



*George Washington University
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Quarks or hadrons?

Duality in electron-nucleon scattering

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Outline

- Introduction: *Bloom-Gilman duality*
- Duality in QCD
 - OPE and higher twists
- Local duality & truncated moments
- Duality in the neutron
 - is duality in proton an accident?
 - extraction of neutron resonance structure from deuterium data
- Duality in pion electroproduction
- Conclusions

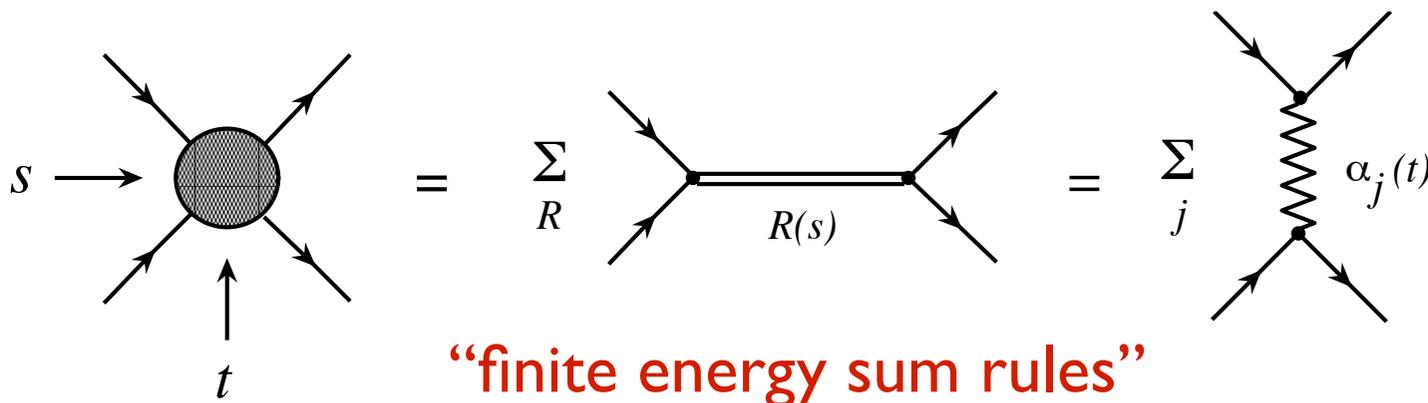
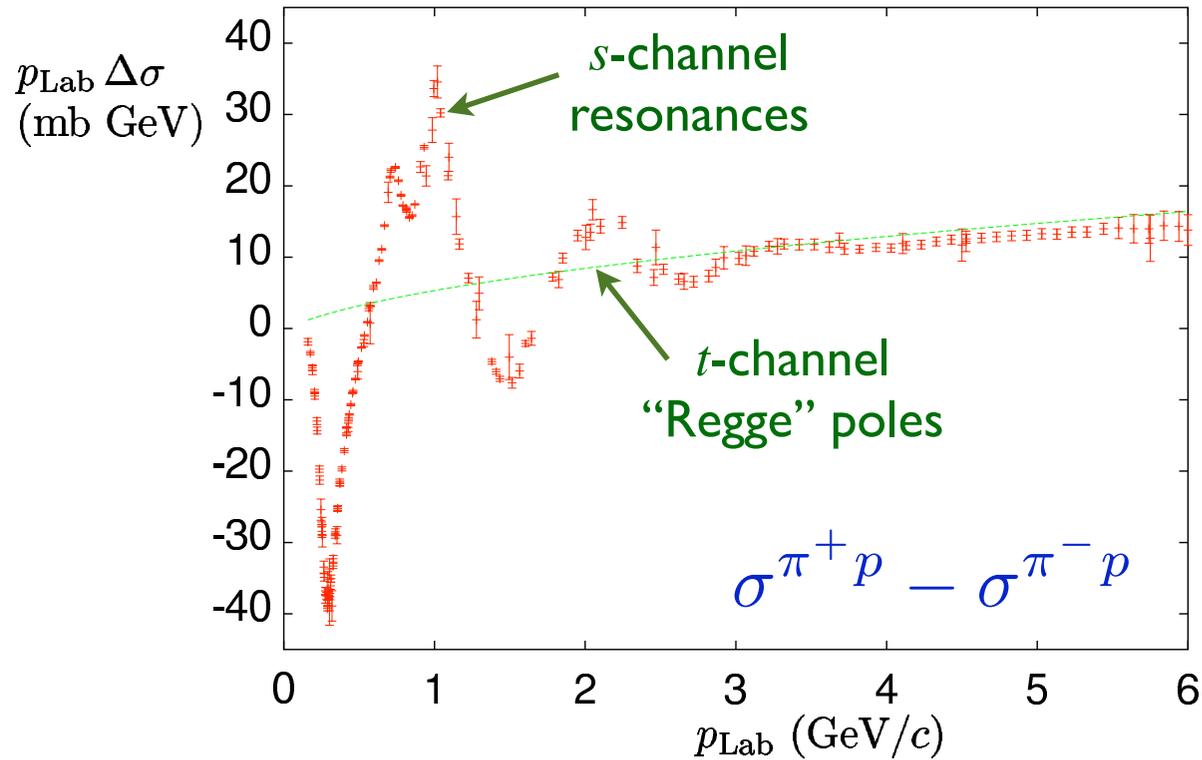
Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{\text{hadrons}} = \sum_{\text{quarks}}$$

Can use either set of complete basis states to describe all physical phenomena

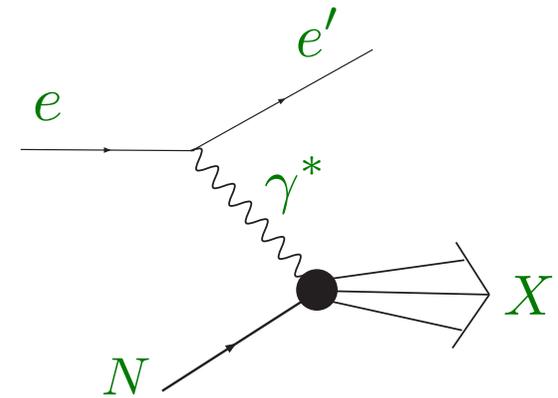
Duality in hadron-hadron scattering



Electron-nucleon scattering

■ Inclusive cross section for $eN \rightarrow eX$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left(2 \tan^2 \frac{\theta}{2} \frac{F_1}{2M} + \frac{F_2}{\nu} \right)$$



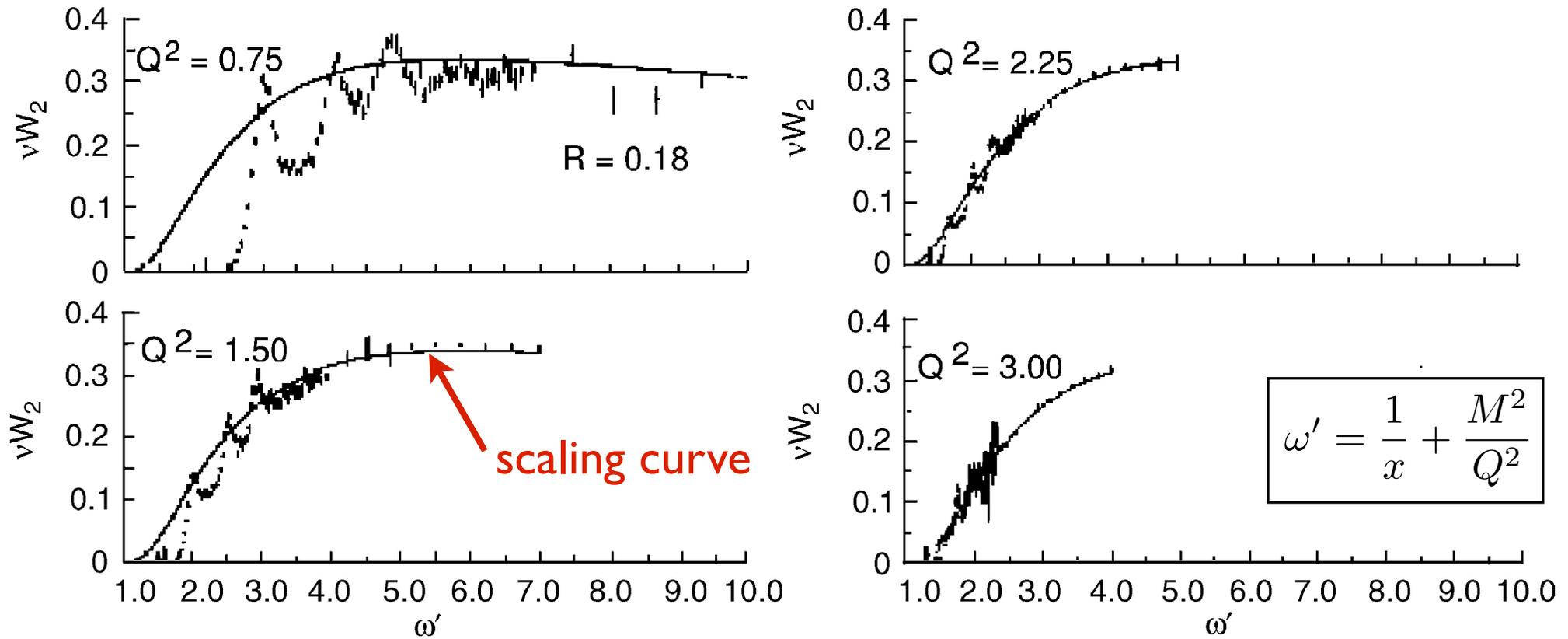
$$\left. \begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^2 - \nu^2 = 4EE' \sin^2 \frac{\theta}{2} \end{aligned} \right\} x = \frac{Q^2}{2M\nu} \quad \text{“Bjorken scaling variable”}$$

■ F_1, F_2 “structure functions”

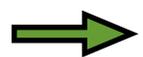
→ contain all information about structure of nucleon

→ functions of x, Q^2 in general

Bloom-Gilman duality



Bloom, Gilman, PRL 85, 1185 (1970)



