Strange Baryon Physics in Full Lattice QCD

Huey-Wen Lin

INT Seminar
INT & University of Washington
2008 Jan. 14
Outline

- **Lattice QCD**
  - Background, actions, observables, …

- **Two-point Green functions**
  - Group theory, operator design, spectroscopy results

- **Three-point Green functions**
  - Hyperon axial coupling constants
  - Strangeness in nucleon magnetic and electric moments
  - Hyperon semi-leptonic decays
Lattice QCD

Lattice QCD is a discrete version of continuum QCD theory

\[ \psi(x) \]

\[ U_\mu(x) \]

\[ \psi(x+\mu) \]
**Lattice QCD**

- Lattice QCD is a discrete version of continuum QCD theory

\[ \psi(x+\mu) \]
\[ U_\mu(x) \]
\[ \psi(x) \]

- Physical observables are calculated from the path integral

\[
\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int [dA][d\bar{\psi}][d\psi] \ O(\bar{\psi}, \psi, A) e^{i \int d^4x \mathcal{L}_{QCD}(\bar{\psi}, \psi, A)}
\]

- Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.

- Take \( a \to 0 \) and \( V \to \infty \) in the continuum limit
Lattice QCD

A wide variety of first-principles QCD calculations can be done:
- In 1970, Wilson started off by writing down the first actions.
- Progress is limited by computational resources.
  - But assisted by advances in algorithms.
T.D. Lee uses an “analog computer” to calculate stellar radiative transfer equations.
Lattice QCD

2007: The 13 Tflops cluster at Jefferson Lab

Other joint lattice resources within the US: Fermilab, BNL. Non-lattice resources open to USQCD: ORNL, LLNL, ANL.
Lattice QCD

- Lattice QCD is computationally intensive
  \[
  \text{Cost} \approx \left( \frac{L}{\text{fm}} \right)^5 L_s \left( \frac{\text{MeV}}{M_\pi} \right) \left( \frac{\text{fm}}{a} \right)^6 \left[ C_0 + C_1 \left( \frac{\text{fm}}{a} \right) \left( \frac{\text{MeV}}{M_K} \right)^2 + C_2 \left( \frac{a}{\text{fm}} \right)^2 \left( \frac{\text{MeV}}{M_\pi} \right)^2 \right]
  \]
  Norman Christ, LAT2007

- Current major US 2+1-flavor gauge ensemble generation:
  - MILC: staggered, \( a \sim 0.06 \text{ fm}, L \sim 3 \text{ fm}, M_\pi \sim 250 \text{ MeV} \)
  - RBC+UKQCD: DWF, \( a \sim 0.09 \text{ fm}, L \sim 3 \text{fm}, M_\pi \sim 330 \text{ MeV} \)
  - Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass may be expected in 2011

- But for now…
  need a pion mass extrapolation \( M_\pi \rightarrow (M_\pi)_{\text{phys}} \)
  (use chiral perturbation theory, if available)
**Lattice Fermion Actions**

**Chiral fermions (e.g., Domain-Wall/Overlap):**
- Automatically $O(a)$ improved, good for spin physics and weak matrix elements
- Expensive

\[
D_{x,s; x', s'} = \delta_{x,x'} D_{s,s'}^\perp + \delta_{s,s'} D_{x,x'}^\parallel
\]

\[
D_{s,s'}^\perp = \frac{1}{2} [(1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2 \delta_{s,s'}]
\]

\[
- \frac{m_f}{2} [(1 - \gamma_5) \delta_{s,L_{s}-1} \delta_{0,s'} + (1 + \gamma_5) \delta_{s,0} \delta_{L_{s}-1,s'}],
\]

**(Improved) Staggered fermions (asqtad):**
- Relatively cheap for dynamical fermions (good)
- Mixing among parities and flavors or “tastes”
- Baryonic operators a nightmare — not suitable

