Parameter Tuning of Three-Flavor Dynamical Anisotropic Clover Action

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Outline

- Background/Motivation
  - Why do we make our life troublesome?
- Methodology and Setup
  - How do we tune them?
- Numerical Results
  - Believe it or not
- Summary/Outlook
  - Cannot wait?

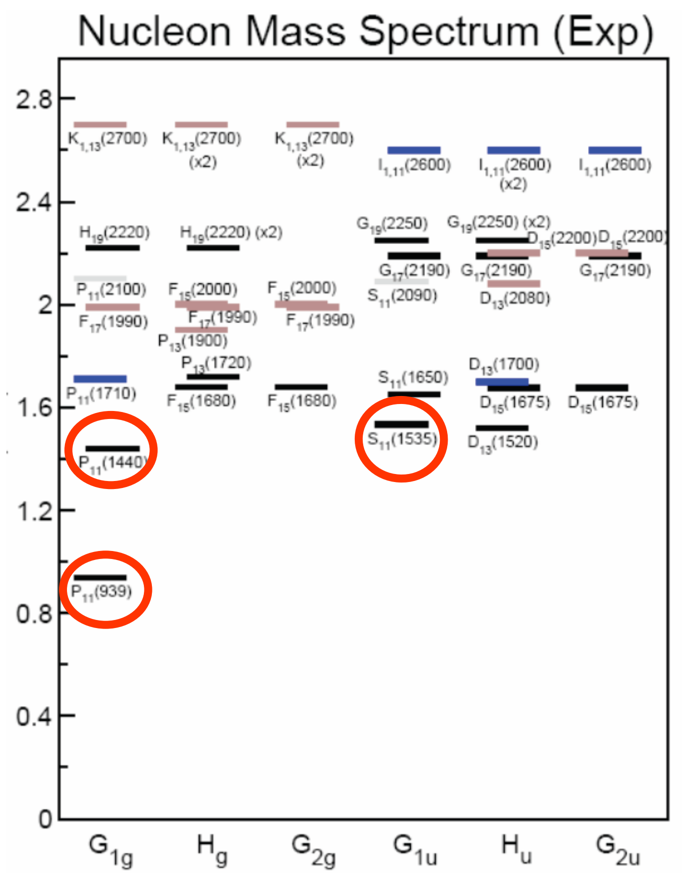
Motivation

Beneficial for excited-state physics,
as well as ground-state

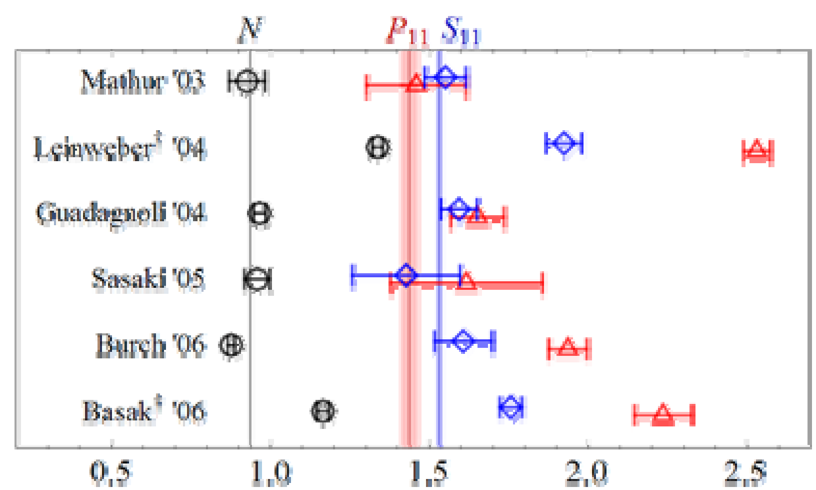
Excited-State Physics

Lattice QCD spectrum

- Successfully calculates many ground states (*Nature*,…)
- Nucleon spectrum, on the other hand… not quite



Example: N, P_{11}, S_{11} spectrum



Excited-State Physics

Lattice QCD spectrum

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- Nucleon spectrum, on the other hand… not quite
- Difficult to see excited states with current dynamical simulation lattice spacing (~2 GeV)

Anisotropic lattices ($a_t < a_{x,y,z}$)

- 2f Wilson excited baryons in progress

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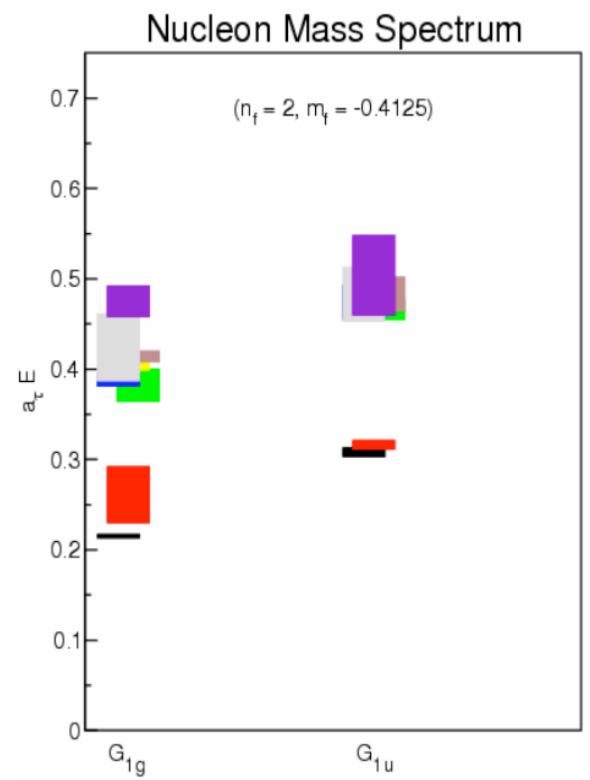
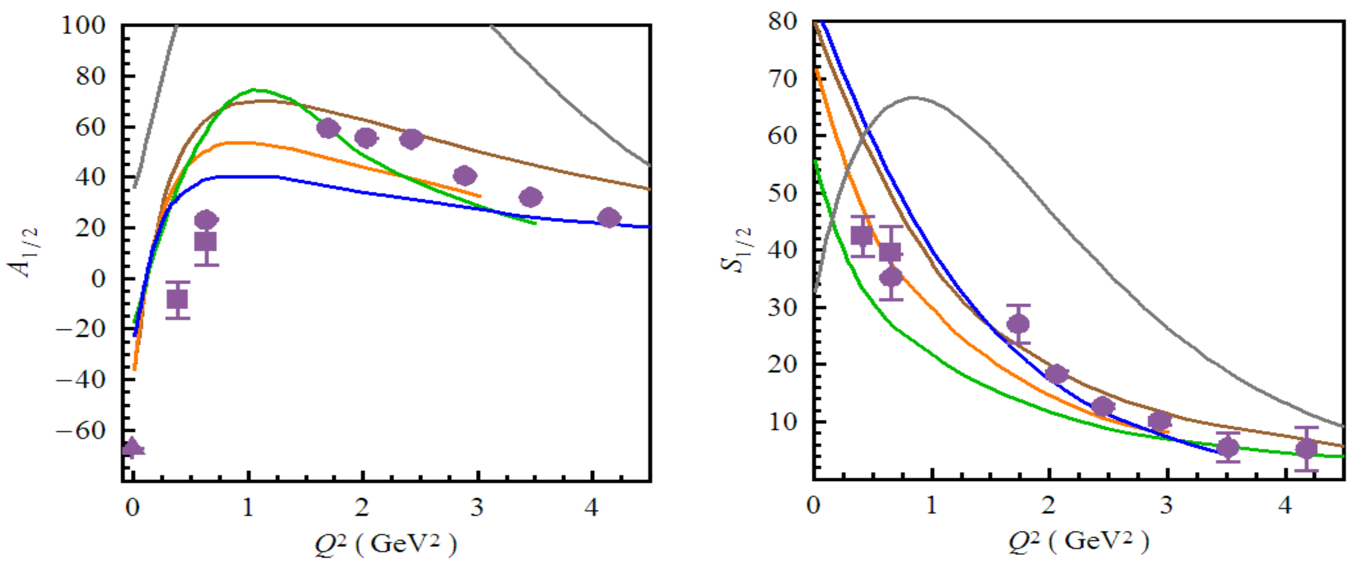
Anisotropic lattices ($a_t < a_{x,y,z}$)

