Structure of the Nucleon with electroweak probes

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Jefferson Lab
Outline

1. Introduction
   - QCD and the strong nuclear force
   - electron scattering

2. Quark distributions in the nucleon
   - sea quarks and flavour asymmetries
   - valence quarks at large $x$
   - nuclear effects
   - lattice QCD
Outline

3. **Quark-hadron duality**
   - structure functions in the resonance region
   - duality and QCD
   - global vs. local duality

4. **Electromagnetic form factors**
   - two-photon exchange
   - strangeness form factors
Thursday, February 2, 2006
U.S. Department of Energy Requests $4.1 Billion Investment
As Part of the American Competitiveness Initiative

Nuclear Physics Program ($454.1 million)

This is an $87 million increase over FY 2006. This funding supports research to provide new insights and knowledge of the structure and interaction of atomic nuclei and the primary forces of particles of nature in nuclear matter. The funding increase restores operations at both the Thomas Jefferson National Accelerator Facility (TJNAF) and the Relativistic Heavy Ion Collider (RHIC). In addition, new funding is requested for a TJNAF power upgrade and a new injector for RHIC.

High Energy Physics Program ($775.1 million)

This is a $58.4 million increase over FY 2006. This funding for grants and full experimental facility operations will be used to further explore basic research to explore the laws of nature governing the most basic constituents of matter and the forces binding them. These are fundamental principles at the heart of physics and the physical sciences. Project engineering and design funding of $10.3 million is requested for the new Electron Neutrino Appearance project.

“puts DOE's Office of Science on the path to doubling its budget by 2016”
1.
Introduction
- QCD and the strong nuclear force
Building Blocks of the Universe

<table>
<thead>
<tr>
<th>FERMIONS</th>
<th>Quarks</th>
<th>matter constituents spin = 1/2, 3/2, 5/2, ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptons</td>
<td>spin = 1/2</td>
<td></td>
</tr>
<tr>
<td>Flavor</td>
<td>Mass GeV/c²</td>
<td>Electric charge</td>
</tr>
<tr>
<td>$\nu_e$ electron neutrino</td>
<td>$&lt;1 \times 10^{-8}$</td>
<td>0</td>
</tr>
<tr>
<td>$e$ electron</td>
<td>0.000511</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_\mu$ muon neutrino</td>
<td>$&lt;0.0002$</td>
<td>0</td>
</tr>
<tr>
<td>$\mu$ muon</td>
<td>0.106</td>
<td>-1</td>
</tr>
<tr>
<td>$\nu_\tau$ tau neutrino</td>
<td>$&lt;0.02$</td>
<td>0</td>
</tr>
<tr>
<td>$\tau$ tau</td>
<td>1.7771</td>
<td>-1</td>
</tr>
<tr>
<td>Quarks</td>
<td>spin = 1/2</td>
<td></td>
</tr>
<tr>
<td>Flavor</td>
<td>Approx. Mass GeV/c²</td>
<td>Electric charge</td>
</tr>
<tr>
<td>$u$ up</td>
<td>0.003</td>
<td>2/3</td>
</tr>
<tr>
<td>$d$ down</td>
<td>0.006</td>
<td>-1/3</td>
</tr>
<tr>
<td>$c$ charm</td>
<td>1.3</td>
<td>2/3</td>
</tr>
<tr>
<td>$s$ strange</td>
<td>0.1</td>
<td>-1/3</td>
</tr>
<tr>
<td>$t$ top</td>
<td>175</td>
<td>2/3</td>
</tr>
<tr>
<td>$b$ bottom</td>
<td>4.3</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

- Each quark comes in 3 “colours”: red, green and blue.
- Leptons do not carry color charge.
### Force Carriers of the Universe

#### BOSONS

<table>
<thead>
<tr>
<th>Unified Electroweak</th>
<th>Spin = 1</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>γ</strong> photon</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>W⁻</strong></td>
<td>80.4</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td><strong>W⁺</strong></td>
<td>80.4</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td><strong>Z⁰</strong></td>
<td>91.187</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strong (color)</th>
<th>Spin = 1</th>
<th>Mass GeV/c²</th>
<th>Electric charge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>g</strong> gluon</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- The massless photon mediates the long-range e.m. interactions.
- Gluons carry color and mediate the strong interaction.
- The very massive W⁻, W⁺, and Z⁰ bosons mediate the weak interaction.
Quantum Chromodynamics (QCD)

- Photons do not carry electric charge.
- Gluons do carry colour charge!
- Gluons can directly interact with other gluons!
- This is new!

A red quark emitting a red anti-blue gluon to leave a blue quark.

Quark-quark force grows WEAKER as quarks come close.

“Asymptotic Freedom”
Existence or otherwise is a CRUCIAL question in strong interaction physics.

Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD.

Jefferson Lab is the ideal facility to definitively answer this question!

2004 Nobel Prize for discovery of asymptotic freedom (Gross, Politzer, Wilczek)

Calculate observables using perturbation theory as power series in small expansion parameter $\alpha_s$.
Operated by the Southeastern Universities Research Association for the U.S. Department of Energy

Thomas Jefferson National Accelerator Facility

Page 16

Pentaquark

Summary

• Existence or otherwise is a CRUCIAL question in strong interaction physics

• Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD

• Jefferson Lab is the ideal facility to definitively answer this question!

BUT - only half of the story...

at low energy \( \rightarrow \textit{confinement} \)!

\( \alpha_s \sim 1 \) so cannot use perturbative expansion

here QCD said to be “nonperturbative”
QCD and the Origin of Mass

\[ u + u + d = \text{proton} \]

mass: \[ 0.003 + 0.003 + 0.006 \neq 0.938 \text{ MeV} \]

HOW does the rest of the proton mass arise?
QCD: Unsolved in Nonperturbative Regime

• 2004 Nobel Prize awarded for “asymptotic freedom”

• BUT in nonperturbative regime QCD is still unsolved

• One of the top 10 challenges for physics!

• Is it right/complete?

• Do glueballs, exotics and other apparent predictions of QCD in this regime agree with experiment?

central to answering these questions is the need to understand how quarks form hadrons
Looking for quarks in the nucleon is like looking for the Mafia in Sicily - everybody *knows* they’re there, but it’s hard to find the evidence!

Anonymous
"Quarks, neutrinos, mesons. All those damn particles you can't see. That's what drove me to drink. But now I can see them."
How to probe the structure of hadrons?

collide hadrons

probe with leptons
1. Introduction
   - electron scattering
Electron scattering

Electron Scattering Provides an Ideal Microscope for Nuclear Physics

- Electrons are point-like
- The interaction (QED) is well-known
- The interaction is weak
Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab (JLab)
Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab (JLab)

0.6 GeV electrons / linac
$\times 10 \rightarrow 6$ GeV
Experimental Halls

Hall A

Hall B

Hall C

Hall D
Experimental Halls

**Hall A**

**Hall C**

High luminosity

$> 10^{38} \text{ cm}^{-2} \text{ s}^{-1}$

Very high precision measurements

High $Q^2$ form factors, parity-violating $e$ scattering, precision structure functions, ...
large acceptance
lower luminosity
\[ \sim 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \]
collect all data “at once”

\( N^* \) spectroscopy
(multi-hadron final states),
structure function moments,
...
proposed new Hall as part of 12 GeV upgrade

$4\pi$ acceptance

photon beam

exotic meson spectroscopy (GlueX Collaboration) “origins of confinement”
JLab Central to all of Nuclear Science

- Quark-Gluon Structure Of Nucleons and Nuclei
- Nature of Confinement
  - Exotic mesons and baryons
  - Precise few-nucleon calculations
  - Correlations n-radii: N ≠ Z
  - Hypernuclei
  - Hadrons in-medium
  - Effective NN (+ HN) force
- \( \ldots n\text{-stars} \)
Electron scattering

Inclusive cross section for $eN \rightarrow eX$

one-photon exchange approximation
Electron scattering

Inclusive cross section for $eN \rightarrow eX$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{M} + \frac{F_2}{\nu} \right)$$

$$\nu = E - E'$$

$$Q^2 = \vec{q}^2 - \nu^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

$F_1, F_2$ “structure functions”

contain all information about structure of nucleon

functions of $x, Q^2$ in general
Electron scattering

Inclusive cross section for $eN \rightarrow eX$

$$\frac{d^2 \sigma}{d\Omega dE'} \sim L^{\mu\nu} W_{\mu\nu}$$

leptonic tensor

Hadronic tensor

$$W_{\mu\nu} = \sum_X \langle X | J_\mu(z) | N \rangle \langle N | J_\nu(0) | X \rangle \delta^4(p + q - p_X)$$

$$= \int d^4z \ e^{i q \cdot z} \langle N | J_\mu(z) J_\nu(0) | N \rangle$$

using completeness (sum over ALL states $X$)

$$\sum_X |X\rangle\langle X| = 1$$

“duality”
in general, $N \rightarrow X$ transition matrix element very complicated

at large $Q^2$ and large $\nu$ ("Bjorken limit") things simplify ...

