Parameter Tuning of Three-Flavor Dynamical Anisotropic Clover Action

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Outline

- Background/Motivation
  - Why do we make our life troublesome?

- Methodology and Setup
  - How do we tune them?

- Numerical Results
  - Believe it or not

- Summary/Outlook
  - Cannot wait?
Motivation

Beneficial for excited-state physics, as well as ground-state
Excited-State Physics

Lattice QCD spectrum

- Successfully calculates many ground states (*Nature,…*)
- Nucleon spectrum, on the other hand… not quite

Example: $N$, $P_{11}$, $S_{11}$ spectrum
Excited-State Physics

Lattice QCD spectrum

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- Nucleon spectrum, on the other hand… not quite
- Difficult to see excited states with current dynamical simulation lattice spacing (~2 GeV)

**Anisotropic lattices** ($a_t < a_{x,y,z}$)

- 2f Wilson excited baryons in progress
Excited-State Physics

Lattice QCD spectrum

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**Anisotropic lattices** ($a_t < a_{x,y,z}$)

- 2f Wilson excited baryons in progress
- Preliminary result at SciDAC All Hands’ meeting ($m_\pi = 432$ MeV)
Example: $N\cdot P_{11}$ Form Factor

- Experiments at Jefferson Laboratory (CLAS), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
- **Helicity amplitudes** are measured (in $10^{-3}$ GeV$^{-1/2}$ units)
- One of the major tasks given to Excited Baryon Analysis Center (EBAC)
- Many models disagree (a selection are shown below)

\[ A_{1/2} \]

\[ S_{1/2} \]

- **Lattice work in progress at JLab**
Larger-\( t \) solution does not always work well with three-point correlators

Example:
Quark helicity distribution
LHPC & SESAM


50% increase in error budget at \( t_{\text{sep}} = 14 \)

Confronting the excited states might be a better solution than avoiding them.
Methodology and Setup

Anisotropic Lattice
+ Schrödinger Functional
+ Stout-Smearing
Anisotropic Tadpole-ed Lattice Actions

- \(O(a^2)\)-improved Symanzik gauge action

\[
S_G^{\xi} = \frac{\beta}{N_c} \left\{ \frac{u_t}{\xi_0 u_s^3} \sum_{x,s > s'} [c_0 P_{ss'} + c_1 R_{ss'}] + \frac{\xi_0}{u_s^4} \sum_{x,s} [c_0 P_{st} + c_1 R_{st}] \right\}
\]

- \(O(a)\)-improved Wilson fermion (Clover) action

\[
a_t Q_F = \frac{1}{u_t} \left\{ u_t \hat{m}_0 + \nu_t \hat{W}_t + \frac{\nu_s}{\xi_0} \sum_s \hat{W}_s \right. \\
+ \left. \frac{1}{2} \left[ C_{SW}^t \sum_s \sigma_{ts} \hat{F}_{ts} + \frac{C_{SW}^s}{\xi_0} \sum_{s < s'} \sigma_{ss'} \hat{F}_{ss'} \right] \right\}
\]

with \(C_{SW}^s = \frac{\nu}{u_s^3}, \quad C_{SW}^t = \frac{1}{2} \left( \nu + \frac{1}{\xi} \right) \frac{1}{u_t u_s^2}\) (P. Chen, 2001)

- Coefficients to tune: \(\xi_0, \nu, m_0, \beta\)
Schrödinger Functional

Applying a chromoelectric field across the lattice as

\[ U_0^B(x) = 1, \quad U_s^B(x) = e^{-i \frac{\alpha_s}{\pi} \left[ x_0 C_s' + (T - x_0) C_s \right]} \]

Fermionic sector with additional boundary condition:

\[ P_0^+ \psi(x) = \rho(x), \quad \bar{\psi}(x) P_0^- = \bar{\rho}(x) \text{ at } x_0 = 0 \]
\[ P_0^- \psi(x) = \rho'(x), \quad \bar{\psi} P_0^+ = \bar{\rho}'(x) \text{ at } x_0 = T', \]

Fermionic boundary fields are derivative of BC

Boundary counter-terms enter PCAC at \( O(a^2) \);
no further improvement needed

Background field helps with exceptional small eigenvalues

Example: lowest eigenvalue from \( \bar{Q}Q \) (3f anisotropic lattice)

Dynamical 2- and 2+1-flavor isotropic lattice (Alpha, CP-PACS)
**Isotropic Nonperturbative $c_{SW}$**

