Nuclear Corrections to Neutron Structure Functions

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Outline

- Why is neutron structure at large $x$ important?
  - $d/u$ ratio
  - isospin dependence of duality (& higher twists)

- Nuclear corrections at finite $Q^2$
  - generalized nuclear smearing formula

- New method for extracting neutron from inclusive data
  - applicable in DIS and resonance regions
  - future comparison with BONUS data
$d/u$ ratio as $x \to 1$
- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon

- **SU(6) spin-flavor symmetry**

**Proton wave function**

\[
p^\uparrow = -\frac{1}{3} d^\uparrow (uu)_1 - \frac{\sqrt{2}}{3} d^\downarrow (uu)_1 \\
+ \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0
\]
- Ratio of $d$ to $u$ quark distributions particularly sensitive to quark dynamics in nucleon

- **SU(6) spin-flavor symmetry**

proton wave function

\[
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\[+ \frac{\sqrt{2}}{6} u^\uparrow (ud)_1 - \frac{1}{3} u^\downarrow (ud)_1 + \frac{1}{\sqrt{2}} u^\uparrow (ud)_0\]

\[\rightarrow u(x) = 2 \ d(x) \ \text{for all } x\]

\[\frac{F^m_2}{F^p_2} = \frac{2}{3}\]
scalar diquark dominance

\[ M_\Delta > M_N \implies (qq)_1 \text{ has larger energy than } (qq)_0 \]

\[ \implies \text{scalar diquark dominant in } x \to 1 \text{ limit} \]

since only \( u \) quarks couple to scalar diquarks

\[ \frac{d}{u} \to 0 \]

\[ \frac{F_2^n}{F_2^p} \to \frac{1}{4} \]

\text{Feynman 1972, Close 1973, Close/Thomas 1988}
hard gluon exchange

at large $x$, helicity of struck quark = helicity of hadron

$\Rightarrow$ helicity-zero diquark dominant in $x \to 1$ limit

$\Rightarrow$ $q^\uparrow \gg q^\downarrow$

$\Rightarrow$

\[
\begin{align*}
\frac{d}{u} &\to \frac{1}{5} \\
\frac{F_n^2}{F_p^2} &\to \frac{3}{7}
\end{align*}
\]

Farrar, Jackson 1975
Duality in the Neutron?
Bloom-Gilman duality well established for the proton

Niculescu et al., PRL 85 (2000) 1182, 1185

Christy et al. (2005)
$F_2^p$ resonance spectrum

how much of this region is leading twist?


* JLab Hall C
- **truncated moments** allow study of restricted regions in $x$ within pQCD in well-defined, systematic way

\[
\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \ x^{n-2} \ F_2(x, Q^2)
\]

- obey DGLAP-like evolution equations, similar to PDFs

\[
\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left( P'_{(n)} \otimes \overline{M}_n \right)(\Delta x, Q^2)
\]

where modified splitting function is

\[
P'_{(n)}(z, \alpha_s) = z^n \ P_{NS,S}(z, \alpha_s)
\]

→ can follow evolution of specific resonance (region) with $Q^2$ in pQCD framework!
entire resonance region

analysis in terms of “truncated moments”
higher twists < 10–15% for $Q^2 > 1$ GeV$^2$
Minimum condition for duality


In NR Quark Model, even and odd parity states correspond to 56 \((L=0)\) and 70 \((L=1)\) multiplets of spin-flavor SU(6)

<table>
<thead>
<tr>
<th>(SU(6))</th>
<th>([56, 0^+]^28)</th>
<th>([56, 0^+]^410)</th>
<th>([70, 1^-]^28)</th>
<th>([70, 1^-]^48)</th>
<th>([70, 1^-]^210)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F^p_1)</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>(F^n_1)</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>

Proton sum saturated by lower-lying resonances


Close, Isgur, \textit{PLB} 509 (2001) 81
Is duality in the proton a coincidence?