- 2f Wilson excited baryons in progress
- Preliminary result at SciDAC All Hands' meeting (m_π = 432 MeV)



Example: N-P_{11} Form Factor

- Experiments at Jefferson Laboratory (**CLAS**), MIT-Bates, LEGS, Mainz, Bonn, GRAAL, and Spring-8
- Helicity amplitudes are measured (in 10^{-3} $\mathrm{GeV}^{-1/2}$ units)
- One of the major tasks given to Excited Baryon Analysis Center (EBAC)
- Many models disagree (a selection are shown below)



- Lattice work in progress at JLab

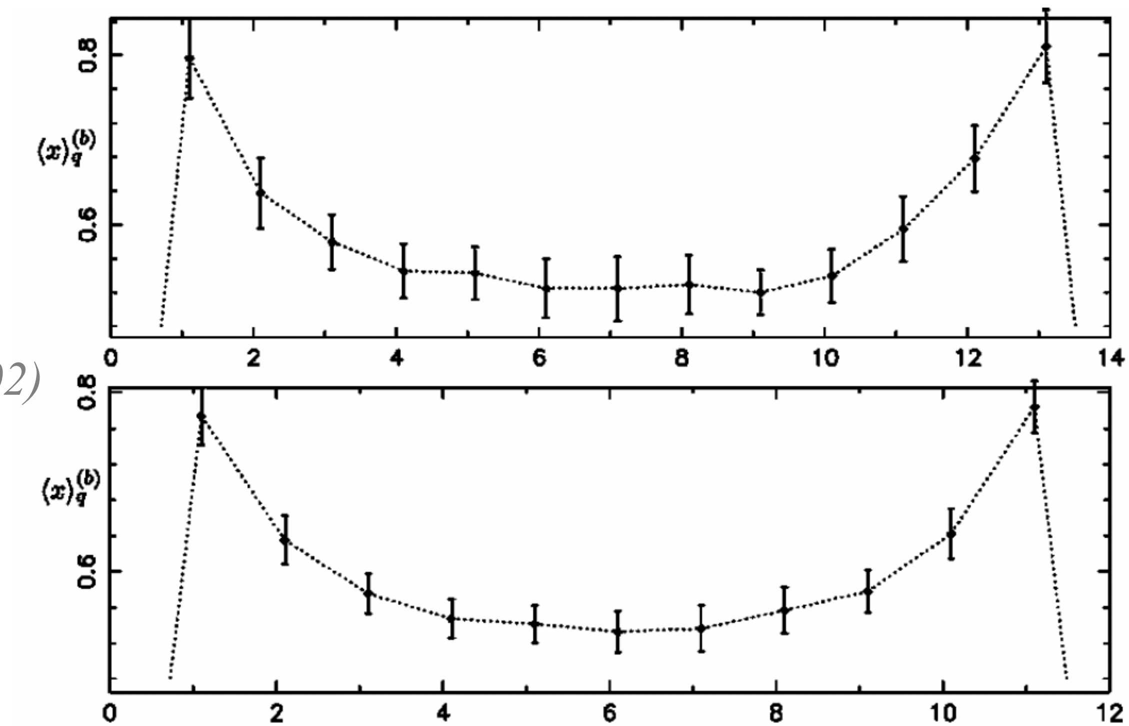
Only Interested in Ground State?

- Larger-t solution does not always work well with three-point correlators
- Example:

  Quark helicity distribution

  LHPC & SESAM

  Phys. Rev. D ***66****, 034506 (2002)*



50% increase in error budget at $t_{sep} = 14$

- Confronting the excited states might be a better solution than avoiding them.

Methodology and Setup

Anisotropic Lattice
+ Schrödinger Functional
+ Stout-Smearing

Anisotropic Tadpole-ed Lattice Actions

- $O(a^2)$-improved Symanzik gauge action

$$S_G^{\xi} = \frac{\beta}{N_c}\left\{\frac{u_t}{\xi_0 u_s^3}\sum_{x,s>s'}\left[c_0\mathcal{P}_{ss'} + c_1\mathcal{R}_{ss'}\right] + \frac{\xi_0}{u_s^4}\sum_{x,s}\left[c_0\mathcal{P}_{st} + c_1\mathcal{R}_{st}\right]\right\}$$

- $O(a)$-improved Wilson fermion (Clover) action

$$\begin{aligned} a_t Q_F &= \frac{1}{u_t}\left\{u_t\hat{m}_0 + \nu_t\hat{W}_t + \frac{\nu_s}{\xi_0}\sum_s \hat{W}_s \right. \\ &\left. + \frac{1}{2}\left[C_{\mathrm{SW}}^t\sum_s \sigma_{ts}\hat{F}_{ts} + \frac{C_{\mathrm{SW}}^s}{\xi_0}\sum_{s<s'}\sigma_{ss'}\hat{F}_{ss'}\right]\right\} \end{aligned}$$

with $C_{\mathrm{SW}}^s = \frac{\nu}{u_s^3}, \quad C_{\mathrm{SW}}^t = \frac{1}{2}\left(\nu + \frac{1}{\xi}\right)\frac{1}{u_t u_s^2}$ (P. Chen, 2001)

- Coefficients to tune: ξ_0, v_s, m_0, β

Schrödinger Functional

- Applying a chromoelectric field across the lattice as

$$U_0^B(x) = 1, \quad U_s^B(x) = e^{-i\frac{a_s}{T}\left[x_0 C_s' + (T - x_0) C_s\right]}$$

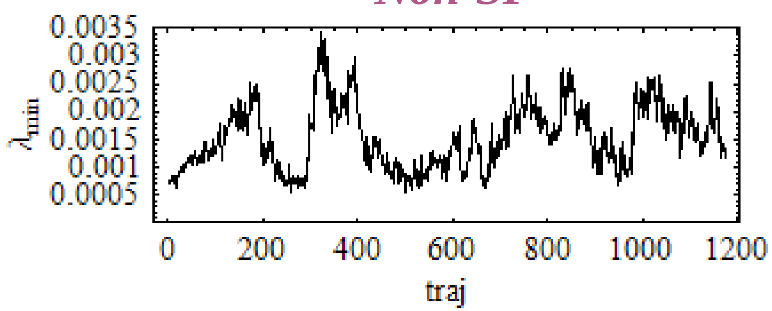
- Fermionic sector with additional boundary condition:

$$P_0^+\psi(x) = \rho(x), \quad \overline{\psi}(x)P_0^- = \overline{\rho}(x) \text{ at } x_0 = 0$$

