Nucleon and Hyperon Form Factors from Lattice QCD

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Outline

- Background on Lattice QCD
- Nucleon Properties
  - Axial coupling charge
  - Structure functions
  - Charge radius and Goldberger-Treiman relation ($N_f = 2$)
  - Summary/Outlook: I
- From Hyperon Analysis
  - Proton strangeness magnetic moment
  - $\Sigma$ and $\Xi$ axial coupling prediction
  - Summary/Outlook: II
Background: Lattice 101
Physical observables are calculated from the path integral

$$\langle \Omega | O | \Omega \rangle = \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] O(U, \psi, \bar{\psi}) e^{i \int d^4x [S_F(U, \bar{\psi}, \psi) + S_G(U)]}$$

Lattice QCD is a discrete version of continuum QCD theory

Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.

Take $a \to 0$ and $V \to \infty$ for the “continuum limit”
Lattice Gauge Theory

Physical observables are calculated from the path integral

\[ \langle \Omega | O | \Omega \rangle = \frac{1}{Z} \int [dU][d\bar{\psi}][d\psi]O(U, \bar{\psi}, \psi)e^{i \int d^4x [S_F(U, \bar{\psi}, \psi) + S_G(U)]} \]

Lattice QCD is a discrete version of continuum QCD theory

Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.

Chiral extrapolation \( m_\pi \rightarrow (m_\pi)_{\text{phys}} \)
**Lattice Fermion Actions**

**Chiral fermions (e.g., Domain-Wall/Overlap):**
- Automatically $O(a)$ improved, suitable for spin physics and weak matrix elements
- Expensive

$$D_{x,s,x',s'} = \delta_{x,x'}D_{s,s'}^{\perp} + \delta_{s,s'}D_{x,x'}^{\|}$$

$$D_{s,s'}^{\perp} = \frac{1}{2}[(1 - \gamma_5)\delta_{s+1,s'} + (1 + \gamma_5)\delta_{s-1,s'} - 2\delta_{s,s'}]$$

- $$\frac{m_f}{2}[(1 - \gamma_5)\delta_{s,L_s-1}\delta_{0,s'} + (1 + \gamma_5)\delta_{s,0}\delta_{L_s-1}$$

**(Improved) Staggered fermions (asqtad):**
- Relatively cheap for dynamical fermions (good)
- Mixing among parities and flavours or “tastes”
- Baryonic operators a nightmare — not suitable

**Mixed actions:**
- Staggered sea (cheap) with Domain-Wall valence (chiral)
- Match the sea Goldstone pion mass to the DWF pion
Green Functions

- Interpolating field for a baryon
  \[ J_{\alpha} (\vec{p}, t) = \sum_{\vec{x}, a, b, c} e^{i\vec{p} \cdot \vec{x}} \epsilon^{abc} [u_a^T(y_1, t) \gamma_5 d_b(y_2, t)] u_{c, \alpha}(y_3, t) \phi(y_1 - x) \phi(y_2 - x) \phi(y_3 - x) \]

- Two-point function with projection
  \[ C_{2pt}(\vec{p}, t) = \sum_{\alpha, \beta} \left( \frac{1 + \gamma_4}{2} \right) \alpha\beta \langle J_\beta(\vec{p}, t) \bar{J}_\alpha(\vec{p}, 0) \rangle \]

- Three-point function with connected piece only
  \[ C_{3pt}^{\Gamma, \mathcal{O}} (\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta(\vec{p}, t) \mathcal{O}(\tau) \bar{J}_\alpha(\vec{p}, 0) \rangle \]
Ratio Construction

Two–point function

\[ \Gamma_{NN}^{NN}(t_i, t_f, \vec{p}; T) \rightarrow \]
\[ a^6 Z_B^N(p) Z_A^N(p) \sum_s T_{\alpha \beta} \bar{u}_\alpha(\vec{p}, s) u_\beta(\vec{p}, s) \]
\[ \times \frac{m_N}{E_N(\vec{p})} e^{-(t_f - t_i)E_N(\vec{p})} \]
\[ = \left( \frac{E_N(\vec{p}) + m_N}{2E_N(\vec{p})} \right) e^{-(t_f - t_i)E_N(\vec{p})} \]