**Mixed action:***
- Staggered sea (cheap) with domain-wall valence (chiral)
- Match the sea Goldstone pion mass to the DWF pion
- Only mixes with the “scalar” taste of sea pion
- Free light quark propagators (LHPC & NPLQCD)
Lattice QCD: Observables

- **Two-point Green function**
  - e.g. spectroscopy

\[ \sum_{\alpha, \beta} \Gamma_{\alpha, \beta} \langle J(X_{\text{snk}}) J(X_{\text{src}}) \rangle_{\alpha, \beta} \]

- **Three-point Green function**
  - e.g. form factors, structure functions, …

\[ \sum_{\alpha, \beta} \Gamma_{\alpha, \beta} \langle J(X_{\text{snk}}) O(X_{\text{int}}) J(X_{\text{src}}) \rangle_{\alpha, \beta} \]
Lattice QCD: Observables

- **Two-point Green function**
  - e.g. spectroscopy
  - \( \sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}) J(X_{\text{src}}) \rangle_{\alpha,\beta} \)

- **Three-point Green function**
  - e.g. form factors, structure functions, ...
  - \( \sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}) O(X_{\text{int}}) J(X_{\text{src}}) \rangle_{\alpha,\beta} \)

After taking spin and momentum projection
(ignoring the variety of boundary condition choices)

Two-point correlator

\( \sum_n Z_{n,B} e^{-E_n(\vec{P})t} \)

Three-point correlator

\( \sum_n \sum_{n'} Z_{n',B}(p_f) Z_{n,A}(p_i) \)

\( \times \text{FF's} \times e^{-(t_f-t)E_{n}(\vec{P}_f)} e^{-(t-t_i)E_{n}(\vec{P}_i)} \)

At large enough \( t \), the ground-state signal dominates
Two-Point Green Functions

work with

Lattice Hadron Physics Collaboration (LHPC)

Strange quark propagators from NPLQCD
Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, …)
- Nucleon spectrum, on the other hand… not quite

Example: $N$, $P_{11}$, $S_{11}$ spectrum
Strange Baryons

Strange baryons are of special interest; challenging even to experiment

Example from **PDG Live:**

### Ξ Baryons ($S = -2, I = 1/2$)

<table>
<thead>
<tr>
<th>Ξ</th>
<th>$J^P$</th>
<th>$I^G$</th>
<th>Mass</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi^0$</td>
<td>1/2(1/2$^+$)</td>
<td>***</td>
<td>$\Xi(1820)$</td>
<td>$D_{13}$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1/2(1/2$^+$)</td>
<td>***</td>
<td>$\Xi(1950)$</td>
<td>$\Xi(2370)$</td>
</tr>
<tr>
<td>$\Xi(1530) P_{13}$</td>
<td>1/2(3/2$^+$)</td>
<td>***</td>
<td>$\Xi(2030)$</td>
<td>$\Xi(2500)$</td>
</tr>
<tr>
<td>$\Xi(1680)$</td>
<td>1/2(?)</td>
<td>•*</td>
<td>$\Xi(2120)$</td>
<td>$\Xi(2250)$</td>
</tr>
<tr>
<td>$\Xi(1690)$</td>
<td>1/2(?)</td>
<td>***</td>
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</tr>
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</table>

### Ω Baryons ($S = -3, I = 0$)

<table>
<thead>
<tr>
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<th>$J^P$</th>
<th>$I^G$</th>
<th>Mass</th>
<th>Width</th>
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<tbody>
<tr>
<td>$\Omega^-$</td>
<td>0(3/2$^+$)</td>
<td>****</td>
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<tr>
<td>$\Omega(2250)^-$</td>
<td>0(?)</td>
<td>***</td>
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Huey-Wen Lin — INT & UW
Operator Design

- All baryon spin states wanted: $j = 1/2, 3/2, 5/2, \ldots$
- Rotation symmetry is reduced due to discretization rotation $O(3) \Rightarrow$ octahedral $O_h$ group