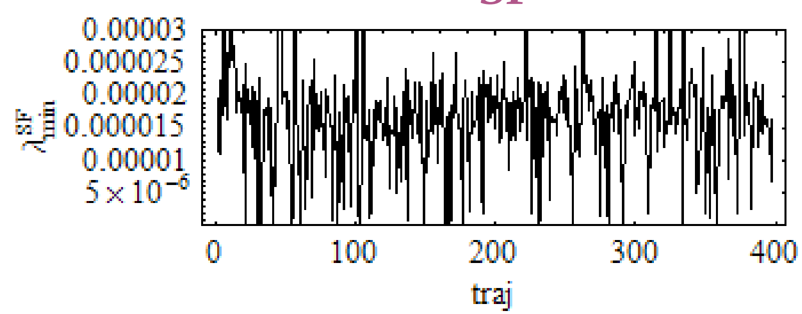
$$P_0^-\psi(x) = \rho'(x), \quad \overline{\psi}P_0^+ = \overline{\rho}'(x) \text{ at } x_0 = T',$$

- Fermionic boundary fields are derivative of BC
- Boundary counter-terms enter PCAC at $O(a^2)$; no further improvement needed
- Background field helps with exceptional small eigenvalues
    - Example: lowest eigenvalue from $Q^\dagger Q$ *(3f anisotropic lattice)*

Non-SF



SF



- Dynamical 2- and 2+1-flavor isotropic lattice (Alpha, CP-PACS)

Isotropic Nonperturbative c_{SW}

- $O(a)$ improved axial current $A^a_{\mathrm{Imp},\mu} = A^a_\mu + a c_A \frac{1}{2}\left(\partial_\mu + \partial^*_\mu\right) P^a$
- PCAC tells us $\frac{1}{2}\left(\partial_\mu + \partial^*_\mu\right) A^a_{\mathrm{Imp},\mu} = 2am_q P^a + \mathcal{O}(a^2)$
- Green function with boundary fields $f_{O_\Gamma}(x_0) = -a^6 \sum_a \langle O_\Gamma(x)^a \mathcal{O}^a \rangle$
- PCAC implies $m(x_0) = r(x_0) + a c_A s(x_0)$

  where

$$
\begin{aligned}
r(x_0) &= 0.25\left(\partial_0 + \partial^*_0\right) f_A(x_0)/f_P(x_0) \\
s(x_0) &= 0.5a\, \partial_0 \partial^*_0 f_P(x_0)/f_P(x_0).
\end{aligned}
$$

- Redefined the mass through algebra exercise

$$
\begin{aligned}
M(x_0, y_0) &= r(x_0) - \hat{c}_A(y_0) s(x_0) \\
M'(x_0, y_0) &= r'(x_0) - \hat{c}_A(y_0) s'(x_0)
\end{aligned}
\quad \text{with} \quad \hat{c}_A(y_0) = \frac{1}{a}\frac{r'(y_0) - r(y_0)}{s'(y_0) - s(y_0)}
$$

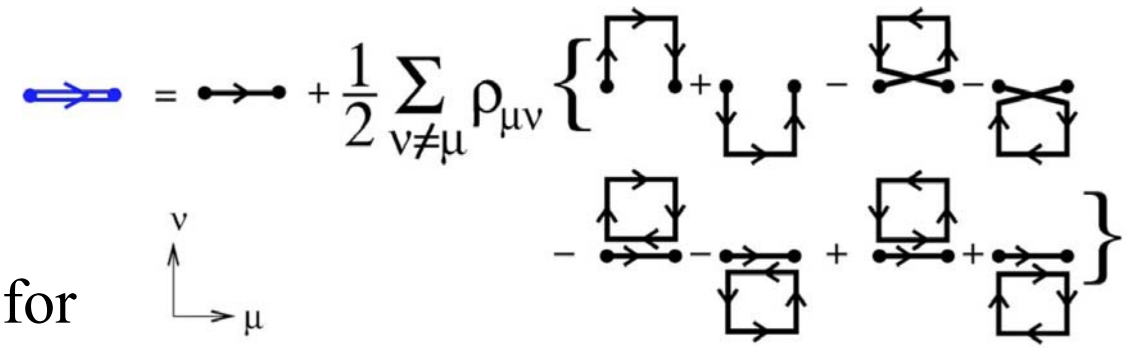
- Nonperturbative c_{SW} from

$$\Delta M = M(x_0, y_0) - M'(x_0, y_0) = \Delta M^{(0)}$$

Stout-Link Smearing

Morningstar, Peardon'04

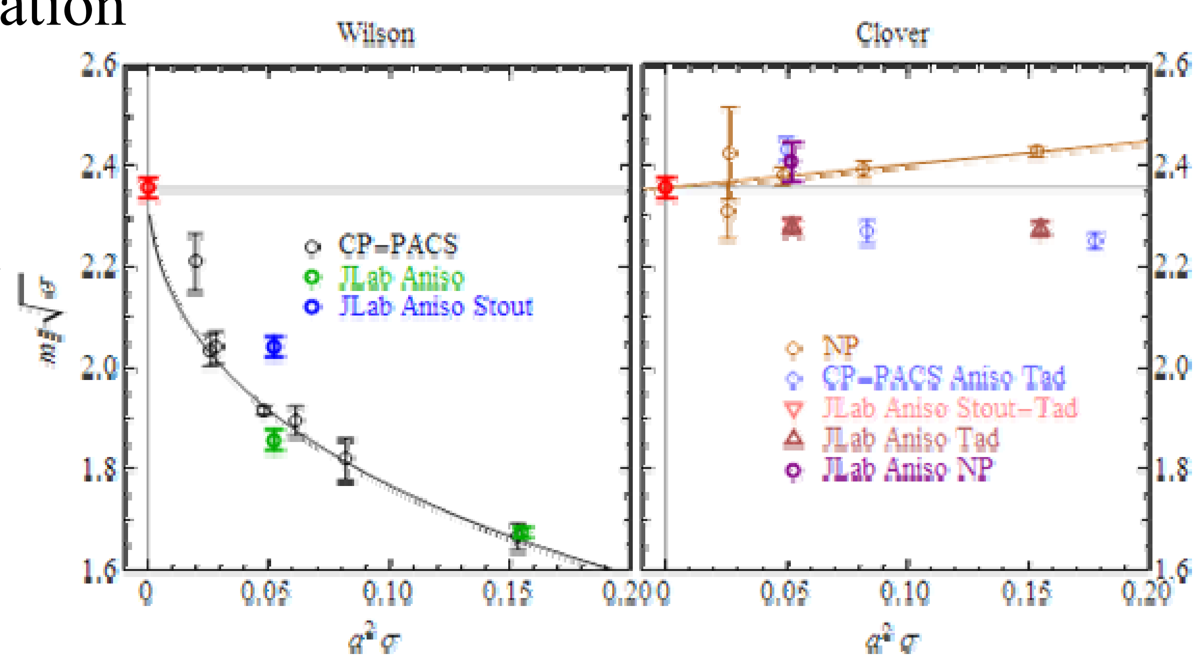
- Smoothes out dislocations; impressive glueball results
- Updating *spatial* links only
- Differentiable!
  Direct implementation for dynamical simulation



Better scaling!

