\[ \gamma - Z^0 \] Contributions to the Parity-Violating Asymmetry

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(also Peter Blunden (Manitoba) & John Arrington (Argonne))
Proton $G_E/G_M$ Ratio

Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

**LT method**

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$\rightarrow$ $G_E$ from slope in $\varepsilon$ plot

$\rightarrow$ suppressed at large $Q^2$

**PT method**

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1 + \varepsilon)}{2\varepsilon}} \frac{P_T}{P_L}$$

$\rightarrow$ $P_{T,L}$ recoil proton polarization in $\vec{e} \ p \rightarrow e \ \vec{p}$
possible reason – QED Radiative Corrections

- cross section modified by $\gamma \gamma$ loop effects

$$d\sigma = d\sigma_0 (1 + \delta)$$

$\delta$ contains additional $\varepsilon$ dependence, mostly from box diagrams (most difficult to calculate)
Two-photon exchange

- interference between Born and two-photon exchange amplitudes

\[
\delta^{(2\gamma)} = \frac{2 \text{Re} \left\{ M_0^\dagger M_{\gamma\gamma} \right\}}{|M_0|^2}
\]

- contribution to cross section:

- standard “soft photon approximation” (used in most data analyses)
  - approximate integrand in \( M_{\gamma\gamma} \) by values at \( \gamma^* \) poles
  - neglect nucleon structure (no form factors) \( Mo, Tsai (1969) \)
Two-photon exchange

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4k}{(2\pi)^4} \frac{N(k)}{D(k)} \]

where

\[ N(k) = \bar{u}(p_3) \gamma_{\mu} (p_1 - k + m_e) \gamma_{\nu} u(p_1) \times \bar{u}(p_4) \Gamma^\mu(q - k) (p_2 + k + M) \Gamma^\nu(k) u(p_2) \]

and

\[ D(k) = (k^2 - \lambda^2) ((k - q)^2 - \lambda^2) \times ((p_1 - k)^2 - m^2) ((p_2 + k)^2 - M^2) \]

with \( \lambda \) an IR regulator, and e.m. current is

\[ \Gamma^{\mu}(q) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_2(q^2) \]
Two-photon exchange

- “exact” calculation of loop diagram (including $\gamma^*NN$ form factors)

$$
\delta_{\text{full}}^{(2\gamma)} - \delta_{\text{Mo–Tsai}}^{(2\gamma)}
$$

$\Delta (\varepsilon, Q^2)$

few % magnitude
positive slope
non-linearity in $\varepsilon$

Blunden, Melnitchouk, Tjon
PRL 91 (2003) 142304;
PRC 72 (2005) 034612
What about higher-mass intermediate states?

Lowest mass excitation is $P_{33}$ $\Delta (1232)$ resonance

relativistic $\gamma^* N\Delta$ vertex

$$
\Gamma_{\gamma \Delta \rightarrow N}^{\nu \alpha}(p, q) \equiv iV_{\Delta in}^{\nu \alpha}(p, q) = i \frac{eF_\Delta(q^2)}{2M_\Delta^2} \left\{ g_1 \left[ g^{\nu \alpha} q \not{\! p} - p^{\nu} \gamma^\alpha q - \gamma^\nu \gamma^\alpha p \cdot q + \gamma^\nu p q^\alpha \right] + g_2 \left[ p^{\nu} q^\alpha - g^{\nu \alpha} p \cdot q \right] + (g_3/M_\Delta) \left[ q^2(p^{\nu} \gamma^\alpha - g^{\nu \alpha} p) + q^\nu(q^\alpha p - \gamma^\alpha p \cdot q) \right] \right\} \gamma_5 T_3
$$

- coupling constants
  - $g_1$ magnetic $\rightarrow 7$
  - $g_2 - g_1$ electric $\rightarrow 9$
  - $g_3$ Coulomb $\rightarrow -2 \ldots 0$
Higher-mass intermediate states have also been calculated

more model dependent, since couplings & form factors
not well known (especially at high $Q^2$)

