Mysteries of nucleon structure at large $x$

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Outline

- Open questions at large $x$
- Valence quarks in parity-violating DIS
  - finite-$Q^2$ corrections
  - Xiaochao Zheng’s talk
- Target mass corrections
- Resonances & quark-hadron duality
  - truncated moments
- Extraction of neutron structure from nuclear data
  - new method for unpolarized & polarized SFs
- Summary
Open questions

What is the structure of valence quarks at large $x$?
- how does $d/u$ ratio behave as $x \to 1$
- how is spin distributed amongst valence quarks

To what extent are low $Q^2$ data dominated by leading twist?
- can JLab data be used to constrain global PDFs
  (joint analysis with CTEQ under way)

How large are higher twists?
- how to quantify duality violation

Can we reliably extract neutron structure functions from nuclei?
- can we recover neutron resonance structure?
Valence quarks
Valence quarks

- Most direct connection between quark distributions and models of the nucleon is through *valence* quarks.
- Nucleon structure at intermediate & large $x$ dominated by valence quarks.
Valence quarks

- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon

- SU(6) spin-flavoursymmetry

**proton wave function**

$$p^\uparrow = -\frac{1}{3} d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3} d^\downarrow (uu)_1$$

$$+ \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0$$

![Diagram](image)
Valence quarks

- **Ratio of** $d$ **to** $u$ **quark distributions particularly sensitive to quark dynamics in nucleon**

- **SU(6) spin-flavour symmetry**

**Proton wave function**

$$p^\uparrow = -\frac{1}{3}d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow (uu)_1$$

$$+ \frac{\sqrt{2}}{6}u^\uparrow (ud)_1 - \frac{1}{3}u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow (ud)_0$$

$$\rightarrow u(x) = 2 \ d(x) \ \text{for all} \ x$$

$$\frac{F_2^n}{F_2^p} = \frac{2}{3}$$
Valence quarks

**scalar diquark dominance**

\[ M_\Delta > M_N \implies (qq)_1 \text{ has larger energy than } (qq)_0 \]

\[ \implies \text{ scalar diquark dominant in } x \to 1 \text{ limit} \]

since only \( u \) quarks couple to scalar diquarks

\[ \frac{d}{u} \to 0 \]

\[ \frac{F_2^n}{F_2^p} \to \frac{1}{4} \]

Valence quarks

- **hard gluon exchange**

At large $x$, helicity of struck quark = helicity of hadron

$\Rightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

$\frac{d}{u} \to \frac{1}{5}$

$\frac{F_2^n}{F_2^p} \to \frac{3}{7}$

Farrar, Jackson 1975
Quark polarization at large $x$

**SU(6) symmetry**

\[ A_1^p = \frac{5}{9}, \quad A_1^n = 0 \]

\[ \Delta u = \frac{2}{3}, \quad \Delta d = -\frac{1}{3} \]

**Scalar diquark dominance**

\[ A_1^p \rightarrow 1, \quad A_1^n \rightarrow 1 \]

\[ \Delta u \rightarrow 1, \quad \Delta d \rightarrow -\frac{1}{3} \]

**pQCD (helicity conservation)**

\[ A_1^p \rightarrow 1, \quad A_1^n \rightarrow 1 \]

\[ \Delta u \rightarrow 1, \quad \Delta d \rightarrow 1 \]
Valence quarks

- At large $x$, valence $u$ and $d$ distributions extracted from $p$ and $n$ structure functions

\[ F_2^p \approx \frac{4}{9} u_v + \frac{1}{9} d_v \]
\[ F_2^n \approx \frac{4}{9} d_v + \frac{1}{9} u_v \]

- $u$ quark distribution well determined from $p$

- $d$ quark distribution requires $n$ structure function

\[ \frac{d}{u} \approx \frac{4 - F_2^n / F_2^p}{4F_2^n / F_2^p - 1} \]
large uncertainty at large $x$ in $d/u$ ratio
Nuclear effects

- no free neutron targets
  (neutron half-life ~ 12 mins)

  → use deuteron as "effective" neutron target

- **BUT** deuteron is a nucleus, and \( F_2^d \neq F_2^p + F_2^n \)

  → nuclear effects (nuclear binding, Fermi motion, shadowing)
  *obscure neutron structure* information

  → “nuclear EMC effect”
without EMC effect in $d$ $F_2^d$ underestimated at large $x$!
“Cleaner” methods of determining $d/u$