**resonance – scaling duality in
proton $\nu W_2 = F_2$ structure function**

Bloom-Gilman duality

- Average over (strongly Q^2 dependent) resonances $\approx Q^2$ independent scaling function
- Finite energy sum rule for eN scattering

$$\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

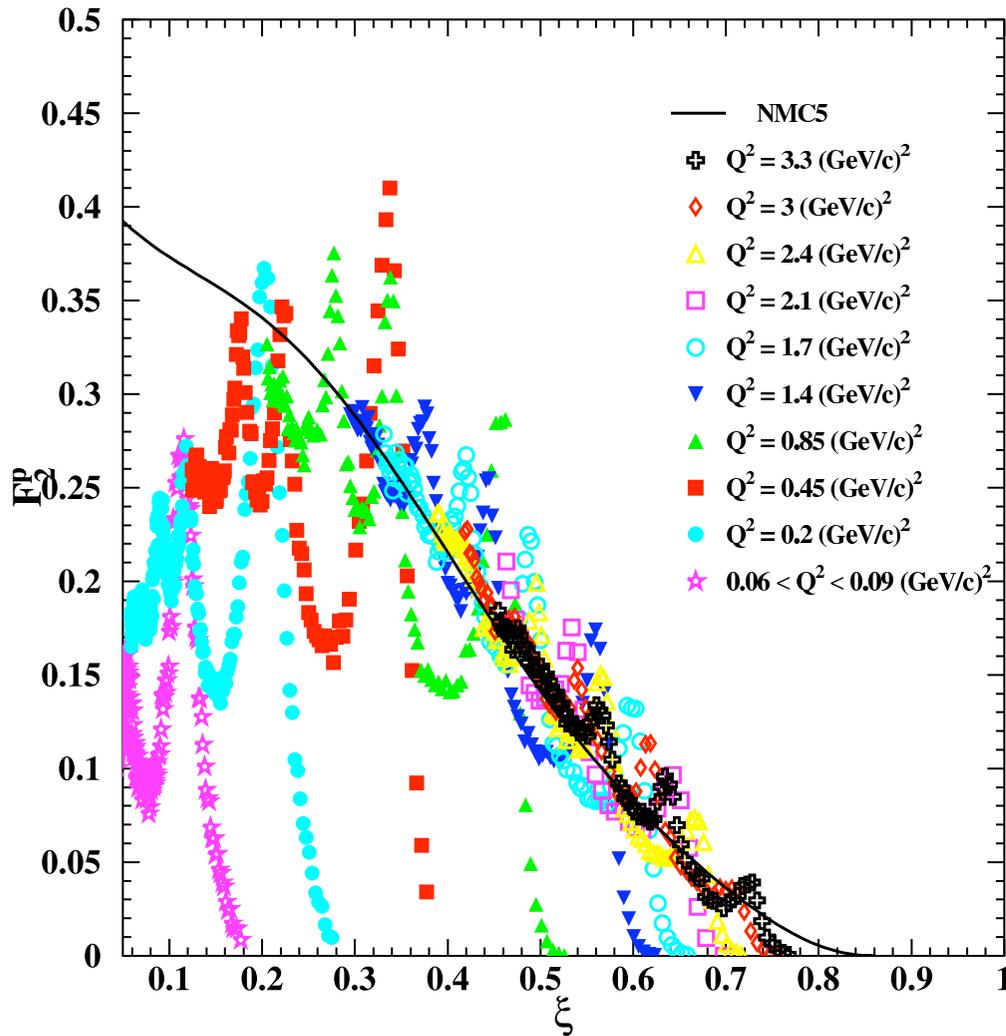
“hadrons”

$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

scaling function
(function of ω' only)

“quarks”

Bloom-Gilman duality



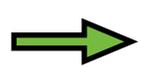
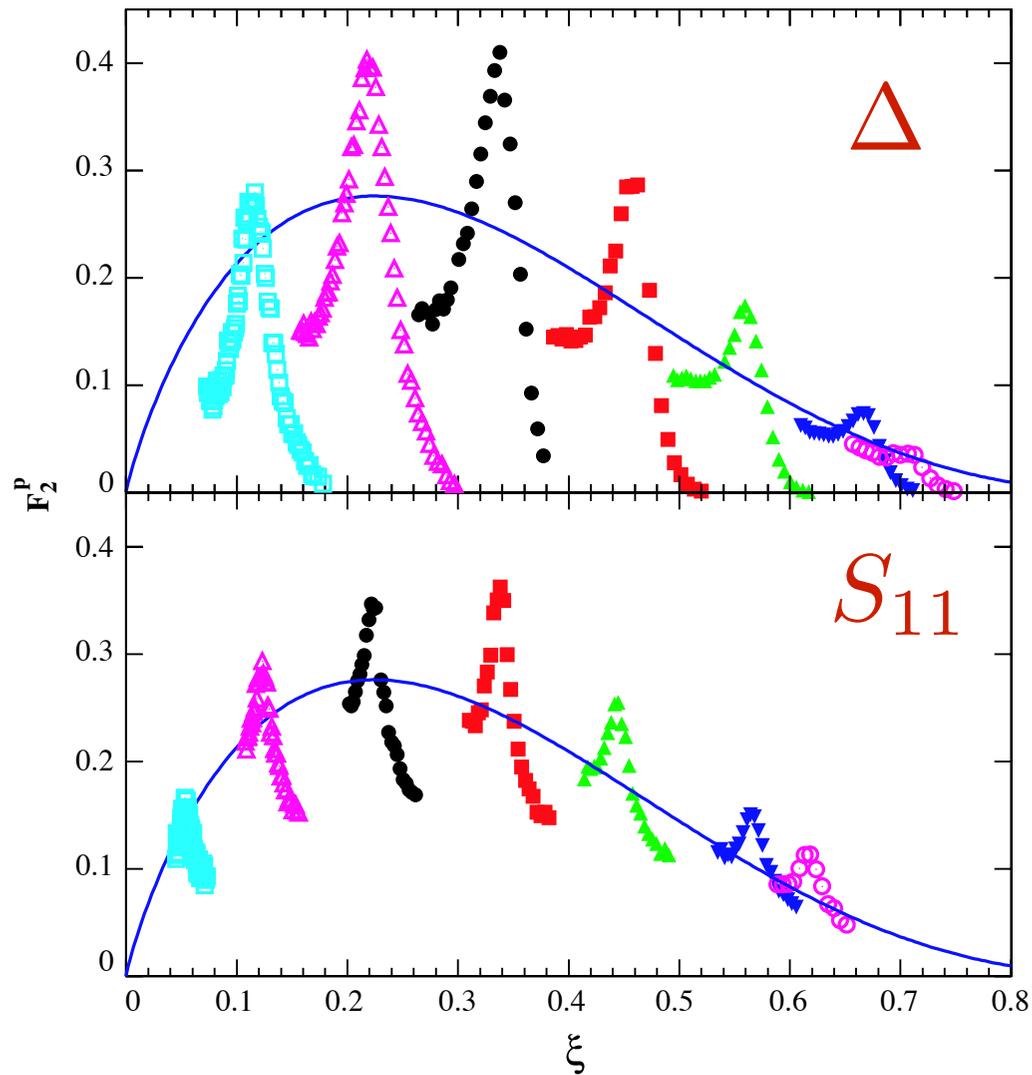
Niculescu et al., PRL 85, 1182 (2000)

Average over
(strongly Q^2 dependent)
resonances
 $\approx Q^2$ independent
scaling function

“Nachtmann scaling variable”

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}}$$

- Duality exists also in local regions, around individual resonances



local Bloom-Gilman duality

Duality in QCD

(“global duality”)

Duality in QCD

■ Operator product expansion

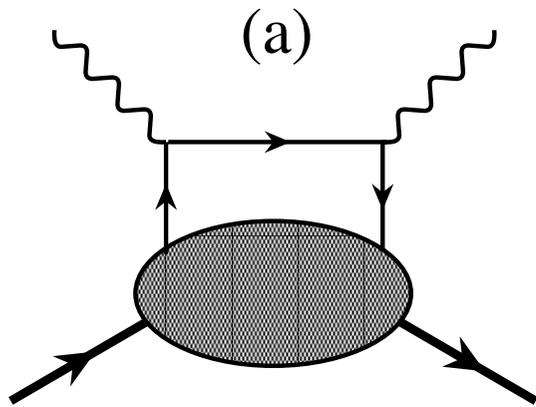
→ expand *moments* of structure functions
in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

matrix elements of operators with
specific “twist” τ

$\tau = \text{dimension} - \text{spin}$

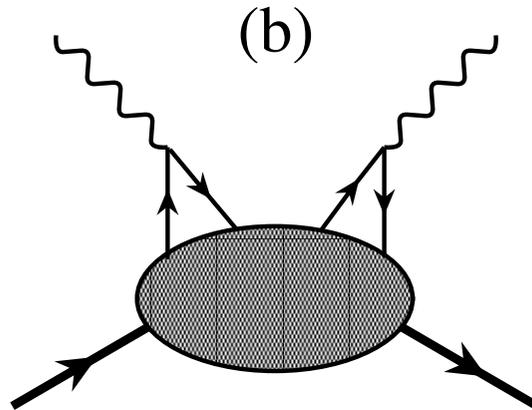
Duality in QCD



$$\tau = 2$$

single quark
scattering

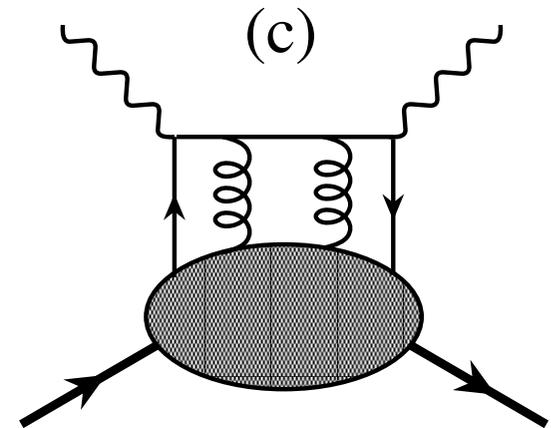
e.g. $\bar{\psi} \gamma_\mu \psi$



$$\tau > 2$$

qq and *qg*
correlations

e.g. $\bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma_\nu \psi$
or $\bar{\psi} \tilde{G}_{\mu\nu} \gamma^\nu \psi$



Duality in QCD

■ Operator product expansion

→ expand *moments* of structure functions
in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

■ If moment \approx independent of Q^2

→ higher twist terms $A_n^{(\tau > 2)}$ small

Duality in QCD

■ Operator product expansion

→ expand *moments* of structure functions
in powers of $1/Q^2$

$$\begin{aligned} M_n(Q^2) &= \int_0^1 dx x^{n-2} F_2(x, Q^2) \\ &= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \dots \end{aligned}$$

