GPDs in high–energy $ep$ and $pp$ scattering

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- Explore nucleon structure  
  JLab 12 GeV, EIC
  - Quark core, pion cloud, polarization
  - Small–$x$ diffusion, $\alpha'$, . . .

- Understand $ep$ at small $x$  
  HERA, EIC
  - Dipole picture
  - Unitarity limit: “Black–disk regime”

- Model $pp$ with hard processes  
  LHC, Tevatron, RHIC
  - Impact parameter dependence
  - Central vs. peripheral collisions
  - Exclusive diffraction $pp \to p + H + p$, . . .
Factorization: Inclusive $ep$ scattering

\[ \sigma^{nN} = Q^2 \rightarrow W \rightarrow \]

\[ Q^2 W^2 \gg \mu^2_{\text{had}} = \]

\[ \text{PDF} \]

- Factorization of amplitude
  - Quark subprocess \( \sim 1/Q \) short distance
  - Parton distribution in nucleon (PDF) \( \sim 1/\mu_{\text{had}} \) long distance

- PDF as matrix element of QCD operator
  \( \langle p | \bar{\psi}(0) \ldots \psi(z) | p \rangle_{z^2=0} \leftrightarrow f(x) \)

- Space–time interpretation:
  Density of quarks with longitudinal momentum fraction \( x \) in wave function of fast–moving nucleon \( (P \gg \mu_{\text{had}}) \)

[Concept: Gribov 70's]
Factorization: Exclusive processes in $ep$

- Meson/$\gamma$ produced in reaction with quasi-free parton in target

- GPD $H(x, x', t)$ combines properties of parton distribution and elastic FF

- Factorization implies
  - GPDs universal, process–independent
  - Leading $1/Q^2$ power behavior

\[
\begin{align*}
\gamma^* p &\rightarrow \gamma + p \quad \text{Deeply virtual Compton} \\
\gamma_{LP}^* &\rightarrow \rho + p \quad \text{Vector meson} \\
J/\psi + p &\rightarrow \text{Heavy } Q\bar{Q} \quad \text{(gluon GPD)} \\
\pi + N &\rightarrow \text{pseudoscalar}
\end{align*}
\]

[D. Müller et al. 94; Brodsky et al. 94; Collins et al. 96; Radyushkin 96, Ji 96]
GPDs: Transverse spatial distribution of partons

- Transverse coordinate representation ($x' = x$)

$$H(x, t) = \int d^2 \rho \ e^{-i \Delta_T \cdot \rho} \ f(x, \rho)$$

FF of partons with mom. $xP$ transverse spatial distribution

$$\int d^2 \rho \ f(x, \rho) = f(x)$$ longitud. momentum density

- Transverse size of nucleon ($x$–dep.)

$$\langle \rho^2 \rangle_f = 4 \ \frac{\partial}{\partial t} \left. \frac{H(x, t)}{H(x, t = 0)} \right|_{t = 0}$$

“Tomographic image” of nucleon at fixed $x$

[Burkardt 02; Diehl 02]
GPDs: Polarization in quark distributions

Quarks unpolarized:

\[ \gamma_\mu \]

\[ \text{GPD} \]

\[ = \]

\[ H, \]

\[ E \]

Dirac

Pauli

polarized:

\[ \gamma_\mu \gamma_5 \]

\[ \tilde{H}, \]

\[ \tilde{E} \]

axial

pseudoscalar

- \[ E(x) \]: Distortion of longitudinal motion of quarks due to transverse nucleon spin

[Burkardt 03]

\[ \rightarrow \] JLab 12 GeV exp. program
GPDs: Transverse gluon distribution from $J/\psi$

- Test of factorization: Universality of $t$–slopes at high $Q^2$

- Spatial distribution from $t$–dependence (unnormalized)

\[
\frac{d\sigma}{dt} \propto \left[ \frac{H_g(x, t)}{H_g(x, 0)} \right]^2 \quad \text{FT} \quad \text{spatial distribution}
\]

- Also: $J/\psi$ fixed–target data
  [FNAL, SLAC, Cornell, CERN]