- Wilson Operator Product Expansion

  Expand product of currents $J(z)J(0)$ in a series of (nonperturbative) local operators $\hat{O}$
  and (perturbative) coefficient functions $C_n$

  $$J(z)J(0) \sim \sum_n C_n(z^2) \, z^{\mu_1}z^{\mu_2} \cdots z^{\mu_n} \, \hat{O}_{\mu_1\mu_2\cdots\mu_n}$$

- Matrix elements of $\hat{O}_{\mu_1\mu_2\cdots\mu_n}$

  $$\langle N|\hat{O}_{\mu_1\mu_2\cdots\mu_n}|N \rangle = A_n(\mu^2) \, p_{\mu_1}p_{\mu_2} \cdots p_{\mu_n} - \text{traces}$$

  $M^2/Q^2$ corrections
• Moments of structure function $F_2$

$$M_n(Q^2) \equiv \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2)$$

$$= \sum_i \tilde{C}_n^i(Q^2) \, A_n^i(Q^2/\mu^2)$$

where $\tilde{C}_n(Q^2)$ is Fourier transform of $C_n(z^2)$

• Reconstruct structure function from moments via inverse Mellin transform

• Parton model: $F_2(x, Q^2) = x \sum_q e_q^2 \, q(x, Q^2)$

probability to find quark type “$q$” in nucleon, carrying (light-cone) momentum fraction $x = \frac{p_q^+}{p_N^+} = \frac{p_q^0 + p_q^z}{p_N^0 + p_N^z}$
Fourier transform of \( J_\mu(z)J_\nu(0) \)

\[
\rightarrow \text{series in } \left( \frac{1}{Q^2} \right)^{d-n-2}, \text{ where } \tau \equiv d-n \\
\text{“twist”}
\]

- Twist expansion of moments

\[
M_n(Q^2) = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \ldots
\]

leading twist \((\tau = 2)\)

e.g. \(\bar{\psi} \gamma_\mu \psi\)

\(\rightarrow\) free quark scattering
• Fourier transform of \( J_\mu(z) J_\nu(0) \)

\[ \to \text{series in } \left( \frac{1}{Q^2} \right)^{d-n-2}, \text{ where } \tau \equiv d - n \]

“twist”

• Twist expansion of moments

\[ M_n(Q^2) = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

higher twists

\( \text{e.g. } \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma_\nu \psi \)

or \( \bar{\psi} \tilde{G}_{\mu\nu} \gamma^\nu \psi \)

\( \to \text{multi-quark or quark-gluon correlations} \)
2. 
Quark distributions
Parton distributions functions (PDFs)  
*(leading twist)*

- PDFs provide basic information on structure of bound states in QCD
  - *momentum, flavour, spin ... distributions of quarks and gluons in hadrons*

- Integrals of PDFs ("moments") test fundamental sum rules (*Adler, Bjorken ...*)
  - *relate high-energy observables to low-energy hadron properties*
Parton distributions functions (PDFs) *(leading twist)*

provide input into nuclear physics and astrophysics calculations

\[ \rightarrow e.g. \text{relativistic heavy ion collisions} \]

needed to understand backgrounds in searches for “new physics” beyond the Standard Model in high-energy colliders

\[ \rightarrow e.g. \text{neutrino oscillations} \]
Structure function data

Structure function data

Parton distributions functions (PDFs) (leading twist)

PDFs extracted in global analyses of structure function data from electron, muon & neutrino scattering (also from Drell-Yan & W-boson production in hadronic collisions)

parameterized using some functional form, e.g.

\[ xq(x, Q^2) = A_0 \ x^{A_1} (1 - x)^{A_2} \ e^{A_3 x} (1 + e^{A_4 x})^{A_5} \]

determined over several orders of magnitude in \( x \) and \( Q^2 \)

\[ 10^{-6} < x < 1 \]
\[ 1 < Q^2 < 10^8 \ \text{GeV}^2 \]
$Q^2 = 25 \text{ GeV}^2$

- $u_V$
- $d_V$
- $\bar{u}$
- $\bar{d}$
- $s$
- $g/15$

sea quarks & gluons

$\bar{q} = \bar{u}, \bar{d}, \bar{s}...$

$q = u, d, s...$

valence quarks

$p$

$\bar{p}$

$u$

$u$

$d$
Virtual sea of $q\bar{q}$ pairs and gluons dominate small-$x$ region
2. Quark distributions
   - sea quarks
Sea quarks

- Because sea quarks & antiquarks are produced "radiatively" (by $g \rightarrow q\bar{q}$ radiation)

$\rightarrow$ expect flavour-symmetric sea

IF quark masses are the same

$\rightarrow$ e.g. since $m_s \gg m_d \implies \bar{d}(x) > \bar{s}(x)$

- BUT since $m_u \approx m_d \implies$ expect $\bar{d}(x) \approx \bar{u}(x)$
Fermilab E866 Drell-Yan experiment

$q\bar{q}$ annihilation in hadron-hadron collisions

$q\bar{q} \rightarrow \gamma^* \rightarrow \mu^+\mu^-$

$$\frac{d^2\sigma}{dx_bdx_t} = \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2 \left( q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t) \right)$$

For $x_b \gg x_t$

$$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2} \left( 1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)} \right)$$

Drell, Yan, Phys. Rev. Lett. 25 (1970) 316
Sea quarks

- Large $d - u$ asymmetry in proton observed in DIS (NMC) and Drell-Yan (CERN NA51 and FNAL E866) experiments

\[
\int_0^1 dx \left( \bar{d}(x) - \bar{u}(x) \right) = 0.118 \pm 0.012
\]


why is $d \gg u$?
Sea quarks

- Pauli Exclusion Principle

It's not worth worrying about. There's nothing you can do about it. No two quarks in a small region can occupy the same quantum-mechanical state.

BARTENDER WITH PH.D. IN PHYSICS
Sea quarks

- Pauli Exclusion Principle

- Since proton has more valence $u$ than $d$
- Easier to create $d\bar{d}$ than $u\bar{u}$

  *Field, Feynman, Phys. Rev. D15 (1977) 2590*

- Explicit calculations of antisymmetrization effects in $g \rightarrow u\bar{u}$ and $g \rightarrow d\bar{d}$

- $\bar{u} > \bar{d}$
- Asymmetry tiny

  *Ross, Sachrajda, Nucl. Phys. B149 (1979) 497*
  *Steffens, Thomas, Phys. Rev. 55 (1997) 900*
"But, Heisenberg - you must be certain about something!"
Sea quarks

- Pion cloud

  - some of the time the proton looks like a neutron & $\pi^+$
    
    \[(Heisenberg \ Uncertainty \ Principle)\]

    \[p \rightarrow \pi^+ \quad n \rightarrow p\]

  - at the quark level

    \[uud \rightarrow (u dd)(\bar{d}u) \rightarrow u ud\]

    \[\bar{d} > \bar{u}!\]

\[Thomas, \ Phys. \ Lett. \ 126B \ (1983) \ 97\]
$$p(uud) \rightarrow \pi^-(du) + \Delta^{++}(uuu)$$

$$\Rightarrow \bar{u} > \bar{d}$$

→ difficult to understand quantitatively large $x$ behavior

→ JLab can significantly improve uncertainties at large $x$
\begin{itemize}
\item FNAL E866
\item FNAL E906 Projection
\item JLab Hall A Projected 60 days with 11 GeV/c (statistical uncertainties only)
\end{itemize}
Polarization asymmetry of proton sea  
(aside...)  

Neither gluon radiation nor pion cloud contribute to $\Delta \bar{d} - \Delta \bar{u}$  

Pauli Exclusion Principle (antisymmetrization)  

\[ \Delta \bar{u} - \Delta \bar{d} \approx \frac{5}{3} (\bar{d} - \bar{u}) \]  

also contributes to $\bar{d} - \bar{u}$  

Disentangle origin of unpolarized and polarized asymmetries in sea via semi-inclusive DIS
Polarization asymmetry of proton sea  
(aside...)

\[ \Delta \bar{u} - \Delta \bar{d} \]

HERMES preliminary (1996-2000)

\[ Q^2 = 2.5 \text{ GeV}^2 \]
Polarization asymmetry of proton sea

(aside...)
Sea quarks

Strange asymmetry

$s \neq \bar{s}$ can similarly be generated by nonperturbative kaon cloud

\[ \int_0^1 dx (s - \bar{s}) = 0 \]


net number of strange quarks must be zero
Sea quarks

- Strange asymmetry

Shape very sensitive to details of $Kp\Lambda$

Sea quarks

Strange asymmetry

shape from global fits also not well constrained

B. Portheault, hep-ph/0406226
S. Kretzer, hep-ph/0408287
Strange asymmetry

can also be generated perturbatively by higher-order (3-loop) gluon radiation

... though cannot predict shape

$s \not= \bar{s}$ can have significant impact on extraction of $\sin^2 \theta_W$ from $\nu$, $\bar{\nu}$ data
Sea quarks

- Charm structure function

Photon-gluon fusion

![Graphs showing charm structure functions](image)