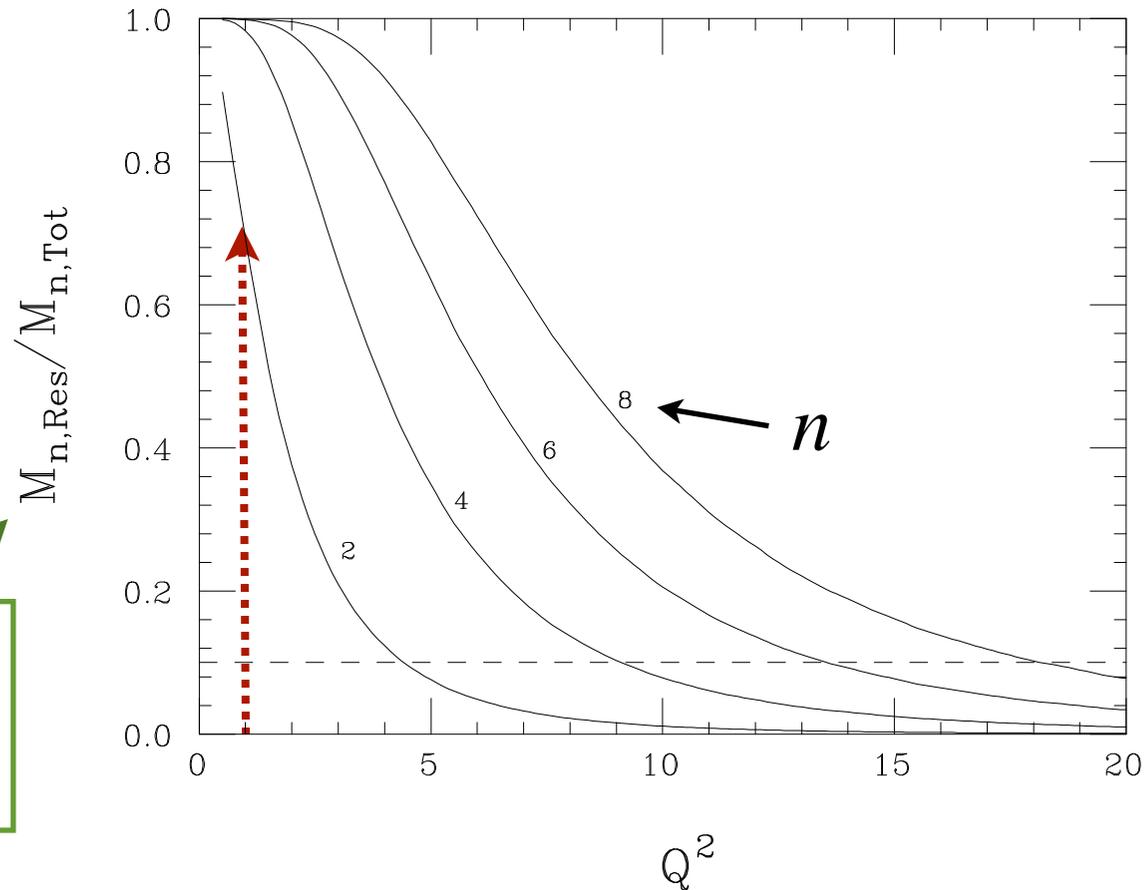
■ Duality \longleftrightarrow suppression of higher twists

de Rujula, Georgi, Politzer,
Ann. Phys. **103**, 315 (1975)

Duality in QCD

- Much of recent new data is in resonance region, $W < 2 \text{ GeV}$
 - *common wisdom*: pQCD analysis not valid in resonance region
 - *in fact*: partonic interpretation of moments does include resonance region
- Resonances are an integral part of deep inelastic structure functions!
 - implicit role of quark-hadron duality

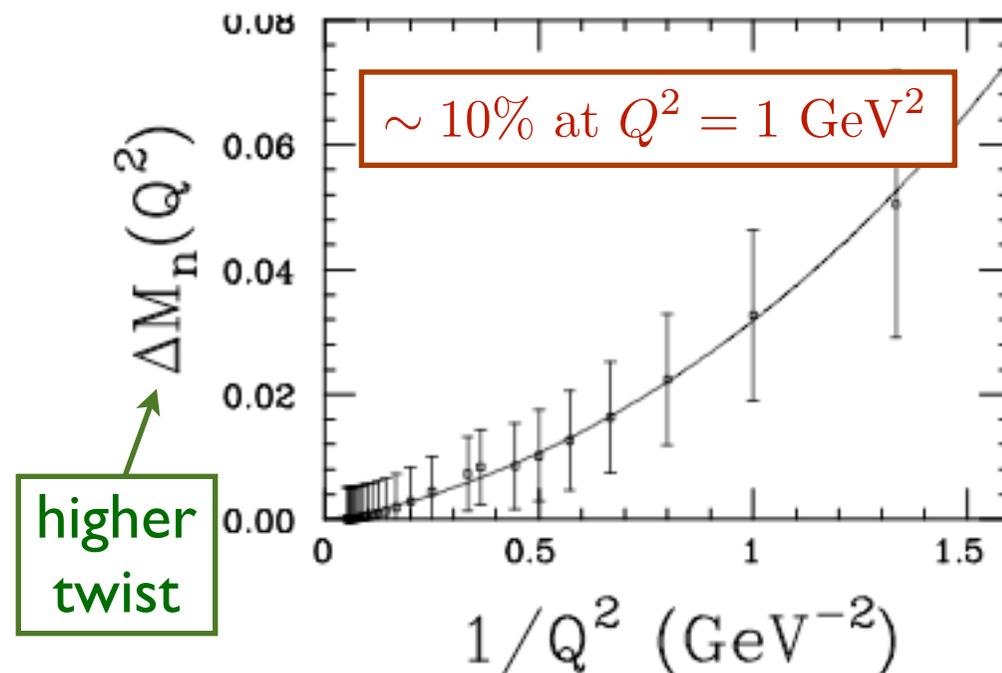
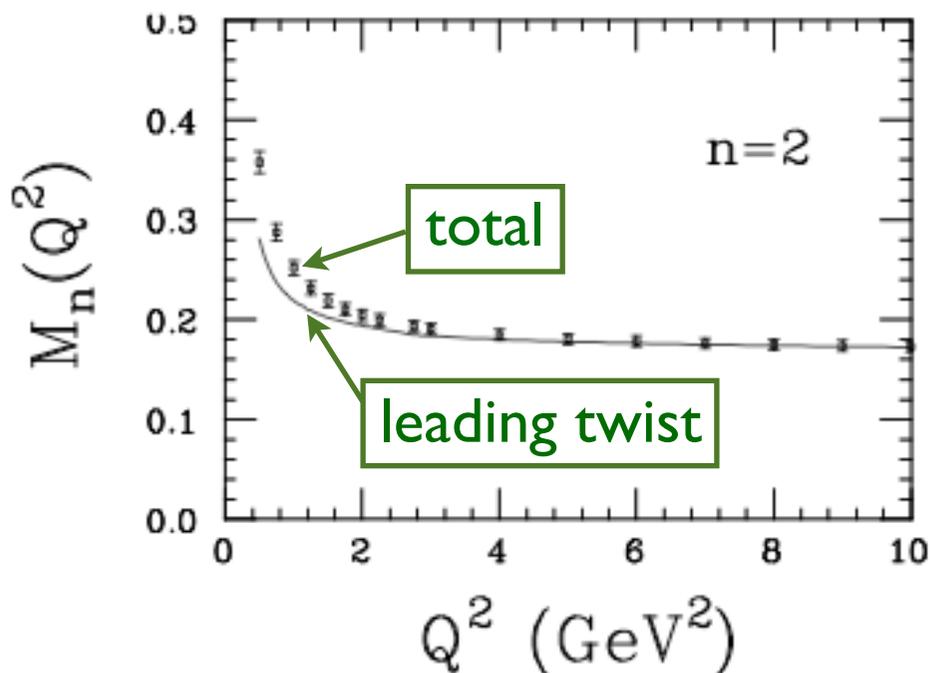
Proton moments



relative contribution
of resonance region
to n -th moment

➔ At $Q^2 = 1 \text{ GeV}^2$, \sim 70% of lowest moment of F_2^p
comes from $W < 2 \text{ GeV}$

Proton moments



➔ BUT resonances and DIS continuum conspire to produce only ~ 10% higher twist contribution!

→ total higher twist small at $Q^2 \sim 1 - 2 \text{ GeV}^2$

- on average, nonperturbative interactions between quarks and gluons not dominant at these scales
- suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*
- OPE does not tell us why higher twists are small
 - need more detailed information (*e.g.* about individual resonances) to understand behavior dynamically

Local Duality & Truncated Moments

Truncated moments

- complete moments can be studied via twist expansion
 - Bloom-Gilman duality has a precise meaning
(*i.e.*, duality violation = higher twists)
- rigorous connection between local duality & QCD difficult
 - need prescription for how to average over resonances
- *truncated* moments allow study of restricted regions in x (or W) within pQCD in well-defined, systematic way

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx x^{n-2} F_2(x, Q^2)$$

Truncated moments

- truncated moments obey DGLAP-like evolution equations, similar to PDFs

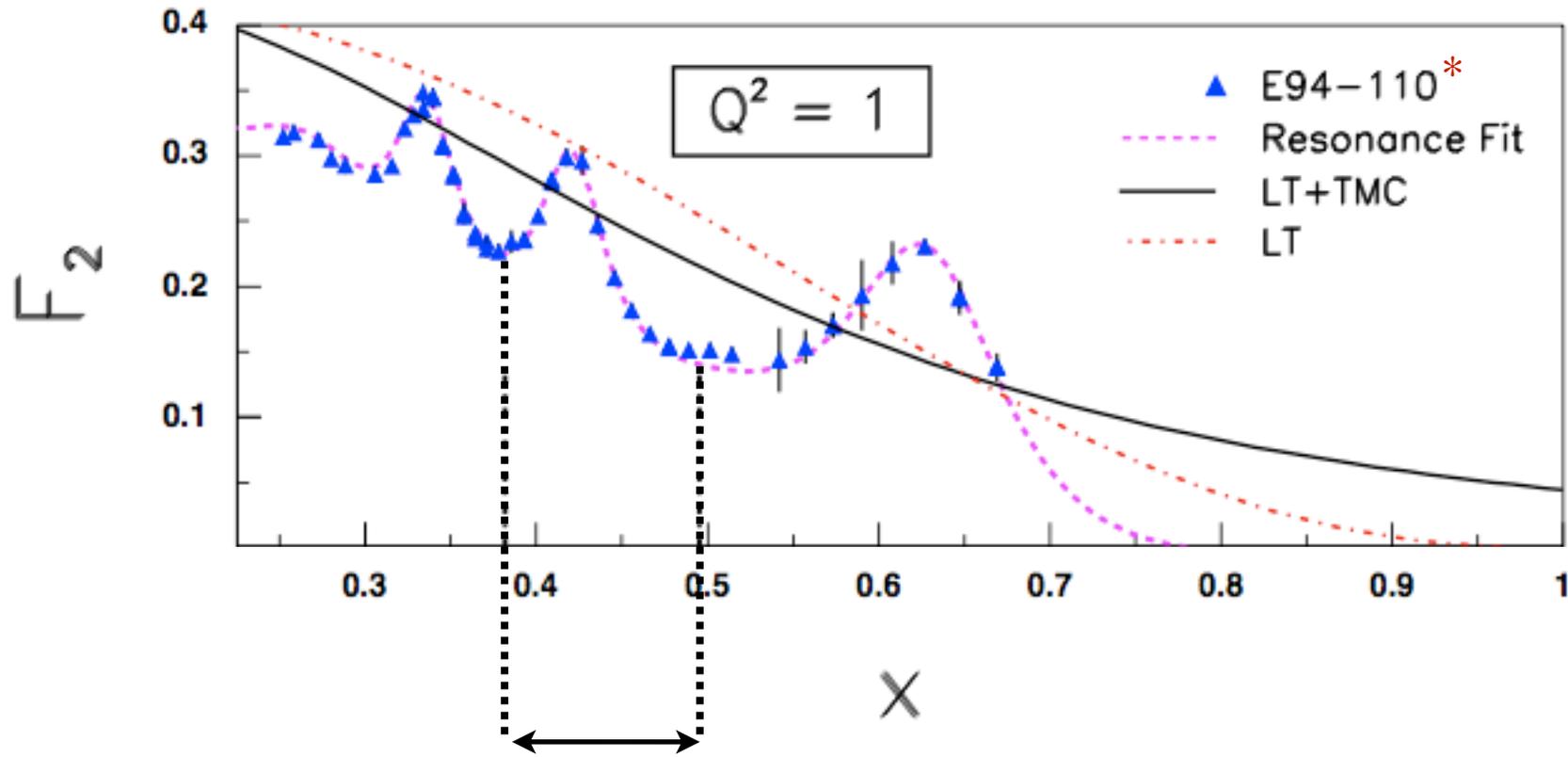
$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left(P'_{(n)} \otimes \overline{M}_n \right) (\Delta x, Q^2)$$

where modified splitting function is

$$P'_{(n)}(z, \alpha_s) = z^n P_{NS,S}(z, \alpha_s)$$

- can follow evolution of specific resonance (region) with Q^2 in pQCD framework!
- suitable when complete moments not available