→ consider symmetric nucleon wave function

**cat’s ears diagram**  
(4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i e_j \sim \left( \sum_i e_i \right)^2 - \sum_i e_i^2$$

- **proton**  
  \[ HT \sim 1 - \left( 2 \times \frac{4}{9} + \frac{1}{9} \right) = 0 ! \]

- **neutron**  
  \[ HT \sim 0 - \left( \frac{4}{9} + 2 \times \frac{1}{9} \right) \neq 0 \]

need to test duality in the neutron!
No **FREE** neutron targets
(neutron half-life $\sim 12$ mins)

use deuteron as “effective” neutron target

**BUT** deuteron is a nucleus, and $F_2^d \neq F_2^p + F_2^n$

**nuclear effects** (nuclear binding, Fermi motion, shadowing) *obscure neutron structure* information

need to correct for “nuclear EMC effect”
Nuclear Effects in the Deuteron
nuclear “impulse approximation”

→ incoherent scattering from individual nucleons in $d$
  (good approx. at $x \gg 0$)

$\gamma^* \rightarrow N p \rightarrow F_2^d(x, Q^2) = \int dx \ f(y, \gamma) \ F_2^N(x/y, Q^2) + \delta^{(\text{off})} F_2^d$

nucleon momentum distribution in $d$
  (“smearing function”)

→ at finite $Q^2$, smearing function depends also on parameter

$\gamma = |q|/q_0 = \sqrt{1 + 4M^2x^2/Q^2}$
weak binding approximation (WBA):
expand amplitudes to order $\vec{p}^2/M^2$

\[
f(y, \gamma) = \int \frac{d^3 p}{(2\pi)^3} |\psi_d(p)|^2 \ \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right) \times \frac{1}{\gamma^2} \left[1 + \frac{\gamma^2 - 1}{y^2} \left(1 + \frac{2\varepsilon}{M} + \frac{\vec{p}^2}{2M^2}(1 - 3\hat{p}_z^2)\right)\right]
\]

\[\rightarrow\] deuteron wave function $\psi_d(p)$

\[\rightarrow\] deuteron separation energy $\varepsilon = \varepsilon_d - \frac{\vec{p}^2}{2M}$

\[\rightarrow\] approaches usual nonrelativistic momentum distribution in $\gamma \to 1$ limit
$N$ momentum distributions in $d$

for most kinematics $\gamma \lesssim 2$

Kahn, WM, Kulagin, PRC 79, 035205 (2009)
Off-shell correction

\[ \delta^{(\text{off})} F_2^d \rightarrow \delta^{(\Psi)} F_2^d \quad \text{negative energy components of } \psi_d \]

\[ \rightarrow \delta^{(p^2)} F_2^d \quad \text{off-shell } N \text{ structure function} \]

\[ \leq 1 - 2 \% \text{ effect} \]

WM, Schreiber, Thomas
PLB 335 (1994) 11
EMC effect in deuteron

$Q^2 = 2 \text{ GeV}^2$

- larger EMC effect (smaller $d/N$ ratio) at $x \sim 0.5-0.6$
- with binding + off-shell corrections
- can significantly affect neutron extraction

* Frankfur, Strikman light-cone model (no binding)

* Kulagin, Petti NPA765 (2006)126
EMC effect in deuteron
deuteron wave function dependence

\[ Q^2 = 2 \text{ GeV}^2 \]

\[
\frac{R_{uN}/R_{dN}}{\text{Paris}}
\]

- Bonn
- AV18
- Gross
- Van Orden
- JISP

\[ x \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \]

\[ 0.9 \quad 0.91 \quad 1 \quad 1.1 \]

→ mild dependence for $x < 0.8-0.85$
most PDFs assume no nuclear corrections

large uncertainty from nuclear effects in deuteron
(range of nuclear models*)
beyond $x \sim 0.5$

symmetry breaking mechanism remains unknown!