- $O(a)$ improved axial current
  \[ A_{\text{Imp},\mu}^a = A_\mu^a + ac_A \frac{1}{2} (\partial_\mu + \partial^*_\mu) P^a \]

- PCAC tells us
  \[ \frac{1}{2} (\partial_\mu + \partial^*_\mu) A_{\text{Imp},\mu}^a = 2am_q P^a + O(a^2) \]

- Green function with boundary fields
  \[ f_{\Omega}(x_0) = -a^6 \sum_a \langle \Omega(x)^a \Omega^a \rangle \]

- PCAC implies
  \[ m(x_0) = r(x_0) + ac_A s(x_0) \]
  where
  \[ r(x_0) = 0.25 (\partial_0 + \partial^*_0) f_A(x_0) / f_P(x_0) \]
  \[ s(x_0) = 0.5a \partial_0 \partial^*_0 f_P(x_0) / f_P(x_0). \]

- Redefined the mass through algebra exercise
  \[ M(x_0, y_0) = r(x_0) - \hat{c}_A(y_0) s(x_0) \]
  \[ M'(x_0, y_0) = r'(x_0) - \hat{c}_A(y_0) s'(x_0) \]
  with
  \[ \hat{c}_A(y_0) = \frac{1}{a} \frac{r'(y_0) - r(y_0)}{s'(y_0) - s(y_0)} \]

- Nonperturbative $c_{SW}$ from
  \[ \Delta M = M(x_0, y_0) - M'(x_0, y_0) = \Delta M^{(0)} \]
Stout-Link Smearing

Morningstar, Peardon’04

- Smoothes out dislocations; impressive glueball results
- Updating spatial links only
- Differentiable!
- Direct implementation for dynamical simulation

Better scaling!

with $n_\rho = 2$ and $\rho = 0.22$

Quenched Wilson gauge comparison
Tadpole Factors

- Mostly use either tree-level or one-loop PT value
- Question: How good is it on dynamical anisotropic lattices?
- Take the tadpole value from the 1/4 root of the plaquette
- Without link-smearing (<2% discrepancy is observed)

With stout-link smearing (25% in spatial plaquette)

We modify the tadpole factors from numerical runs to have consistency to within 2%
**Numerical Setup**

- **Chroma** HMC code with RHMC for the 3\textsuperscript{rd} flavor and multi-timescale integration
- Create *additional* Schrödinger Functional world with background fields in the “z” direction
- Question: *What would be an ideal spatial dimension?*

![Graph showing optimal $L_z$](image)
Numerical Setup

- **Chroma** HMC code with RHMC for the 3rd flavor and multi-timescale integration
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Optimal $L_z$

Optimal $L_{x,y}$

**Optimal dimension: $12^2 \times 16$**
Conditions to Tune

Traditionally, conditions for anisotropic clover action

- Gauge anisotropy $\xi_0$ ratios of static quark potential (Klassen)
- Fermion anisotropy $\nu_s$ meson dispersion relation

The above two are done in the non-SF world, big volume

- NP Clover coeffs. ($c_{SW}$) from PCAC mass difference only in isotropic (Alpha,CP-PACS)

Implement background fields in two directions: $t$ and “$z$”

Proposed conditions:

- Gauge anisotropy $\xi_0$ ratios of static quark potential
- Fermion anisotropy $\nu_s$ PCAC mass ratio

Done in the SF world, small volume

- 2 Clover coeffs. ($c_{SW}$)
  - $\xi_G(\xi_0', \nu', m_0') = \xi$
  - $M_s(\xi_0', \nu', m_0') = a_s m_q$
  - $M_t(\xi_0', \nu', m_0') = a_s m_q / \xi$

Set to stout-smeared tadpole coefficient
Check the PCAC mass difference
Numerical Results
Gauge Anisotropy

- **Klassen Method**: ratio of Wilson loops
  \[
  R_{ss}(x, y) = \frac{W_{ss}(x, y)}{W_{ss}(x + 1, y)} \xrightarrow{\text{asym.}} e^{-a_s V_s(ya_s)},
  \]
  \[
  R_{st}(x, t) = \frac{W_{st}(x, t)}{W_{st}(x + 1, t)} \xrightarrow{\text{asym.}} e^{-a_s V_s(ta_t)}.
  \]

- **Diff**: measurement done with BF in z direction

- **Wanted**: \( V_s(ya_s) = V_s(ta_s/\zeta_R) \) \(\Rightarrow\) **Condition**: \( R_{ss}(x, y) = R_{st}(x, t) \)