- Disagreement with perturbative charm??
Sea quarks

Intrinsic (nonperturbative) charm

\[ |p\rangle = c_0 |uud\rangle + c_1 |uudc\bar{c}\rangle \quad \text{1\% normalisation} \]

\[ c^{IC1}(x) = 6x^2 ((1 - x)(1 + 10x + x^2) - 6x(1 + x) \log 1/x) \]

Meson cloud model

\[ c^{IC2}(x) = \int_x^{1} \frac{dz}{z} f_{\Lambda_c/N}(z) \, c^{\Lambda_c}(x/z) \]

\[ \approx \frac{3}{2} f_{\Lambda_c/N}(3x/2) \]

\[ \bar{c}^{IC2}(x) = \int_x^{1} \frac{dz}{z} f_{\bar{D}/N}(z) \, \bar{c}^{D-}(x/z) \]

\[ \approx f_{\bar{D}/N}(x) \]
Sea quarks

- Perturbative + intrinsic charm

\[ F_2^c \]

\[ q^2 = 45 \text{GeV}^2 \]

\[ q^2 = 60 \text{GeV}^2 \]

- need more data at large \( x \)!
2. Quark distributions
   - valence quarks
Valence quarks

- Most direct connection between quark distributions and models of the nucleon is through *valence* quarks
- Nucleon structure at intermediate & large $x$ dominated by valence quarks
Valence quarks

- At large $x$, valence $u$ and $d$ distributions extracted from $p$ and $n$ structure functions

$$F_{2}^{p} \approx \frac{4}{9}u_{v} + \frac{1}{9}d_{v}$$

$$F_{2}^{n} \approx \frac{4}{9}d_{v} + \frac{1}{9}u_{v}$$

- $u$ quark distribution well determined from $p$

- $d$ quark distribution requires $n$ structure function

$$\frac{d/2}{u} \approx \frac{4 - F_{2}^{n} / F_{2}^{p}}{4F_{2}^{n} / F_{2}^{p} - 1}$$
Valence quarks

- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon

- SU(6) spin-flavour symmetry

proton wave function

$$p^\uparrow = -\frac{1}{3}d^\uparrow(uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow(uu)_1$$

$$+ \frac{\sqrt{2}}{6}u^\uparrow(ud)_1 - \frac{1}{3}u^\downarrow(ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow(ud)_0$$

- Interacting quark
- Diquark spin
- Spectator diquark
Valence quarks

- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon
- SU(6) spin-flavour symmetry

Proton wave function

\[ p^\uparrow = -\frac{1}{3} d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3} d^\downarrow (uu)_1 \]
\[ + \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0 \]

$\rightarrow u(x) = 2 \ d(x) \ \text{for all} \ x$

$\frac{F_n^m}{F_p^p} = \frac{2}{3}$
Valence quarks

**scalar diquark dominance**

\[ M_\Delta > M_N \implies (qq)_1 \text{ has larger energy than } (qq)_0 \]

\[ \implies \text{ scalar diquark dominant in } x \to 1 \text{ limit} \]

since only \( u \) quarks couple to scalar diquarks

\[
\frac{d}{u} \to 0
\]

\[
\frac{F^p_2}{F^n_2} \to \frac{1}{4}
\]

Valence quarks

- **hard gluon exchange**

at large $x$, helicity of struck quark = helicity of hadron

$\Rightarrow$ **helicity-zero diquark dominant in** $x \rightarrow 1$ limit

\[
\begin{align*}
&\frac{d}{u} \rightarrow \frac{1}{5} \\
&\frac{F_{2}^{n}}{F_{2}^{p}} \rightarrow \frac{3}{7}
\end{align*}
\]

Farrar, Jackson 1975
Valence quarks

**BUT**  no free neutron targets!
(neutron half-life ~ 12 mins)

- use deuteron as “effective neutron target”

**However:** deuteron is a nucleus, and $F_{2}^{d} \neq F_{2}^{p} + F_{2}^{n}$

- nuclear effects (nuclear binding, Fermi motion, shadowing)
  *obscure neutron structure information*

- “nuclear EMC effect”
2. Quark distributions
   - nuclear effects
Nuclear “EMC effect”

\[ F_2^A(x, Q^2) \neq A F_2^N(x, Q^2) \]

Original EMC data

Later SLAC data


Nuclear “EMC effect”

- **anti-shadowing pions?**
- **Fermi motion**
- **shadowing**
- **multiple scattering**

what about $d / N$?

“EMC effect” binding, $N$ off-shell
EMC effect in deuteron

Nuclear “impulse approximation”

\[ F_2^d(x) = \int dy \ f_{N/d}(y) \ F_2^N(x/y) + \delta^{(\text{off})} F_2^d(x) \]

nucleon momentum distribution

off-shell correction
EMC effect in deuteron

Nucleon momentum distribution in deuteron

\[ f_{N/d}(y) = \frac{1}{4} M_d y \int_{-\infty}^{p_{\text{max}}^2} dp^2 \frac{E_p}{p_0} |\Psi_d(p^2)|^2 \]

momentum fraction of deuteron carried by nucleon
EMC effect in deuteron

Nucleon momentum distribution in deuteron

relativistic $dNN$ vertex function

\[ f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}^2} dp^2 \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^2) \right|^2 \]
EMC effect in deuteron

Nucleon momentum distribution in deuteron

\[ f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}} dp^2 \frac{E_p}{p_0} |\Psi_d(p^2)|^2 \]

Wave function dependence only at large \(|y-1/2|\)

- sensitive to large \(p\) components of wave function
- not very well known
**EMC effect in deuteron**

Nucleon momentum distribution in deuteron

→ relativistic $dNN$ vertex function

$$ f_{N/d}(y) = \frac{1}{4} M_d \ y \int_{-\infty}^{p_{\text{max}}^2} dp^2 \frac{E_p}{p_0} \left| \Psi_d(\vec{p}^2) \right|^2 $$

Nucleon off-shell correction

$$ \delta^{(\text{off})} F_2^d \quad \rightarrow \quad \delta^{(\Psi)} F_2^d \quad \text{negative energy components of } d \text{ wave function} $$

$$ \delta^{(p^2)} F_2^d \quad \rightarrow \quad \delta^{(\vec{p}^2)} F_2^d \quad \text{off-shell } N \text{ structure function} $$
Off-shell correction

\[\leq 1 - 2\%\] effect

\[\delta(p^2) F_2^d\]

\[\delta(\Psi) F_2^d\]
EMC effect in deuteron

![Graph showing the ratio of $F_2^D/F_2^N$ as a function of $x$ for different models. The full model and the light-cone model are compared, with annotations for Fermi motion only and with binding + off-shell. The graph indicates a larger EMC effect (smaller $d/N$ ratio) with $F_2^n$ underestimated at large $x$.](image)
Unsmearing

Note: calculated $d/N$ ratio depends on input $F_2^n$

extracted $n$ depends on input $n$ ... cyclic argument

Solution: iteration procedure

0. subtract $\delta^{(off)} F_2^d$ from d data: $F_2^d \rightarrow F_2^d - \delta^{(off)} F_2^d$

1. smear $F_2^p$ with $f_{N/d}$: $f_{N/d} \otimes F_2^p \equiv S_p F_2^p$

2. extract neutron via $F_2^n = S_n (F_2^d - F_2^p / S_p)$
   starting with e.g. $S_n = S_p$

3. smear $F_2^n$ with $f_{N/d}$ to get new $S_n$

4. repeat 2-3 until convergence
Unsmearing

\[
\frac{F_2^n}{F_2^p} \quad x
\]


- good convergence after several iterations
- resulting \( F_2^n \) independent of starting assumptions
- depends only on smearing function \( f_{N/d} \)
Effect on $n/p$ ratio

$F_2 n/F_2 p$

with binding & off-shell

Fermi motion only

uncertainty due to nuclear effects in neutron (full range of nuclear models)

$d$ distribution poorly known beyond $x \sim 0.5$
“Cleaner” methods of determining $d/u$

\[ e^\pm \, p \rightarrow \nu(\bar{\nu})X \] need high luminosity

\[ \nu(\bar{\nu}) \, p \rightarrow l^\mp \, X \] low statistics

\[ p \, p(\bar{p}) \rightarrow W^\pm \, X \] need large lepton rapidity

\[ \bar{e}_L(\bar{e}_R) \, p \rightarrow e \, X \] low count rate

\[ e \, p \rightarrow e \, \pi^\pm \, X \] need $z \sim 1$, factorization

\[ e \, ^3\text{He}(^3\text{H}) \rightarrow e \, X \] tritium target
“Cleaner” methods of determining $d/u$

\[ e \, d \rightarrow e \, p \, X \]

- Target $d$
- Recoil $p$
- Slow backward $p$
- Neutron nearly on-shell
- Minimize rescattering

JLab Hall B experiment (“BoNuS”) completed run Dec. 2005
2. Quark distributions
   - nuclear shadowing
Nuclear shadowing

Interference of multiple scattering amplitudes

For deuteron:

Nuclear impulse approximation

Double scattering

*e.g. Piller, Weise, Phys. Rep. 330 (2000) 1*
Nuclear shadowing

Interference of multiple scattering amplitudes

For deuteron:

\[
F_2^d = F_2^p + F_2^n + \delta^{(\text{shad})} F_2^d
\]

*Fig. 5.1. Single (a) and double (b) scattering contribution to virtual ... part of this amplitude. In fact, we expect \( \text{Re} \, T_{\gamma^*N \to XN} < \sim 0.15 \, \text{Im} \, T_{\gamma^*N \to XN} \) by analogy with high-energy hadron-hadron interactions.*

*e.g. Piller, Weise, Phys. Rep. 330 (2000) 1*
Space-time view of shadowing

propagation length of hadronic fluctuation of mass $\mu$ (in lab frame)

$$\lambda \sim \frac{1}{\Delta E} = \frac{2\nu}{\mu^2 + Q^2} \rightarrow \frac{1}{2xM} , \quad \mu^2 \sim Q^2$$

if propagation length exceeds average distance between nucleons $\lambda > d \approx 2$ fm

coherent multiple scattering can occur

$$x < 0.05$$
Shadowing in deuterium

\[ Q^2 = 4 \text{ GeV}^2 \]

vector meson dominance
(higher twist)
Shadowing in deuterium

Pomeron exchange

\[ \delta F_{2D}(x) \]

-0.006
-0.004
-0.002
-0.000
0.000
0.002
0.004

\[ x \]

Bochum
VMD
Pomeron
Meson
\( (\Lambda_{\mu} = 1.3 \text{GeV}) \)
Anti-shadowing in deuterium

meson (pion) exchange
VMD important even at moderate $Q^2$
Shadowing in nuclei

Perturbative or nonperturbative origin of \( Q^2 \) dependence?

\[ Q^2 = 1 - 140 \text{ GeV}^2 \]

\( \frac{d}{d \ln Q^2} (\frac{F_{2n}}{F_{2c}}) \)

**Graphs:**
- IP + VMD
- IP

**References:**
Comparison with data

- see also Badelek, Kwicinski (1992), Nikolaev, Zoller (1992)
Effect on neutron structure function at small $x$

\[
\frac{F_2^n}{(F_2^n)_{\text{exp}}} = 1 - \frac{\delta F_2^d}{F_2^d} \left( 1 + \frac{(F_2^n / F_2^p)_{\text{exp}}}{(F_2^n / F_2^p)_{\text{exp}}} \right)
\]

where “experimental” $n/p$ ratio is defined as

\[
\left. \frac{F_2^n}{F_2^p} \right|_{\text{exp}} \equiv \frac{F_2^d}{F_2^p} - 1
\]
Effect on neutron structure function at small $x$

1-2% enhancement at $x \sim 0.01$
Gottfried sum rule

Integrated difference of $p$ and $n$ structure functions

$$S_G = \int_0^1 dx \frac{F_2^p(x) - F_2^n(x)}{x}$$

$$= \frac{1}{3} + \frac{2}{3} \int_0^1 dx \left( \bar{u}(x) - \bar{d}(x) \right)$$

Experiment: $S_G = 0.235 \pm 0.026$

NMC, Phys. Rev. D 50 (1994) 1

\[ \bar{d}(x) \neq \bar{u}(x) \quad \text{flavor asymmetric sea} \]
Saturation of Gottfried sum rule

\[ S_G(x, 1) = \int_x^1 dx' \frac{F^p_2(x') - F^n_2(x')}{x'} \]

\[ S_G(x, 1) = \int_1^x dx' \frac{F^p_2(x') - F^n_2(x')}{x'} \]
Saturation of Gottfried sum rule

$S_G(0, 1) \approx -0.02$

~ 10% decrease due to shadowing
2. Quark distributions - from lattice QCD
Lattice QCD

Solve QCD equations of motion *numerically* on discretized space-time grid

\[ U_\mu(x) = \exp i g a \int_0^1 dt A_\mu(x + t\mu) \]

- quarks on lattice nodes
- gluons as links between nodes
Observables calculated from path integrals in Euclidean space

\[ \langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \ O(U)e^{-S_G(U)} \]

generating functional

\[ Z = \int \mathcal{D}U \ \det M(U) \ e^{-S_G(U)} \]

Fermion mass matrix

\[ M(x, y, U) = m \delta_{x,y} + \frac{1}{2} \sum_{\mu} \gamma_{\mu} (U_{\mu}(x) \delta_{y,x+\hat{\mu}} - U_{\mu}^\dagger(x - \hat{\mu}) \delta_{y,x-\hat{\mu}}) \]

Approximations

- finite lattice spacing \( a \) \( \to 0 \)
- finite lattice volume \( V \) \( \to \infty \)
- large quark mass \( m_q \) \( \to m_q^{\text{phys}} \) \( \implies \text{cost} \propto m_q^{-4} \)
- “quenching” - suppression of background \( q\bar{q} \) loops \( \implies \det M \to 1 \)
PDFs from Lattice QCD

Cannot calculate $x$-distribution on lattice
(no light-cone in Euclidean space) - only moments

$$\langle x^n \rangle_q = \int_0^1 dx \ x^n \left( q(x) + (-1)^{n+1} \bar{q}(x) \right)$$

use OPE to relate moments of PDFs to matrix elements of local operators

$$\langle x^n \rangle \ p_{\mu_1} \cdots p_{\mu_{n+1}} = \langle N | \mathcal{O}_{\mu_1 \cdots \mu_{n+1}} | N \rangle$$

twist-2 operators
overestimates lowest moment of $u-d$ by $\sim 50\%$!
Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies, their moments have chiral expansion.

FIG. 3: Contributions to the \( \pi \) moments.
Chiral extrapolation of lattice moments

Even though structure functions are measured at high energies their moments have chiral expansion

\[
\langle x^n \rangle_{u-d} = a_n \left( 1 + c_{\text{LNA}} m^2_\pi \log \frac{m^2_\pi}{m^2_\pi + \mu^2} \right) + b_n \frac{m^2_\pi}{m^2_\pi + m^2_{b,n}}
\]


Leading non-analytic coefficient (non-analytic in \( m_q \sim m^2_\pi \))

\[
c_{\text{LNA}} = -(1 + 3g^2_A)/(4\pi f_\pi)^2
\]

calculated from chiral perturbation theory

Arndt, Savage (2001)
Ji, Chen (2001)
PDF in heavy quark limit

\[ u(x) - d(x) \xrightarrow{m_q \to \infty} \delta(x - \frac{1}{3}) \]

Moment

\[ \left\langle x^n \right\rangle_{u-d} \xrightarrow{m_q \to \infty} \frac{1}{3^n} \]

Coefficient ensures correct \( m_\pi \to \infty \) behavior

\[ b_n = \frac{1}{3^n} - a_n \left( 1 - \mu^2 c_{\text{LNA}} \right) \]

Parameter \( \mu \) determines amount of curvature at low \( m_\pi^2 \)

\[ (m_\pi^2 \propto m_q) \]
Chiral physics *vital* for understanding lattice data
Odds and evens

For unpolarized parton distributions

- $n$ even $\rightarrow$ total $q + \bar{q}$

- $n$ odd $\rightarrow$ valence $q - \bar{q}$

If have sufficient number of moments

- fit odd and even moments separately to obtain both valence and total

- subtract $2 \times$ empirical sea from odd moments

$$q_v \equiv q - \bar{q} = q + \bar{q} - 2\bar{q}$$
Chiral extrapolation of valence moments

Moments of $u_v - d_v$ (scaled by $3^n$)
How well can one reconstruct PDFs from a few moments?

Test case:

\[ xq(x) = ax^b(1 - x)^c(1 + \epsilon \sqrt{x} + \gamma x) \]

\[ \text{fit}(i) : 4 \text{ unconstrained parameters (} b, c, \epsilon, \gamma) \]

\[ \text{fit}(vii) : 2 \text{ unconstrained parameters (} b, c) \]
Reconstructed distribution

\[ xq(x) = ax^b(1 - x)^c(1 + \epsilon \sqrt{x} + \gamma x) \]
Quark mass dependence of PDFs

Looks like “constituent quark” distribution in heavy quark limit!
Connecting models with lattice QCD

Young, Wright, Leinweber, Thomas et al.
Connecting models with lattice QCD

• At large quark masses, observables display “constituent quark” behavior

\[ M_{\text{baryon}} \sim 3m_q \]
\[ M_{\text{meson}} \sim 2m_q \]

suggests new approach to modeling QCD

- construct “constituent quark” model at large quark masses
- extrapolate to physical quark mass using known chiral behavior
Summary - quark distributions

- **Sea quarks**
  - Asymmetry $\bar{d} > \bar{u}$ arises from nonperturbative QCD effects such as pion cloud of the nucleon
  - Similarly, strong indications that $s \neq \bar{s}$

- **Valence quarks**
  - $d$ quark poorly known at large $x$
  - $n$ structure obscured by nuclear effects in deuteron (also nuclear shadowing at small $x$)

- **Progress in extracting quark distributions from lattice QCD**
  - Need to extrapolate lattice data to physical regime
3. Quark-hadron duality
Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

\[ \sum \text{hadrons} = \sum \text{quarks} \]

