Recent Results for Hadron Structure from Lattice QCD

Anthony W. Thomas

Few Body 18, Santos Brazil : August 26th 2006
Outline

• Quantum Chromodynamics within the Standard Model

• Lattice QCD:
  there are problems $\Rightarrow$ new opportunities!

  (and, by the way, some things CAN be calculated ACCURATELY)

• $M_N$, $M_\Delta$, QQCD $\leftrightarrow$ QCD $\leftrightarrow$ pQQCD, $M_\rho$; $g_A$, $\mu_N$, $G_{E,M}$
Advances in Lattice QCD

Inclusion of Pion Cloud

\( \chi \) PT allows accurate extrapolation

(needed because \( t \sim m_\pi^4 V^{1.25} \))

Precise computations at Physical Pion Mass

Improvements in algorithms

e.g. DWF \( \Rightarrow \) Exact Chiral Symmetry

Advances in high-performance computing

from D. Richards
USQCD and the World

- **Asqtad (Staggered) fermions:**
  - Large scale generation on-going by MILC Collaboration.
  - Lattice spacing: $a \sim 0.13\text{fm}, 0.09\text{fm}, 0.06\text{fm}$
  - Suitable for valence Domain Wall (spin-physics) via partially quenched chiral perturbation theory
  - **Not** suitable for baryon spectrum program

- **Clover (anisotropic):**
  - Suitable for spectrum and simple form-factors
  - Anisotropy requires new calculation

- **Chiral fermions (e.g., Domain-Wall/Overlap):**
  - Algorithm investigations on-going at JLab
  - Large scale production by UKQCD and RBC
  - Too coarse lattice for JLab spectra

from R. Edwards
Anisotropic Clover: dynamical generation

Estimated cost of $N_f=2+1$ production (in TFlop-yrs) using $z_\pi=4$

Cost (TFlop-yr) = $\text{const} \left( \frac{m_{PS}}{m_V} \right)^{-4} V(\text{fm})^{5/4} a(\text{fm})^{-7}$

- Phase I – initial production + 10% analysis overhead
  - Hybrid photo-couplings
    - cost = 1.1 TF-yr + 10% analysis
- Phase II – all of 0.10fm and 0.125fm lattices
  - Baryon spectra
    - cost = 4.8 TF-yr + 50% analysis
- Phase III – $a=0.08$fm
  - Light pion mass and continuum limit
    - cost = 23 TF-yr + 50% analysis

from R. Edwards

<table>
<thead>
<tr>
<th>Lattice Spacing</th>
<th>$m_\pi$ (MeV)</th>
<th>2.4fm</th>
<th>3.2fm</th>
<th>4.0fm</th>
<th>Total (TFlop-yr)</th>
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<td>$a = 0.08$ fm</td>
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<td>254</td>
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<td>Sub-total = 0.1 TF-yr Total = 0.76 TF-yr</td>
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</tbody>
</table>
By the way…. It's NOT $\exp(-m_\pi L)$ that matters!

We must have

$m_\pi (L/2 - R) \gg 1$

with $R \sim 0.8$ fm

i.e. $L > 2R + 4/m_\pi$

or $L \sim 6$ fm

when $m_\pi \sim 200$ MeV

Thomas et al., hep-lat/0502002
χ’al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon

FRR give same answer to <<1% systematic error!

Leinweber et al., PRL 92 (2004) 242002
Extrapolation of Masses

At “large $m_\pi$” preserve observed linear (constituent-quark-like) behaviour: $M_H \sim m_\pi^2$

As $m_\pi \sim 0$: ensure LNA & NLNA behaviour:

( **BUT** must die as $(\Lambda / m_\pi)^2$ for $m_\pi > \Lambda$)

Hence use:

$$M_H = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + \sigma_{\text{LNA}}(m_\pi, \Lambda) + \sigma_{\text{NLNA}}(m_\pi, \Lambda)$$

- Evaluate self-energies with form factor, “finite range regulator”, FRR, with $\Lambda \sim 1/\text{Size of Hadron}$
χ’al Extrapolation Under Control when Coefficients Known – e.g. for the nucleon

FRR give same answer to <<1% systematic error!