F_2^p resonance spectrum

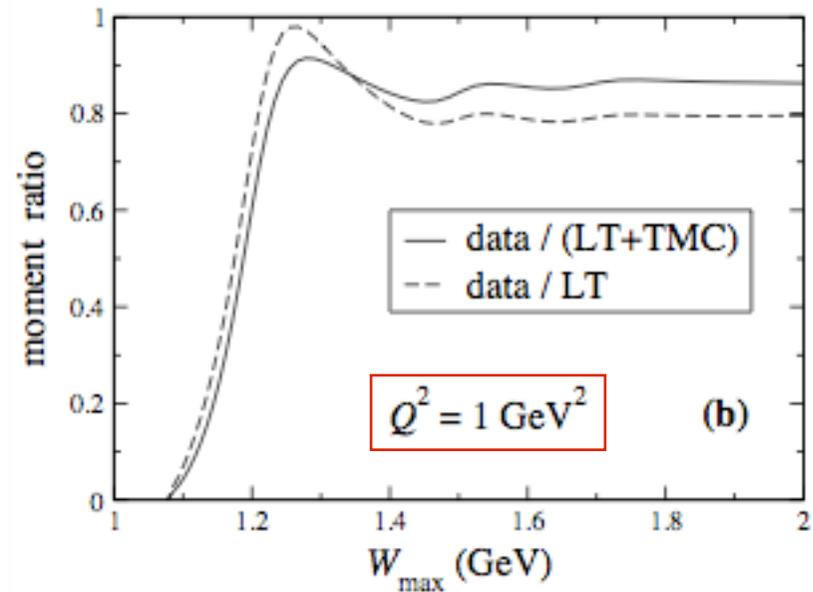
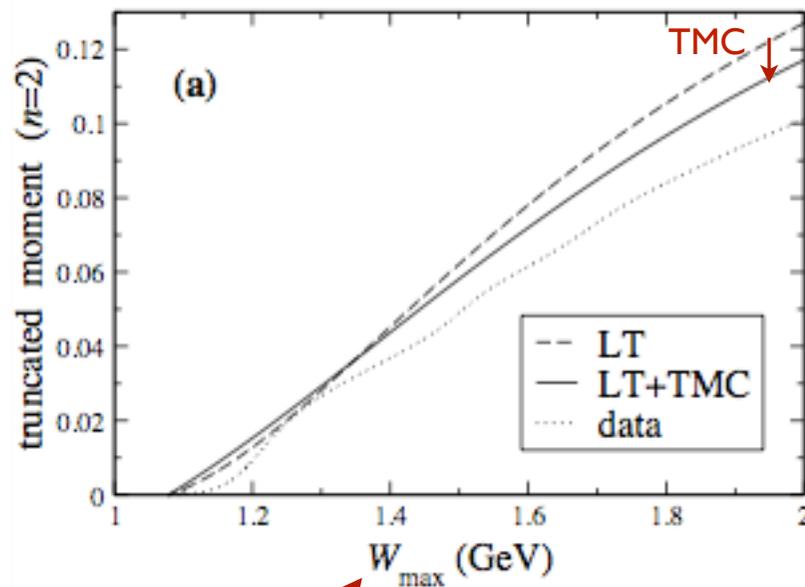


*JLab Hall C

how much of this region is leading twist ?

Data analysis

- assume data at large enough Q^2 are *entirely* leading twist
- evolve fit to data at large Q^2 down to lower Q^2
- apply target mass corrections and compare with low- Q^2 data



$$W^2 = M^2 + \frac{Q^2}{x}(1-x)$$

Psaker, WM et al.
PRC 78, 025206 (2008)

Data analysis

■ consider individual resonance regions:

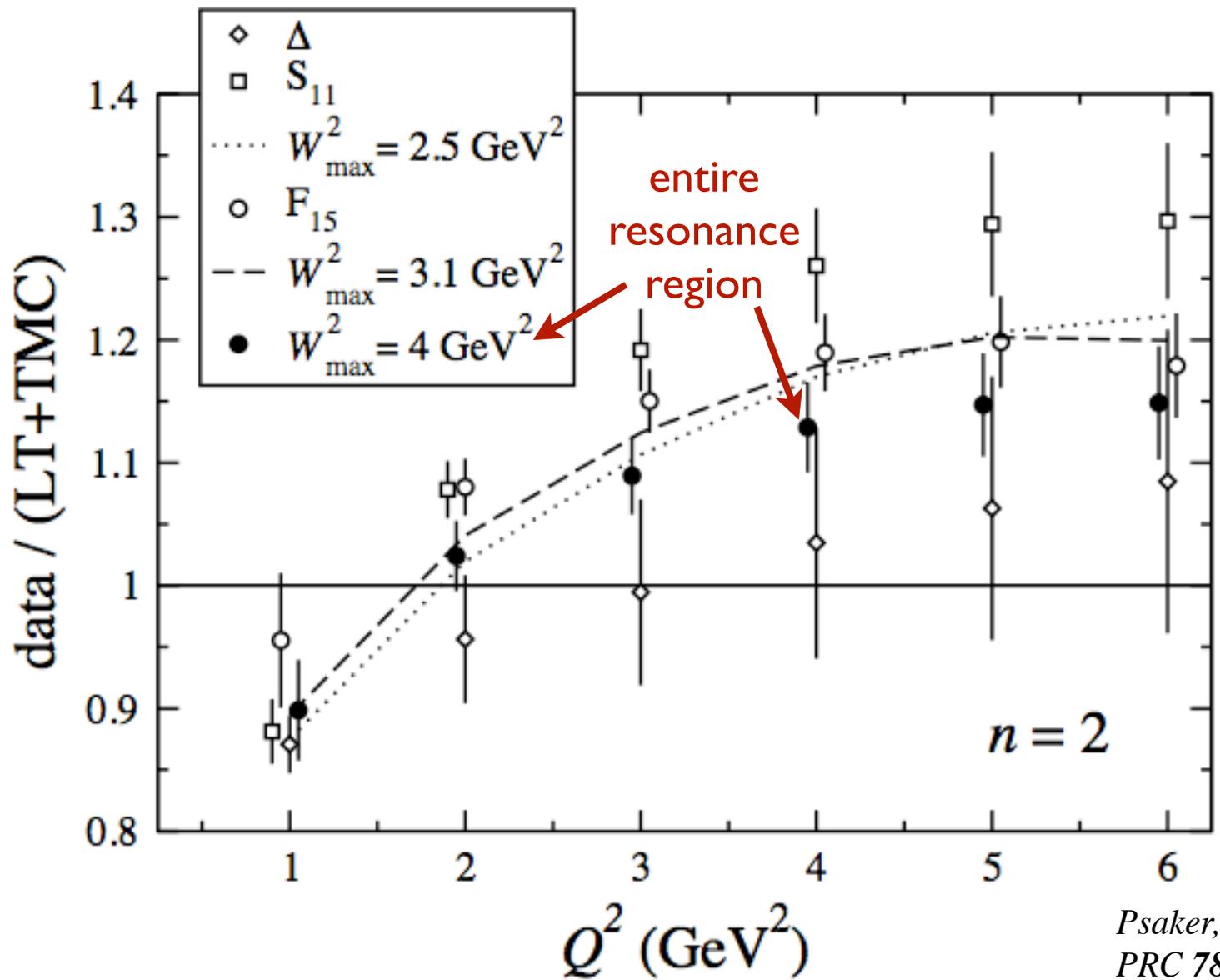
$$\rightarrow W_{\text{thr}}^2 < W^2 < 1.9 \text{ GeV}^2 \quad \text{“}\Delta(1232)\text{”}$$

$$\rightarrow 1.9 < W^2 < 2.5 \text{ GeV}^2 \quad \text{“}S_{11}(1535)\text{”}$$

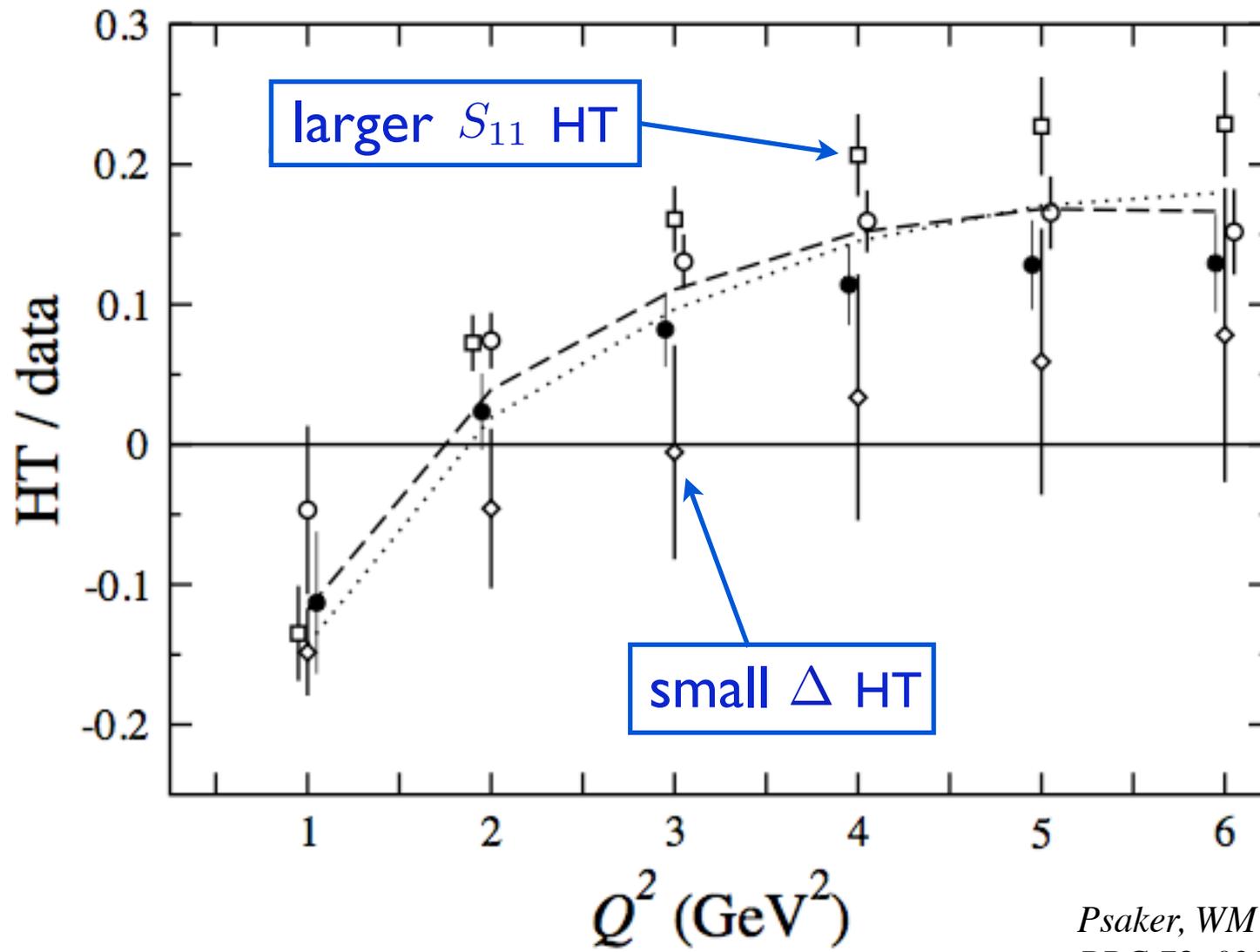
$$\rightarrow 2.5 < W^2 < 3.1 \text{ GeV}^2 \quad \text{“}F_{15}(1680)\text{”}$$