Can use either set of complete basis states to describe all physical phenomena
Duality in Nature

- Duality between quarks (high energy) and hadrons (low energy) manifests itself in many processes

- $e^+ e^-$ annihilation
  - total hadronic cross section at high energy averages resonance cross section

- Heavy meson decays
  - duality between hadronic & quark descriptions of decays in $m_Q \to \infty$ limit

- Duality between $s$-channel resonances and $t$-channel (Regge) poles in hadronic reactions
\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]
\[ B \rightarrow D \ell \nu \]

\[ b \rightarrow c \ell \nu \]

Finite energy sum rules

Igi (1962), Dolen, Horn, Schmidt (1968)
3. Quark-hadron duality
   - Bloom-Gilman duality
Resonances

As $W$ decreases, DIS region gives way to region dominated by nucleon resonances

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$
scaling curve

resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Quark-hadron duality

Average over (strongly $Q^2$ dependent) resonances
$\approx Q^2$ independent scaling function

“Finite energy sum rule” for $eN$ scattering

$$\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \; F_2(\nu, Q^2) = \int_1^{\omega'} d\omega' \; F_2(\omega')$$

$$\omega' = 1/x + M^2/Q^2$$

Bloom-Gilman duality

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \]

Average over (strongly \( Q^2 \) dependent) resonances
\[ \approx Q^2 \text{ independent scaling function} \]

Jefferson Lab (Hall C)
Local Bloom-Gilman duality

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \]

Nachtmann scaling variable
Local Bloom-Gilman duality

Duality in $F_2$ and $F_L$ structure functions
(from longitudinal-transverse separation)

importance of target mass corrections

E. Christy et al. (2005)
Integrated strength

~10% agreement for $Q^2 > 1$ GeV$^2$

Niculescu et al,
Moments

data from longitudinal-transverse separation!
Nuclear structure functions

for larger nuclei, Fermi motion does resonance averaging automatically!
Neutron \( ^3\text{He} \) \( g_1 \) structure function

\[ g_1 \]

\[ \xi \]

PLATE 2: \( g_1 \) vs. \( \xi \) for different values of \( Q^2 \). The data points represent measurements from the JLab Hall A experiment, with error bars indicating uncertainties. The graph is preliminary and subject to further analysis and validation.
3. Quark-hadron duality - duality in QCD
Duality and QCD

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific “twist” $\tau$

$$\tau = \text{dimension} - \text{spin}$$
Higher twists

\( \tau = 2 \)  

single quark scattering

\( \tau > 2 \)  

qq and qg correlations
Duality and QCD

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of $Q^2$

higher twist terms $A_n^{(\tau>2)}$ small
Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\iff$ suppression of higher twists

*de Rujula, Georgi, Politzer, Ann. Phys. 103 (1975) 315*
Applications of duality

If higher twists are small (duality “works”)
- can use single-parton approximation to describe structure functions
- extract leading twist parton distributions

If duality is violated, and if violations are small
- can use duality violations to extract higher twist matrix elements
- learn about nonperturbative $qq$ or $qg$ correlations
**Example:**

The lowest moment of $g_1$

\[ \Gamma_1(Q^2) = \int_0^1 dx \, g_1(x, Q^2) \]

\[ = \mu_2 + \frac{\mu_4}{Q^2} + \frac{\mu_6}{Q^4} + \cdots \]

**Twist 2**

\[ \mu_2^{p(n)} = \left( \pm \frac{1}{12} g_A + \frac{1}{36} a_8 \right) C_{ns}(Q^2) + \frac{1}{9} \Delta \Sigma C_s(Q^2) \]

- **triplet**
- **octet**
- **RGI singlet axial charge**
Higher twist terms

$1/Q^2$ correction to $g_1$ moment

$$\mu_4 = \frac{1}{9} M^2 (a_2 + 4d_2 + 4f_2)$$

target mass correction

quark-gluon correlations
Higher twist terms

$1/Q^2$ correction to $g_1$ moment

$$\mu_4 = \frac{1}{9} M^2 (a_2 + 4d_2 + 4f_2)$$

$d_2 \to \langle N | \bar{\psi} \tilde{G}^{\mu\{\nu\gamma^{\alpha}\}} \psi | N \rangle$

twist 3

$f_2 \to \langle N | \bar{\psi} \tilde{G}^{\mu\nu} \gamma^{\nu} \psi | N \rangle$

twist 4
Color polarizabilities

\[ 1/Q^2 \] correction to \( g_1 \) moment

\[ \mu_4 = \frac{1}{9} M^2 (a_2 + 4d_2 + 4f_2) \]

**color electric polarizability**

\[ \chi_E = \frac{1}{3} (4d_2 + 2f_2) \sim \langle \vec{j}_a \times \vec{E}_a \rangle_z \]

**color magnetic polarizability**

\[ \chi_B = \frac{1}{3} (4d_2 - f_2) \sim \langle \vec{j}_a^0 \cdot \vec{B}_a \rangle_z \]

\[ j_a^\mu = g_s \psi \gamma^\mu t_a \psi \]

Ji (1995), Schafer, Mankiewicz, ... (1995)
Color polarizabilities

response of collective color electric and magnetic fields to spin of nucleon
Nachtmann moment

\[ M_1 = \int_0^1 dx \frac{x^2 \xi^2}{x^2} \left[ g_1 \left( \frac{x}{\xi} - \frac{M^2 x \xi}{9Q^2} \right) - g_2 \frac{4M^2 x^2}{3Q^2} \right] = \mu_2 + \frac{4M^2}{9Q^2} f_2 + \cdots \]

\[ \chi^p_E = 0.026 \pm 0.015 \text{ (stat)} \pm 0.021 \text{ (sys)} \]

\[ \chi^p_B = -0.013 \pm 0.007 \text{ (stat)} \pm 0.011 \text{ (sys)} \]

**Compare with theoretical calculations:**

<table>
<thead>
<tr>
<th>Theory</th>
<th>( \chi^p_E )</th>
<th>( \chi^p_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD sum rules</td>
<td>−0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>MIT bag</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>Instanton</td>
<td>−0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Lattice</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
Neutron $g_1$ moment

$\Gamma_1^n$ extracted from $\Gamma_1^{3\text{He}}$ data correcting for nuclear effects

\[ \chi^n_E = +0.033 \pm 0.029 \]
\[ \chi^n_B = -0.001 \pm 0.016 \]

Compare with theoretical calculations:

<table>
<thead>
<tr>
<th>Method</th>
<th>( \chi^n_E )</th>
<th>( \chi^n_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD sum rules</td>
<td>-0.04</td>
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</tr>
<tr>
<td>Lattice</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
Higher twist contribution to neutron moment
Total higher twist $\sim$ zero at $Q^2 \sim 1 - 2$ GeV$^2$

- nonperturbative interactions between quarks and gluons not dominant at these scales

- suggests strong cancellations between resonances, resulting in dominance of leading twist

- OPE does not tell us why higher twists are small!
Can we understand this behavior dynamically?

*How* do cancellations between *coherent* resonances produce *incoherent* scaling function?
3.

Quark-hadron duality

- local duality
Coherence vs. incoherence

Exclusive form factors

\[ d\sigma \sim \left( \sum_i e_i \right)^2 \]

Inclusive structure functions

\[ d\sigma \sim \sum_i e_i^2 \]

How can the square of a sum become the sum of squares?
Pedagogical model

Two quarks bound in a harmonic oscillator potential

\[ F(\nu, q^2) \sim \sum_{n} |G_{0,n}(q^2)|^2 \delta(E_n - E_0 - \nu) \]

Charge operator \( \sum_i e_i \exp(iq \cdot r_i) \) excites

\textit{even} partial waves with strength \( \propto (e_1 + e_2)^2 \)

\textit{odd} partial waves with strength \( \propto (e_1 - e_2)^2 \)
Pedagogical model

Resulting structure function

\[ F(\nu, q^2) \sim \sum_n \{(e_1 + e_2)^2 G^2_{0,2n} + (e_1 - e_2)^2 G^2_{0,2n+1}\} \]

If states degenerate, cross terms \((\sim e_1 e_2)\) cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

- at least one complete set of even and odd parity resonances must be summed over

Even and odd parity states generalize to $56^+ (L=0)$ and $70^- (L=1)$ multiplets of spin-flavor SU(6)

Scaling occurs if contributions from $56^+$ and $70^-$ have equal overall strengths

Simplified case: magnetic coupling of $\gamma^*$ to quark

Expect dominance over electric at large $Q^2$
Quark model

Even and odd parity states generalize to $56^+ (L=0)$ and $70^- (L=1)$ multiplets of spin-flavor SU(6) scaling occurs if contributions from $56^+$ and $70^-$ have equal overall strengths