- $e \ d \rightarrow e \ p_{\text{spec}} \ X$
  
- $e \ ^3\text{He}(^3\text{H}) \rightarrow e \ X$
  
- $e \ p \rightarrow e \ \pi^{\pm} \ X$

- $e^{\mp}\ p \rightarrow \nu(\bar{\nu})X$
  
- $\nu(\bar{\nu}) \ p \rightarrow l^{\mp} \ X$
  
- $p \ p(\bar{p}) \rightarrow W^{\pm} \ X$
  
- $\vec{e}_L(\vec{e}_R) \ p \rightarrow e \ X$

“BONUS”

mirror-symmetric nuclei

semi-inclusive DIS as flavor tag

weak current as flavor probe
“Cleaner” methods of determining $d/u$

- $e \, d \rightarrow e \, p_{\text{spec}} \, X$
- $e \, ^3\text{He} (^3\text{H}) \rightarrow e \, X$
- $e \, p \rightarrow e \, \pi^\pm \, X$
- $e^\mp \, p \rightarrow \nu (\bar{\nu}) \, X$
  \[ \nu (\bar{\nu}) \, p \rightarrow l^\mp \, X \]
  \[ p \, p (\bar{p}) \rightarrow W^\pm \, X \]
  \[ \vec{e}_L (\vec{e}_R) \, p \rightarrow e \, X \]

“BONUS”

- mirror-symmetric nuclei
- semi-inclusive DIS as flavor tag
- weak current as flavor probe
Parity-violating DIS
(with Tim Hobbs)
Parity-violating $e$ scattering

- **Left-right polarization asymmetry in** $\vec{e} \ p \rightarrow e \ X$
  \[
  A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}
  \]
  
  measure interference between e.m. and weak currents

- **In terms of structure functions**
  
  \[
  A^{PV} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left( g_A^e Y_1 \frac{F_1^\gamma Z}{F_1^\gamma} + \frac{g_V^e}{2} Y_3 \frac{F_3^\gamma Z}{F_1^\gamma} \right)
  \]
  
  \[
  Y_1 = \frac{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma))}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{1 + R^\gamma Z}{1 + R^\gamma} \right)
  \]
  
  \[
  Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{r^2}{1 + R^\gamma} \right)
  \]
  
  where $y = \nu/E$ and $r^2 = 1 + 4M^2x^2/Q^2$
Parity-violating $e$ scattering

- **Left-right polarization asymmetry in** $\vec{e} \; p \to e \; X$

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

→ **measure interference between e.m. and weak currents**

- **In terms of structure functions**

$$A^{PV} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left( g_A^e Y_1 \ \frac{F_1^{\gamma Z}}{F_1^\gamma} + \frac{g_V^e}{2} \ Y_3 \ \frac{F_3^{\gamma Z}}{F_1^\gamma} \right)$$

$$Y_1 = \frac{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^{\gamma Z})) - 2xyM/E}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{1 + R^{\gamma Z}}{1 + R^\gamma} \right)$$

$$Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{r^2}{1 + R^\gamma} \right)$$

**where** $y = \nu/E$ **and** $r^2 = 1 + 4M^2 x^2 / Q^2$
Parity-violating $e$ scattering

- Longitudinal-transverse interference cross section ratio

$$R^\gamma Z = \frac{\sigma^\gamma Z_L}{\sigma^\gamma Z_T} \quad \rightarrow \text{unknown phenomenology}$$

- At large $Q^2$: $Y_1 \rightarrow 1, \quad Y_3 \rightarrow \frac{1 - (1 - y)^2}{1 + (1 - y)^2}$

$$A^{PV} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \right) (Y_1 \ a_1 \ + \ Y_3 \ a_3)$$

where $a_1 = \frac{2 \sum_q C_{1q} \ (q + \bar{q})}{\sum_q e_q^2 \ (q + \bar{q})}$ \hspace{1cm} $a_3 = \frac{2 \sum_q C_{2q} \ (q - \bar{q})}{\sum_q e_q^2 \ (q - \bar{q})}$

$$C_{1q} = g_A^e \ g_V^q \quad \quad C_{2q} = g_V^e \ g_A^q$$
Parity-violating $e$ scattering

Proton asymmetry sensitive to $d/u$ ratio

\[ a_1^p = \frac{12C_{1u} - 6C_{1d}}{4 + d/u} \]

* $d/u \rightarrow 0.2$ as $x \rightarrow 1$

Hobbs, Melnitchouk
Parity-violating $e$ scattering

- **Sensitivity to $R^\gamma$**

![Graph showing relative change to Bjorken limit asymmetry](image)

- $Q^2 = 5 \text{ GeV}^2$

- **Hobbs & Melnitchouk**
  *Phys. Rev. D 77, 114023 (2008)*

- **Relative change to Bjorken limit asymmetry**

- **Uncertainty due to $R^\gamma$ smaller than $d/u$ differences at large $x$**
Parity-violating $e$ scattering

