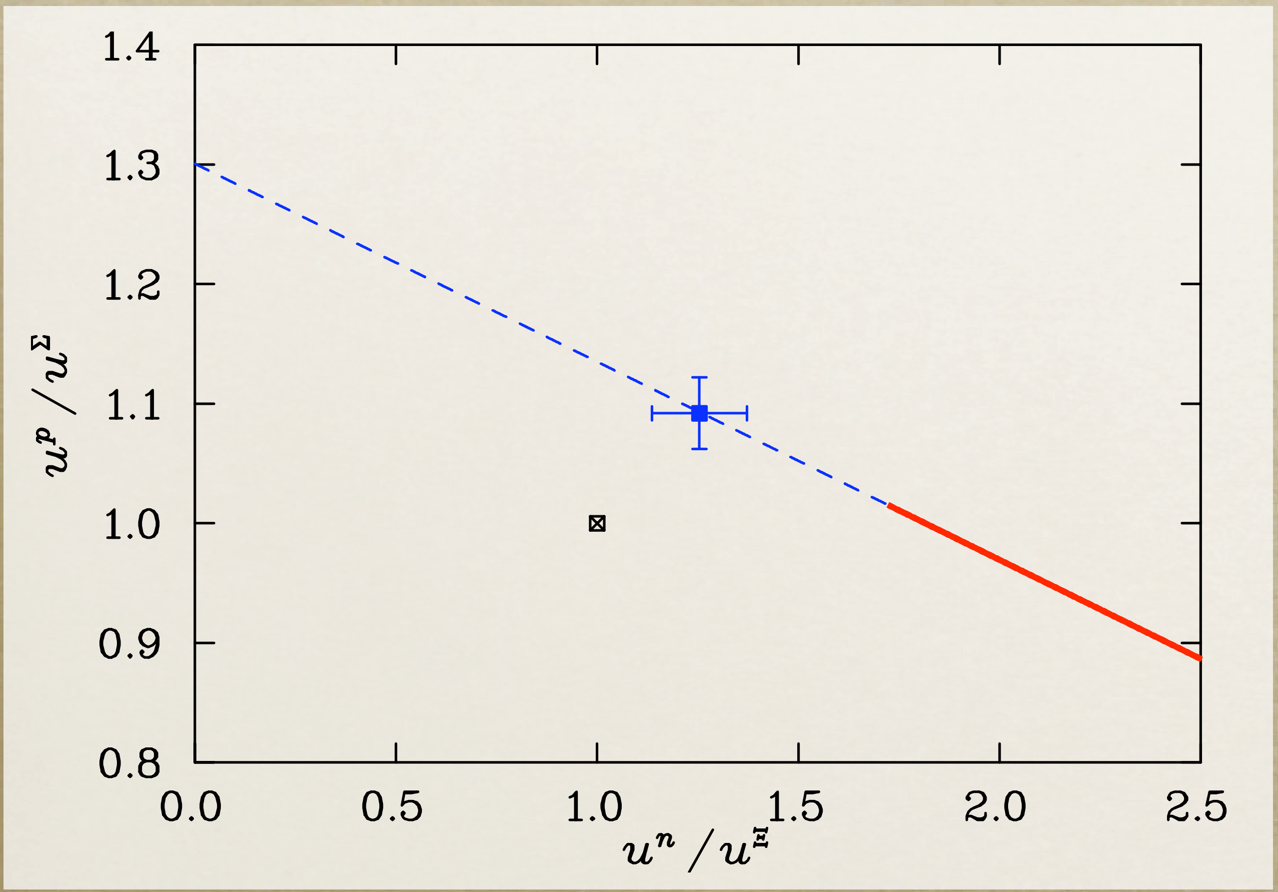
u-quark in the proton



u-quark in the Sigma



Final Result

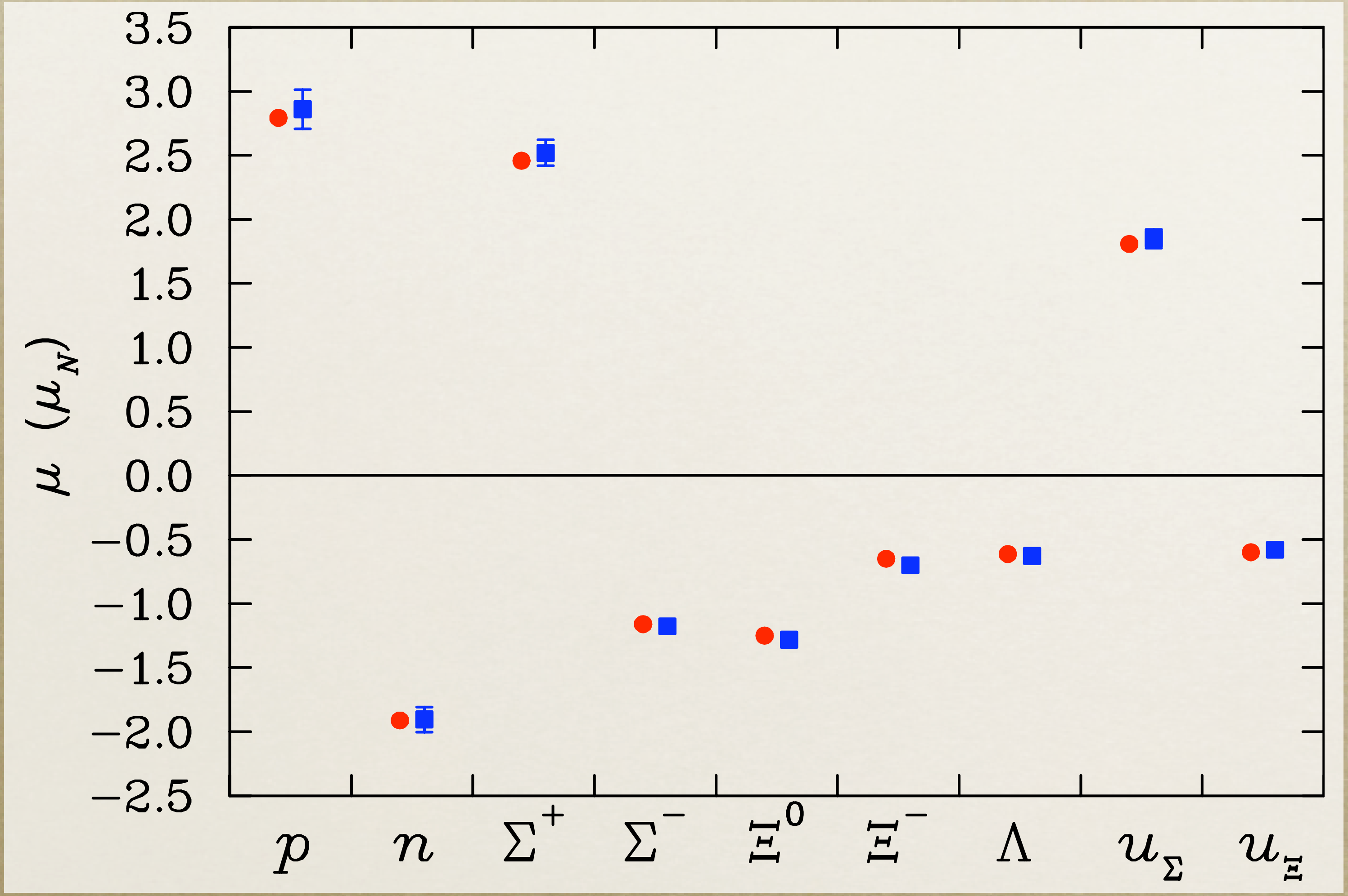


$$\frac{u^p}{u^\Sigma} = 1.092 \pm 0.030$$

$$\frac{u^n}{u^\Xi} = 1.254 \pm 0.124$$

$$G_M^s = -0.046 \pm 0.019\, \mu_N$$

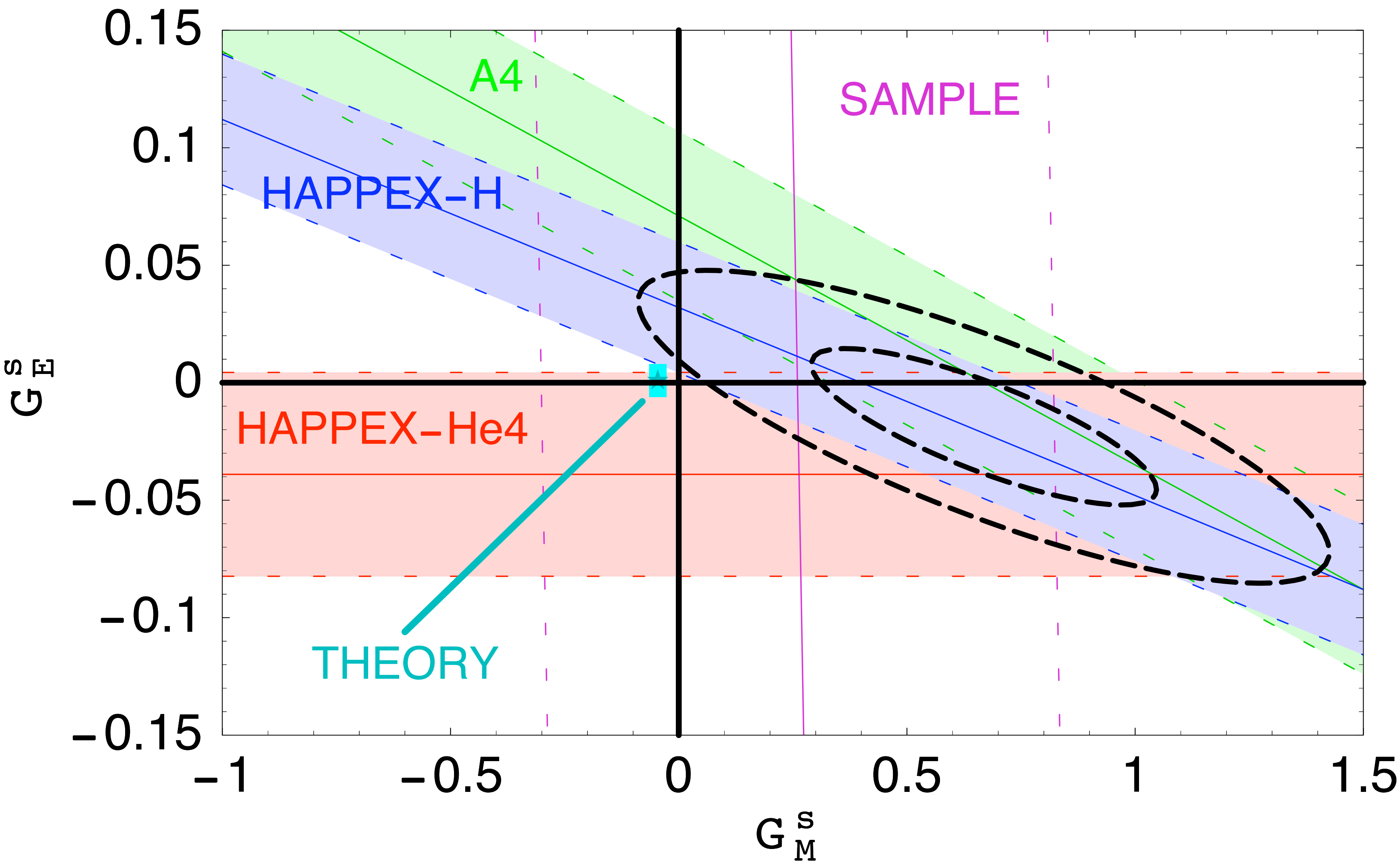
Magnetic Moments



Leinweber *et al.* PRL(2005)

Repeat analysis for strange electric form factor

$$\Rightarrow \quad G_E^s(Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.003$$



Summary - Form Factors

- Surprisingly different behavior for G_E^p and G_M^p

  $\longrightarrow$ different charge and magnetization distributions

- 2γ exchange needed to resolve discrepancy between LT and PT measurements of G_E^p/G_M^p

  $\longrightarrow$ reached limit of applicability of 1γ exchange in elastic eN scattering

- Strange magnetic moment large and positive

  $\longrightarrow$ *cf.* lattice QCD/phenomenology, which gives very small and negative value

  $\longrightarrow$ G0 backward angle run in 2006-2007 will determine G_E^s and G_M^s separately slides at www.jlab.org/div_dept/theory/talks/index.html

Thank you students - good luck!

Thank you Bruce!