as well as total resonance region:

$$\rightarrow W^2 < 4 \text{ GeV}^2$$



Psaker, WM et al.
 PRC 78, 025206 (2008)



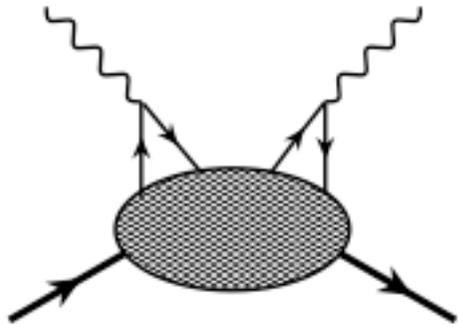
*Psaker, WM et al.
PRC 78, 025206 (2008)*

→ higher twists < 10–15% for $Q^2 > 1 \text{ GeV}^2$

Duality in the Neutron

■ Is duality in the proton a coincidence?

→ consider model with symmetric nucleon wave function



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i e_j \sim \left(\sum_i e_i \right)^2 - \sum_i e_i^2$$

↑ coherent ↑ incoherent

■ *proton* HT $\sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9} \right) = 0 !$

■ *neutron* HT $\sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9} \right) \neq 0$

Brodsky, hep-ph/0006310

➡ need to test duality in the neutron!

■ How can the square of a sum become the sum of squares?

→ in *hadronic* language, duality is realized by summing over at least one complete set of even and odd parity resonances

Close, Isgur, PLB 509, 81 (2001)

→ in NR Quark Model, even and odd parity states generalize to **56** ($L=0$) and **70** ($L=1$) multiplets of spin-flavor SU(6)

■ assume magnetic coupling of photon to quarks
(better approximation at high Q^2)

■ in this limit Callan-Gross relation valid $F_2 = 2xF_1$

■ structure function given by squared sum of transition FFs

$$F_1(\nu, \vec{q}^2) \sim \sum_R |F_{N \rightarrow R}(\vec{q}^2)|^2 \delta(E_R - E_N - \nu)$$

■ How can the square of a sum become the sum of squares?

→ in *hadronic* language, duality is realized by summing over at least one complete set of even and odd parity resonances

Close, Isgur, PLB 509, 81 (2001)

→ in NR Quark Model, even and odd parity states generalize to **56** ($L=0$) and **70** ($L=1$) multiplets of spin-flavor SU(6)

representation	${}^2\mathbf{8}[\mathbf{56}^+]$	${}^4\mathbf{10}[\mathbf{56}^+]$	${}^2\mathbf{8}[\mathbf{70}^-]$	${}^4\mathbf{8}[\mathbf{70}^-]$	${}^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2 + 9\lambda^2$
F_1^n	$(3\rho + \lambda)^2/4$	$8\lambda^2$	$(3\rho - \lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2 + 27\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wfn.

Close, WM, PRC 68, 035210 (2003)

■ **SU(6) limit** $\longrightarrow \lambda = \rho$

\longrightarrow relative strengths of $N \rightarrow N^*$ transitions:

$SU(6) :$	$[56, 0^+]^2 8$	$[56, 0^+]^4 10$	$[70, 1^-]^2 8$	$[70, 1^-]^4 8$	$[70, 1^-]^2 10$	<i>total</i>
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

■ summing over all resonances in 56^+ and 70^- multiplets

$\longrightarrow \frac{F_1^n}{F_1^p} = \frac{2}{3}$ as in quark-parton model (for $u=2d$) !

■ proton sum saturated by lower-lying resonances

\longrightarrow expect duality to appear *earlier* for p than n

Neutron structure functions

- Problem: no free neutron targets!

(neutron half-life ~ 12 mins)

→ use deuteron as “effective neutron target”

→ extract F_2^n from F_2^d and F_2^p data

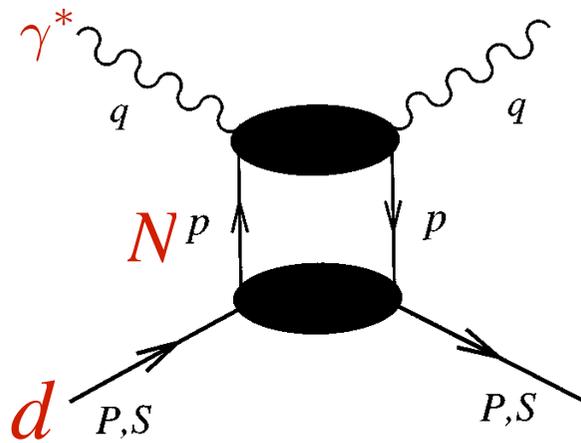
- But: deuteron is a nucleus, and $F_2^d \neq F_2^p + F_2^n$

→ nuclear effects (nuclear binding, Fermi motion, shadowing)
obscure neutron structure information

→ need to correct for “nuclear EMC effect”

■ nuclear “impulse approximation”

→ incoherent scattering from individual nucleons in d
(good approx. at $x \gg 0$)



$$F_2^d(x, Q^2) = \int_x^1 dy f(y, \gamma) F_2^N(x/y, Q^2) + \delta^{(\text{off})} F_2^d$$

nucleon momentum
distribution in d
 (“smearing function”)

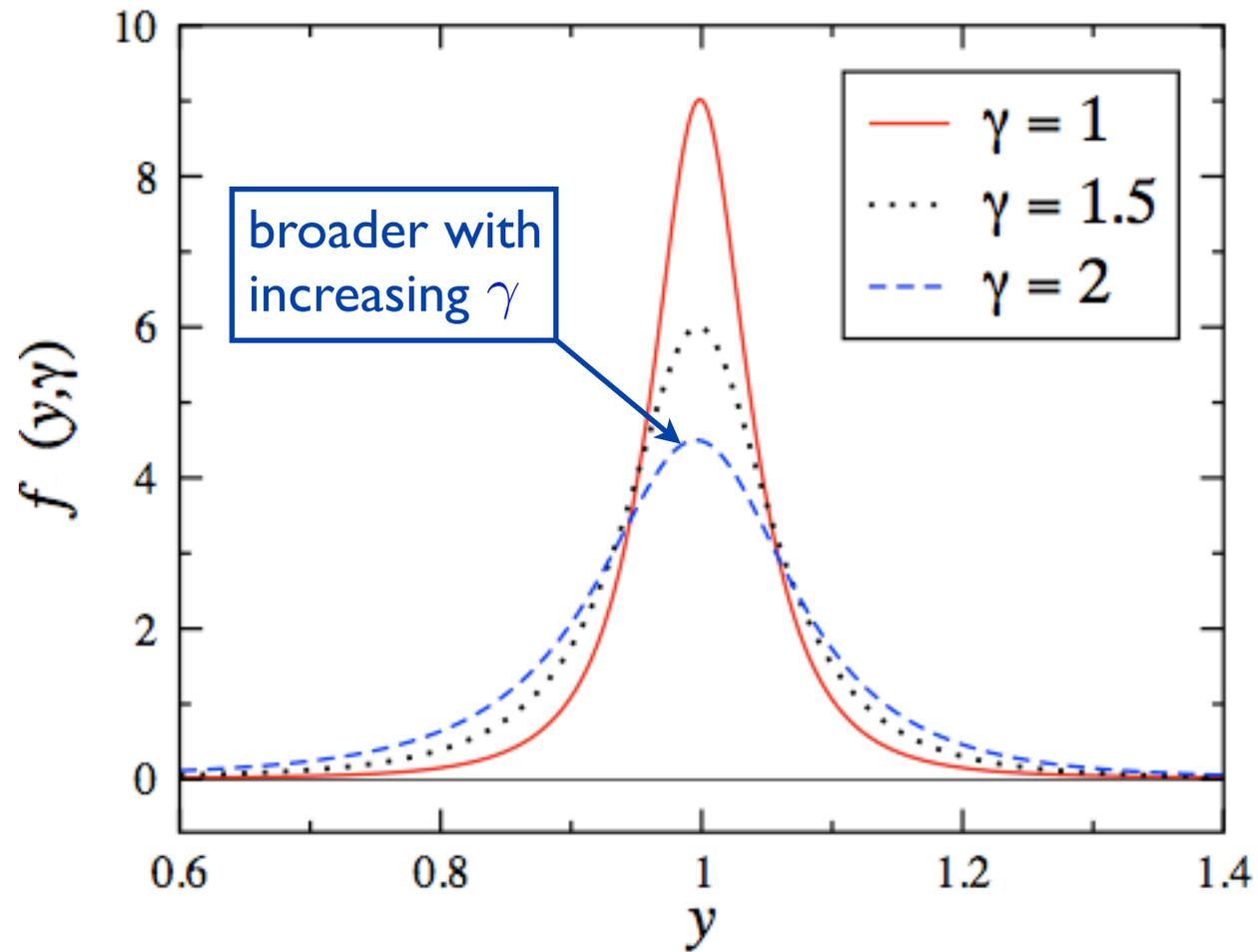
off-shell
correction
(~1%)

$$N=p+n$$

→ at finite Q^2 , smearing function depends also on parameter

$$\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2x^2/Q^2}$$

N momentum distributions in d



→ for most kinematics $\gamma \lesssim 2$

Unsmearing – additive method

- calculated F_2^d depends on input F_2^n
→ extracted n depends on input n ... cyclic argument

- solution: (additive) iteration procedure

0. subtract $\delta^{(\text{off})} F_2^d$ from d data: $F_2^d \rightarrow F_2^d - \delta^{(\text{off})} F_2^d$