Three–point function

\[ \Gamma_{\mu,AB}^{NN}(t_i, t, t_f, \vec{p}_i, \vec{p}_f, T) \rightarrow \]
\[ = \frac{m_N^2}{E_N(\vec{p}_f)E_N(\vec{p}_i)} e^{-(t_f - t)E_N(\vec{p}_f)} e^{-(t - t_i)E_N(\vec{p}_i)} \]
\[ \sum_{s, s'} T_{\alpha \beta} Z_B(p_f) u_\beta(\vec{p}_f, s') \]
\[ \langle N(\vec{p}_f, s') | j_\mu(0) | N(\vec{p}_i, s) \rangle \bar{u}_\alpha(\vec{p}_i, s) Z_A(p_i) \]

Ratio cancels out \( t \) and \( Z \) dependence

\[ R_{j_\mu} = \frac{Z \Gamma^{(3), P}_{\mu, GG}(t_{\text{src}}, t, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})}{\Gamma^{(2), P+}_{GG}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}})} \]
\[ \times \sqrt{\frac{\Gamma^{(2), P+}_{LG}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}})}{\Gamma^{(2), P+}_{LG}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}})}} \]
\[ \sqrt{\frac{\Gamma^{(2), P+}_{LG}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}})}{\Gamma^{(2), P+}_{LG}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}})}} \]
Nucleon Structure
(work in progress)
in collaboration with

Tom Blum, Shigemi Ohta, Kostas Orginos,
Shoichi Sasaki and Takeshi Yamazaki
2+1-flavor RBC/UKQCD Ensembles

- Generated jointly by RBC and UKQCD
- Iwasaki gauge action \((c_1 = -0.331)\)
- Pion mass range: 300–625 MeV
- \(V_{\text{lat}} = 24^3 \times 64, \ a \approx 0.125 \ \text{fm}, \ L_s = 16, \ M_5 = 1.8\)

FSE (RBCK quenched study)

<table>
<thead>
<tr>
<th>(m_{\text{sea}})</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
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<tbody>
<tr>
<td>(t_{\text{snk}} - t_{\text{src}})</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>(t_{\text{src}})</td>
<td>0, 16, 32, 48</td>
<td>0, 16, 32, 48</td>
<td>0, 16, 32, 48</td>
</tr>
<tr>
<td># of conf.</td>
<td>119</td>
<td>49</td>
<td>54</td>
</tr>
<tr>
<td>(m_\pi) (GeV)</td>
<td>0.399(3)</td>
<td>0.535(3)</td>
<td>0.625(3)</td>
</tr>
<tr>
<td>(m_N) (GeV)</td>
<td>1.169(19)</td>
<td>1.204(13)</td>
<td>1.474(18)</td>
</tr>
</tbody>
</table>

TABLE I: Gaussian-smeared source parameters

C. Allton, et al., RBC/UKQCD collaborations, coming soon

Huey-Wen Lin @ Baryon07, Seoul, Korea
Axial Charge Coupling

**Isovector** vector and axial-vector current

\[
\langle p | V^{\dagger}_\mu (0) | n \rangle = \bar{u}_p [\gamma_\mu g_V (q^2) - q_\mu \sigma_{\mu\nu} g_T (q^2)] u_n
\]

\[
\langle p | A^{\dagger}_\mu (0) | n \rangle = \bar{u}_p [\gamma_\mu \gamma_5 g_A (q^2) - i q_\mu \gamma_5 g_P (q^2)] u_n
\]

Chiral symmetry gives \( Z_A = Z_V = 1/g_V \)