<table>
<thead>
<tr>
<th>representation</th>
<th>$^28[56^+]$</th>
<th>$^410[56^+]$</th>
<th>$^28[70^-]$</th>
<th>$^48[70^-]$</th>
<th>$^210[70^-]$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9\rho^2$</td>
<td>$8\lambda^2$</td>
<td>$9\rho^2$</td>
<td>0</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 + 9\lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$8\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$4\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 + 27\lambda^2)/2$</td>
</tr>
<tr>
<td>$g_1^p$</td>
<td>$9\rho^2$</td>
<td>$-4\lambda^2$</td>
<td>$9\rho^2$</td>
<td>0</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 - 3\lambda^2$</td>
</tr>
<tr>
<td>$g_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$-4\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$-2\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 - 9\lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda (\rho) = \text{(anti) symmetric component of ground state wfn.}$
Quark model

SU(6) limit \( \lambda = \rho \)

<table>
<thead>
<tr>
<th>( SU(6) ) :</th>
<th>([56, 0^+]^2 )</th>
<th>([56, 0^+]^4 )</th>
<th>([70, 1^-]^2 )</th>
<th>([70, 1^-]^4 )</th>
<th>([70, 1^-]^2 )</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1^p )</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>( F_1^n )</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>( g_1^p )</td>
<td>9</td>
<td>-4</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>( g_1^n )</td>
<td>4</td>
<td>-4</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Summing over all resonances in \( 56^+ \) and \( 70^- \) multiplets

\[
R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3} \quad A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9} \quad A_1^n = \frac{g_1^n}{F_1^n} = 0
\]

as in quark-parton model!
**Quark model**

**SU(6) limit**  \[ \lambda = \rho \]

<table>
<thead>
<tr>
<th>SU(6) :</th>
<th>[56, 0+]^28</th>
<th>[56, 0+]^10</th>
<th>[70, 1−]^28</th>
<th>[70, 1−]^48</th>
<th>[70, 1−]^210</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^p_1 )</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>( F^n_1 )</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>( g^p_1 )</td>
<td>9</td>
<td>−4</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>( g^n_1 )</td>
<td>4</td>
<td>−4</td>
<td>1</td>
<td>−2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

- expect duality to appear earlier for \( F^p_1 \) than \( F^n_1 \)
- earlier onset for \( g^n_1 \) than \( g^p_1 \)
- cancellations **within** multiplets for \( g^n_1 \)
Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

*But* significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No ( ^410 )</th>
<th>No ( ^210, ^410 )</th>
<th>No ( S_{3/2} )</th>
<th>No ( \sigma_{3/2} )</th>
<th>No ( \psi_\lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^{np} )</td>
<td>2/3</td>
<td>10/19</td>
<td>1/2</td>
<td>6/19</td>
<td>3/7</td>
<td>1/4</td>
</tr>
<tr>
<td>( A_1^p )</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A_1^n )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \Delta u/u > 1 \] inconsistent with duality
Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

But significant deviations at large $x$

which combinations of resonances reproduce behavior of structure functions at large $x$?

<table>
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<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No $^410$</th>
<th>No $^210, ^410$</th>
<th>No $S_{3/2}$</th>
<th>No $\sigma_{3/2}$</th>
<th>No $\psi_\lambda$</th>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A_1^n$</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

$^410 [56^+]$ and $^48 [70^-]$ suppressed
Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

**But** significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No ( {}^4!10 )</th>
<th>No ( {}^2!10, {}^4!10 )</th>
<th>No ( S_{3/2} )</th>
<th>No ( \sigma_{3/2} )</th>
<th>No ( \psi_\lambda )</th>
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<td>3/7</td>
<td>1/4</td>
</tr>
<tr>
<td>( A_1^p )</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A_1^n )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

helicity 3/2 suppression
$N \rightarrow N^*$ transitions for helicity-1/2 dominance

<table>
<thead>
<tr>
<th>SU(6) representation</th>
<th>$^28[56^+]$</th>
<th>$^410[56^+]$</th>
<th>$^28[70^-]$</th>
<th>$^48[70^-]$</th>
<th>$^210[70^-]$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p = g_1^p$</td>
<td>9</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>$F_1^n = g_1^n$</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>9</td>
</tr>
</tbody>
</table>

polarization asymmetries $A_1^N \rightarrow 1$

cf. pQCD “counting rules”

hard gluon exchange between quarks

neutron to proton ratio $F_2^n / F_2^p \rightarrow 3/7$

cf. “helicity retention” model

Farrar, Jackson, Phys. Rev. Lett. 35 (1975) 1416
Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

**But** significant deviations at large $x$

which combinations of resonances reproduce behavior of structure functions at large $x$?

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>$^4\text{10}$</th>
<th>$^2\text{10}, ^4\text{10}$</th>
<th>$S_{3/2}$</th>
<th>$\sigma_{3/2}$</th>
<th>$\psi_\lambda$</th>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A_1^n$</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

e.g. through $\vec{S}_i \cdot \vec{S}_j$ interaction between quarks

suppression of symmetric part of spin-flavor wfn.
Fit to \( \{ \text{SU}(6) \text{ symmetry at } x \sim 1/3 \}
\]
\( \text{SU}(6) \text{ breaking at } x \sim 1 \)

\[ R^{np} \]

uncertainty in \( F_2^m \) due to nuclear effects in deuteron
Polarization asymmetry $A_1^p$

![Graph showing polarization asymmetry $A_1^p$ vs. $x$]
Polarization asymmetry $A_1^\pi$

$$
\begin{align*}
A_1^\pi(x) &= 0.5 \quad \text{for} \quad 0 < x < 0.2 \\
A_1^\pi(x) &= 0.8 \quad \text{for} \quad 0.2 < x < 1
\end{align*}
$$

For $x < 0.2$, the asymmetry $A_1^\pi$ is constant at 0.5, and for $x > 0.2$, it increases to 0.8. The plot shows the behavior of $A_1^\pi$ as a function of $x$. The curves $\sigma_{1/2}$, $S_{1/2}$, and $\psi_\rho$ represent different scenarios, with $\sigma_{1/2}$ being the dominant contribution in the range of $0 < x < 0.2$. The SU(6) model is also indicated on the plot.
\[
\Delta \frac{d}{d} = \frac{4}{15} A_1^n \left( 4 + \frac{u}{d} \right) - \frac{1}{15} A_1^p \left( 1 + 4 \frac{u}{d} \right)
\]

\[
\frac{u}{d} = \frac{4 - R_{np}}{4R_{np} - 1}
\]

\[
\Delta d = \frac{4}{15} A_1^n \left( 4 + \frac{u}{d} \right) - \frac{1}{15} A_1^p \left( 1 + 4 \frac{u}{d} \right)
\]

\[
u \frac{d}{d} = \frac{4 - R_{np}}{4R_{np} - 1}
\]

Summary - quark-hadron duality

- Remarkable confirmation of quark-hadron duality in structure functions
  → higher twists “small” down to low $Q^2(\sim 1 \text{ GeV}^2)$

- Use duality violations to extract higher twist matrix elements → color polarizabilities

- Quark models provide clues to origin of resonance cancellations → local duality

- Practical applications
  → broaden kinematic region for studying
  (leading and higher twist) quark-gluon structure of nucleon
4. Form factors
Elastic $eN$ scattering

Elastic cross section

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \frac{\tau}{\varepsilon (1 + \tau)} \sigma_R$$

$$\tau = \frac{Q^2}{4M^2}$$

$$\varepsilon = \left(1 + 2(1 + \tau) \tan^2 \left(\frac{\theta}{2}\right)\right)^{-1}$$

$$\sigma_{\text{Mott}} = \frac{\alpha^2 E' \cos^2 \frac{\theta}{2}}{4E^3 \sin^4 \frac{\theta}{2}}$$

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$G_E$, $G_M$ Sachs electric and magnetic form factors
Elastic $eN$ scattering

In Breit frame

\[ \nu = 0, \quad Q^2 = \vec{q}^2 \]

electromagnetic current is

\[ \bar{u}(p', s') \Gamma^\mu u(p, s) = \chi_{s'}^\dagger \left( G_E + \frac{i\vec{\sigma} \times \vec{q}}{2M} G_M \right) \chi_s \]

cf. classical (\textbf{Non-Relativistic}) current density

\[ J^{NR} = \left( e \rho_E^{NR}, \mu \vec{\sigma} \times \vec{\nabla} \rho_M^{NR} \right) \]

\[ \rho_E^{NR}(r) = \frac{2}{\pi} \int_0^\infty dq \ \vec{q}^2 \ j_0(qr) \ G_E(\vec{q}^2) \quad \text{charge density} \]

\[ \mu \rho_M^{NR}(r) = \frac{2}{\pi} \int_0^\infty dq \ \vec{q}^2 \ j_0(qr) \ G_M(\vec{q}^2) \quad \text{magnetisation density} \]
Until recently...

Proton Neutron

Electric

Magnetic

Before JLab

JLab data on the EM form factors provide a testing ground for theories constructing nucleons from quarks and glue.

Until recently...
Latest data...