Leinweber et al., PRL 92 (2004) 242002

<table>
<thead>
<tr>
<th>Regulator</th>
<th>Bare Coefficients</th>
<th>Renormalized Coefficients</th>
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<tr>
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<td>$a_0^\Lambda$</td>
<td>$a_2^\Lambda$</td>
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<tr>
<td>Monopole</td>
<td>1.74</td>
<td>1.64</td>
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<tr>
<td>Dipole</td>
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<tr>
<td>Gaussian</td>
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<td>Sharp cutoff</td>
<td>1.06</td>
<td>1.47</td>
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<tr>
<td>Dim. Reg. (BP)</td>
<td>0.79</td>
<td>4.15</td>
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</tbody>
</table>
Power Counting Regime

Ensure coefficients $c_0, c_2, c_4$ all identical to 0.8 GeV fit

Leinweber, Thomas & Young, hep-lat/0501028
Convergence from LNA to NLNA is Rapid – Using Finite Range Regularization

<table>
<thead>
<tr>
<th>Regulator</th>
<th>LNA</th>
<th>NLNA</th>
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<tbody>
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<td>Sharp</td>
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<td>961</td>
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<tr>
<td>Monopole</td>
<td>964</td>
<td>960</td>
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<tr>
<td>Dipole</td>
<td>963</td>
<td>959</td>
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<tr>
<td>Gaussian</td>
<td>960</td>
<td>960</td>
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<tr>
<td>Dim Reg</td>
<td>784</td>
<td>884</td>
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</table>

$M_N$ in MeV
Axial Charge of the Nucleon

- Hybrid Computation of Hadron Structure using MILC asqtad lattices and domain-wall-fermion valence quarks
- Has enabled computations to be performed in full QCD at $m_\pi$ approaching 350 MeV

LHPC (Edwards et al.), PRL 96 (2006) 052001
FRR Yields Essentially Identical Result

$O(m_\pi^2)$ only

$O(m_\pi^4)$ and $g_{\Delta\Delta}$ from width

$\Rightarrow$ preferred value $g_A = 1.21 \pm 0.07$

$O(m_\pi^2)$ only

Data: LHPC

Young & Thomas, 2006
Proton EM Form Factors

- Lattice QCD computes the isovector form factor
- Hence obtain Dirac charge radius $\langle r^2 \rangle_{\text{u-d, ch}}$ assuming dipole form
- Chiral extrapolation. Using LNA and LA terms and FRR
- As the pion mass approaches the physical value, the size approaches the correct value

\[
\langle r^2 \rangle_{\text{u-d, ch}} = a_0 - \frac{2}{(4\pi f_\pi)^2} \log \left( \frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right)
\]

Data from LHPC Collaboration (Edwards et al.)

Leinweber, Thomas, Young, PRL86, 5011

from D. Richards

Thomas Jefferson National Accelerator Facility

Operated by Jefferson Science Associates for the U.S. Department of Energy
Isovector Form Factor at Higher $Q^2$

- Preliminary calculation with $m_{\pi} \sim 600$ MeV enabling us to reach $Q^2 \sim 4$ GeV$^2$
- Fits of experimental data suggest $G_E^{u-d}$ vanishes at $Q^2 \sim 4$ GeV$^2$
- Tantalizing suggestion of such behavior in lattice data.