with $n_\rho = 2$ and $\rho = 0.22$

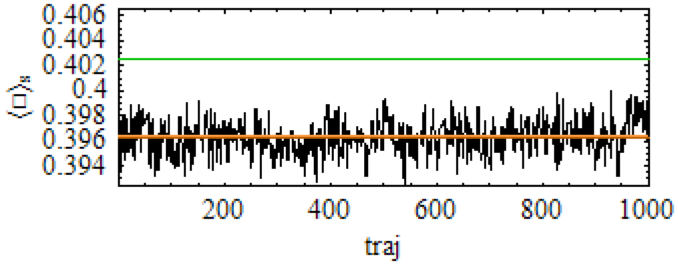
Quenched Wilson gauge comparison



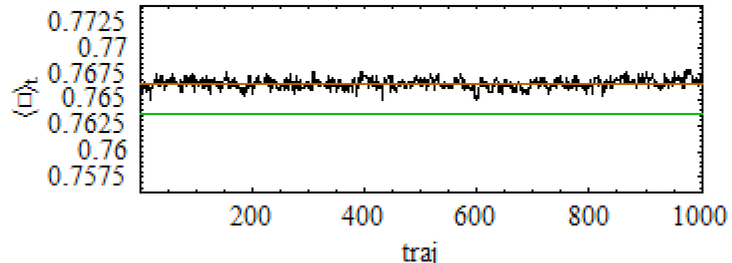
Tadpole Factors

- Mostly use either tree-level or one-loop PT value
- Question: *How good is it on dynamical anisotropic lattices?*
- Take the tadpole value from the 1/4 root of the plaquette
- Without link-smearing (<2% discrepancy is observed)

Space

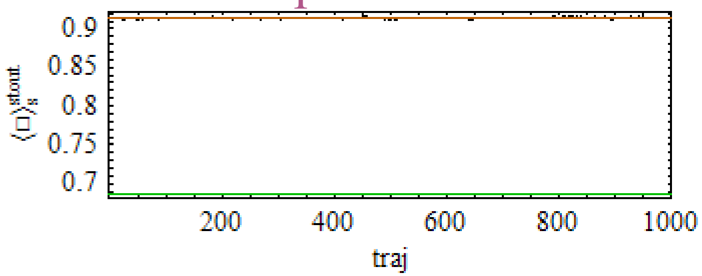


Time



- With stout-link smearing (25% in spatial plaquette)

Space



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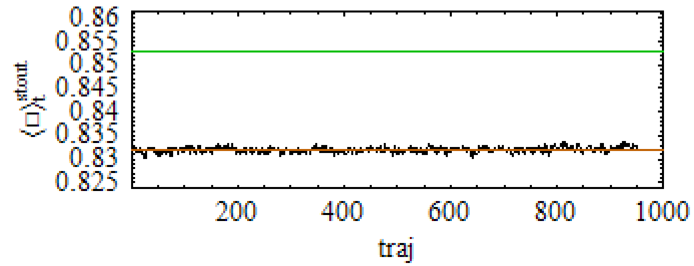
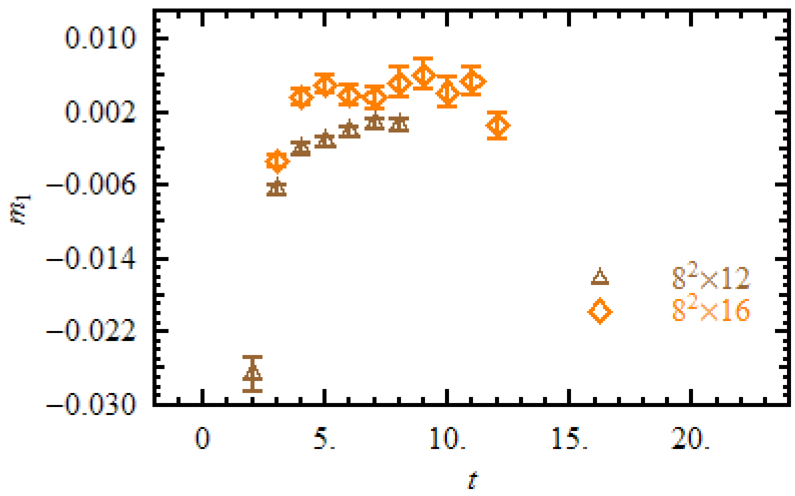


- We modify the tadpole factors from numerical runs to have consistency to within 2%

Numerical Setup

- *Chroma* HMC code with RHMC for the 3rd flavor and multi-timescale integration
- Create *additional* Schrödinger Functional world with background fields in the "*z*" direction
- Question: *What would be an ideal spatial dimension?*

Optimal L_z



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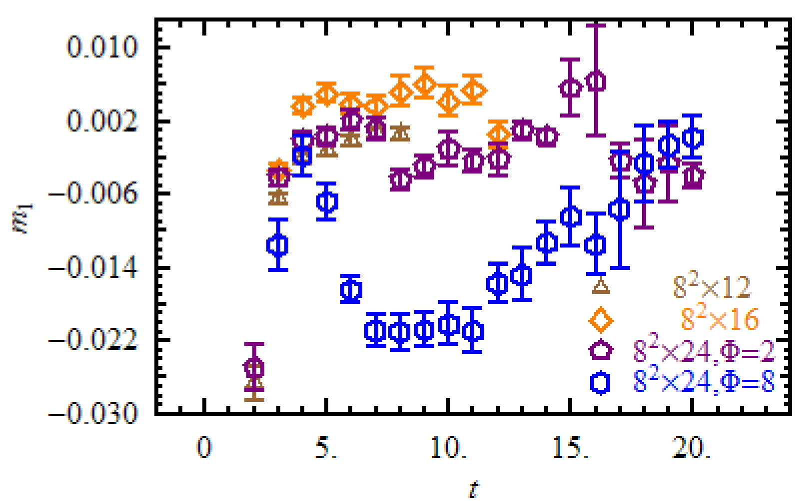
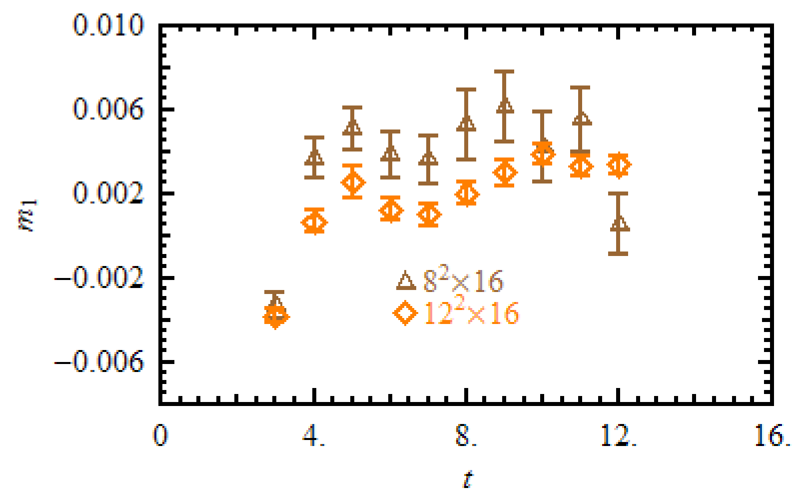


Optimal $L_{x,y}$



- Optimal dimension: $12^2\times16$

Conditions to Tune

- Traditionally, conditions for anisotropic clover action
  - Gauge anisotropy ξ_0 — ratios of static quark potential (Klassen)
  - Fermion anisotropy ν_s — meson dispersion relation

  The above two are done in the non-SF world, big volume

  - NP Clover coeffs. (c_{SW}) from PCAC mass difference only in isotropic (Alpha,CP-PACS)

- Implement background fields in two directions: *t* and "*z*"
- Proposed conditions:
  - Gauge anisotropy ξ_0 — ratios of static quark potential
  - Fermion anisotropy ν_s — PCAC mass ratio