- **Sensitivity to $R^{\gamma Z}$**

> correction from $R^{\gamma Z}$ needs further investigation

Hobbs & Melnitchouk  
Target mass corrections

- Additional corrections from kinematical $Q^2/\nu^2$ effects

  → “target mass corrections” (TMC)

- Important at large $x$ and low $Q^2$

  → but not unique – depend on formalism (e.g. OPE, collinear factorization)

  → most implementations exhibit “threshold problem”

  \[ F(x = 1) \neq 0 \]

  → uncertainties not overwhelming, except at very large $x$

  → new (“Nachtmann”) scaling variable \[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}} \]
Target mass corrections

\[ F_1^{\gamma, \text{TMC}} \text{ from CTEQ6 PDFs} \]

\[ Q^2 = 2 \text{ GeV}^2 \]

- Collinear factoriz’n
- \( \approx \) to full OPE
- \( 1/Q^2 \) expansion

Hobbs & Melnitchouk (2008)
Target mass corrections

Hobbs & Melnitchouk (2008)

leading twist analysis breaks down

\( F_1^{\gamma, \text{TMC}} \) from CTEQ6 PDFs

\( Q^2 = 2 \text{ GeV}^2 \)

collinear factoriz’n

\( \approx \) to full OPE

\( 1/Q^2 \) expansion
TMC effects ~ 1–2% in PV asymmetry
larger in absolute structure functions
Target mass corrections

collinear factorization

$Q^2 = 2 \text{ GeV}^2$

$Q^2 = 25 \text{ GeV}^2$

$F_2/F_2^{(0)}$
$F_{2P}/F_2^{(0)}$
$F_{2n}/F_2^{(0)}$

$F_2/F_2^{(0)}$
$F_{2P}/F_2^{(0)}$

$x_B = 0.8$

$x_B = 0.4$

→ TMC important at large $x$ even for large $Q^2$

Accardi & Qiu,
JHEP 0807, 090 (2008)
Target mass corrections

- Important to implement in pQCD data analyses, if large-$x$ (low-$W$) & low-$Q^2$ data incorporated into global PDF fits

  → greatly expanded data set, especially with high-precision JLab data

- Currently working with CTEQ (J. Owens) to study effects of TMCs on $W$ and $Q^2$ cuts on data (A. Accardi, E. Christy, C. Keppel, P. Monaghan)

  → crucial for neutrino scattering and oscillations

  → important for “new physics” searches at colliders
Duality & truncated moments
(with Ales Psaker et al.)
Bloom-Gilman duality

Average over (strongly $Q^2$ dependent) resonances
≈ $Q^2$ independent scaling function

Jefferson Lab (Hall C)
Truncated moments

- complete moments can be studied in QCD via twist expansion
  - Bloom-Gilman duality has a precise meaning
    (i.e., duality violation = higher twists)

- for local duality, difficult to make rigorous connection with QCD
  - e.g. need prescription for how to average over resonances

- truncated moments allow study of restricted regions in $x$ (or $W$) within QCD in well-defined, systematic way

$$
\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \ x^{n-2} \ F_2(x, Q^2)
$$
Truncated moments

truncated moments obey DGLAP-like evolution equations, similar to PDFs

\[ \frac{d \overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left( P'_n(\Delta x, Q^2) \otimes M_n(\Delta x, Q^2) \right) \]

where modified splitting function is

\[ P'_n(z, \alpha_s) = z^n P_{NS,S}(z, \alpha_s) \]

can follow evolution of specific resonance (region) with \( Q^2 \) in pQCD framework!

suitable when complete moments not available
how much of this region is leading twist?

Data analysis

- Assume data at large $Q^2$ is entirely leading twist
- Evolve fit to data (as NS) at large $Q^2$ down to lower $Q^2$
  
  \[\rightarrow\] Apply TMC, and compare with data at lower $Q^2$

Data analysis

Psaker et al., arXiv:0803.2055

entire resonance region
higher twists less than $10-15\%$ for $n=2$ moment

also study higher twists in higher moments

Psaker et al., arXiv:0803.2055
Extracting neutron SFs from nuclear data
(with Yoni Kahn)
EMC effect in deuteron

\[ F_2^d(x) = \int dy \, f_{N/d}(y) \, F_2^N(x/y) + \delta^{(\text{off})} F_2^d(x) \]

- **Nucleon momentum distribution** ("smearing function")
- **off-shell correction** (very small in \( d \))

**Nuclear “impulse approximation”**

incoherent scattering from individual nucleons in deuteron

\( \gamma^* \to N p \to p p \to d P, S \quad \text{and} \quad P, S \to d P, S \)
EMC effect in deuteron