Continuum \( \chi PT \) extrapolation (SSE scheme)

\[
g_A^{\text{SSE}} (m^2_\pi) = g_A^0 + \left[ 4 C_{\text{SSE}}^0 (\lambda) - \frac{(g_A^0)^3}{16 \pi^2 f^2_\pi} - \frac{25 c_A^2 g_1}{324 \pi^2 f^2_\pi} + \frac{19 c_A^2 g_A^0}{108 \pi^2 f^2_\pi} \right] m^2_\pi
\]

\[
- \frac{m^2_\pi}{4 \pi^2 f^2_\pi} \left[ (g_A^0)^3 + \frac{1}{2} g_A^0 \right] \ln \frac{m_\pi}{\lambda} + \frac{4 c_A^2 g_A^0}{27 \pi \Delta_0 f^2_\pi} m^3_\pi
\]

\[
+ \left[ 25 c_A^2 g_1 \Delta_0^2 - 57 c_A^2 g_A^0 \Delta_0^2 - 24 c_A^2 g_A^0 m^2_\pi \right] \frac{\sqrt{m^2_\pi - \Delta_0^2}}{81 \pi^2 f^2_\pi \Delta_0} \arccos \frac{\Delta_0}{m_\pi}
\]

\[
+ \frac{25 c_A^2 g_1}{162 \pi^2 f^2_\pi} \left[ 2 \Delta_0^2 - m^2_\pi \right] \ln \frac{2 \Delta_0}{m_\pi} + \frac{c_A^2 g_A^0}{54 \pi^2 f^2_\pi} \left[ 3 m^2_\pi - 38 \Delta_0^2 \right] \ln \frac{2 \Delta_0}{m_\pi} + \mathcal{O}(e^4)
\]
Axial Charge Coupling

**Isovector** vector and axial-vector current

\[
\langle p | V^i_\mu (0) | n \rangle = \bar{u}_p [\gamma_\mu g_V(q^2) - q_\mu \sigma_{\mu\nu} g_T(q^2)] u_n
\]

\[
\langle p | A^i_\mu (0) | n \rangle = \bar{u}_p [\gamma_\mu \gamma_5 g_A(q^2) - i q_\mu \gamma_5 g_P(q^2)] u_n
\]

**Chiral symmetry gives** \( Z_A = Z_V = 1 / g_V \)

**Continuum \( \chi \)PT extrapolation (SSE scheme)**

\[
\delta_L (\Gamma_{NN}) \equiv \delta g_A = \frac{m^2}{3 \pi^2 f^2} \left[ g_A^3 F_1 + \left( g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta \Delta} \right) F_2 + g_A F_3 + g_{\Delta N}^2 g_A F_4 \right]
\]

\[
F_2(m, \Delta, L) = -\sum_{n \neq 0} \left[ \frac{K_1(mL|n|)}{mL|n|} - \frac{\Delta}{m^2} \int_{m}^{\infty} d\beta \frac{\beta K_0(\beta L|n|) + (\Delta^2 - m^2)L|n| K_1(\beta L|n|)}{\sqrt{\beta^2 + \Delta^2 - m^2}} \right]
\]

\[
F_1(m, L) = \sum_{n \neq 0} \left[ K_0(mL|n|) - \frac{K_1(mL|n|)}{mL|n|} \right]
\]

\[
F_3(m, L) = -\frac{3}{2} \sum_{n \neq 0} \frac{K_1(mL|n|)}{mL|n|} ;
\]

\[
F_4(m, \Delta, L) = \frac{8}{9} \sum_{n \neq 0} \left[ \frac{K_1(mL|n|)}{mL|n|} - \frac{\pi e^{-mL|n|}}{2\Delta L|n|} \right] - \Delta^2 - m^2 \int_{m}^{\infty} d\beta \frac{\beta K_0(\beta L|n|)}{\sqrt{\beta^2 + \Delta^2 - m^2}} ,
\]


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Axial Charge Coupling

- **Isovector** vector and axial-vector current

\[
\langle p|V_\mu^\dagger(0)|n\rangle = \bar{u}_p [\gamma_\mu g_V(q^2) - q_\mu \sigma_{\mu\nu} g_T(q^2)] u_n
\]

\[
\langle p|A_\mu^\dagger(0)|n\rangle = \bar{u}_p [\gamma_\mu \gamma_5 g_A(q^2) - i q_\mu \gamma_5 g_P(q^2)] u_n
\]

- Chiral symmetry gives \( Z_A = Z_V = 1/g_V \)

- Continuum \( \chi \)PT extrapolation (SSE scheme)
Axial Charge Coupling: Global View