* most PDFs assume *no* nuclear corrections
Extraction of Neutron Structure Function
Fermi smearing in the deuteron

can one reconstruct ("unsmear") neutron resonance structure from deuteron data?

usual "multiplicative" unsmearing method does not work for "bumpy" data or which change sign (spin-dep. SFs)
Unsmearing – additive method

- calculated $F_2^d$ depends on input $F_2^n$

$\rightarrow$ extracted $n$ depends on input $n$ ... cyclic argument

Solution: iteration procedure

0. subtract $\delta^{\text{(off)}}F_2^d$ from $d$ data: $F_2^d \rightarrow F_2^d - \delta^{\text{(off)}}F_2^d$

1. define difference $\Delta$ between smeared and free SFs

$$F_2^d - \widetilde{F}_2^p = \widetilde{F}_2^n \equiv f \otimes F_2^n \equiv F_2^n + \Delta$$

2. first guess for $F_2^n(0)$ $\rightarrow$ $\Delta^{(0)} = \widetilde{F}_2^n(0) - F_2^n$

3. after one iteration, gives

$$F_2^n(1) = F_2^n(0) + (\widetilde{F}_2^n - \widetilde{F}_2^n(0))$$

4. repeat until convergence obtained
Unsmearing – test of convergence

- $F_2^d$ constructed from known $F_2^p$ and $F_2^n$ inputs
  (using leading twist MRST parameterization)

Kahn, WM, PRC 79 (2009) 035205

rapid convergence in DIS region

initial guess $F_2^{n(0)} = 0$
$F_2^d$ constructed from known $F_2^p$ and $F_2^n$ inputs

(using MAID resonance parameterization)

Unsmearing – test of convergence

$F_2^n(0) = 0$ *

* even faster convergence if choose $F_2^n(0) = F_2^p$

Kahn, WM, 
PRC 79 (2009) 035205

→ can reconstruct almost arbitrary shape
Unsmearing – $Q^2$ dependence

- Important to use correct $\gamma$ dependence in extraction

important also in DIS region
(do not have resonance “benchmarks”)

Kahn, WM, PRC 79 (2009) 035205
Unsmearing spin-dependent structure functions
Data extraction

1 iteration

\[ Q^2 = 1.7 \text{ GeV}^2 \]

- \[ F_2 \]
- \[ F_2^d \]
- \[ F_2^p \]
- Extracted \[ F_2^d, F_2^p \]
- First guess
- Reconstructed \[ F_2^d \]

\* Malace et al. (E00-116)

arXiv:0905.2374 [nucl-ex]

neutron errors → vary \( d \) data points by Gaussians
(proton data smeared, so errors very small)

→ run 50 sample extractions, calculate RMS error
Data extraction

relatively stable results after only 2 iterations!

excellent agreement of reconstructed $d$ with data
Data extraction

1 iteration

\[ Q^2 = 4.5 \text{ GeV}^2 \]
Data extraction

2 iterations

$Q^2 = 4.5 \text{ GeV}^2$

→ clear neutron resonance structure visible
Data extraction

$Q^2 = 6.4 \text{ GeV}^2$

1 iteration
Data extraction

$Q^2 = 6.4 \text{ GeV}^2$

2 iterations
dependence on initial guess for $n$

results converge eventually, but errors increase for more iterations
Duality test

Comparison with leading twist (MRST) parameterization + target mass corrections
neutron HT indeed larger than proton!
consistent with quark model expectations
Limitations of method

- Need data up to $x = 1$
  → usually not a problem – unless cut $d$ quasi-elastic tail

- Difficult to use on sparse data sets
  → discontinuities in $d$ data sharply magnified in $n$

- Some dependence on starting point for iteration
  → convergence faster with judicious first guess for $n$

- Method limited to convolution representation
  → corrections beyond convolution to be evaluated
Summary

- Nuclear corrections in deuteron computed at finite $Q^2$ through generalized convolution

- New unsmeearing method for extracting neutron SFs
  - first(?) extraction in resonance and DIS regions

- Test of duality in the neutron
  - violations *larger* in neutron than in proton (as expected from quark models)
  - need to estimate systematic errors from nuclear corrections

- Comparison with BONUS data will test methodology
The End