Meson spectrum and coupling to photons from Lattice QCD

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applications to charmonium
meson photocouplings

radiative transitions

CLEO-c, BES III

basic object: $\langle \gamma m' | m \rangle$
meson photocouplings

radiative transitions

CLEO-c, BES III

basic object: $\langle \gamma m' | m \rangle$

$\langle m' | \bar{\psi} \gamma^\mu \psi | m \rangle \langle \gamma | A_\mu | 0 \rangle$
extract from three-point correlators

\[ C(t_f, t, t_i) = \langle 0 | \Phi'(t_f) \left[ \bar{\psi} \gamma^\mu \psi \right](t) \Phi(t_i) | 0 \rangle \]

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C(t_f, t, t_i) = \sum_{n,m} \langle 0 | \Phi'(0) | n \rangle e^{-E_n(t_f - t)} \langle n | \left[ \bar{\psi} \gamma^\mu \psi \right](0) | m \rangle e^{-E_m(t - t_i)} \langle m | \Phi(0) | 0 \rangle
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spectrum of eigenstates of \( H_{\text{QCD}} \)
- i.e. the meson spectrum

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in Euclidean time (\( t \to -i t \))
meson photocouplings

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e.g. pseudoscalars can be ‘made’ with

\[ \bar{\psi} \gamma^5 \psi \]

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so we need to know the spectrum & vacuum matrix elements of operators first
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Basic object is two-point correlator

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each operator will have different ‘overlap’ on to the tower of pseudoscalar states

sampling the ‘wavefunction’ of the states
meson spectrum

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Sampling the ‘wavefunction’ of the states

Some linear combination of the operators is optimal for a certain state

\[ \Omega_n = \nu_1^n \Phi_1 + \nu_2^n \Phi_2 + \ldots \]
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\[ \Omega_2 \]

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within a finite basis of operators, our best estimate is from a variational solution
variational analysis

matrix of correlators

\[ C(t) = \begin{bmatrix} 
\langle 0| \Phi_1(t) \Phi_1(0)|0 \rangle & \langle 0| \Phi_1(t) \Phi_2(0)|0 \rangle & \cdots \\
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variational solution = generalised eigenvalue problem

\[ C(t) v^n = \lambda_n(t) C(t_0) v^n \]

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\[ C(t)\nu^n = \lambda_n(t)C(t_0)\nu^n \]

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eigenvectors give the ‘optimal’ operators

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orthogonality of eigenvectors - required to extract near degenerate states

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* How big does the basis need to be?
vector spectrum

e.g. charmonium vector spectrum

near deg. states are tough to fit

using multi-exponential fits

\[ C(t) = \sum_{n} e^{-E_{n}t} \langle 0 | \Phi'(0) | n \rangle \langle n | \Phi(0) | 0 \rangle \]

e.g. in two dimensions: \[ \psi_{J}(\theta) = e^{iJ\theta} \]

so under the allowed \( \pi/2 \) rotations, \( J=0,4,8... \) indistinguishable
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more spectrum

e.g. $J^{++}$

$a_s = 0.1 \text{ fm}$
more spectrum

\( \alpha_s = 0.1 \text{ fm} \)

e.g. \( J^{++} \)

\[ X_{c(0,1,2)} \left[ ^2 \! \! P_J \right] \]

\[ X_{c(2,3,4)} \left[ ^3 F_J \right] \]

\[ X_{c(0,1,2)} \left[ ^3 P_J \right] \]
e.g. $J^{++}$

$\alpha_s = 0.1 \text{ fm}$

somewhat limited by the size of operator basis - have subsequently expanded
the calculation

first attempt - little systematic control

*quenched* - no light quarks at all (like models)

one lattice spacing \( a = 0.1 \text{ fm} \) (anisotropic, \( a_t = 0.033 \text{ fm} \))

box possibly too small for highly excited states (1.2 fm)

only connected diagrams

allowed us to get **high statistics** (1000 gauge field configs) and most importantly to ‘try things out’