1. define difference between smeared and free SFs

$$F_2^d - \tilde{F}_2^p = \tilde{F}_2^n \equiv f \otimes F_2^n \equiv F_2^n + \Delta$$

2. first guess for $F_2^{n(0)} \rightarrow \Delta^{(0)} = \tilde{F}_2^{n(0)} - F_2^n$

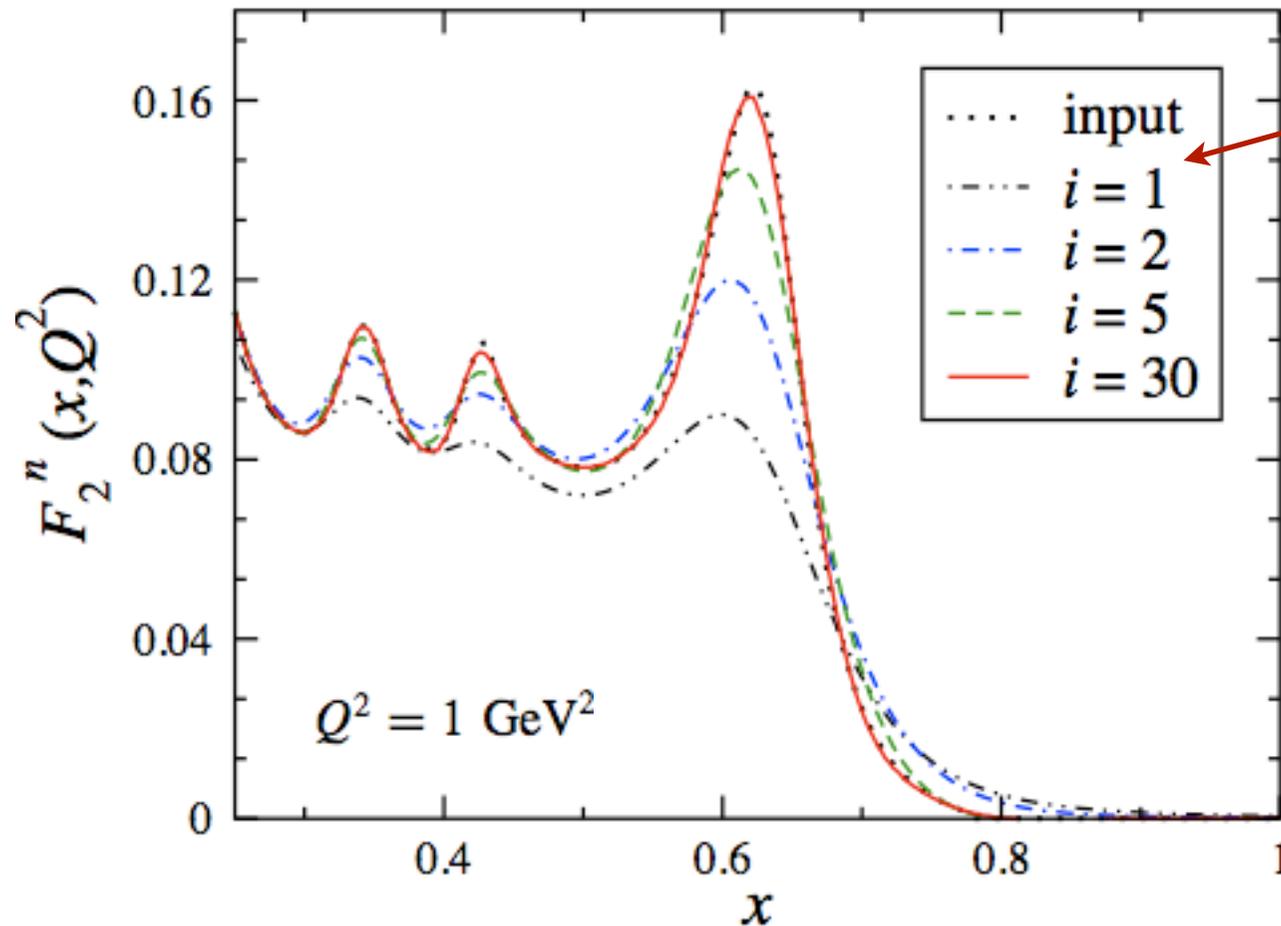
3. after one iteration, gives

$$F_2^{n(1)} = F_2^{n(0)} + (\tilde{F}_2^n - \tilde{F}_2^{n(0)})$$

4. repeat until convergence

Unsmearing – test of convergence

- F_2^d constructed from known F_2^p and F_2^n inputs
(using MAID resonance parameterization)



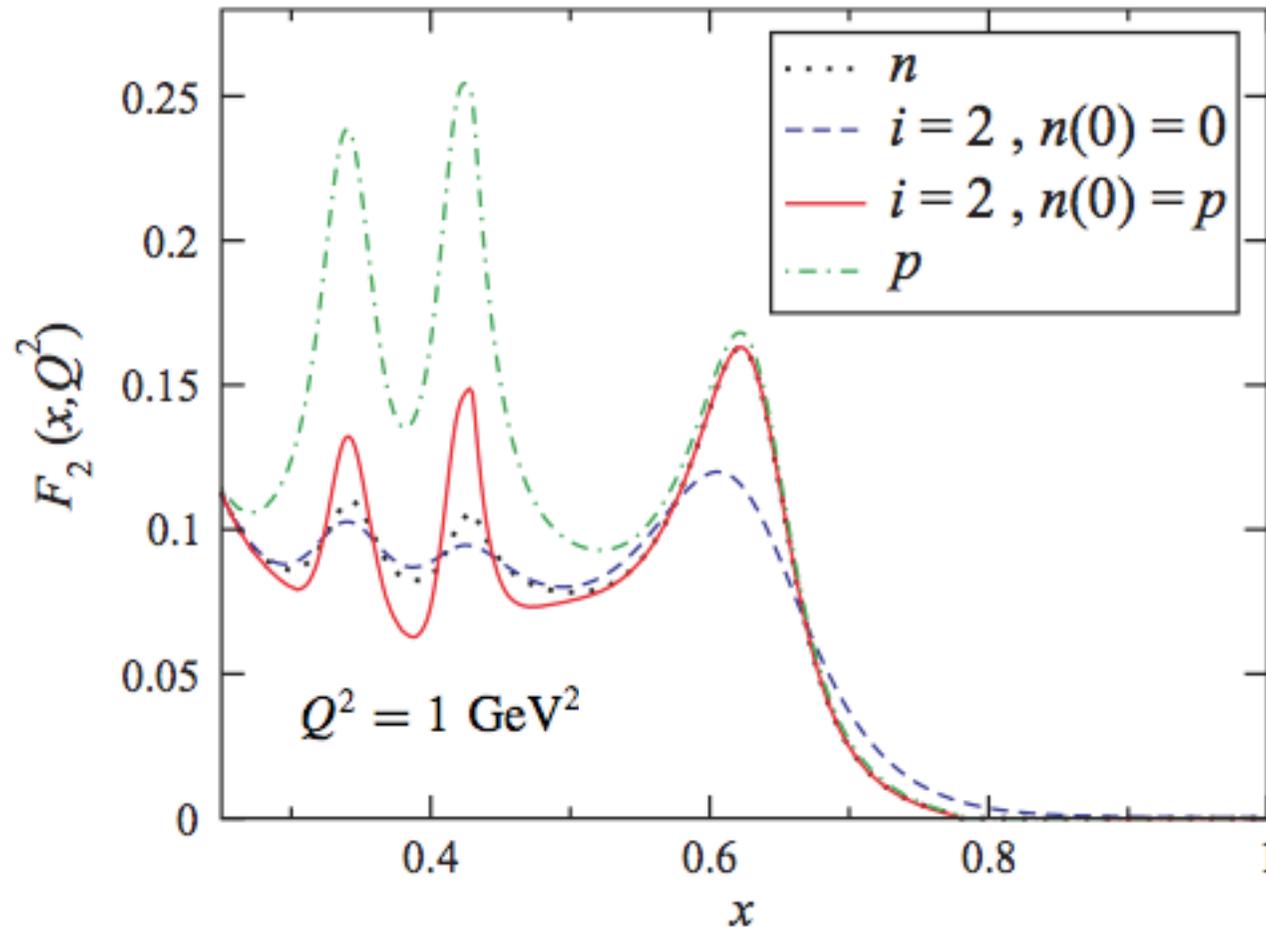
initial guess
 $F_2^{n(0)} = 0$

*Kahn, WM, Kulagin
PRC 79, 035205 (2008)*

→ can reconstruct almost arbitrary shape

Unsmearing – test of convergence

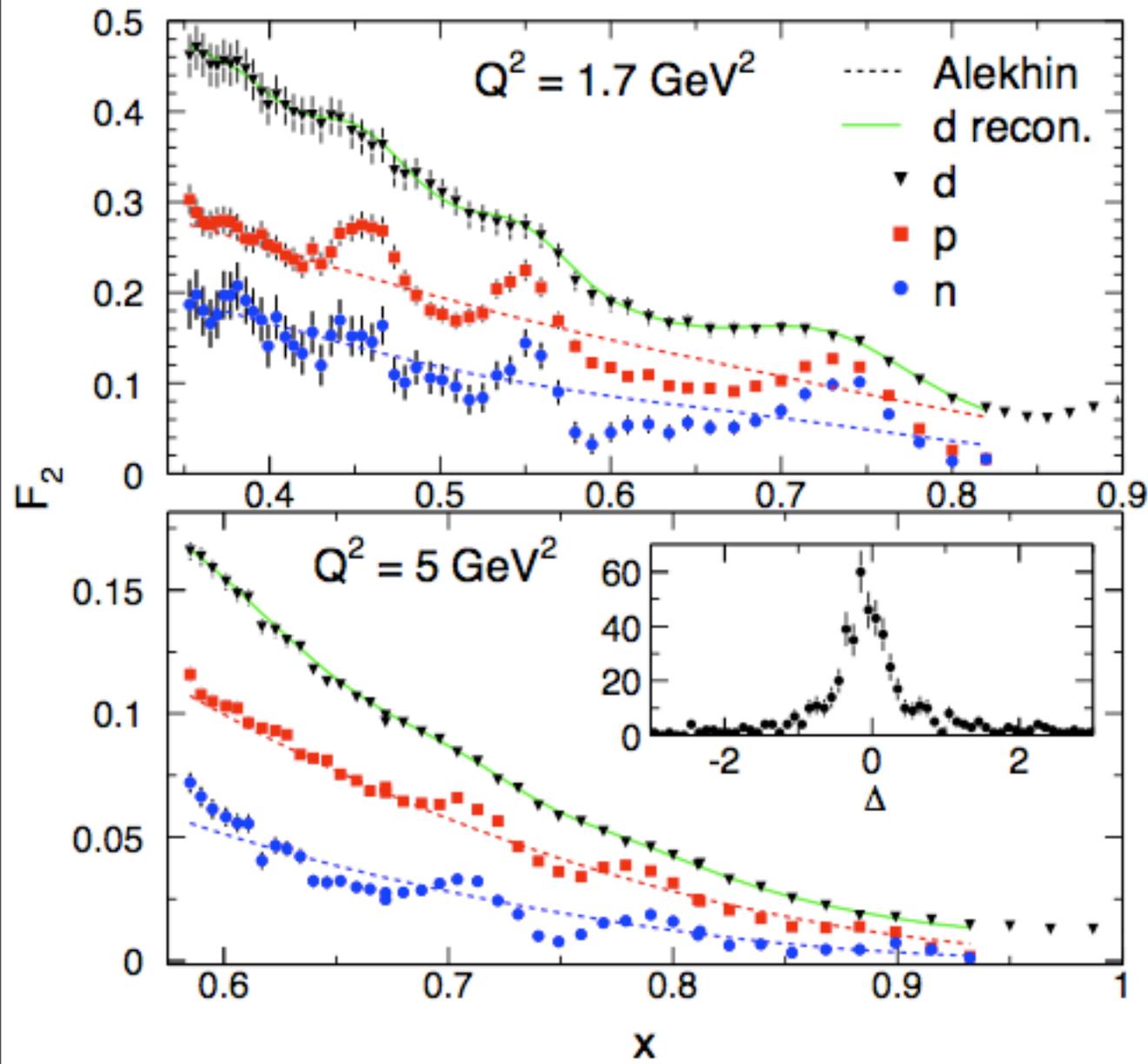
- F_2^d constructed from known F_2^p and F_2^n inputs
(using MAID resonance parameterization)



