Lecture IV

Electromagnetic Interactions: gauge invariance and effective current operators

- General method for doing gauge invariant calculations in systems composed of composite particles.
- What can be learned from high energy electron scattering experiments?

Outline

- General method for obtaining a gauge invariant current
- Gauge invariant two-body current operator in CS theory
- Gauge invariant three-body current operator in CS theory
- Implications of energy dependent interactions
- Construction of off-shell single nucleon current
- Applications: deuteron form factor
- Applications: deuteron photodisintegration
- Conclusions
Gauge invariance with point particles

Proof that the ladder sum is gauge invariant, order by order (Feynman’s proof). First, couple the photon to ALL places where there is a charged particle (only the blue particle below) and keep the labeling of the neutral particles (red) unchanged:

\[ J^\mu = \int dp_2 dp_3 C(p_1 p_2 p_3 p_4) \left\{ \gamma^\mu S_4 S_3 S_2 + S_3 S_2 \gamma^\mu S_2 + S_3 S_2 \gamma^\mu S_2 + S_3 S_2 \gamma^\mu S_1 \right\} \]

where \( S_i = S(P - p_i); ~ \bar{S}_i = S(P' - p_i); ~ P' = P + q \)

Use the Ward-Takahashi (WT) identity

\[ q_\mu \gamma^\mu = \left( (P' - p_i) - (P - p_i) \right) \gamma^\mu = (m - (P' - p_i)) - (m - (P - p_i)) = S_i^{-1} - \bar{S}_i^{-1} \]

to reduce the result

\[ q_\mu J^\mu = \int dp_2 dp_3 C(p_1 p_2 p_3 p_4) \left\{ S_3 S_2 + (\bar{S}_3 S_2 - S_3 S_2) + (\bar{S}_3 S_2 - S_3 S_2) \right\} = 0 \]
Generalization to particles with form factors

★ Require that form factors at vertices have the factorized form
\[ F(k, p, q) = f(k) f(p) g(q) \]
with universal form factors for each particle

★ Three steps
- Reinterpret vertex form factors as dressings of the particle propagators (can be done if they are universal)
- Construct one body currents satisfying a WT identity for the dressed propagators. Construct contact interactions satisfying WT identities.
- Using relativistic equations, derive current operator (that effectively couples photon to all charged particles and all "charged" contact interactions)

★ Off-shell nucleon current operator is the solution of a WT identity with a nucleon propagator "dressed" by the form factor \( f \)

\[
S_N(p) = \frac{(m + p) f^2(p)}{m^2 - p^2 - i\varepsilon}
\]


Gauge invariant two body current operator in the CS© theory

★ Inelastic Scattering

★ Elastic Scattering

★ Interaction current

**F. G. and D. O. Riska, PRC 36, 1928 (1987)**
The gauge invariant* three-body breakup current in the spectator theory (with on-shell particles labeled by an $\times$) requires many diagrams where the FSI term is

Implications of energy dependent interactions

* Lessons from the bubble sum (in 1+1 d for simplicity) suppose the $NN$ interaction is an energy dependent four-point coupling:

* then the scattering amplitude is a geometric sum of bubble diagrams:

* the bound state condition fixes $a$, but the energy dependent parameter $\lambda$ is undetermined
Lessons from the bubble sum (2)

★ the deuteron wave function is independent of $\lambda$,
\[
\Psi(p,M_d) = \frac{N}{\left(m^2 - \left(\frac{1}{2} P + p\right)^2\right)^2 \left(m^2 - \left(\frac{1}{2} P - p\right)^2\right)^2}; \quad P^2 = M_d^2
\]

★ but the NN cross section is not:
\[
\sigma(s) = \frac{1}{\sqrt{s}} |M(s)|^2; \quad M = \frac{a + \lambda (s - M_d^2)}{1 - B(s) \left[ a + \lambda (s - M_d^2) \right]}
\]

Lessons from the bubble sum (3)

“energy dependence comes with a price”

★ the deuteron form factor is the sum of two terms:
\[
J_{\text{RIA}}^\mu + J_{\text{IAC}}^\mu
\]

★ the energy dependence of the interaction generates an interaction current (IAC) which depends on $\lambda$.