Comparison among various lattice results (Lattice 2006)
Add in the lightest pion mass point (310 MeV)

\[ g_A / g_V \]

\[ m_{\pi}^2 (\text{GeV}^2) \]

D. Renner et al., PoS(LAT2006)121
# Nucleon Structure Functions

## List of Operators: lowest moments only

<table>
<thead>
<tr>
<th>Operator</th>
<th>Description</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle x \rangle_q$</td>
<td>Momentum fraction</td>
<td>$\mathcal{O}^{q}_{44} = \bar{q} \left[ \gamma_4 \vec{D}_4 - \frac{1}{3} \sum_k \gamma_k \vec{D}_k \right] q$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_{\langle x \rangle_q} = \frac{C_{3pt}^{q(34)}}{C^{2pt}} = m_N \langle x \rangle_q$</td>
</tr>
<tr>
<td>$\mathcal{P}^{q-1}_{44}$</td>
<td></td>
<td>$\mathcal{P}^{q-1}<em>{44} = \gamma_4 p_4 - \frac{1}{3} \sum</em>{i=1,3} \gamma_i p_i$</td>
</tr>
<tr>
<td>$\langle 1 \rangle_{\delta q}$</td>
<td>Transversity</td>
<td>$\mathcal{O}^{\sigma q}<em>{34} = \bar{q} \gamma_5 \sigma</em>{34q}$</td>
</tr>
<tr>
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<td></td>
<td>$R_{\langle 1 \rangle_{\delta q}} = \frac{C_{3pt}^{q(34)}}{C^{2pt}} = \langle 1 \rangle_{\delta q}$</td>
</tr>
<tr>
<td>$\mathcal{P}^{\sigma q-1}_{34}$</td>
<td></td>
<td>$\mathcal{P}^{\sigma q-1}<em>{34} = \gamma_5 \sigma</em>{34}$</td>
</tr>
<tr>
<td>$\langle x \rangle_{\Delta q}$</td>
<td>Helicity distribution</td>
<td>$\mathcal{O}^{5q}_{{34}} = i \bar{q} \gamma_5 \left[ \gamma_3 \vec{D}_4 + \gamma_4 \vec{D}_3 \right] q$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_{\langle x \rangle_{\Delta q}} = \frac{C_{3pt}^{q,5q(34)}}{C^{2pt}} = m_N \langle x \rangle_{\Delta q}$</td>
</tr>
<tr>
<td>$\mathcal{P}^{5q-1}_{34}$</td>
<td></td>
<td>$\mathcal{P}^{5q-1}_{34} = i \gamma_5 \left( \gamma_3 p_4 + \gamma_4 p_3 \right)$</td>
</tr>
<tr>
<td>$\langle 1 \rangle_{d_1}$</td>
<td>Twist-3 matrix element</td>
<td>$\mathcal{O}^{5q}_{[34]} = i \bar{q} \gamma_5 \left[ \gamma_3 \vec{D}_4 - \gamma_4 \vec{D}_3 \right] q$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_{d_1} = \frac{C_{3pt}^{q,5q[34]}}{C^{2pt}} = d_1$</td>
</tr>
<tr>
<td>$\mathcal{P}^{5q-1}_{[34]}$</td>
<td></td>
<td>$\mathcal{P}^{5q-1}_{[34]} = i \gamma_5 \left( \gamma_3 p_4 - \gamma_4 p_3 \right)$</td>
</tr>
</tbody>
</table>
Chiral extrapolation formulae for each quantity

\[
\langle x \rangle_{u-d} = C \left[ 1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) \right] + \epsilon(\mu^2) \frac{m_\pi^2}{(4\pi f_\pi)^2}
\]

\[
\langle x \rangle_{\Delta u-\Delta d} = \tilde{C} \left[ 1 - \frac{2g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left( \frac{m_\pi^2}{\mu^2} \right) \right] + \tilde{\epsilon}(\mu^2) \frac{m_\pi^2}{(4\pi f_\pi)^2}.
\]

Linear ansatz

RI/MOM-scheme nonperturbative renormalization (except for \(d_1\))
Nucleon Structure Functions

- Chiral extrapolations: lowest moments only

![Graphs showing chiral extrapolations for nucleon structure functions. The graphs plot the functions against $m_{\pi}^2 (\text{GeV}^2)$ for different moments.]