  Done in the SF world, small volume

  - 2 Clover coeffs. (c_{SW}) — Set to *stout-smeared tadpole* coefficient; Check the PCAC mass difference

$$
\begin{aligned}
\xi_G(\xi_0', \nu', m_0') &= \xi \\
M_s(\xi_0', \nu', m_0') &= a_s m_q \\
M_t(\xi_0', \nu', m_0') &= a_s m_q/\xi
\end{aligned}
$$

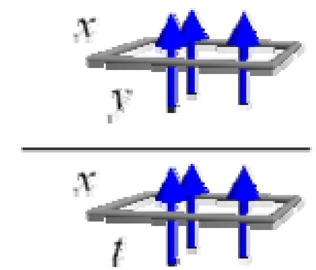
Numerical Results

Gauge Anisotropy

- Klassen Method: ratio of Wilson loops

$$R_{ss}(x,y) = \frac{W_{ss}(x,y)}{W_{ss}(x+1,y)} \xrightarrow{\text{asym.}} e^{-a_s V_s(y a_s)},$$

$$R_{st}(x,t) = \frac{W_{st}(x,t)}{W_{st}(x+1,t)} \xrightarrow{\text{asym.}} e^{-a_s V_s(t a_t)}.$$

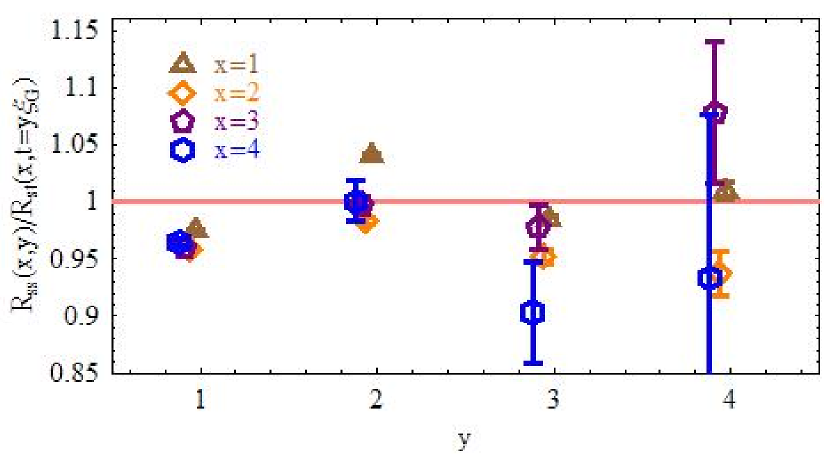


- Diff: measurement done with BF in *z* direction
- Wanted: $V_s(y a_s) = V_s(t a_s/\xi_R)$ ⇨ Condition: $R_{ss}(x,y) = R_{st}(x,t)$

Example

$(\xi_0 = 3.5,\ v_s = 2.0,\ m_0 = -0.0653,\ \beta = 2.0)$

$\xi_R = 3.50(4)$



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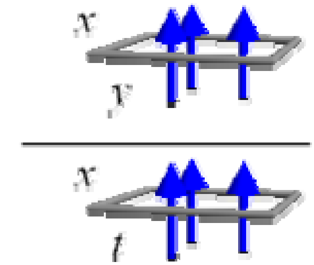

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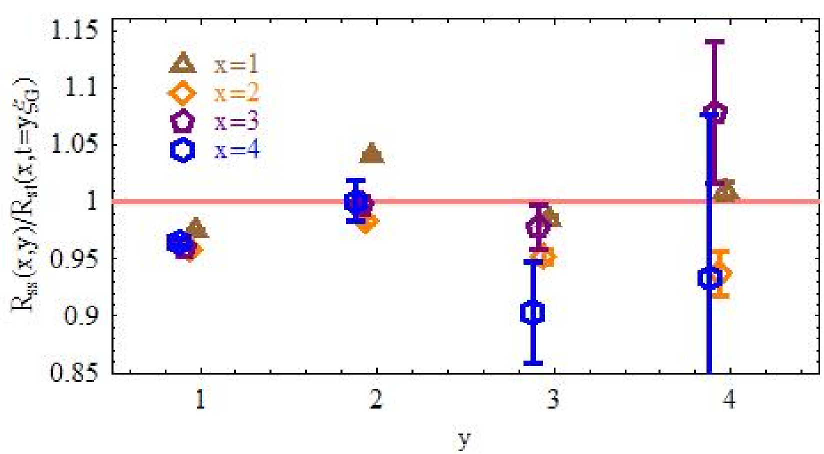
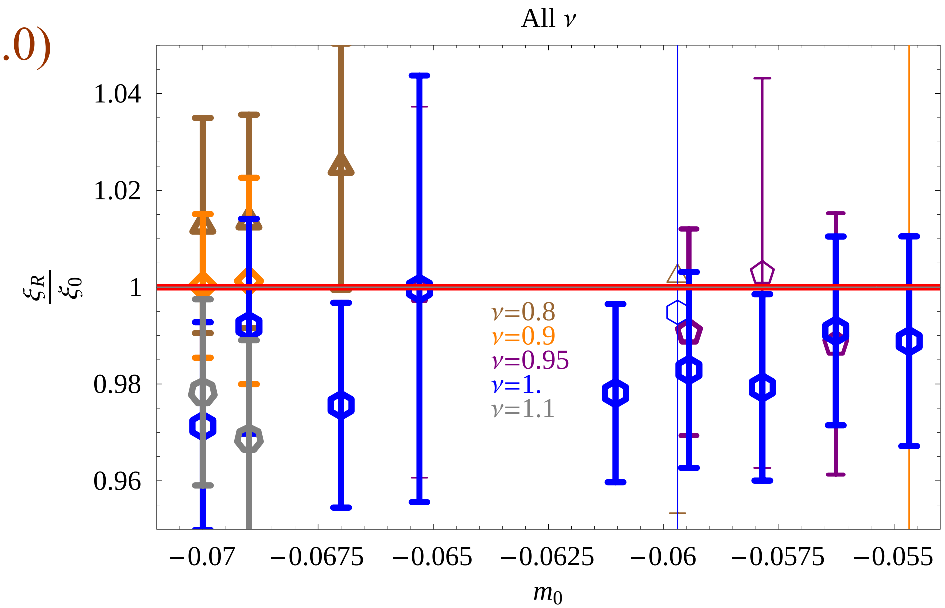
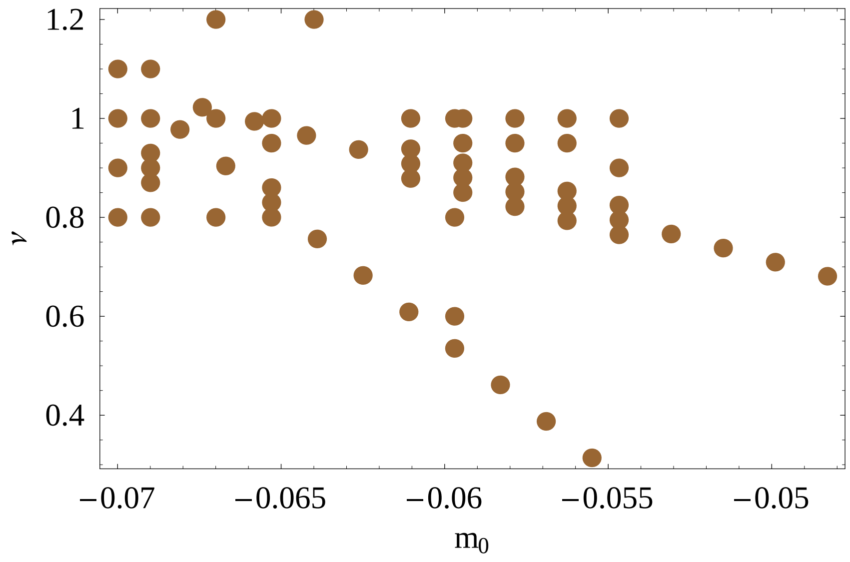