Proton

Neutron

Electric

Magnetic

Today JLab data on the EM form factors provide a testing ground for theories constructing nucleons from quarks and glue.
note neutron $\rho_E > 0$ at small $r$, but $< 0$ at larger $r$

same physics which gives $\bar{d} > \bar{u}$
also gives shape of neutron $\rho_E$

- pion cloud

Surprising result for $G_E^p/G_M^p$

→ expect $G_E^p/G_M^p \rightarrow$ constant at high $Q^2$

→ implies very different proton charge and magnetization densities at small $r$

Are the $G_E^p/G_M^p$ data consistent?
Proton $G_E/G_M$ Ratio

Rosenbluth (Longitudinal-Transverse) Separation

**LT method**

\[
\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)
\]

\[
\tau = \frac{Q^2}{4M^2}
\]

\[
\varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}\right]^{-1}
\]

$G_E/G_M$ from slope in $\varepsilon$ plot
Proton $G_E/G_M$ Ratio

Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau = Q^2/4M^2$$

$$\varepsilon = \left[1 + 2(1 + \tau) \tan^2 \theta/2\right]^{-1}$$

$G_E/G_M$ from slope in $\varepsilon$ plot

PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

$P_{T,L}$ polarization of recoil proton
Proton $G_E/G_M$ Ratio

Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

PT method

Why is there a discrepancy between the two methods?
4. Form factors
- two photon exchange
QED Radiative Corrections

cross section modified by $1\gamma$ loop effects

$$d\sigma = d\sigma_0 (1 + \delta)$$

$\delta$ contains additional $\varepsilon$ dependence

mostly from box (and crossed box) diagram

infrared divergences cancel
Box diagram

\[ M_{\gamma\gamma} = e^4 \int \frac{d^4 k}{(2\pi)^4} \frac{N(k)}{D(k)} \]

where

\[ N(k) = \bar{u}(p_3) \gamma_\mu (p_1 - k + m_e) \gamma_\nu u(p_1) \times \bar{u}(p_4) \Gamma_\mu (q - k) (p_2 + k + M) \Gamma_\nu (k) u(p_2) \]

and

\[ D(k) = (k^2 - \lambda^2) \left( ((k - q)^2 - \lambda^2) \times \left( ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2) \right) \right) \]

with \( \lambda \) an IR regulator, and e.m. current is

\[ \Gamma_\mu (q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \]
Various approximations to $M_{\gamma\gamma}$ used

- **Mo-Tsai:** soft $\gamma$ approximation
  - integrand most singular when $k = 0$ and $k = q$
  - replace $\gamma$ propagator which is not at pole by $1/q^2$
  - approximate numerator $N(k) \approx N(0)$
  - neglect all structure effects

- **Maximon-Tjon:** improved loop calculation
  - exact treatment of propagators
  - still evaluate $N(k)$ at $k = 0$
  - first study of form factor effects
  - additional $\varepsilon$ dependence

- **Blunden-WM-Tjon:** exact loop calculation
  - no approximation in $N(k)$ or $D(k)$
  - include form factors
Two-photon correction

\[ \delta(2\gamma) \rightarrow \frac{2\text{Re}\{M_0^\dagger M_{\gamma\gamma}\}}{|M_0|^2} \]

\[ \delta_{\text{full}}^{(2\gamma)} - \delta_{\text{Mo-Tsai}}^{(2\gamma)} \]

\[ \varepsilon \]

\[ \Delta(\varepsilon, Q^2) \]

\[ Q^2 \]

few % magnitude

positive slope

non-linearity in \( \varepsilon \)

Blunden, WM, Tjon
PRL 91 (2003) 142304;
PRC72 (2005) 034612
Effect on cross section

(a) SLAC

(b) JLab

Born cross section with PT form factors
including TPE effects

* Super-Rosenbluth
Qattan et al.,
PRL 94, 142301 (2005)
**Comparison of Elastic Cross Sections**

- **1γ Exchange** changes sign under $e^+ \leftrightarrow e^-$
- **2γ Exchange** is invariant under $e^+ \leftrightarrow e^-$
- Ratio of $e^+p/e^-p$ elastic cross sections sensitive to $\Delta(\varepsilon, Q^2)$

\[
R^{e^+e^-} = \frac{d\sigma^{e^+}}{d\sigma^{e^-}} \approx 1 - 2\Delta
\]

**Graphical Description**
- Dot-dashed line: $Q^2 = 1$ GeV$^2$
- Dash-dotted line: $Q^2 = 3$ GeV$^2$
- Solid line: $Q^2 = 6$ GeV$^2$

**Additional Notes**
- Simultaneous $e^-p/e^+p$ measurement using tertiary $e^+/e^-$ beam planned in Hall B (to $Q^2 \sim 1$ GeV$^2$)
estimate effect of TPE on $\varepsilon$ dependence

approximate correction by linear function of $\varepsilon$

$$1 + \Delta \approx a + b\varepsilon$$

reduced cross section is then

$$\sigma_R \approx a \frac{G_M^2}{\mu^2} \left[ 1 + \frac{\varepsilon}{\mu^2 \tau} \left( R^2 (1 + \varepsilon b/a) + \mu^2 \tau b/a \right) \right]$$

where “true” ratio is

$$R^2 = \frac{\tilde{R}^2 - \mu^2 \tau b/a}{1 + \varepsilon b/a}$$

“effective” ratio contaminated by TPE

average value of $\varepsilon$ over range fitted
resolves much of the form factor discrepancy
how does TPE affect polarization transfer ratio?

\[ \tilde{R} = R \left( \frac{1 + \Delta_T}{1 + \Delta_L} \right) \]

where \( \Delta_{L,T} = \delta_{\text{full}} - \delta_{\text{Mo-Tsai}}^{\text{IR}} \) is finite part of \( 2\gamma \) contribution relative to IR part of Mo-Tsai

experimentally measure ratio of polarized to unpolarized cross sections

\[ \frac{P_{L,T}^{1\gamma + 2\gamma}}{P_{L,T}^{1\gamma}} = \frac{1 + \Delta_{L,T}}{1 + \Delta} \]
Longitudinal & transverse polarizations

* Note scales!

- Small effect on $P_L$
- Large effect on $P_T$
Does 2γ exchange affect polarization transfer data?

\[ \frac{G_E^p}{G_M^p} \text{ ratio} \]

Rosenbluth separation

polarization transfer corrected for 2γ exchange

large \( Q^2 \) data typically at large \( \varepsilon \)

\(< 3\% \) suppression at large \( Q^2 \)
4. Form factors
- excited intermediate states
Lowest mass excitation is $P_{33} \Delta$ resonance

→ relativistic $\gamma^* N\Delta$ vertex

$$\Gamma_{\gamma\Delta \rightarrow N}^{\nu\alpha}(p, q) \equiv iV_{\Delta in}^{\nu\alpha}(p, q) = i\frac{eF_\Delta(q^2)}{2M_\Delta} \left\{ g_1 \left[ g^{\nu\alpha}p^\beta - p^\nu \gamma^\alpha q^\beta - \gamma^\nu \gamma^\alpha p \cdot q + \gamma^\nu \gamma q^\alpha \right] 
+ g_2 \left[ p^\nu q^\alpha - g^{\nu\alpha} p \cdot q \right] + (g_3/M_\Delta) \left[ q^2 (p^\nu \gamma^\alpha - g^{\nu\alpha} q^\beta) + q^\nu (q^\alpha q^\beta - \gamma^\alpha p \cdot q) \right] \right\} \gamma_5 T_3$$

form factor

$$\frac{\Lambda_\Delta^4}{(\Lambda_\Delta^2 - q^2)^2}$$

→ coupling constants

$$g_1 \text{ magnetic} \quad \rightarrow \quad 7$$

$$g_2 - g_1 \text{ electric} \quad \rightarrow \quad 9$$

$$g_3 \text{ Coulomb} \quad \rightarrow \quad -2 \ldots 0$$
Two-photon exchange amplitude with $\Delta$ intermediate state

\[
\mathcal{M}_\Delta^{\gamma\gamma} = -e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^\Delta_{\text{box}}(k)}{D^\Delta_{\text{box}}(k)} - e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N^\Delta_{\text{out-box}}(k)}{D^\Delta_{\text{out-box}}(k)}
\]

**Numerators**

\[
N^\Delta_{\text{box}}(k) = \overline{U}(p_4) V^{\mu\alpha}_{\Delta\text{in}}(p_2 + k, q - k) [\not{\psi}_2 + k^\mu + M_\Delta] \mathcal{P}^{3/2}_{\alpha\beta} (p_2 + k) V_{\Delta\text{out}}^{\beta\nu}(p_2 + k, k) U(p_2)
\]

\[
\times \overline{u}(p_3) \gamma_\mu [\not{\psi}_1 - k^\mu + m_e] \gamma_\nu u(p_1)
\]

\[
N^\Delta_{\text{out-box}}(k) = \overline{U}(p_4) V^{\mu\alpha}_{\Delta\text{in}}(p_2 + k, q - k) [\not{\psi}_2 + k^\mu + M_\Delta] \mathcal{P}^{3/2}_{\alpha\beta} (p_2 + k) V_{\Delta\text{out}}^{\beta\nu}(p_2 + k, k) U(p_2)
\]

\[
\times \overline{u}(p_3) \gamma_\nu [\not{\psi}_3 + k^\nu + m_e] \gamma_\mu u(p_1)
\]