Huey-Wen Lin @ Baryon07, Seoul, Korea
Nucleon Structure Functions

Comparison among calculations of the first moment of the momentum fraction

D. Renner et al., PoS(LAT2006)121
Nucleon Structure Functions

Comparison among calculations of the first moment of the helicity distribution

D. Renner et al., PoS(LAT2006)121
Nucleon Isovector Form Factor \( (F_{1}^{u-d}) \)

- **2+1-flavor analysis not yet confirmed**

- **2-flavor** DWF data with pion mass: 500–700 MeV

\[ \langle r^2 \rangle = c_1 - \frac{1 + 5g_A^2}{(4\pi f_\pi)^2} \log \frac{m_{\pi}^2}{m_{\pi}^2 + \Lambda^2} \]

\( G_{1}(q^2)/G_{1}(0) \)

\( m_{\pi}^2(GeV^2) \)

\( (r^2)/(m^2) \)

**Dipole-form extrapolation**

\( J. \ J. \ Kelley, \ Phys. \ Rev. \ C70, \ 068202 \ (2004) \)

\( G. \ V. \ Dunne \ et \ al., \ Phys. \ Lett. \ B531, \ 77 \ (2002) \)
Goldberger-Treiman Relation

The Goldberger-Treiman relation states:

\[ q^2 \frac{G_P(q^2)}{2m_N} - 2m_N G_A(q^2) = -\frac{g_{\pi NN} F_\pi m_\pi^2}{q^2 + m_\pi^2} \]

A measure of the discrepancy

\[ \Delta_{GT}(q^2) = 1 - \frac{q^2 + m_{\pi,\text{lat}}^2}{2m_{N,\text{lat}}} \frac{q^2 G_{P,\text{lat}}(q^2) - 4m_{N,\text{lat}}^2 G_{A,\text{lat}}(q^2)}{g_{\pi NN,\text{lat}} F_{\pi,\text{lat}} m_\pi^2} \]

with \( g_{\pi NN}^{\text{lat}} = \frac{2m_{N,\text{lat}} g_A^{\text{lat}}}{F_{\pi}^{\text{lat}}} \)
Summary/Outlook: I

Nucleon structure functions and form factors

- Work in progress: Full-QCD calculation with pion masses 400–600 MeV
- Preliminary study shows good agreement with experiment, even for notoriously difficult quantities, such as momentum fraction and helicity distribution

In the near future

- 300 MeV pion analysis is on the way (Lattice 2007)
- Taking more statistics at each pion mass
- Finite-volume studied (combined UKQCD $16^3 \times 32$ data)
- Lattice discretization effects will be examined
  ($32^3 \times 64$, $a \sim 0.09$ fm lattices are on the way)

Within a few years or so

- < 200 MeV full-QCD gauge generation proposal
Hyperon Channel (preliminary)

in collaboration with

Kostas Orginos
Parameters

This calculation:

- Mixed action (staggered sea with DWF valence), 2+1-flavor
- Pion mass range: 360–700 MeV
- Strange-strange Goldstone fixed at 763(2) MeV
- Volume fixed at 2.6 fm
- \( a \approx 0.125 \text{ fm}, \ L_s = 16, \ M_5 = 1.7 \)
- HYP-smeared gauge, box size of \( 20^3 \times 32 \)

<table>
<thead>
<tr>
<th>Label</th>
<th>( m_\pi ) (MeV)</th>
<th>( m_K ) (MeV)</th>
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<td>m010</td>
<td>358(2)</td>
<td>605(2)</td>
</tr>
<tr>
<td>m020</td>
<td>503(2)</td>
<td>653(2)</td>
</tr>
<tr>
<td>m030</td>
<td>599(1)</td>
<td>688(2)</td>
</tr>
<tr>
<td>m040</td>
<td>689(2)</td>
<td>730(2)</td>
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Strangeness Magnetic Moment of Nucleon