Kahn, WM, Kulagin
PRC 79, 035205 (2008)

→ fast convergence with $n(0)=p$ initial condition

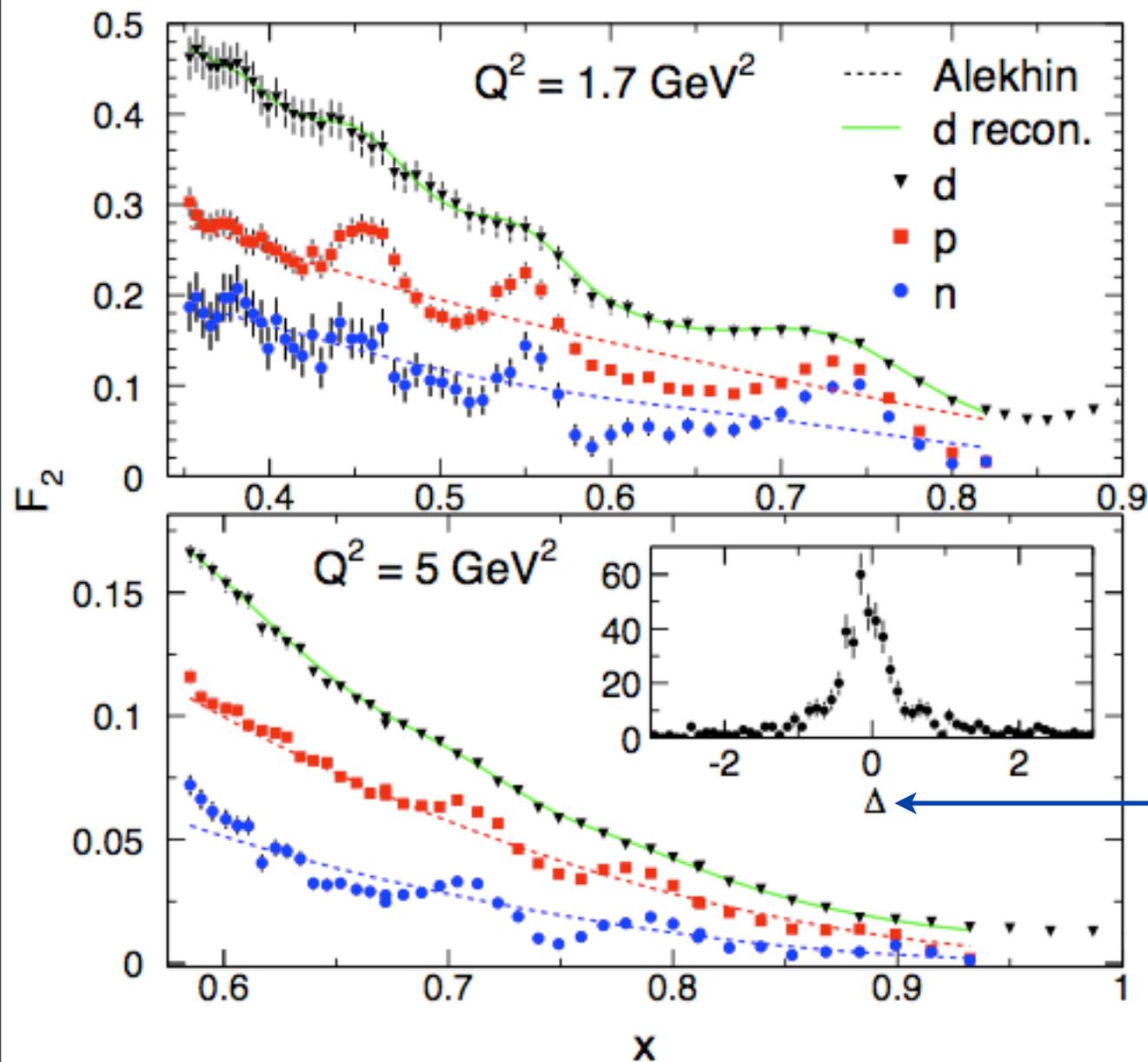
Extracted neutron data



→ JLab & SLAC data

→ 2 iterations
with $n(0)=p$

Extracted neutron data



→ JLab & SLAC data

→ 2 iterations
with $n(0)=p$

■ good agreement with
reconstructed d

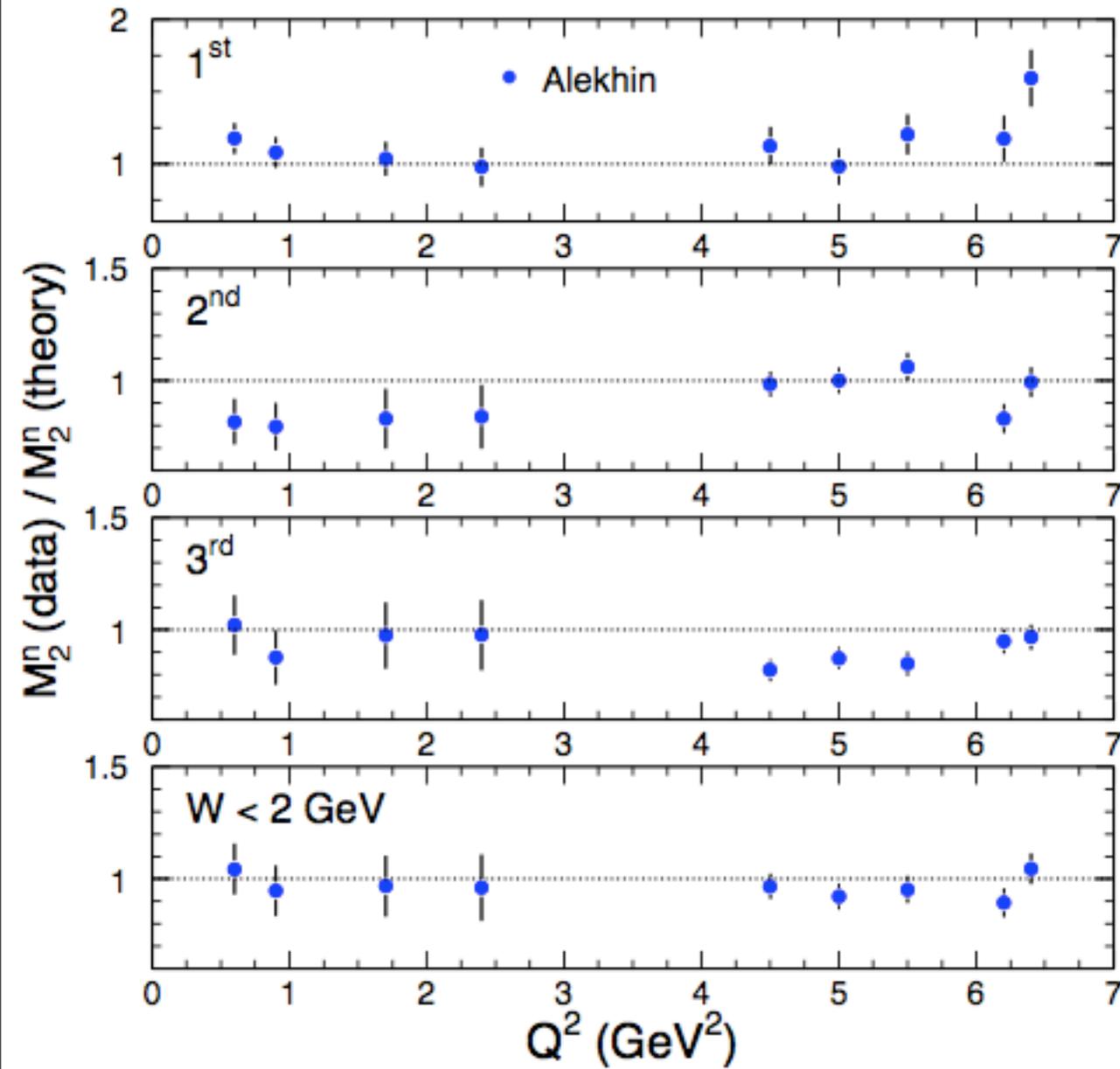
■ weak dependence
on input neutron

$[F_2^n(n(0) = p) - F_2^n(n(0) = p/2)] / \sigma(F_2)$

■ clear neutron resonance
structure observed

■ striking similarity with QCD fit to DIS data!