$\xi_R/\xi_0 \approx 1$



2D Parameter/Data Space

- Fix ξ_0 at 3.5 → $\xi_R \approx 3.5$
- Simplify tuning in 2D parameter space
- List of trial parameters →



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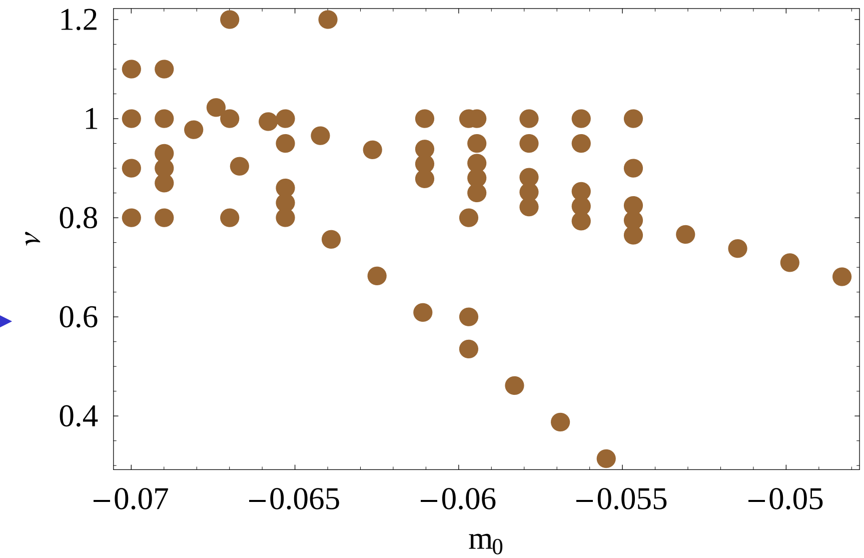

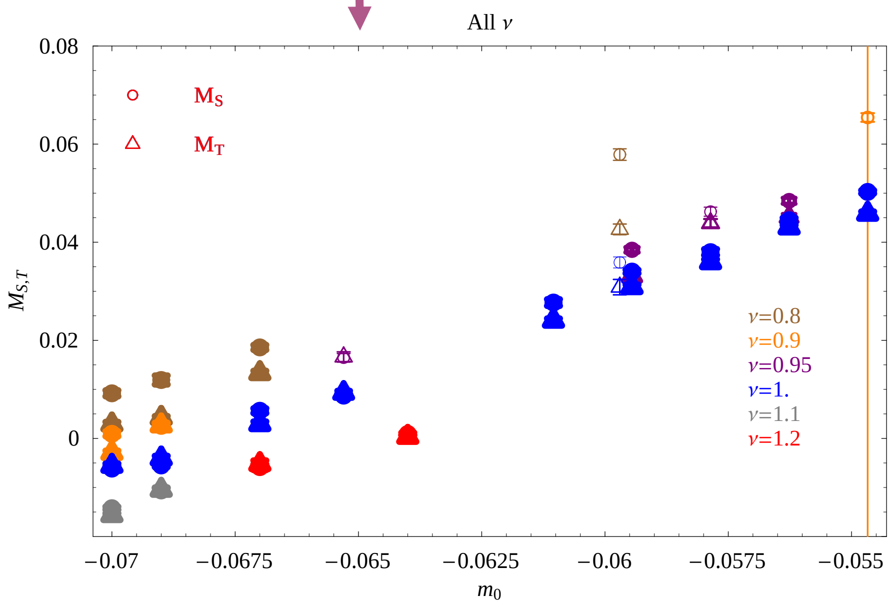

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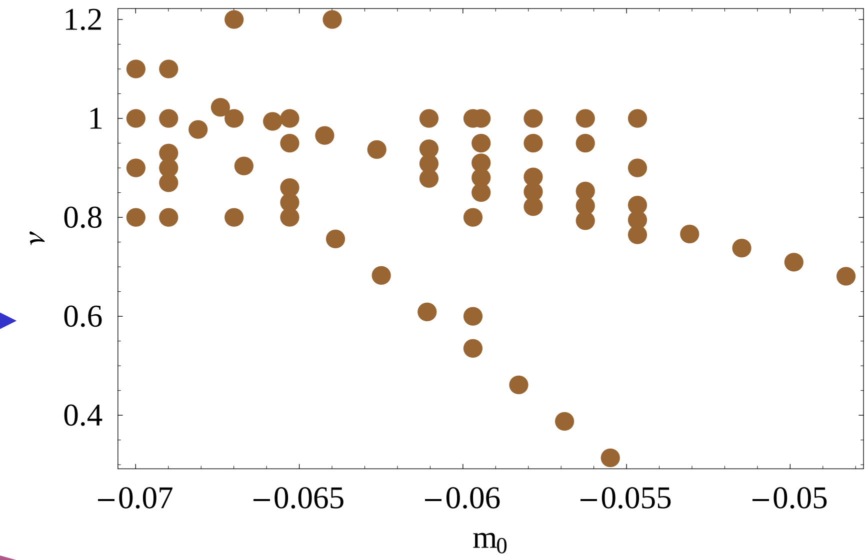
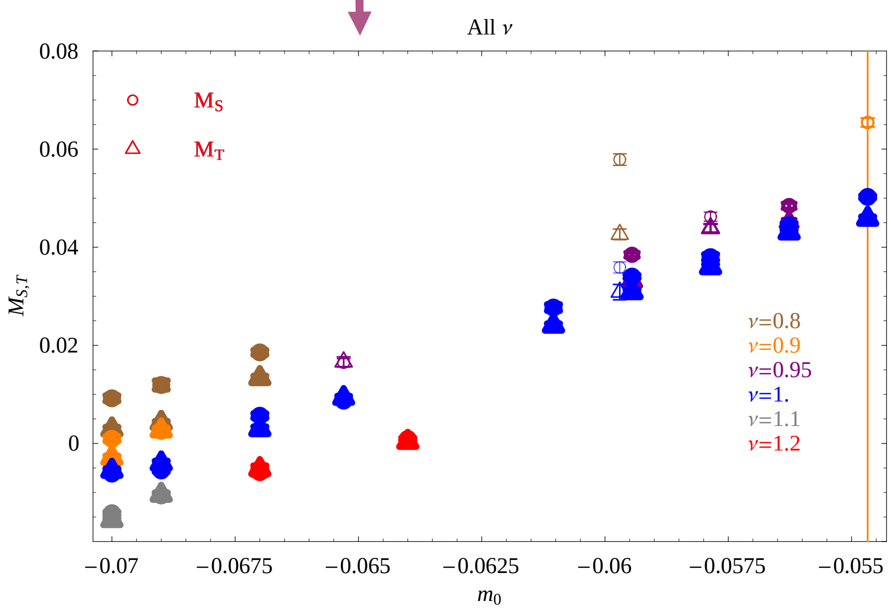
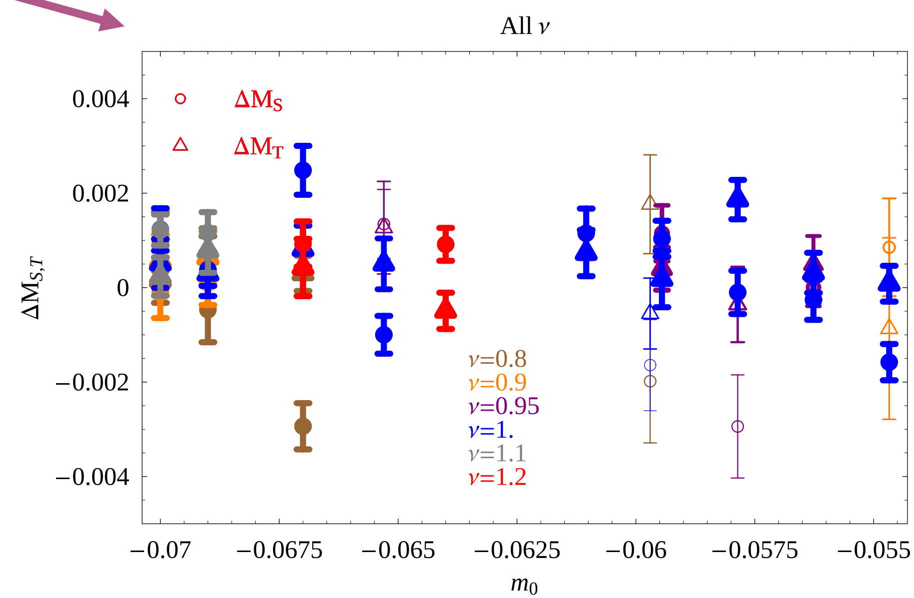
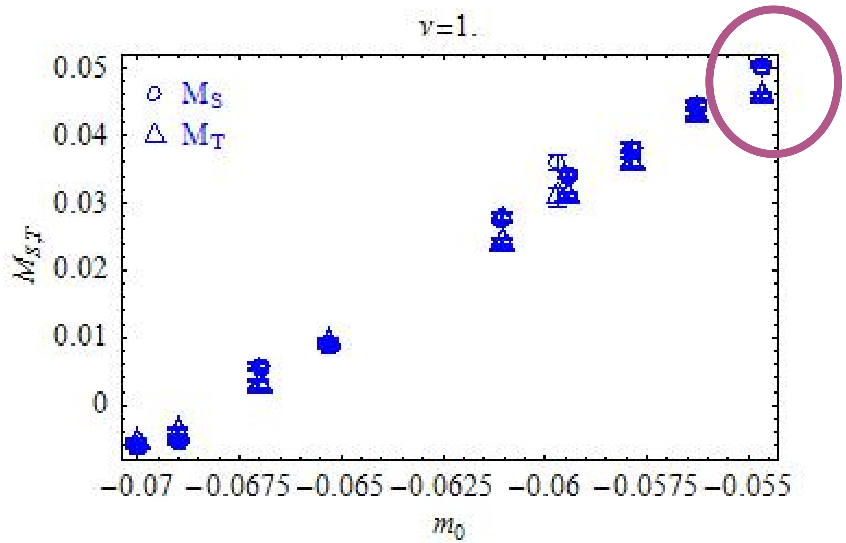