AIM: Form factor at $Q^2 > 10$ GeV$^2$, at pion masses down to 254 MeV in Asqtad/DWF Computation.
Analysis of pQQCD $\rho$ data from CP PACS

i.e. $m_{val} \neq m_{sea}$

Fit with:

$$\sqrt{(M_V^{\text{deg}})^2 - \Sigma_{TOT}} = (a_0^{\text{cont}} + X_1 a + X_2 a^2) + a_2 (M_{PS}^{\text{deg}})^2 + a_4 (M_{PS}^{\text{deg}})^4 + a_6 (M_{PS}^{\text{deg}})^6$$
FRR Mass (in $\Sigma_{TOT}$) well determined by data

$$\sqrt{(M^\text{deg}_V)^2 - \Sigma_{TOT}} = (a_0^{\text{cont}} + X_1 a + X_2 a^2) + a_2 (M^\text{deg}_{PS})^2 + a_4 (M^\text{deg}_{PS})^4 + a_6 (M^\text{deg}_{PS})^6$$
Infinite Volume Unitary Results

\[ a \to 0 \text{ and } m_{\text{sea}} = m_{\text{val}} \]

All 80 data points drop onto single, well defined curve!

\[ 777 \pm 7 \text{ MeV} \]

Allton, Young et al., hep-lat/0504022
Baryon Masses in Quenched QCD

Chiral behaviour in QQCD quite different from full QCD

η' is an additional Goldstone Boson, so that:

\[ m_N = m_0 + c_1 m_\pi + c_2 m_\pi^2 + c_3 m_\pi^3 + c_4 m_\pi^4 + m_\pi^4 \ln m_\pi + \ldots \]

LNA term now \( \sim m_q^{1/2} \)

origin is \( \eta' \) double pole
• Lattice data (from MILC Collaboration) : red triangles
• Green boxes: fit evaluating $\sigma$’s on same finite grid as lattice
• Lines are exact, continuum results

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_N$</th>
<th>$\beta_N$</th>
<th>$\alpha_\Delta$</th>
<th>$\beta_\Delta$</th>
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</thead>
<tbody>
<tr>
<td>FULL</td>
<td>1.24 (2)</td>
<td>0.92 (5)</td>
<td>1.43 (3)</td>
<td>0.75 (8)</td>
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<tr>
<td>QQCD</td>
<td>1.23 (2)</td>
<td>0.85 (8)</td>
<td>1.45 (4)</td>
<td>0.71 (11)</td>
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</table>

Young et al., hep-lat/0111041; Phys. Rev. D66 (2002) 094507

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$\Delta$ in QQCD

LNA term linear in $m_\pi$

$\Delta \rightarrow N \pi$ contribution has opposite sign in QQCD (repulsive)

Overall $\sigma_{QQCD}$ is repulsive!
Confirmation of Predicted Behavior of $\Delta$

Zanotti et al., hep-lat/0407039
These results suggest following conjecture:

IF lattice scale is set using static quark potential (e.g. Sommer scale) (insensitive to chiral physics)

Suppression of Goldstone loops for $m_\pi > \Lambda$ implies:

**Analytic terms** (e.g. $\alpha + \beta m_\pi^2 + \gamma m_\pi^4$) representing “hadronic core” are the same in QQCD & QCD

Can then correct QQCD results by replacing LNA & NLNA behaviour in QQCD by corresponding terms in full QCD

Quenched QCD is then no longer an “uncontrolled approximation”!
Strangeness Widely Believed to Play a Major Role – Does It?

• As much as 100 to 300 MeV of proton mass:

\[ M_N = \langle N(P) | -\frac{9\alpha_s}{4\pi} \text{Tr}(G_{\mu\nu}G^{\mu\nu}) + m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d + m_s \bar{\psi}_s \psi_s | N(P) \rangle \]

\[ \Delta M_{-\text{quarks}} = \frac{ym_s}{m_u + m_d} \sigma_N \]

hence 110 ± 110 MeV (increasing to 180 for higher \( \sigma_N \))