Disconnected diagrams are challenging
Much effort has been put into resolving this difficulty
Alternative approach:

\[\begin{align*}
\Sigma^+ &= e_u u^u + e_s s^u + O_{\Sigma}^+, \\
\Sigma^- &= e_d u^d + e_s s^d + O_{\Sigma}^-, \\
\Xi^0 &= e_s s^\Xi + e_u u^\Xi + O_{\Xi}^0, \\
\Xi^- &= e_s s^\Xi + e_d u^\Xi + O_{\Xi}^-. \\
\end{align*}\]

Assume charge symmetry:

\[\begin{align*}
p &= e_u u^p + e_d d^p + O_N, \\
n &= e_d u^p + e_u d^p + O_N,
\end{align*}\]

The disconnected piece for the proton is

\[O_N = \frac{2}{3} lG_M^u - \frac{1}{3} lG_M^d - \frac{1}{3} lG_M^s.\]

The strangeness contribution is

\[\begin{align*}
G_M^s &= \left( \frac{lR_d^s}{1 - lR_d^s} \right) \left[ 2p + n - \frac{u^p}{u^\Xi} (\Sigma^+ - \Sigma^-) \right], \\
G_M^s &= \left( \frac{lR_d^s}{1 - lR_d^s} \right) \left[ p + 2n - \frac{u^n}{u^\Xi} (\Xi^0 - \Xi^-) \right] \quad \text{with} \quad lR_d^s \equiv lG_M^s/lG_M^d.
\end{align*}\]
Disconnected diagrams are challenging
Much effort has been put into resolving this difficulty
Alternative approach:

Assume charge symmetry:

\[ p = e_u u^p + e_d d^p + O_N; \quad n = e_d u^p + e_u d^p + O_N, \]

\[ \Sigma^+ = e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; \quad \Sigma^- = e_d u^\Sigma + e_s s^\Sigma + O_\Sigma, \]

\[ \Xi^0 = e_s s^\Xi + e_u u^\Xi + O_\Xi; \quad \Xi^- = e_s s^\Xi + e_d u^\Xi + O_\Xi. \]

The disconnected piece for the proton is

\[ O_N = \frac{2}{3} l_G^u - \frac{1}{3} l_G^d - \frac{1}{3} l_G^s. \]

The strangeness contribution is

\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ 3.673 - \frac{u^p}{u^\Sigma} (3.618) \right] \mu_N \]

\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N \]

Need better statistics

\[ l R_d^s \equiv l G_M^s / l G_M^d \]
Dipole-form extrapolation to $q^2 = 0$

Magnetic-moment ratios (linear extrapolation, for now)

\[
\frac{\mu^p}{\mu^\Sigma} = 1.01(20)
\]

\[
\frac{\mu^n}{\mu^\Xi} = 1.02(21)
\]

Strangeness Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

\[
G_M^s = \left( \frac{iR_d^s}{1 - iR_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N
\]
Strangeness Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

![Graphs showing magnetic moment ratios](image)


$$R_d^s = 0.139(42)$$

We find

$$G_M^s = -0.07(3)$$
Axial Coupling Constants: $g_{\Xi \Xi}$ and $g_{\Sigma \Sigma}$

- Has applications such as hyperon scattering, non-leptonic decays, etc.
- Cannot be determined by experiment
- Existing predictions from $\chi$PT and large-$N_c$ calculations

M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

0.18 < $-g_{\Xi \Xi}$ < 0.36
0.30 < $g_{\Sigma \Sigma}$ < 0.55

- We find consistent numbers with much smaller errors
Summary/Outlook: II

From hyperon analysis

- Preliminary estimate of the proton strange magnetic moment directly from full QCD: $-0.07(3)$

- Predictions for $g_{\Sigma\Sigma} = 0.441(14)$ and $g_{\Xi\Xi} = -0.277(11)$

- DWF are too expensive to get small errors with light pion masses, especially as the $< 300$ MeV era approaches

In the future

- Anisotropic 2+1-flavor clover lattices will be started soon
- Proposed light pion mass $< 200$ MeV in a few years
- Obvious cost benefit right away; share propagators
- Anisotropic clover is good for separating excited contributions; thus cleaner ground-state signal for precision calculation