**Monte-Carlo** - statistical error from finite number of samples

all of these ‘lattice issues’ are systematically improvable:

see papers by *Fermilab/MILC & HPQCD*
## Vector States

<table>
<thead>
<tr>
<th>Level</th>
<th>Mass/MeV</th>
<th>Suggested state</th>
<th>Model assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3106(2)</td>
<td>$J/\psi$</td>
<td>$1^3S_1$</td>
</tr>
<tr>
<td>1</td>
<td>3746(18)</td>
<td>$\psi'(3686)$</td>
<td>$2^3S_1$</td>
</tr>
<tr>
<td>2</td>
<td>3846(12)</td>
<td>$\psi_3$</td>
<td>Lattice artifact</td>
</tr>
<tr>
<td>3</td>
<td>3864(19)</td>
<td>$\psi''(3770)$</td>
<td>$1^3D_1$</td>
</tr>
<tr>
<td>4</td>
<td>4283(77)</td>
<td>$\psi(\text{&quot;4040&quot;})$</td>
<td>$3^3S_1$</td>
</tr>
<tr>
<td>5</td>
<td>4400(60)</td>
<td>$Y?$</td>
<td>Hybrid</td>
</tr>
</tbody>
</table>

- Masses systematically high: quenched? finite volume?
- Vacuum matrix elements compared to potential model: [PRD78:094504 (2008)](https://doi.org/10.1103/PhysRevD.78.094504)
extract from three-point correlators

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eigenvectors give the 'optimal' operators

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compute for multiple operators & project with eigenvectors

\[ \nu^p C(t_f, t, t_i) = \sum_{m} \langle 0 | \Omega^p(0) | p \rangle e^{-E_p(t_f - t)} \langle p | [\bar{\psi} \gamma^\mu \psi](0) | m \rangle e^{-E_m(t - t_i)} \langle m | \Phi(0) | 0 \rangle \]

now just a single state \( p \) contributing - can be an excited state
$\hat{V}(Q^2) \propto \langle J/\psi | \bar{\psi} \gamma^\mu \psi | \eta_c \rangle \propto \Gamma_{J/\psi \rightarrow \eta_c \gamma}^{1/2}$

$\gamma, \psi''', \psi''$}

$J/\psi \rightarrow \eta_c \gamma$

CLEO '08

$p_y = 000$

$p_y = 100$
vector - pseudoscalar (M1)

\[ J/\psi \rightarrow \eta_c \gamma \]

in quark-potential models:

\[ \hat{V} \propto \langle J/\psi | \bar{\psi} \gamma_\mu \psi | \eta_c \rangle \propto \Gamma_{J/\psi \rightarrow \eta_c \gamma}^{1/2} \]

\[ V \sim \frac{1}{m_c} \int r^2 dr \, R_f(r) j_0(|\vec{q}|r) R_i(r) \]

\( \Gamma \sim 2.4 - 2.9 \text{ keV} \) vs. exptal (CLEO-c) = 1.85(30) keV

quark spin flip \( \sim \frac{\sigma}{m_c} \)

‘Higher Charmonia’ (Barnes, Godfrey, Swanson)
vector - pseudoscalar (M1)

\[ \psi' \rightarrow \eta_c \gamma \]

\[ \hat{\nu}(Q^2) \]

\[ Q^2 / \text{GeV} \]

\[ p_{y} = 000 \]
\[ p_{y} = 100 \]

CLEO '08
PDG '08

\[ \gamma \]
\[ \psi''' \]
\[ \psi'' \]
\[ \psi' \]
\[ J/\psi \]
\[ \eta_c' \]
\[ \eta_c \]

0.06 (6)
vector - pseudoscalar (M1)

\[ \psi' \rightarrow \eta_c \gamma \]

in quark-potential models:

- **'hindered'**: orthogonal (2S, 1S) wavefunctions
- relativistic corrections at the same order
- frame (in)dependence of non-rel wavefunctions

in quark-potential models:

\[ V \sim \frac{1}{m_c} \int r^2 dr \, R_f(r) \left( 1 + \mathcal{O}(|q|^2 r^2) \right) R_i(r) \]