• Through proton spin crisis:
   As much as 10% of the spin of the proton

• HOW MUCH OF THE ELECTRIC and MAGNETIC FORM FACTORS?
MIT-Bates & A4 at Mainz
G0 and HAPPEX at Jlab
Magnetic Moments within QCD

\[ \begin{align*}
\Sigma^+ &= 2/3 \ u^\Sigma - 1/3 \ s^\Sigma + O_\Sigma \\
\Sigma^- &= -1/3 \ u^\Sigma - 1/3 \ s^\Sigma + O_\Sigma
\end{align*} \]

HENCE: \( O_N = 1/3 \left[ 2p + n - \left( \frac{u^p}{u^\Sigma} \right) (\Sigma^+ - \Sigma^-) \right] \)

OR \( O_N = 1/3 \left[ n + 2p - \left( \frac{u^n}{u^\Sigma} \right) (\Xi^0 - \Xi^-) \right] \)

\( p = 2/3 \ u^p - 1/3 \ d^p + O_N \)

\( n = -1/3 \ u^p + 2/3 \ d^p + O_N \)

\( 2p + n = u^p + 3 \ O_N \)

(and \( p + 2n = d^p + 3 \ O_N \))
Convergence LNA to NLNA Again Excellent
(Effect of Decuplet)
Accurate Final Result for $G_M^s$

$$G_M^s = -0.046 \pm 0.019 \, \mu_N$$

Leinweber et al., (PRL June '05) hep-lat/0406002

Highly non-trivial that intersection lies on constraint line!
GE by similar technique

In this case only know Σ⁻ radius (and p and n)

hence use absolute values of u and d radii:

\[
\begin{align*}
2p + n &= u_p + 3 \Omega_N \\
p + 2n &= d_p + 3 \Omega_N \\
\Rightarrow \langle r^2 \rangle_s &= 0.000 \pm 0.006 \pm 0.007 \text{ fm}^2 ; 0.002 \pm 0.004 \pm 0.004 \text{ fm}^2
\end{align*}
\]

(c.f. using Σ⁻ : -0.007 ± 0.004 ± 0.007 ± 0.021 fm\(^2\))

\[
G_E^s(0.1 \text{ GeV}^2) = +0.001 \pm 0.004 \pm 0.004
\]

(up to order Q^4)

Note consistency and level of precision!

Leinweber, Young et al., hep-lat/0601025 (Jan 2006)
Superimpose NEW HAPPEX Measurement (Dallas APS meeting, April 06)
Include new HAPPEX data: halves errors of previous world data!

- Major success of non-perturbative QCD
- First Determination of “disconnected quark loop”

⇒ Strange quark contributes just tens of MeV to $M_N$

Leinweber et al.: PRL 94, 212001 (2005)
PRL 97, 022001 (2006)
Conclusions

- Wonderful synergy between experimental advances at Jlab and progress using Lattice QCD to solve QCD

- Study of hadron properties as function of $m_q$ using data from lattice QCD is extremely valuable.....
  (major qualitative advance in understanding)
  + TEST BEYOND STANDARD MODEL

- Inclusion of model independent constraints of $\chi$ PT to get to physical quark mass is essential
  FRR $\chi$PT resolves problem of convergence

- Insight enables: accurate, controlled extrapolation of all hadronic observables....
  ( e.g. $m_H$, $\mu_H$, $G_{E,M}^s$, $<r^2>_{ch}$, $G_E$, $G_M$, $<x^n>$....)

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Conclusions

- In case where chiral coefficients are known, FRR enables accurate extrapolation to physical point
- Without chiral coefficients (e.g. spectroscopy of baryons and mesons) need data at very low pion mass (several points below $\sim 0.25$ GeV)
- It is a major challenge to obtain a reliable signal for "disconnected" loops directly in lattice QCD — this is a very important challenge
- For future there is a wonderful synergy with 12 GeV program at JLab and work on GPDs, form factors at high $Q^2$, and higher moments of PDFs just beginning
Special Mentions…….

Derek Leinweber

Ross Young