**Spin-3/2 projection operator**

\[
\mathcal{P}^{3/2}_{\alpha\beta} (p) = g_{\alpha\beta} - \frac{1}{3} \gamma_\alpha \gamma_\beta - \frac{1}{3p^2} (\not{\psi} \gamma_\alpha p_\beta + p_\alpha \gamma_\beta \not{\psi})
\]
\[ \Delta \] has opposite slope to \( N \)

cancels some of TPE correction from \( N \)

\[ \text{Kondratyuk, Blunden, WM, Tjon Phys. Rev. Lett. 2006} \]
The angular dependence of the nucleon and $\Delta$ contributions can be more complicated, especially at forward angles, as can be seen from Fig. ... including the nucleon and $\Delta$ two-photon exchange corrections has the angular dependence which can accommodate the weaker $\varepsilon$ dependence than with $N$ alone and better fit to JLab data!
\[ J^P = \frac{1^+}{2}, \frac{1^-}{2} \quad \text{excited } N^* \text{ states} \]

\[ Q^2 \sim 3 \text{ GeV}^2 \]

higher mass resonance contributions small

enhance nucleon elastic contribution

\[ Tjon, WM, et al. (2005) \]
4. Form factors
- effect on neutron
Neutron correction

since $G^n_E$ is small, effect may be relatively large

full correction - no IR contribution

sign opposite to proton (since $\kappa_n < 0$)

$Q^2 = 6 \text{ GeV}^2$

Blunden, WM, Tjon
Effect on neutron LT form factors

large effect at high $Q^2$ for LT-separation method

LT method unreliable for neutron
Effect on neutron PT form factors

\[ \frac{\mu_n G^n_E}{G^n_M} \]

- small correction for PT

- 4\% (3\%) suppression at \( \varepsilon = 0.3 \) (0.8) for \( Q^2 = 3 \) GeV\(^2\)

- 10\% (5\%) suppression at \( \varepsilon = 0.3 \) (0.8) for \( Q^2 = 6 \) GeV\(^2\)
Next 5 years

Proton

Neutron

Electric

Magnetic

To 9 GeV²

To 3.5 GeV²

JLab data on the EM form factors provide a testing ground for theories constructing nucleons from quarks and glue.
4. Form factors
- strangeness in the nucleon
Strangeness Widely Believed to Play a Major Role – Does It?

- As much as 100 to 300 MeV of proton mass:

\[
M_N = \langle N(P) | -\frac{9\alpha_s}{4\pi} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d + m_s \bar{\psi}_s \psi_s | N(P) \rangle
\]

\[
\Delta M_N^{\text{strange}} = \frac{y m_s}{m_u + m_d} \sigma_N
\]

\[
y = 0.2 \pm 0.2
\]

\[
\sigma_N = 45 \pm 8 \text{ MeV}
\]

\[
\Delta M_N^{\text{strange}} \sim 110 \pm 110 \text{ MeV}
\]

- Through proton spin crisis:
  As much as 10% of the spin of the proton

- HOW MUCH OF THE MAGNETIC FORM FACTOR?
Strangeness in the Nucleon

Proton and neutron electromagnetic form factors give two combinations of 3 unknowns

\[ G^p_{E,M} = \frac{2}{3} G^u_{E,M} - \frac{1}{3} G^d_{E,M} - \frac{1}{3} G^s_{E,M} \]

\[ G^m_{E,M} = \frac{2}{3} G^d_{E,M} - \frac{1}{3} G^u_{E,M} - \frac{1}{3} G^s_{E,M} \]

need 3rd observable to extract \( G^s_{E,M} \)

parity-violating \( e \) scattering (interference of \( \gamma \) and \( Z^0 \) exchange)
Strangeness in the Nucleon

G0 Experiment at Jefferson Lab

- magnet
- beam line
- detectors
- target service vessel

Operated by the Southeastern Universities Research Association for the U.S. Department of Energy

Thomas Jefferson National Accelerator Facility

target service vessel

beam line

magnet

detectors

target service vessel
Strangeness in the Nucleon

Parity-violating $e$ scattering

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[ \frac{-G_F Q^2}{\pi \alpha \sqrt{2}} \right] \frac{\varepsilon G^p_{E} G^p_{E} + \tau G^p_{M} G^p_{M} - \frac{1}{2} \left(1 - 4 \sin^2 \theta_W \right) \varepsilon' G^p_{M} G^p_{A}}{\varepsilon (G^p_{E})^2 + \tau (G^p_{M})^2}$$

$$G^p_{E,M} = \frac{1}{4} \left(G^p_{E,M} - G^p_{E,M} \right) - \sin^2 \theta_W G^p_{E,M} - \frac{1}{4} G^s_{E,M}$$

Strangeness

http://www.npl.uiuc.edu/exp/G0Forward

Armstrong et al. [G0 Collaboration]

nucl-ex/0506021
Strangeness in the Nucleon

Parity-violating $e$ scattering

$\eta = \tau G_M / \varepsilon G_E$

$\sim 0.94 Q^2$

dependence of “zero-point” on e.m. form factors

Armstrong et al. [G0 Collaboration]
ucl-ex/0506021

intriguing $Q^2$ dependence!

trend to positive values at larger $Q^2$
Strangeness in the Nucleon

combined world data at \( Q^2 = 0.1 \) GeV\(^2\)

\[
G_E^s = -0.013 \pm 0.028 \\
G_M^s = +0.62 \pm 0.31 \\
\text{(} \pm 0.62 \text{ 2}\sigma\text{)}
\]

Theories

1. Leinweber, et al.
   PRL 94 (05) 212001
   \text{lattice}
2. Lyubovitskij, et al.
   PRC 66 (02) 055204
   \text{chiral quark model}
3. Lewis, et al.
   PRD 67 (03) 013003
   \text{chiral EFT}
   PRD 65 (01) 014016
   \text{quark soliton model}

→ huge effect!
→ can theory explain result?
Lattice Results

Dong et al. PRD(1998) \[ G^S_M = -0.36 \pm 0.20 \]
Mathur & Dong NPB(2001) \[ G^S_M = -0.27 \pm 0.10 \]
Lewis et al. PRD(2003) \[ G^S_M(0.1\text{GeV}^2) = +0.05 \pm 0.06 \]

Leinweber et al. PRL(2005) \[ G^S_M = -0.046 \pm 0.019 \]
\[ p = \frac{2}{3} u^p - \frac{1}{3} u^n + O_N \]
\[ n = -\frac{1}{3} u^p + \frac{2}{3} u^n + O_N \]
\[ \Sigma^+ = \frac{2}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_{\Sigma} \]
\[ \Sigma^- = -\frac{1}{3} u^\Sigma - \frac{1}{3} s^\Sigma + O_{\Sigma} \]
\[ 3O_N = 2p + n - u^p \]
\[ 3O_N = p + 2n - u^n \]

**Lattice QCD**

\[ \Sigma^+ - \Sigma^- = u^\Sigma \]

Ross Young et al. (JLab/CSSM)
Disconnected Loops

\[ O_N = \]

\[ = \frac{2}{3} G^u_M - \frac{1}{3} G^d_M - \frac{1}{3} G^s_M \]

\[ l G^u_M = l G^d_M \]

\[ QCD \text{ equality for } m_u = m_d \]

\[ l R^s_d = l G^s_M / l G^d_M = 0.139 \pm 0.042 \]

\textit{chiral phenomenology}
Constraint on GMs

$$G^s_M = \left( \frac{l R^s_d}{1 - l R^s_d} \right) \left[ 2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right]$$

$$G^s_M = \left( \frac{l R^s_d}{1 - l R^s_d} \right) \left[ p + 2n - \frac{u^n}{u^{\Xi}} (\Xi^0 - \Xi^-) \right]$$

**Quark model**

$G^s_M < 0$

$Lattice QCD$

$G^s_M > 0$
$u$-quark in the proton

Leinweber, RDY et al. PRL(2005)
$u$-quark in the Sigma

Leinweber, RDY et al. PRL(2005)
Final Result

\[ G_M^S = -0.046 \pm 0.019 \mu_N \]

\[ \frac{u^p}{u^\Sigma} = 1.092 \pm 0.030 \]

\[ \frac{u^n}{u^\Xi} = 1.254 \pm 0.124 \]
Magnetic Moments

Leinweber et al. PRL(2005)
Repeat analysis for strange electric form factor

\[ G_E^s(Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.003 \]
Summary - Form Factors

- **Surprisingly different behavior for** $G_E^p$ and $G_M^p$
  
  → different charge and magnetization distributions

- **$2\gamma$ exchange needed to resolve discrepancy**
  between LT and PT measurements of $G_E^p/G_M^p$

  → reached limit of applicability of $1\gamma$ exchange
  in elastic $eN$ scattering

- **Strange magnetic moment large and positive**

  → *cf.* lattice QCD/phenomenology, which gives very small and negative value

  → G0 backward angle run in 2006-2007 will determine $G_E^s$ and $G_M^s$ separately
Thank you students - good luck!

Thank you Bruce!