**Example**
\[
(\zeta_0 = 3.5, \nu_s = 2.0, m_0 = -0.0653, \beta = 2.0)
\]
\[
\zeta_R = 3.50(4)
\]
Gauge Anisotropy

Klassen Method: ratio of Wilson loops

\[ R_{ss}(x, y) = \frac{W_{ss}(x, y)}{W_{ss}(x + 1, y)} \xrightarrow{\text{asym.}} e^{-\alpha_s V_s(ya_s)}, \]

\[ R_{st}(x, t) = \frac{W_{st}(x, t)}{W_{st}(x + 1, t)} \xrightarrow{\text{asym.}} e^{-\alpha_s V_s(ta_t)}. \]

Diff: measurement done with BF in \( z \) direction

Wanted: \( V_s(ya_s) = V_s(ta_s/\xi_R) \Rightarrow \text{Condition: } R_{ss}(x, y) = R_{st}(x, t) \)

Example

\( (\xi_0 = 3.5, \nu_s = 2.0, m_0 = -0.0653, \beta = 2.0) \)

\( \xi_R = 3.50(4) \)

\( \frac{\xi_R}{\xi_0} \approx 1 \) for all \( \nu \)

\( \nu = 0.8, 0.9, 0.95, 1, 1.1 \)
Fix $\xi_0$ at 3.5 $\rightarrow$ $\xi_R \approx 3.5$

Simplify tuning in 2D parameter space

List of trial parameters
2D Parameter/Data Space

- Fix $\xi_0$ at 3.5 $\Rightarrow$ $\xi_R \approx 3.5$
- Simplify tuning in 2D parameter space
- List of trial parameters and corresponding data

![Graph showing data points in a 2D parameter space with markers for different values of $\nu$.]
2D Parameter/Data Space

Fix $\xi_0$ at 3.5 $\Rightarrow \xi_R \approx 3.5$

Simplify tuning in 2D parameter space

List of trial parameters and corresponding data
Fermion Anisotropy

Question: How does our condition for the fermion anisotropy compare with the conventional dispersion relation in large volume?

Quick local test: $12^3 \times 128$ without background field
3-flavor, $m_0 = -0.054673$, $\nu_s = 1.0$, $a_s = 0.116(3)$ fm

From PCAC $M_s$ and $M_t$, we see about 10% disagreement
Question: *How does our condition for the fermion anisotropy compare with the conventional dispersion relation in large volume?*

Quick local test: $12^3 \times 128$ without background field
3-flavor, $m_0 = -0.054673$, $\nu_s = 1.0$, $a_s = 0.116(3)$ fm

From PCAC $M_s$ and $M_t$, we see about 10% disagreement
Dispersion relation shows similar amount of inconsistency
Implement background fields in two directions: \( t \) and “z”
\[ \Rightarrow 2 \text{ PCAC mass, } M_t, M_s \]
Localized region suitable for linear ansatz
\[ M_{s,t}(\nu, m_0) = b_{s,t} + c_{s,t} \nu + d_{s,t} m_0 \]
Condition: \[ M_s = M_t \]
Parameterization

- Implement background fields in two directions: $t$ and “z”
  $\Rightarrow$ 2 PCAC masses: $M_t$, $M_s$
- Localized region suitable for linear ansatz
  $M_{s,t}(\nu,m_0) = b_{s,t} + c_{s,t} \nu + d_{s,t} m_0$
- Condition: $M_s = M_t$

- More runs in the range $0.95 \leq \nu \leq 1.05$ coming
Nonperturbative $c_{SW}$?

- Nonperturbative condition
  \[ \Delta M = M(2T/4, T/4) - M'(2T/4, T/4) = \Delta M_{\text{Tree}, M=0} \]

- Tree-level $\Delta M$ value obtained from simulation in free-field

Examples:

At points where $M_s = M_t$, the NP condition is satisfied or agrees within $\sigma$
Summary/Outlook

Current Status:

- SF + stout-link smearing show promise in the dynamical runs
- Stout-link smearing + modified tadpole factors make NP $c_{sw}$ tuning condition fulfilled
- Finite-box tuning is as good as conventional large-box runs with gauge and fermion anisotropy but more efficient
- 2f anisotropic ($\zeta_R = 3$) Wilson configurations completed ($L \sim 1.8, 2.6$ fm, $m_\pi \sim 400, 600$ MeV)

In the near future:

- Fine tuning the strange quark points
- Launch 2+1f, $24^3 \times 64$ generation
- $O(a)$-improved coefficients: $c_{V,A}$, $Z_{V,A}$...