[Summary HERA 05 data by A. Levy]
GPDs: Gluonic transverse size of nucleon

- Gluonic transverse size increases with decreasing $x$

- Pion cloud at $x < M_\pi/M_N$
  \[ G(x, \rho) \sim e^{-2M_\pi\rho} \]
  “Yukawa tail”
  [Strikman, CW 03]

- Small $x$: Logarithmic growth with $\alpha'_g \ll \alpha'_\text{soft}$ (“diffusion”)
  [DGLAP: Frankfurt, Strikman, CW 03]

(Scale $Q^2 \approx 3 \text{ GeV}^2$)
Small $x$: GPDs and dipole picture

$A^{dp} \propto d^2 \alpha_s \, x \, G(x, t \mid Q_{\text{eff}}^2)$

Scale $Q_{\text{eff}}^2 \approx \pi^2 \, d^{-2}$

- QCD factorization $\leftrightarrow$ “Color transparency”
  - Gluon GPD $\leftrightarrow$ “Color dipole moment” of proton

- Diagonal approximation $x = x'$ in GPD
  [Frankfurt et al. 97; Shuvaev et al. 99]
Small $x$: Impact parameter representation

- Dipole–proton interaction probes local transverse gluon density $G(x, b)$

- Impact parameter representation of dipole–proton scattering amplitude

$$A^{dp}(s, t) = \frac{is}{4\pi} \int d^2b \ e^{-i\Delta \cdot b} \ \Gamma^{dp}(s, b)$$

profile function

$$\Gamma^{dp} \to 1: \ \text{“Black–disk limit”}$$

Model–independent formulation of unitarity limit in hard interactions

$$\frac{d\sigma_{\text{inel}}^{dp}}{d^2b} = 1 - |1 - \Gamma^{dp}|^2$$

Probability of inelastic scattering
**Small $x$: Approach to black–disk regime**

- $\Gamma^{dp}$ evaluated with LO gluon density + spatial distribution
  - BDR not reached for $\bar{q}q$ dipoles at HERA
  - Definitely reached for $gg$ dipoles in central $pp$ scattering at LHC

- Here BDR reached because of large non-perturbative gluon density (chiral symmetry breaking) and “usual” DGLAP evolution
  - no need for $\log(1/x)$ enhanced radiation [cf. Color Glass Condensate]

[Frankfurt, Guzey, Strikman 02; FS + CW 03–05; FS + Rogers 03]
**pp: Impact parameter dependence**

- **Hard process:** \( x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y} \)

- **Local on scale of soft interactions:** \( M^2 \gg R^{-2} \text{(hadron)} \)

- **Calculate probability as function of pp impact parameter} \( b \)
  in terms of \( f(x, \rho) \) known from \( ep \)

- \( \rightarrow \) Spectator interactions
- \( \rightarrow \) Global event characteristics
- \( \rightarrow \) Diffraction (rapidity gap survival)

\[
P_{\text{hard}}(b) \propto \int d^2 \rho_1 \, d^2 \rho_2 \\
\times \delta(b - \rho_1 + \rho_2) \\
\times f(x_1, \rho_1) \, f(x_2, \rho_2)
\]

"Control" transverse geometry even though \( b \) not observable!
**pp: Hard process selects central collisions**

- Different transverse sizes in hard and soft interactions:
  \[
  R^2 (x \geq 10^{-2}) \ll R^2 \text{(soft)}
  \]
  \[
  \sim 0.3 - 0.4 \text{ fm}^2 \sim 0.8 - 1.0 \text{ fm}^2
  \]
  From excl. \( J/\psi \) \( pp \) elastic

- “Two-scale picture”

- Hard processes (e.g. dijets) as trigger on central collisions

  ... Numerous applications!

  [Frankfurt, Strikman, CW 03]
**pp:** Black–disk limit in hard spectator interactions

- Central collisions: Hard interactions of large–$x_1$ spectators with small–$x_2$ gluons approach black–disk limit
- Max. $p_T$ estimated using dipole model
- Qualitative changes in forward particle production: Large $p_{\perp}$, energy loss, . . .
- Can be studied with LHC detectors

[TOTEM, CMS, TOTEM]

[Frankfurt, Strikman, CW 03/04]
**Diffraction: Gap survival in** $pp \rightarrow p + H + p$

- **Heavy particle produced in**
  hard process (2–gluon exchange)
  $x_{1,2} \sim 10^{-2}$ Higgs at LHC  
  [Khoze et al. 97]

- **Soft spectator interactions**
  must not destroy rapidity gaps!