- **'Higher Charmonia'** (Barnes, Godfrey, Swanson)
- \( \Gamma \sim 4 - 10 \text{ keV} \) vs. expt (CLEO-c) = 1.37(20) keV
first lattice QCD extraction of a radiative transition involving an excited meson

\[ \psi' \rightarrow \eta_c' \gamma \]

\[ V(Q^2) \]

- CLEO '08
- PDG '08

\[ 0.06(6) \]
vector - pseudoscalar (M1)

\[ \psi'' \rightarrow \eta_c \gamma \]

\[ \hat{V}(Q^2) = 0.27(15) \]

\[ \eta_c' \]

\[ J/\psi \]

\[ \eta_c \]
vector - pseudoscalar (M1)

\[ \psi'' \rightarrow \eta_c \gamma \]

\[ \hat{V}(Q^2) \]

\[ 0.27(15) \]

\[ Q^2 / \text{GeV}^2 \]

\[ \psi'' \rightarrow \eta_c' \gamma \]

\[ \hat{V}(Q^2) \]

\[ 0.28(6) \]

\[ Q^2 / \text{GeV}^2 \]
better than we could have hoped for - still a signal for our (technically) 5th excited state
Experimental results from angular dependence of radiative decay events

Suppressed magnetic quadrupole of right sign, but too large in magnitude

Electric octopole consistent with zero

Relativistic correction in quark models - rather model dependent

Has quite simple explanation
experimental results from angular dependence of radiative decay events

suppressed magnetic quadrupole of right sign, but too large in magnitude

relativistic correction in quark models - rather model dependent

electric octopole consistent with zero

has quite simple explanation
tensor - vector (E1,M2,E3)

\[ \chi'_{c2} \rightarrow J/\psi \gamma \]

- \( E_1 \approx 0, E_3 \neq 0 \)
- \( m = 4115(28) \text{ MeV} \)

\[ \chi''_{c2} \rightarrow J/\psi \gamma \]

- \( E_3 \approx 0, E_1 \neq 0 \)
- \( m = 4165(30) \text{ MeV} \)

see Christopher Thomas's talk for explanation
excited tensor states?

**Belle** \( \gamma \gamma \rightarrow D \bar{D} \)

\[ \chi_{c2}(3930) \]

Belle experiment

two-photon fusion
excited tensor states?

Belle $\gamma \gamma \rightarrow D \bar{D}$

$\chi_{c2}(3930)$

$\chi_{c2}(4090)$?

two-photon fusion
excited tensor states?

**Belle**  \( \gamma \gamma \rightarrow D \bar{D} \)

\[ \chi_{c2}(3930) \]

optimism

\[ \chi_{c2}(4090)? \]

two-photon fusion
excited tensor states?

Belle \( \gamma \gamma \rightarrow D \bar{D} \)

\( \chi_{c2}(3930) \)

optimism

\( \chi_{c2}(4090)? \)

g our calculation finds the \( F \)-wave lighter - may be artifact of small box ‘squeezing’
excited tensor states?

Belle

we have the technology to do the appropriate two-photon coupling calculation

\( \chi_{c0} \rightarrow \gamma\gamma^* \)

one pole fit

lattice

expt.

- PDG
- Belle
- CLEO-c

Belle experiment
<table>
<thead>
<tr>
<th>exotics</th>
</tr>
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<tbody>
<tr>
<td>$J^{PC}=1^{-+}$ not accessible to $c\bar{c}$ pair</td>
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HYBRID MESON: excited gluonic field

$\eta_{c1} \rightarrow J/\psi \gamma$
magnetic dipole transition

$\Gamma \sim 100$ keV
$J^{PC}=1^{+}$ not accessible to $c\bar{c}$ pair

we find state at about 4.3 GeV

$\eta_{c1} \rightarrow J/\psi \gamma$

magnetic dipole transition

\[ \eta_{c1}(1^{-+}) \rightarrow J/\psi \gamma \]

$\Gamma \sim 100$ keV

compare with $J/\psi \rightarrow \eta_{c}\gamma \sim 1$ keV

HYBRID MESON: excited gluonic field

quark spin flip $\sim \frac{\sigma}{M_{c}}$
exotics

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we find state at about 4.3 GeV

$\eta_{c1} \rightarrow J/\psi \gamma$
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$\Gamma \sim 100 \text{ keV}$

compare with $J/\psi \rightarrow \eta_c \gamma \sim 1 \text{ keV}$
quark spin flip $\sim \frac{\sigma}{M_C}$

perhaps this is not spin-flip?