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>J</th>
<th>6 $C_4$</th>
<th>8 $C_6$</th>
<th>8 $C_3$</th>
<th>6 $C_2$</th>
<th>6 $C_9$</th>
<th>12 $C_4$</th>
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<td>$H$</td>
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<td>T_2</td>
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<td>H</td>
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$6 C_4(1)$
Operator Design

- All baryon spin states wanted: \( j = 1/2, 3/2, 5/2, \ldots \)
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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<td>1</td>
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<tr>
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Huey-Wen Lin — INT & UW
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Rotation symmetry is reduced due to discretization rotation $O(3) \Rightarrow$ octahedral $O_h$ group

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<tr>
<td>$1/2$</td>
<td>$G_1$</td>
</tr>
<tr>
<td>$3/2$</td>
<td>$H$</td>
</tr>
<tr>
<td>$5/2$</td>
<td>$G_2 \oplus H$</td>
</tr>
<tr>
<td>$7/2$</td>
<td>$G_1 \oplus G_2 \oplus H$</td>
</tr>
<tr>
<td>$9/2$</td>
<td>$G_1 \oplus 2H$</td>
</tr>
<tr>
<td>$11/2$</td>
<td>$G_1 \oplus G_2 \oplus 2H$</td>
</tr>
<tr>
<td>$13/2$</td>
<td>$G_1 \oplus 2G_2 \oplus 2H$</td>
</tr>
<tr>
<td>$15/2$</td>
<td>$G_1 \oplus G_2 \oplus 3H$</td>
</tr>
<tr>
<td>$17/2$</td>
<td>$2G_1 \oplus G_2 \oplus 3H$</td>
</tr>
<tr>
<td>$19/2$</td>
<td>$2G_1 \oplus 2G_2 \oplus 3H$</td>
</tr>
<tr>
<td>$21/2$</td>
<td>$G_1 \oplus 2G_2 \oplus 4H$</td>
</tr>
<tr>
<td>$23/2$</td>
<td>$2G_1 \oplus 2G_2 \oplus 4H$</td>
</tr>
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Baryons
Operator Design

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<table>
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<tbody>
<tr>
<td>1/2</td>
<td>( G_1 )</td>
</tr>
<tr>
<td>3/2</td>
<td>( H )</td>
</tr>
<tr>
<td>5/2</td>
<td>( G_1 \oplus 2 \ G_2 \oplus 3 \ H )</td>
</tr>
<tr>
<td>7/2</td>
<td>( G_1 \oplus 2 \ G_2 \oplus 4 \ H )</td>
</tr>
<tr>
<td>9/2</td>
<td>( 2 \ G_1 \oplus 2 \ G_2 \oplus 4 \ H )</td>
</tr>
</tbody>
</table>

Baryons

- For more details and extended-link operators:
  
Variational Method

- Construct the correlator matrix

\[ C_{\Lambda,m,n}(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 | B_{\lambda,m}^\Lambda(\vec{x}, t) \bar{B}_{\lambda}^{\Lambda,n}(0) | 0 \rangle \]

- Construct the matrix

\[ C_{i,j}(t) = \langle 0 | \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) | 0 \rangle \]

- Solve for the generalized eigensystem of

\[ C(t)\psi = \lambda(t, t_0)C(t_0)\psi \]

with eigenvalues

\[ \lambda_n(t, t_0) = e^{-(t-t_0)E_n(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})}) \]


- At large \( t \), the signal of the desired state dominates.

- Unfortunately, we cannot see a clear radial excited state with the smeared propagators we got for free.
General Spectroscopy

The non-strange baryons ($N$)

Symbols: $J^P = \frac{1}{2}^+ \bigtriangleup, \frac{1}{2}^- \bigtriangledown, \frac{3}{2}^+ \blacklozenge, \frac{3}{2}^- \square$

$N(1535)$
$N(1720)$
$N(1520)$
General Spectroscopy

The non-strange baryons ($N$ and $\Delta$)

Symbols: $J^P = \frac{1}{2}^+$ △, $\frac{1}{2}^-\nabla$, $\frac{3}{2}^+\lozenge$, $\frac{3}{2}^-\square$