$\Delta$ partially cancels $N$ contribution

dominant contribution from $N$
Higher-mass intermediate states have also been calculated.

higher mass resonance contributions small

much better fit to data including TPE
Global analysis

- reanalyze *all* elastic *ep* data (Rosenbluth, PT), including TPE corrections consistently *from the beginning*

- use explicit calculation of $N$ elastic contribution

- approximate higher mass contributions by phenomenological form, based on $N^*$ calculations:

\[ \delta_{\text{high mass}}^{(2\gamma)} = -0.01 (1 - \varepsilon) \log Q^2 / \log 2.2 \]

for $Q^2 > 1 \text{ GeV}^2$, with $\pm 100\%$ uncertainty

- decreases $\varepsilon = 0$ cross section by $1\% (2\%)$ at $Q^2 = 2.2 \ (4.8) \text{ GeV}^2$
Arrington, Melnitchouk, Tjon

LT separation
polarization transfer
with TPE correction
resolves discrepancy (within errors)
Charge density

\[ \rho_{E}^{NR}(r) = \frac{2}{\pi} \int_{0}^{\infty} dq \, q^2 \, j_0(qr) \, G_E(q^2) \]

25% less charge in the center of the proton
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\vec{e} \, p \rightarrow e \, p$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left( \frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

$$A_V = g_A^e \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - (\varepsilon G_{\gamma p} G_{\gamma n} + \tau G_{\gamma p} G_{\gamma n})/\sigma_{\gamma p} \right]$$

radiative corrections, including TBE

using relations between weak and e.m. form factors

$$G_{E,M}^{Zp} = (1 - 4\sin^2 \theta_W)G_{E,M}^{\gamma p} - G_{E,M}^{\gamma n} - G_{E,M}^s$$
Parity-violating $e$ scattering

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→ measure interference between e.m. and weak currents

$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)} \tilde{G}_A^Zp G_{\gamma p}^M / \sigma^\gamma p$$

includes axial RCs + anapole term

$$A_s = -g_A^e \rho (\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s) / \sigma^\gamma p$$

strange electric & magnetic form factors
Two-boson exchange corrections

- Current PDG estimates (of \(\gamma(Z\gamma)\)) computed at \(Q^2 = 0\)
  - Marciano, Sirlin (1980)

- Do not include hadron structure effects
  (parameterized via \(VNN\) form factors)
Two-boson exchange corrections

At tree level, \( \rho = \kappa = 1 \)

Including TBE corrections,

\[
\rho = \rho_0 + \Delta \rho , \quad \kappa = \kappa_0 + \Delta \kappa
\]

Standard RCs

Born-TBE interference

From vector part of asymmetry,

\[
\Delta \rho = \frac{A_p^p + A_n^p}{A_V^{p,\text{tree}} + A_V^{n,\text{tree}}} - \frac{\Delta \sigma^{\gamma(\gamma\gamma)}}{\sigma^{\gamma p}}
\]

\[
\Delta \kappa = \frac{A_p^p}{A_V^{p,\text{tree}}} - \frac{A_p^p + A_n^p}{A_V^{p,\text{tree}} + A_V^{n,\text{tree}}}
\]

Tree level contribution
Two-boson exchange corrections

- some cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections in $\Delta \rho$

- no $\gamma(\gamma\gamma)$ contribution to $\Delta \kappa$

*Tjon, Melnitchouk, PRL 100, 082003 (2008)*
Two-boson exchange corrections

- 2-3% correction at $Q^2 < 0.1$ GeV$^2$
- strong $Q^2$ dependence at low $Q^2$
- cf. Marciano-Sirilin ($Q^2 = 0$): $\Delta \rho = -0.37\%$, $\Delta \kappa = -0.53\%$

Tjon, Melnitchouk, PRL 100, 082003 (2008)
Two-boson exchange corrections

dependence on input form factors

\[ \delta = \frac{A_{\text{TBE}}}{A_{\text{tree}}} \]