HYBRID MESON:
excited gluonic field
**exotics**

- $J^{PC} = 1^+$ not accessible to $c\bar{c}$ pair
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- HYBRID MESON: excited gluonic field

**$\eta_{c1} \rightarrow J/\psi \gamma$**

- magnetic dipole transition

**$\eta_{c1}^{(1^{-+})} \rightarrow J/\psi \gamma$**

- $\Gamma \sim 100$ keV

**Compare with** $J/\psi \rightarrow \eta_c \gamma \sim 1$ keV

**quark spin flip $\sim \frac{\sigma}{m_c}$**

**perhaps this is not spin-flip?**

**$^3H_1 \rightarrow ^3S_1\gamma$**
**exotics**

\( J^{pc}=1^+ \) not accessible to \( c\bar{c} \) pair

we find state at about 4.3 GeV

\( \eta_{c1} \rightarrow J/\psi \gamma \)
magnetic dipole transition

\( \eta_{c1}(1^{-+}) \rightarrow J/\psi \gamma \)

\( \Gamma \approx 100 \text{ keV} \)

compare with \( J/\psi \rightarrow \eta_{c} \gamma \approx 1 \text{ keV} \)

quark spin flip \( \sim \frac{\sigma}{M_c} \)

perhaps this is not spin-flip?

\( ^3H_1 \rightarrow ^3S_1 \gamma \)

supports models in which the exotic has \( S_{q\bar{q}} = 1 \)
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**HYBRID MESON:**
excited gluonic field

\( \eta_{c1} \rightarrow J/\psi \gamma \)
magnetic dipole transition

\[ \Gamma \sim 100 \text{ keV} \]

compare with \( J/\psi \rightarrow \eta_{c} \gamma \sim 1 \text{ keV} \)

quark spin flip \( \sim \frac{\sigma}{m_c} \)

perhaps this is not spin-flip?

\[ 3^{2}H_{1} \rightarrow 3^{1}S_{1} \gamma \]

supports models in which the exotic has \( S_{q\bar{q}} = 1 \)

e.g. flux-tube model, Coulomb gauge ...
exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width
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but ... if this large number is duplicated in the light meson case:

![Diagram of particle interactions](image)

- peripheral photoproduction
- GlueX
- plenty of exotic photoproduction?
exciting?

charmonium exotic (probably) has negligible radiative relative to hadronic width

but ... if this large number is duplicated in the light meson case:

\begin{align*}
\gamma \rightarrow \rho \\
\rho \rightarrow N + \pi_1
\end{align*}

peripheral photoproduction

GlueX

plenty of exotic photoproduction?

our group’s current aim:
perform similar calculations with much lighter quarks - say something useful for GlueX
exciting?

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But... if this large number is duplicated in the light meson case:

Peripheral photoproduction

\[ \gamma \rightarrow \rho \rightarrow N \]

**GlueX**

Plenty of exotic photoproduction?

Our group’s current aim:
Perform similar calculations with much lighter quarks - say something useful for GlueX

As part of larger **HadSpec** collaboration
summary

reliable techniques for extraction of excited states in lattice field theory

now applied to radiative matrix element calculations

initial trials with quenched studies of charmonium - compare with potential models

I have emphasized the exceptions - but actually potential models agree rather well with many results I have not presented

exotic (hybrid?) to conventional meson radiative transitions are large

same techniques can be used in baryon sector - applications to **CLAS** electroproduction program (some first attempts from Huey-Wen Lin et.al.)
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for more information attend Christopher Thomas’s talk at 4.30pm in the ‘Charm Spectroscopy’ parallel session