★ the IAC required by the interaction is unique and separately gauge invariant.
\[
J_{\text{IAC}}^\mu (Q^2) = \lambda N^2 (P + P')^\mu B^2 (Q^2)
\]

★ FSI and IAC must be consistent with the dynamics! Calculations must be consistent.
The current is constrained by the WT identity

To conserve current, the current operator must satisfy the WT identity

\[ q_\mu j_\mu^N(p',p) = S^{-1}(p) - S^{-1}(p') \]

The spectator models use a nucleon form factor, \( f(p) \). This means that the nucleon propagator can be considered to be dressed. One solution (the simplest) is

\[ j^\mu(p',p) = F_0 \gamma^\mu + F_2 \frac{i\sigma^\mu\nu q_\nu}{2m} + G_0 F_3 \Lambda_-(p') \gamma^\mu \Lambda_-(p) \]

\[ F_0 = \frac{f(p)}{f(p')} \left( \frac{m^2 - p'^2}{p'^2 - p^2} \right) \quad F_2 = \frac{f(p)}{f(p')} \left( \frac{m^2 - p^2}{p'^2 - p^2} \right) \]

\[ G_0 = \left( \frac{f(p)}{f(p')} \frac{f(p)}{f(p')} \right) \frac{4m^2}{p'^2 - p^2} \]

\( F_3(Q^2) \) is unknown, except \( F_3(0) = 1 \). EXPLOIT THIS FREEDOM 😊

Applications

Deuteron form factors

- Last calculation using the CS model was done in 1995* using an old model (called IIB) with NO off-shell couplings
- This is a pure isoscalar transition process, with no isoscalar interaction currents (except for small \( p\pi\gamma \) current of dubious significance)

Photodisintegration (a very different story)

There are three form factors that enter the relativistic current:

\[-\langle d' | J'^\mu | d \rangle = \left\{ G_1(Q^2)(\xi^{\ast\mu} \xi) - G_3(Q^2)\frac{(\xi^{\ast\mu} \cdot q)(\xi^{\ast} \cdot q)}{2M_d^2} \right\}(d + d')^\mu + G_M(Q^2)\left[ \xi^{\ast\mu}(\xi^{\ast\mu} \cdot q) - \xi^{\ast\mu}(\xi^{\ast} \cdot q) \right] \]

The physical combinations are:

\[ G_C = G_1 + \frac{2}{3} \eta G_Q \]
\[ G_Q = G_1 - G_M + (1 + \eta)G_3 \]

The observables are:

\[ A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta G_Q^2 + \frac{2}{3} \eta G_M^2 \]
\[ B(Q^2) = \frac{4}{3} \eta(1 + \eta) G_M^2 \]

Choice of a "hard" \( F_3 \) is sufficient for an excellent fit!

The Spectator theory, with a suitable $F_3$, can explain the elastic electron deuteron scattering data!
Extracting free neutron properties

★ Spectator formalism ideal; but still many problems:
  • The impulse term is well defined, and distinct from the other contributions (FSI and MEC)
  • What kinematics will separating out the FSI, the proton contributions, or the contributions for np overlap (quark exchange)?
  • If the RIA diagram dominates, how are off-shell effects controlled?
★ How does this work in other formalisms?

Deuteron Photodisintegration
100's of channels excited in photodisintegration at 4 GeV*

IN DEUTERON PHOTODISINTEGRATION, THE "RIGHT-HAND" RESONANCES ARE EXPOSED


Total NN cross sections

12 GeV photons

High energy photodisintegration probes deep into the inelastic region
Smooth, scaling-like behavior at high energies

Conventional models fail (so far)
Quark-interaction models:

- Radyushkin
- Brodsky and Hiller (RNA)
- Kondratyuk, et al.
- quark-gluon string model
- Frankfurt, Miller, Strikman, and Sargsian (final state \( NN \) scattering)

A quark-exchange diagram:

The QGS model

Regge pole exchange

Conclusions: (form factors vs. photodisintegration)

- The deuteron form factors and deuteron photodisintegration probe completely different physics even though both are done with electrons of a few GeV.

- Form factors:
  - one nucleon is off-shell, but total mass of the final state is \( M_d \)
  - all resonances are frozen out

- Photodisintegration (or electrodisintegration)
  - both nucleons can be on-shell and mass of final state varies up to 8 GeV!
  - resonances and resonance channels are explicitly excited
  - proper description of high energy \( NN \) interaction (Eikonal or GEA) is essential
Conclusions

★ The CS theory works well for the deuteron form factors, showing that hadronic degrees of freedom are sufficient for the description of elastic scattering up to the highest $Q^2$ measured.

★ There is no evidence for the appearance of explicit quark degrees of freedom

★ BUT: these results must be confirmed using the new off-shell OBE models, and calculations of other few-body reactions are needed.

★ Photodisintegration of the deuteron does show the appearance of quark degrees of freedom.