$N$ $N(1535)$ $N(1720)$ $N(1520)$
$\Delta(1620)$ $\Delta$ $\Delta(1700)$

$M_N$ (GeV) $M_\Delta$ (GeV)

$M_\pi^2 / f_\pi^2$ $M_\pi^2 / f_\pi^2$
The singly strange baryons: ($\Sigma$ and $\Lambda$)

Symbols: $J^P = \frac{1}{2}^+, \triangle, \frac{1}{2}^-\bigtriangledown, \frac{3}{2}^+\blacklozenge, \frac{3}{2}^-\square$

- $\Sigma$: $\Sigma(1620)$
- $\Lambda$: $\Lambda(1405)$
- $\Sigma^*$: $\Sigma(1580)$
- $\Lambda$: $\Lambda(1890)$
- $\Lambda$: $\Lambda(1520)$
General Spectroscopy

The less known baryons ($\Xi$)

Symbols: $J^P = 1/2^+ \bigtriangleup, 1/2^- \bigtriangledown, 3/2^+ \bigcirc, 3/2^- \square$

$\Xi \quad \Xi(1690)? \quad \Xi(1530) \quad \Xi(1820)$
General Spectroscopy

The less known baryons ($\Xi$)

Symbols: $J^P = 1/2^+ \bigtriangleup$, $1/2^- \bigtriangledown$, $3/2^+ \lozenge$, $3/2^- \square$

$\Xi$, $\Xi(1690)$?, $\Xi(1530)$, $\Xi(1820)$

Babar at MENU 2007:

$\Xi(1690)^0$ negative parity $-1/2$
The less known baryons ($\Xi$ and $\Omega$)

Symbols: $J^P = 1/2^+ \Delta$, $1/2^- \nabla$, $3/2^+ \Diamond$, $3/2^- \Box$

Could they be $\Omega(2250)$, $\Omega(2380)$, $\Omega(2470)$?
Multiplet Mass Relations

- SU(3) flavor symmetry breaking
  - Gell-Mann-Okubo relation
    \[ \Delta_{GMO} = \frac{3}{4} M_\Lambda + \frac{1}{4} M_\Sigma - \frac{1}{2} M_N - \frac{1}{2} M_\Xi \]
  - Decuplet Equal Spacing relation
    \[ \Delta_{DESI} = \frac{1}{2} (M_{\Sigma^*} - M_\Delta) + \frac{1}{2} (M_\Omega - M_{\Xi^*}) - M_{\Xi^*} + M_{\Sigma^*} \]
- Mass differences are close to experimental numbers
Summary/Outlook — I

What we have done:
- 2+1-flavor calculations with volume around 2.6 fm
- Ground states of $G_{1g/u}$ and $H_{g/u}$ for each flavor
- Preliminary study with lightest pion mass 300 MeV
- Correct mass-ordering pattern is seen

Currently in progress:
- Mixed action chiral extrapolation for octet and decuplet
- Open-minded for extrapolation to physical pion mass for other states

In the future:
- Lower pion masses to confirm chiral logarithm drops
- Finer lattice spacing for excited states
Three-Point Green Functions

in collaboration with

Kostas Orginos

• Hyperon axial coupling constants
• Strangeness in nucleon magnetic and electric moments
• Semi-leptonic decays
Green Functions

- Three-point function with connected piece only

\[ C_{3\text{pt}}^{\Gamma, \mathcal{O}}(\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma_{\alpha, \beta} \langle \mathcal{O}(\tau) J_\beta(\vec{p}, t) J_\alpha(\vec{p}, 0) \rangle \]

- Two constructions:

- Isovector quantities
- Use ratios to cancel out the unwanted factors

\[
\frac{\gamma_{BB,GG}(t_i, t_f, \vec{p}_i, \vec{p}_f; T)}{\gamma_{PG,GG}(t_i, t_f, \vec{p}_f; T)} \sqrt{\frac{\gamma_{BB,GG}(t_i, t_f, \vec{p}_i; T)}{\gamma_{PG,GG}(t_i, t_f, \vec{p}_i; T)}} \sqrt{\frac{\gamma_{BB,GG}(t_i, t_f, \vec{p}_f; T)}{\gamma_{PG,GG}(t_i, t_f, \vec{p}_i; T)}} \sqrt{\frac{\gamma_{BB,GG}(t_i, t_f, \vec{p}_i; T)}{\gamma_{PG,GG}(t_i, t_f, \vec{p}_i; T)}}
\]
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- **Defined**  
  \[
  \langle B | A_\mu(q) | B \rangle = \bar{u}_B(p') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B} \right] u_B(p)e^{-iq\cdot x}
  \]

- **Has applications such as hyperon scattering, non-leptonic decays, …**

- **Cannot be determined by experiment**

- **Existing theoretical predictions:**
  - **Chiral perturbation theory**
    \[
    0.35 \leq g_{\Sigma\Sigma} \leq 0.55 \quad 0.18 \leq -g_{\Xi\Xi} \leq 0.36
    \]
    M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - **Large-$N_c$**
    \[
    0.30 \leq g_{\Sigma\Sigma} \leq 0.36 \quad 0.26 \leq -g_{\Xi\Xi} \leq 0.30
    \]
    R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

- **Loose bounds on the values**

- **Lattice QCD can provide substantial improvement**
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Pion mass: 350–750 MeV
- First lattice calculation of these quantities; mixed-action full-QCD

<table>
<thead>
<tr>
<th>$m_\pi$ (MeV)</th>
<th>m010</th>
<th>m020</th>
<th>m030</th>
<th>m040</th>
<th>m050</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>354.2(8)</td>
<td>493.6(6)</td>
<td>594.2(8)</td>
<td>685.4(19)</td>
<td>754.3(16)</td>
</tr>
<tr>
<td>$m_\pi/f_\pi$</td>
<td>2.316(7)</td>
<td>3.035(7)</td>
<td>3.478(8)</td>
<td>3.822(23)</td>
<td>4.136(20)</td>
</tr>
<tr>
<td>$m_K/f_\pi$</td>
<td>3.951(14)</td>
<td>3.969(10)</td>
<td>4.018(11)</td>
<td>4.060(26)</td>
<td>4.107(21)</td>
</tr>
<tr>
<td>confs</td>
<td>612</td>
<td>345</td>
<td>561</td>
<td>320</td>
<td>342</td>
</tr>
<tr>
<td>$g_{A,N}$</td>
<td>1.22(8)</td>
<td>1.21(5)</td>
<td>1.195(17)</td>
<td>1.150(17)</td>
<td>1.167(11)</td>
</tr>
<tr>
<td>$g_{\Sigma\Sigma}$</td>
<td>0.418(23)</td>
<td>0.450(15)</td>
<td>0.451(7)</td>
<td>0.444(8)</td>
<td>0.453(5)</td>
</tr>
<tr>
<td>$g_{\Xi\Xi}$</td>
<td>-0.262(13)</td>
<td>-0.270(10)</td>
<td>-0.269(7)</td>
<td>-0.257(9)</td>
<td>-0.261(7)</td>
</tr>
</tbody>
</table>

- Combine with $g_A$ for study of
  - SU(3) symmetry breaking
  - SU(3) simultaneous fits among three coupling constants
    $\rightarrow$ $D$, $F$, and other low-energy constants
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- **SU(3) symmetry breaking**

  \[
  \delta_{SU(3)} = g_A - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi} \\
  = \sum_n c_n x^n \quad \text{with} \quad x = (m_K^2 - m_\pi^2) / (4\pi f_\pi^2)
  \]

- **Quadratic behaviour is observed**

- **Not predicted by any theorem nor chiral perturbation theory**

- **20% breaking at physical point**
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