Fermion Anisotropy

- Question: *How does our condition for the fermion anisotropy compare with the conventional dispersion relation in large volume?*
- Quick local test: $12^3 \times 128$ without background field
  3-flavor, $m_0 = -0.054673$, $v_s = 1.0$, $a_s = 0.116(3)$ fm



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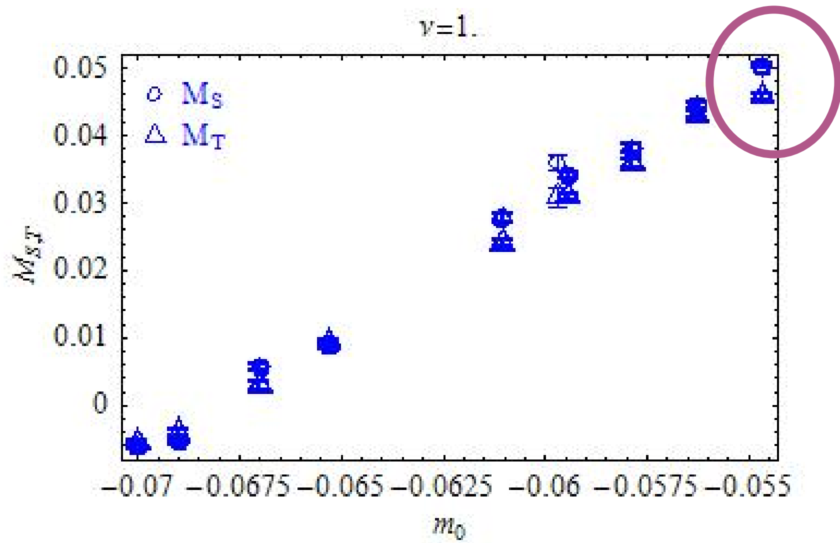
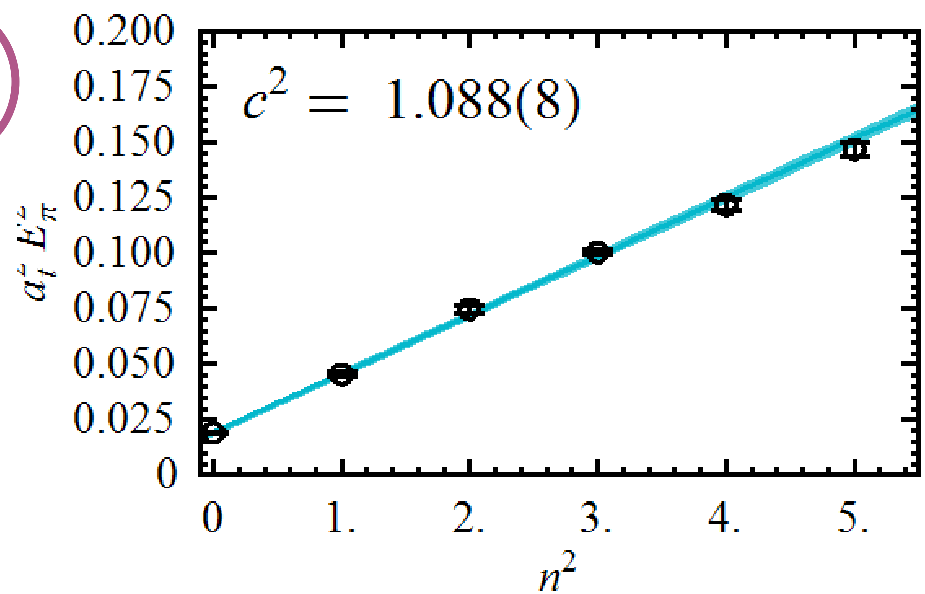
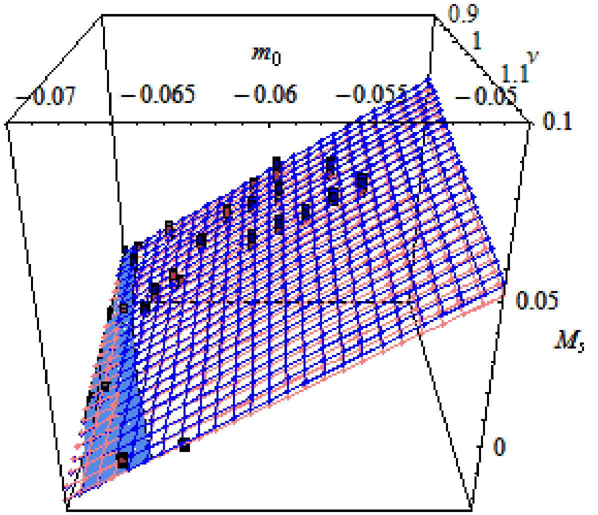