- **Gap survival probability**

\[
S^2 \approx \int d^2 b \, P_{\text{hard}}(b) \left| 1 - \Gamma_{PP}(b) \right|^2 \\
\approx 0.03 \quad (\text{Higgs at LHC})
\]

Calculable, model-independent

[Frankfurt, Hyde, Strikman, CW 07]

**Suppression of small** $b$ from
“blackness” of $pp$ scattering
Diffraction: $p_T$ dependence

Amplitude computed in terms of

- Gluon GPD $t$–dep. $\sim R^2$(hard)
- $pp$ elastic $S$–matrix $t$–dep. $\sim R^2$(soft)

Diffractive minimum

\[
T_{\text{diff}}(p_{1T}, p_{2T}) \propto \int d^2 \Delta_T \\
\times H_g(x_1, p_{1T} - \Delta_T) \\
\times H_g(x_2, p_{2T} + \Delta_T) \\
\times S_{\text{el}}(\Delta_T) \\
\sqrt{1 + iT_{\text{el}}} 
\]

Coordinate representation:
Diffraction of wave packet from “hole” $1 - \Gamma(b)$
Diffraction: $p_T$ dependence

- Pattern determined by two scales $R^2(\text{hard}) \ll R^2(\text{soft})$
- Entangled dependence on $\bm{p}_{1T}$ and $\bm{p}_{2T}$
**Diffraction: Disentangling $p_T$**

- Define CM and relative momentum
  \[ P_T = \frac{(p_{1T} + p_{2T})}{2} \]
  \[ r_T = p_{1T} - p_{2T} \]

- $r_T$ dependence has diffractive minimum: $R^2$(soft) and $R^2$(hard)

- $P_T$ dependence sensitive to $t$–dependence of gluon GPD: $R^2$(hard) only

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Test reaction mechanism and two–scale picture
Diffraction: Beyond the mean–field approximation

- Mean–field approximation:
  \[ G(x, \rho) \]
  \[ \Gamma(s, b) \]
  independent, determined by “average” configurations

- Several effects lead to correlations between parton density and spectator interactions

  \[ \rightarrow \text{lower RGS probability } S^2 \]

  \[ \rightarrow \text{steeper } p_{1T}, p_{2T} \text{ dependence} \]

Diffraction: Hard spectator interactions

- Parent partons \((k^2 \sim \text{few GeV}^2)\) experience absorptive interactions with small--\(x\) gluons in other proton
  “Black–Disk Regime”

- Use estimate of “critical” \(k_T^2\) from dipole model

- Effect reduces RGS probability at LHC by at least factor 2
  . . . much weaker effect at Tevatron

- Larger impact parameters
  \(\rightarrow\) steeper \(p_{1T}, p_{2T}\) dependence!
Outlook: Transverse correlations of partons

- GPDs single–particle distributions
  Next step: Two–particle correlations

- Fermilab CDF data on 3 jet + photon compatible with strong transverse correlations of size $\rho \sim 0.3$ fm
  [Frankfurt, Strikman, CW 04]

  \ldots Constituent quarks?

  cf. Instanton liquid picture of QCD vacuum [Diakonov, Petrov 84]

- Correlations could substantially modify rapidity gap survival in diffraction
  [Frankfurt, Hyde, Strikman, CW 07]
Summary

**ep**
- Future precision measurements of exclusive channels with EIC could substantially improve knowledge of transverse nucleon structure at intermediate and small $x$

- Transverse structure essential for understanding approach to black–disk regime (unitarity limit, saturation)

**pp**
- $pp$ collisions with hard processes much more central than min.bias; very different final–state properties
  → Include transverse structure in MC generators!

- Possible to probe $t$–dependence of GPDs in $p_T$ dependence of central exclusive diffraction

GPDs as unifying concept $ep \leftrightarrow pp$