Quark-Hadron Duality in Electron Scattering

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Goal: QCD-based understanding of structure function data at low $Q^2$ and $W$

Outline:

- Quark-hadron ("Bloom-Gilman") duality
- Duality in QCD
- Local duality & truncated moments
- Conclusions
Quark-Hadron
("Bloom-Gilman")
Duality
Quark-hadron duality

Complementarity between quark and hadron descriptions of observables

\[
\sum_{\text{hadrons}} = \sum_{\text{quarks}}
\]

Can use either set of complete basis states to describe all physical phenomena
Inclusive electron-proton scattering

\[ \nu W_2 = F_2 \] structure function

Bloom-Gilman duality

Average over (strongly $Q^2$ dependent) resonances
$\approx Q^2$ independent scaling function

Finite energy sum rule for $eN$ scattering

$$\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \ \nu W_2(\omega')$$

measured structure function (function of $\nu$ and $Q^2$)

scaling function (function of $\omega'$ only)

“hadrons” $\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$

“quarks”
Bloom-Gilman duality

Average over
(strongly $Q^2$ dependent) resonances
$\approx Q^2$ independent scaling function

Jefferson Lab (Hall C)

Scaling variables

\[ (p + q)^2 = m_q^2 \]
\[ \begin{align*}
& m_q = 0 \\
& p_T = 0 
\end{align*} \]

light-cone fraction of target’s momentum carried by parton

\[ \xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M} \]

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4x^2 M^2 / Q^2}} \quad \rightarrow \quad x \quad \text{as} \quad Q^2 \rightarrow \infty \]

Nachtmann scaling variable
Scaling variables

\begin{equation}
Q^2 = 1 \text{ GeV}^2
\end{equation}
Duality in QCD
Duality in QCD

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \ldots$$

matrix elements of operators with specific “twist” $\tau$

$$\tau = \text{dimension} - \text{spin}$$
Higher twists

\( \tau = 2 \)

single quark scattering
e.g. \( \bar{\psi} \gamma_\mu \psi \)

\( \tau > 2 \)

qq and qg correlations
e.g. \( \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma_\nu \psi \) or \( \bar{\psi} \tilde{G}_{\mu\nu} \gamma^\nu \psi \)
Duality in QCD

Operator product expansion

expand moments of structure functions in powers of \( 1/Q^2 \)

\[
M_n(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2) \\
= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots
\]

If moment \( \approx \) independent of \( Q^2 \)

higher twist terms \( A_n^{(\tau > 2)} \) small
Duality in QCD

Operator product expansion

expand moments of structure functions
in powers of $1/Q^2$

\[ M_n(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2) = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

Duality $\iff$ suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. 103 (1975) 315
Duality exists also in local regions, around individual resonances.

“local Bloom-Gilman duality”
ratio of integrated resonance to DIS contributions

~10% agreement for $Q^2 > 1 \text{ GeV}^2$

Niculescu et al.,
Truncated Moments
Truncated moments

- Complete moments can be studied in QCD via twist expansion
  - Bloom-Gilman duality has a precise meaning
    (i.e., duality violation = higher twists)

- For local duality, difficult to make rigorous connection with QCD
  - E.g. need prescription for how to average over resonances

- Truncated moments allow study of restricted regions in $x$ (or $W$) within QCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \ x^{n-2} \ F_2(x, Q^2)$$
Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

\[
\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left( P'_n \otimes \overline{M}_n \right)(\Delta x, Q^2)
\]

where modified splitting function is

\[
P'_n(z, \alpha_s) = z^n \ P_{NS,S}(z, \alpha_s)
\]

→ can follow evolution of specific resonance (region) with \( Q^2 \) in pQCD framework!

→ suitable when complete moments not available
Truncated moments

- truncated moment evolution equations exist for singlet (S) and nonsinglet (NS) separately

- for analysis of data, do not know much of experimental structure function is NS and how much is S

→ for lowest \((n=2)\) truncated moment, assumption that total \(\approx\) NS is good to few \% for \(x_{\text{min}} > 0.2\)

→ for higher moments, small-\(x\) region is further suppressed, so that NS is a very good approximation to total
$n = 2$ truncated moment of $F_2^p$

$\Delta x = [x_{min}, 1]$

Psaker, WM, Christy, Keppel (2007)
Parameterization of $F_2^p$ data

$Q^2 = 1 \text{ GeV}^2$

how much of this region is leading twist?

Psaker et al. (2007)
Parameterization of $F_2^{p}$ data

$Q^2 = 4 \text{ GeV}^2$

how much of this region is leading twist?

Psaker et al. (2007)
Parameterization of $F_2^p$ data

$Q^2 = 9 \text{ GeV}^2$

how much of this region is leading twist?

Psaker et al. (2007)
Analysis of JLab data

- assume data at highest $Q^2$ ($Q^2 = 9 \text{ GeV}^2$) is entirely leading twist

- evolve (as NS) fit to data at $Q^2 = 9 \text{ GeV}^2$ down to lower $Q^2$

  → apply TMC, and compare with data at lower $Q^2$

$M^2(x_{\text{min}}, Q^2)$

$Q^2 = 1 \text{ GeV}^2$

Psaker et al. (2007)
ratio of data to leading twist

\[ Q^2 = 1 \text{ GeV}^2 \]
consider individual resonance regions:

\[ W_{\text{thr}}^2 < W^2 < 1.9 \text{ GeV}^2 \quad \text{“}\Delta(1232)\text{”} \]

\[ 1.9 < W^2 < 2.5 \text{ GeV}^2 \quad \text{“}S_{11}(1535)\text{”} \]

\[ 2.5 < W^2 < 3.1 \text{ GeV}^2 \quad \text{“}F_{15}(1680)\text{”} \]

as well as total resonance region:

\[ W^2 < 4 \text{ GeV}^2 \]
method breaks down for low $x$ (high $W$) at low $Q^2$
$Q^2 > 1 \text{ GeV}^2$

$n=2$

higher twists < 10% for $Q^2 > 1 \text{ GeV}^2$
higher moments

higher twists < 5-10% for $Q^2 > 2-3$ GeV$^2$
in resonance region

$n=4$

$n=6$
Summary

- **Observation of quark-hadron duality in structure functions**
  - higher twists “small” down to low $Q^2 \approx 1 \text{ GeV}^2$
  - global duality understood within QCD moments

- **Local duality**
  - duality exists in local regions of $x$ (or $W$)
  - difficult to understand within QCD

- **Truncated moments**
  - firm foundation for study of local duality in QCD
  - higher twists < 10% for $Q^2 > 1 \text{ GeV}^2$ in resonance region