- From PCAC M_s and M_t, we see about 10% disagreement
- Dispersion relation shows similar amount of inconsistency

Parameterization

- Implement background fields in two directions: t and "z"
  ⇒ 2 PCAC mass, M_t, M_s
- Localized region suitable for linear ansatz
  $$M_{s,t}(v,m_0) = b_{s,t} + c_{s,t}v + d_{s,t}m_0$$
- Condition: $M_s = M_t$



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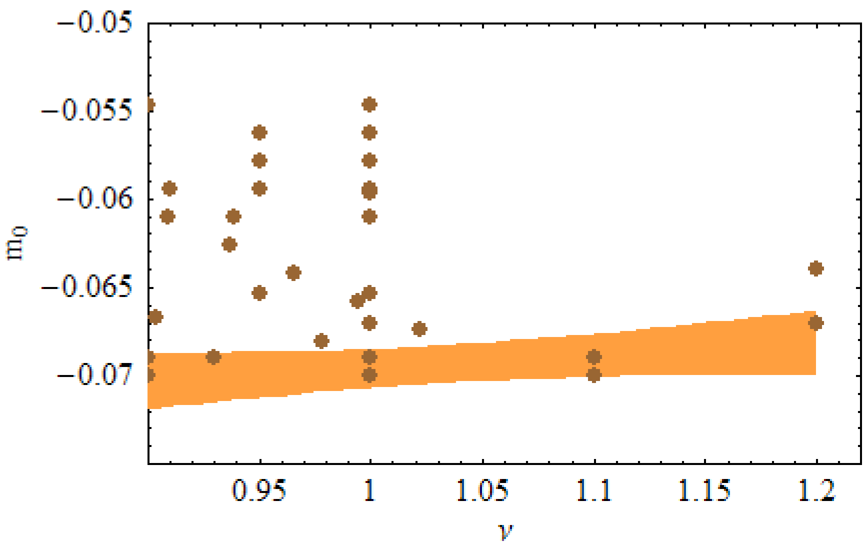
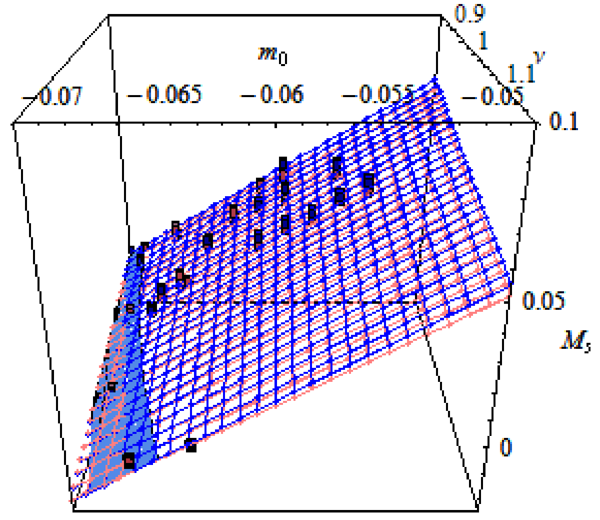




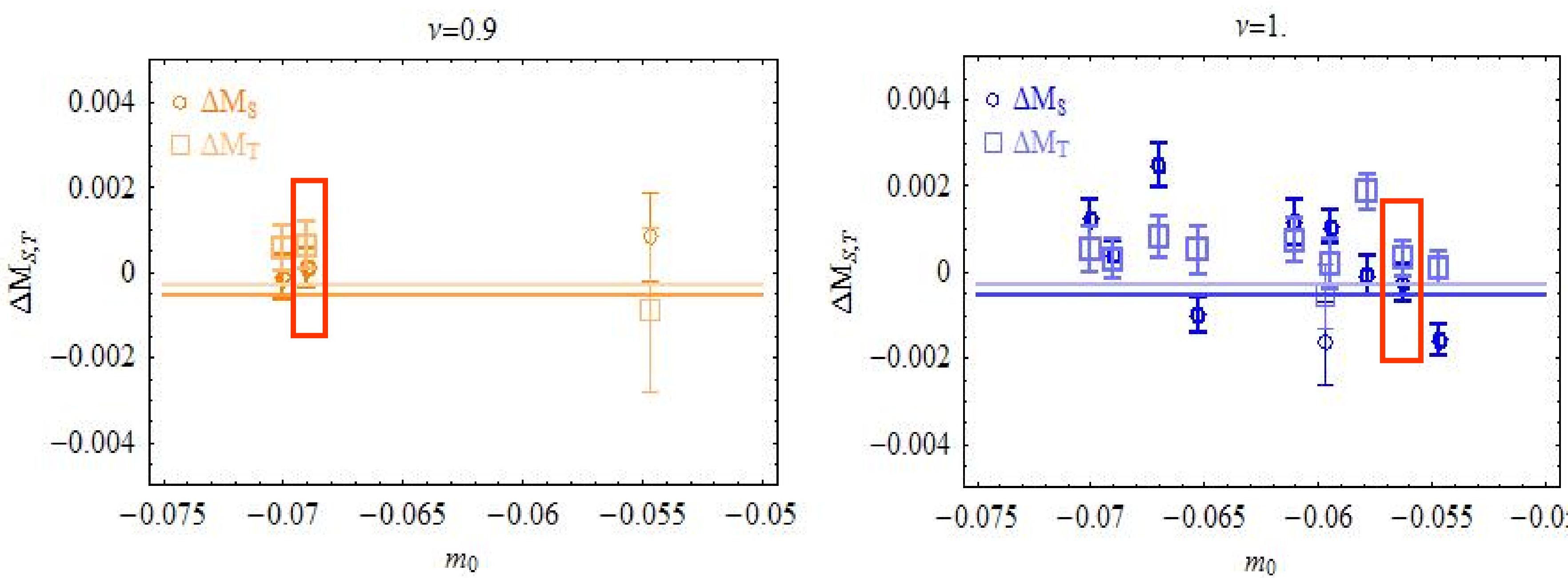
- More runs in the range $0.95 \le \nu \le 1.05$ coming

Nonperturbative c_{SW}?

- Nonperturbative condition

$$\Delta M = M(2T/4, T/4) - M'(2T/4, T/4) = \Delta M^{\text{Tree}, M=0}$$

- Tree-level ΔM value obtained from simulation in free-field
- Examples:



- At points where $M_s = M_t$, the NP condition is satisfied or agrees within σ

Summary/Outlook

Current Status:

- SF + stout-link smearing show promise in the dynamical runs
- Stout-link smearing + modified tadpole factors make NP c_{sw} tuning condition fulfilled
- Finite-box tuning is as good as conventional large-box runs with gauge and fermion anisotropy but more efficient
- 2f anisotropic ($\xi_R = 3$) Wilson configurations completed ($L \sim 1.8$, 2.6 fm, $m_\pi \sim 400$, 600 MeV)

In the near future:

- Fine tuning the strange quark points
- Launch 2+1f, $24^3 \times 64$ generation
- $O(a)$-improved coefficients: $c_{V,A}$, $Z_{V,A}\ldots$