Simultaneous SU(3) fit

SU(3) chiral perturbation theory (with 8 parameters)


which fails to describe the data

Simple chiral form

$$g_A = D + F + \sum_n C_N^{(n)} x^n$$

$$g_{\Xi\Xi} = F - D + \sum_n C_{\Xi}^{(n)} x^n$$

$$g_{\Sigma\Sigma} = F + \sum_n C_{\Sigma}^{(n)} x^n$$

Systematic errors:
finite volume + finite $a$

$$g_A = 1.18(4)_{\text{stat}}(6)_{\text{syst}}$$

$$g_{\Sigma\Sigma} = 0.450(21)_{\text{stat}}(27)_{\text{syst}}$$

$$g_{\Xi\Xi} = -0.277(15)_{\text{stat}}(19)_{\text{syst}}$$

$$D = 0.715(6)_{\text{stat}}(29)_{\text{syst}}$$

$$F = 0.453(5)_{\text{stat}}(19)_{\text{syst}}$$
Strange Magnetic Moment of Nucleon

- Purely sea-quark effect
- First strange magnetic moment was measured by SAMPLE
  \[ G_M^s(Q^2 = 0.1 \, GeV^2) = 0.23(37)(25)(29) \]
- More data is being collected today
  *HAPPEX and G0 collaborations at Jefferson Lab, SAMPLE at MIT-BATES, and A4 at Mainz*
- Lattice calculations
  \[ \langle B | V_\mu | B \rangle(q) = \bar{u}_B(p') \left[ \gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_B} \right] u_B(p)e^{-iq\cdot x} \]
  the disconnected diagram is a must
- Noisy estimator
  \[-0.28(10) \text{ to } +0.05(6)\]
  Kentucky Field Theory group (1997–2001)
- Help with chiral perturbation theory
  \[-0.046(19)\]
  Adelaide-JLab group (2006)
  in quenched approximation
Quenched Approximation

Full QCD:
\[
\langle O \rangle = \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] e^{-S_F(U,\psi,\bar{\psi}) - S_G(U)} O(U, \psi, \bar{\psi})
\]
\[
= \frac{1}{Z} \int [dU] \det M e^{-S_G(U)} O(U)
\]

Quenched: Take \( \det M = \) constant.

Historically used due to the lack of computation power

Bad: Uncontrollable systematic error

Good? Cheap exploratory studies to develop new methods
Strange Magnetic Moment of Nucleon

Disconnected diagrams are challenging
Much effort has been put into resolving this difficulty
Alternative approach:

Assume charge symmetry:

\[
\begin{align*}
p &= e_u u^p + e_d d^p + O_N; \\
n &= e_d u^p + e_u d^p + O_N; \\
\Sigma^+ &= e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; \\
\Sigma^- &= e_d u^\Sigma + e_s s^\Sigma + O_\Sigma; \\
\Xi^0 &= e_s s^\Xi + e_u u^\Xi + O_\Xi; \\
\Xi^- &= e_s s^\Xi + e_d u^\Xi + O_\Xi.
\end{align*}
\]

The disconnected piece for the proton is

\[
O_N = \frac{2}{3} \{ G_M^u - \frac{1}{3} G_M^d - \frac{1}{3} G_M^s \}
\]

The strangeness contribution is

\[
\begin{align*}
G_M^s &= \left(\frac{\hat{R}_d^s}{1 - \hat{R}_d^s}\right) \left[ 2p + n - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right] \\
G_M^s &= \left(\frac{\hat{R}_d^s}{1 - \hat{R}_d^s}\right) \left[ p + 2n - \frac{u^n}{u^\Xi} (\Xi^0 - \Xi^-) \right] \quad \text{with} \quad \hat{R}_d^s = \frac{l G_M^s}{l G_M^d}
\end{align*}
\]

Strange Magnetic Moment of Nucleon

Disconnected diagrams are challenging
Much effort has been put into resolving this difficulty
Alternative approach:

Assume charge symmetry:

\[ p = e_u u^p + e_d d^p + O_N; \quad n = e_d u^p + e_u d^p + O_N, \]

\[ \Sigma^+ = e_u u^\Sigma + e_s s^\Sigma + O_{\Sigma}; \quad \Sigma^- = e_d u^\Sigma + e_s s^\Sigma + O_{\Sigma}, \]

\[ \Xi^0 = e_s s^\Xi + e_u u^\Xi + O_{\Xi}; \quad \Xi^- = e_s s^\Xi + e_d u^\Xi + O_{\Xi}. \]

The disconnected piece for the proton is

\[ O_N = \frac{2}{3} G^u_M - \frac{1}{3} G^d_M - \frac{1}{3} G^s_M \]

The strangeness contribution is

\[ G^s_M = \left( \frac{\l R^s_d}{1 - \l R^s_d} \right) \left[ 3.673 - \frac{u^p}{u^\Sigma} (3.618) \right] \mu_N \]

\[ G^s_M = \left( \frac{\l R^s_d}{1 - \l R^s_d} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N \]

with \( \l R^s_d \equiv \l G^s_M / \l G^d_M \)

Needs better statistics

Strange Magnetic Moment of Nucleon

- Magnetic moment $\mu_B = F_2(q^2=0)$
- Dipole-form extrapolation to $q^2 = 0$

Example: $u$-quark contribution in $\Sigma$ form factor $F_2(q^2)$

\[
\mu_B = F_2(q^2=0)
\]

$m_\pi = 359$ MeV
Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

\[ \frac{\mu_p^u}{\mu_\Sigma^u} \]

\[ \frac{\mu_p^u}{\mu_\Xi^u} \]

Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi}(-0.599) \right] \mu_N \]
Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

\[
R_d^s = 0.139(42)
\]


We find

\[
G_M^s = -0.066(12)_{\text{stat}}(23)_{\text{sys}} R_d^s
\]

H WL, arXiv:0707.3844 [hep-lat]
Strange Electric Moment of Nucleon

- $G_E^s$ is proportional to $Q^2 \langle r^2 \rangle^s$
- Charge symmetry: 
  
  \[
  \langle r^2 \rangle^s = \frac{r_d^s}{1 - r_d^s} \left[ 2 \langle r^2 \rangle^p + \langle r^2 \rangle^n - \langle r^2 \rangle^u \right] \\
  r_d^s = 0.16(4)
  \]
- $u$-quark form contribution of vector form factors

Need more literature research on chiral extrapolation
Using an extrapolation of the form

\[
\langle r^2 \rangle^u = a_0 - \frac{1 + 5g_A^2}{(4\pi f_\pi)^2} \log \left( \frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right)
\]

We find

\[
G_E^s(Q^2 = 0.1 \text{ GeV}) = 0.022(61)
\]

Preliminary
Strangeness

$G_E^s - G_M^s$ plots

Hyperon Decays

Matrix element of the hyperon $\beta$-decay process $B_1 \to B_2 e^- \bar{\nu}$

\[ \mathcal{M} = \frac{G_s}{\sqrt{2}} u_{B_2} (O^V_\alpha + O^A_\alpha) u_{B_1} \bar{u}_e \gamma^\alpha (1 + \gamma_5) v_\nu \]

with

\[ O^V_\alpha = f_1(q^2) \gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{f_3(q^2)}{M_{B_2}} q_\alpha \]

\[ O^A_\alpha = \left( g_1(q^2) \gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{g_3(q^2)}{M_{B_2}} q_\alpha \right) \gamma_5 \]
Hyperon Decay Experiments

Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48

Summary N. Cabibbo et al. 2003
with $f_2/f_1$ and $f_1$ at the SU(3) limit

<table>
<thead>
<tr>
<th>Decay</th>
<th>Rate ($\mu$s$^{-1}$)</th>
<th>$g_1/f_1$</th>
<th>$V_{us}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \rightarrow p\gamma\bar{\nu}$</td>
<td>3.161(58)</td>
<td>0.718(15)</td>
<td>0.2224 ± 0.0034</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n\gamma\bar{\nu}$</td>
<td>6.88(24)</td>
<td>−0.340(17)</td>
<td>0.2282 ± 0.0049</td>
</tr>
<tr>
<td>$\Xi^- \rightarrow \Lambda\gamma\bar{\nu}$</td>
<td>3.44(19)</td>
<td>0.25(5)</td>
<td>0.2367 ± 0.0099</td>
</tr>
<tr>
<td>$\Xi^0 \rightarrow \Sigma^+\gamma\bar{\nu}$</td>
<td>0.876(71)</td>
<td>1.32(+.22/−.18)</td>
<td>0.209 ± 0.027</td>
</tr>
<tr>
<td>Combined</td>
<td>—</td>
<td>—</td>
<td><strong>0.2250 ± 0.0027</strong></td>
</tr>
</tbody>
</table>

PDG 2006 number

Better $g_1/f_1$ from lattice calculations?
| $V_{us}$ | from Hyperons Decays |

Two quenched calculations, different channels

- Pion mass $> 700$ MeV
- $f_1(0) = -0.988(29)_{\text{stat.}}$
- $|V_{us}| = 0.230(5)_{\text{exp}(7)_{\text{lat}}}$
  
Guadagnoli et al.

- Pion mass $\approx 530$–$650$ MeV
- $f_1(0) = 0.953(24)_{\text{stat.}}$
- $|V_{us}| = 0.219(27)_{\text{exp}(5)_{\text{lat}}}$
  
Sasaki et al.

No systematic error estimate from quenching effects!
Ademollo-Gatto Theorem

Chiral extrapolation:
- SU(3) symmetry-breaking Hamiltonian
  \[ H' = \frac{1}{\sqrt{3}} \left( m_s - \frac{m_d + m_u}{2} \right) \bar{q} \lambda^8 q \]
- There is no first-order correction \( O(H') \) to \( f_1(0) \); thus
  \[ f_1(0) = f_1^{SU(3)}(0) + O(H'^2) \]
- Common choice of observable for \( H' : M_K^2 - M_\pi^2 \)
  \[ R(M_K, M_\pi) = \frac{1 - |f'(0)|}{\alpha^4(M_K^2 - M_\pi^2)^2} \]
- Step I: \[ R(M_K, M_\pi) = b_0 + b_1 a^2(M_K^2 + M_\pi^2) \]
  
Obtain \( |V_{us}| \) from
\[ \Gamma = \frac{G_F^2 |V_{us}|^2}{60\pi^3} \frac{\Delta m^5}{(1 + \delta_{\text{rad}})} \times \left[ \left( 1 - \frac{3}{2} \beta \right) (|f_1|^2 + |g_1|^2) + \frac{6}{7} \beta^2 \left( |f_1|^2 + 2|g_1|^2 + \text{Re}(f_1 f_2^*) + \frac{2}{3} |f_2|^2 \right) + \delta_{q^2} \right] \]
with \( g_1/f_1(\text{exp}) \) and \( f_2/f_1(\text{SU(3 value)}) \)
Simultaneous Fit

Combined momentum and mass dependence

\[ f_1(0) = -0.88(15) \]  (Preliminary)
Summary/Outlook — II

🔵 From hyperon analysis

- Predictions for $g_{\Sigma \Sigma} = 0.450(21)(27)$ and $g_{\Xi \Xi} = -0.277(15)(19)$
- Preliminary **proton strange magnetic and electric moments** directly from full QCD: $-0.066(12)(23)$ and $-0.022(61)$
- Looking for improvements in $G_E^s$

🔵 More work to be done in hyperon semi-leptonic decay

- First dynamical calculation
- Preliminary result from Lin-Orginos is consistent with the previous calculation
- We need much higher statistics for a lighter-pion mass calculation (compared with the quenched one)
- Higher precision $g_1/f_1$:
  Will make the $|V_{us}|$ equivalent to or better than the one from $K_{l3}$ channel