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>$\theta$</th>
<th>Empirical</th>
<th>Dipole</th>
<th>Monopole</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>144.0°</td>
<td>1.62</td>
<td>1.52</td>
<td>1.72</td>
</tr>
<tr>
<td>0.23</td>
<td>35.31°</td>
<td>0.63</td>
<td>0.58</td>
<td>0.84</td>
</tr>
<tr>
<td>0.477</td>
<td>12.3°</td>
<td>0.16</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>0.997</td>
<td>20.9°</td>
<td>0.22</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td>0.109</td>
<td>6.0°</td>
<td>0.20</td>
<td>0.16</td>
<td>0.32</td>
</tr>
<tr>
<td>0.23</td>
<td>110.0°</td>
<td>1.39</td>
<td>1.33</td>
<td>1.52</td>
</tr>
<tr>
<td>0.03</td>
<td>8.0°</td>
<td>0.58</td>
<td>0.47</td>
<td>0.86</td>
</tr>
</tbody>
</table>

“dipole” results ~ 5-10% smaller than “empirical” \([1]\)

“monopole” \([2]\) results ~ 50% larger than “empirical” \([1]\)

Effects on strange form factors

- global analysis of all PVES data at $Q^2 < 0.3$ GeV$^2$

\[ G^s_E = 0.0025 \pm 0.0182 \]
\[ G^s_M = -0.020 \pm 0.254 \]

at $Q^2 = 0.1$ GeV$^2$

Young et al., PRL 97, 102002 (2006)

- including TBE corrections:

\[ G^s_E = 0.0023 \pm 0.0182 \] (* fixed mainly by $^4$He data)
\[ G^s_M = -0.020 \pm 0.254 \]

at $Q^2 = 0.1$ GeV$^2$
TBE in nuclei

scatter from individual nucleons \((\text{quasi-elastic})\), or whole nuclei?

assume nucleus is \(Z\) protons and \((A-Z)\) neutrons

\((\text{i.e. nuclear corrections in } A^A_{PV} \rightarrow A^N_{PV} \text{ have already been removed})\)

<table>
<thead>
<tr>
<th>(\gamma(\gamma \gamma))</th>
<th>(-0.11)</th>
<th>(\Delta \kappa) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z(\gamma \gamma))</td>
<td>(0.05)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>(\gamma(Z\gamma))</td>
<td>(0.61)</td>
<td>(-0.04)</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>0.56</strong></td>
<td><strong>-0.04</strong></td>
</tr>
</tbody>
</table>

Tjon, Melnitchouk (2008)
TBE in nuclei

- at the nuclear level, consider TBE with elastic intermediate state

- assume dipole form factor with cut-off \( \Lambda_{\text{Pb}} = \sqrt{12/\langle r^2 \rangle} \approx 0.12 \text{ GeV} \)

\[
\begin{align*}
\delta_{\gamma(\gamma\gamma)} &\quad 0.052 \\
\delta_{Z(\gamma\gamma)} &\quad -0.026 \\
\delta_{\gamma(Z\gamma)} &\quad 0.018 \\
\end{align*}
\]

\[
1 + \delta_{\gamma(Z\gamma)} + \delta_{Z(\gamma\gamma)} - \delta_{\gamma(\gamma\gamma)} \quad \frac{A_{PV}}{A^{(0)}_{PV}} \quad 0.944
\]

Tjon, Melnitchouk (2008)
Summary

- TPE corrections resolve most of Rosenbluth vs. PT $G_E^p / G_M^p$ discrepancy
  - “25% less charge” in the center of the proton
  - first consistent form factor fit at order $\alpha^3$

- $\gamma(Z\gamma)$ and $Z(\gamma\gamma)$ contributions give $\sim 2\%$ corrections to PVES at small $Q^2$
  - strong $Q^2$ dependence at low $Q^2$
  - affects extraction of strange form factors

- First results on TBE in nuclei ($^{208}$Pb)
  - at nucleon level, correction $< 1\%$ ($\Delta \rho$)
  - larger effect at nuclear level (elastic intermediate state only)
The End