Nucleon momentum distribution in deuteron

computed from $d$ wave function

$$f_{N/d}(y) = \frac{1}{4} M_d \ y \ \int_{-\infty}^{p_{\text{max}}^2} dp^2 \ \frac{E_p}{p_0} \ |\Psi_d(p^2)|^2$$
EMC effect in deuteron

- At finite $Q^2$, smearing function depends also on parameter
  \[ \gamma = |q|/q_0 = \sqrt{1 + 4M^2x^2/Q^2} \]

  \[ \rightarrow \text{simple factorization of convolution formula breaks down} \]

- For polarized SFs, have mixing between $g_1$ & $g_2$ at finite $Q^2$
  \[ g_i^d(x, Q^2) = \int dy \frac{dy}{y} f_{ij}(y, \gamma) \, g_j^N(x/y, Q^2) , \quad i, j = 1, 2 \]

  \[ \rightarrow \text{for most kinematics } \gamma \lesssim 2 \]

  \[ \rightarrow \text{off diagonal functions small} \quad |f_{12}|, |f_{21}| \ll f_{11}, f_{22} \]

Kulagin, Melnitchouk
N momentum distributions in $d$
Unsmearing – multiplicative method

- calculated $d/N$ ratio depends on input $F^n_2$

  $\Rightarrow$ extracted $n$ depends on input $n$ ... cyclic argument

Solution: iteration procedure

0. subtract $\delta^{\mathrm{off}} F^d_2$ from d data: $F^d_2 \rightarrow F^d_2 - \delta^{\mathrm{off}} F^d_2$

1. smear $F^p_2$ with $f_{N/d}$: $f_{N/d} \otimes F^p_2 \equiv S_p^{-1} F^p_2$

2. extract neutron via $F^n_2 = S_n (F^d_2 - F^p_2 / S_p)$
   starting with e.g. $S_n = S_p$

3. smear $F^n_2$ with $f_{N/d}$ to get new $S_n$

4. repeat 2-3 until convergence
Unsmearing – multiplicative method

$F_2^d$ constructed from $F_2^P$ and $F_2^n$ inputs

(using Bosted/Christy parameterizations)

Kahn, WM (2008)
Unsmearing – additive method

- since $g_1$ & $g_2$ are not positive-definite, expect multiplicative method to fail for spin-dependent SFs

Solution: additive iteration procedure (avoids zeros)

0. subtract $\delta^{(\text{off})}F_2^d$ from $d$ data: $F_2^d \rightarrow F_2^d - \delta^{(\text{off})}F_2^d$

1. define difference between smeared and free SFs

$$\tilde{F}_2^n = f_{N/d} \otimes F_2^n = F_2^n + \delta$$

2. first guess for $F_2^{n(0)}$ \[\delta^{(0)} = \tilde{F}_2^{n(0)} - F_2^{n(0)}\]

3. after one iteration, gives

$$F_2^{n(1)} = F_2^{n(0)} + (\tilde{F}_2^n - \tilde{F}_2^{n(0)})$$

4. repeat until convergence
Unsmearing – additive vs. multiplicative

Comparison of additive and multiplicative methods at $Q^2 = 1 \text{ GeV}^2$

5 iterations

- Test function $F_2^n$
- Multiplicative method
- Additive method

Both methods work well for unpolarized SFs

Kahn, WM (2008)
Unsmearing – additive vs. multiplicative

Comparison of additive and multiplicative methods at $Q^2 = 1 \text{ GeV}^2$

- Multiplicative method
- Additive method

5 iterations

Test function $xg_i^n$

Multiplicative method

Additive method

Kahn, WM (2008)

→ multiplicative method problematic for polarized SFs
Unsmearing – additive method

additive method works well for polarized SFs

Kahn, WM (2008)
Unsmearing – additive method

Preliminary extraction of $xg^n_1$ at $Q^2 = 10 \text{ GeV}^2$

5 iterations
- Extracted $xg^n_1$
- $xg^p_1$
- $xg^d_1$

Kahn, WM (2008)

→ additive method works well for polarized SF data at large $Q^2$
Unsmearing – additive method

Preliminary extraction of $xg_1^n$ at $Q^2 = 2 \text{ GeV}^2$

- Extraction sensitive to discontinuities in $d$ data
- cf. future RSS data
Summary

- Fundamental questions remain to be addressed at large $x$
- Need to account for finite-$Q^2$ effects in PVDIS
  - quantify effects of $R_{\gamma Z}$, as well as higher twists
- Target mass corrections
  - joint analysis with CTEQ of global data under way
  - TMCs in polarized structure functions
- Truncated moments
  - firm foundation for study of local duality in QCD
  - higher twists $< 10\%$ for $Q^2 > 1$ GeV$^2$ in resonance region
- New method for extracting neutron SFs from nuclear data
  - await higher-precision nuclear data!
The End