Truncated moment ratio

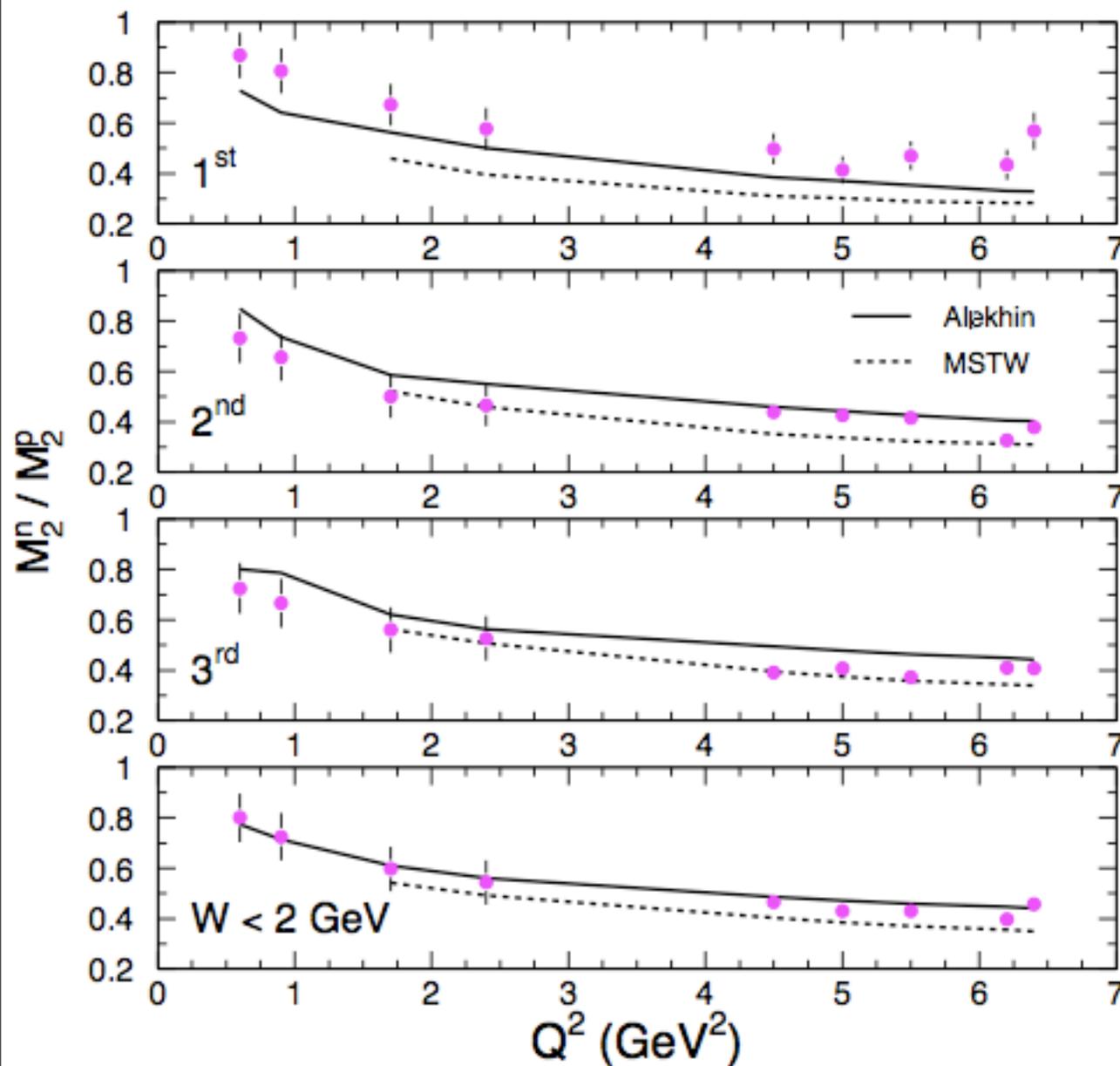


→ “theory” is QCD fit
to $W > 2 \text{ GeV}$ data

Alekhin et al., 0908.2762 [hep-ph]

- globally, deviations from unity generally $< 10\%$
- locally, deviations in individual resonance regions $< 15\text{--}20\%$
- largest deviations in Δ resonance region (fits least constrained at high x)

Isospin dependence



→ MSTW fit to
W > 4 GeV data

Martin et al., 0905.3531 [hep-ph]

■ good agreement overall

■ maximal duality violation
in Δ region

→ I=1 res. transition: $p=n$

→ DIS: $p \gg n$

→ QCD fit underestimates
resonance data

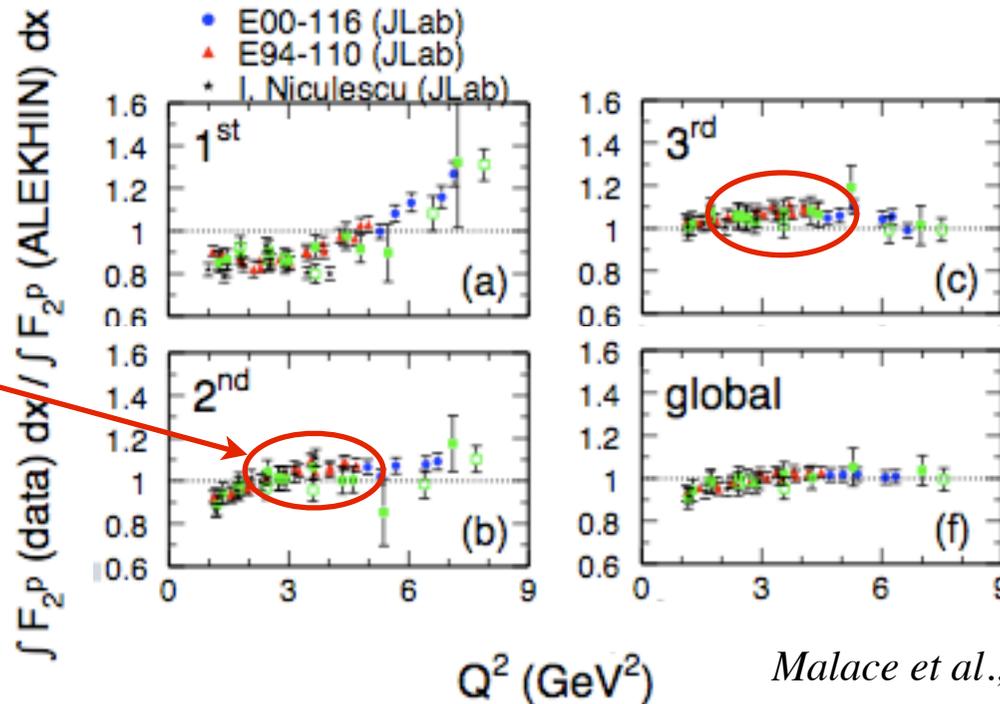
Quark model comparison

- Quark model predicts systematic deviations of resonance data from local duality

$SU(6)$:	$[56, 0^+]^{28}$	$[56, 0^+]^{410}$	$[70, 1^-]^{28}$	$[70, 1^-]^{48}$	$[70, 1^-]^{210}$	total
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- Proton data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)

data exceeds
DIS function



Malace et al., PRC 80, 035207 (2009)

Quark model comparison

- Quark model predicts systematic deviations of resonance data from local duality

$SU(6) :$	$[56, 0^+]^{28}$	$[56, 0^+]^{410}$	$[70, 1^-]^{28}$	$[70, 1^-]^{48}$	$[70, 1^-]^{210}$	total
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

- Proton data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)
- Neutron data predicted to lie *below* DIS function in 2nd region

➡ Patterns borne out by data!

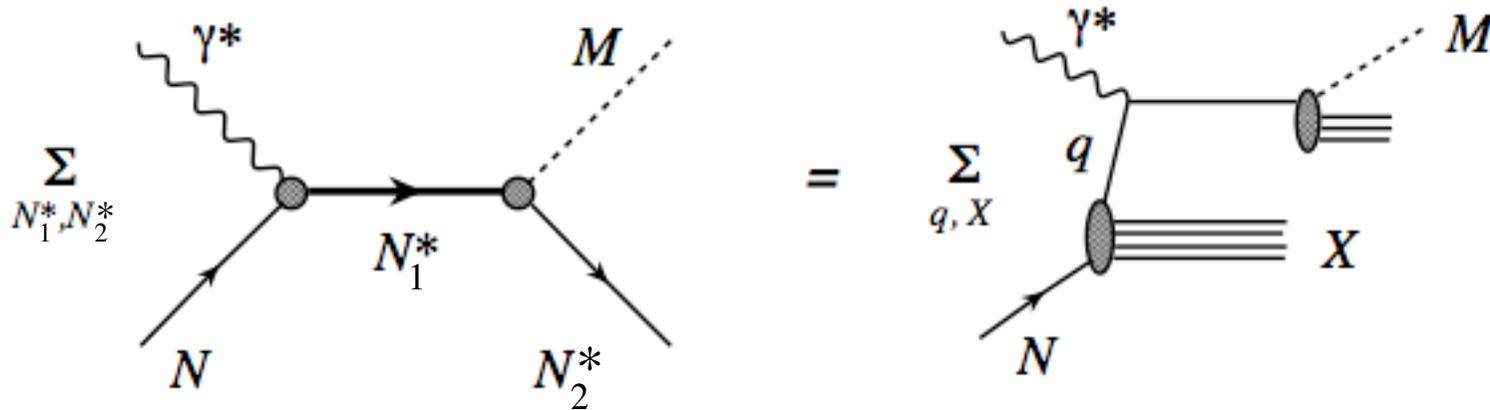
➡ Suggests duality is *not accidental*, but a general feature of resonance–scaling transition

Duality in Semi-Inclusive Meson Production

- Duality expected to work better for inclusive observables (e.g. structure functions)

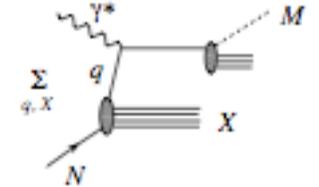
→ what about for *semi-inclusive scattering*?

- Hypothesis: equivalent descriptions afforded by scattering from partons or via N^* excitations



→ test whether hypothesis is consistent with *models* and *data*

■ Partonic description

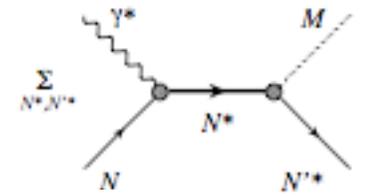


$$\mathcal{N}_N^\pi(x, z) = e_u^2 u^N(x) D_u^\pi(z) + e_d^2 d^N(x) D_d^\pi(z)$$

$q \rightarrow \pi$ fragmentation function

$z = E_\pi/\nu$ fractional energy carried by pion

■ Hadronic description

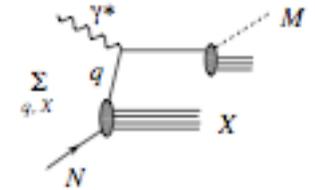


$$\mathcal{N}_N^\pi(x, z) = \sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \rightarrow N_1^*}(Q^2, M_1^*) \mathcal{D}_{N_1^* \rightarrow N_2^* \pi}(M_1^*, M_2^*) \right|^2$$

transition
form factor

decay function

■ Partonic description

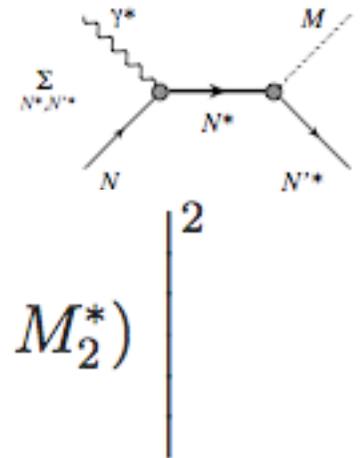


$$\mathcal{N}_N^\pi(x, z) = e_u^2 u^N(x) D_u^\pi(z) + e_d^2 d^N(x) D_d^\pi(z)$$

→ ratios given by quark charges

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = \frac{e_u^2}{e_d^2} = 4$$

■ Hadronic description

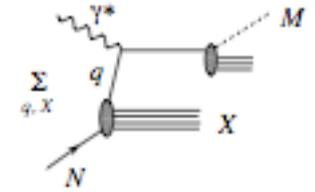


$$\mathcal{N}_N^\pi(x, z) = \sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \rightarrow N_1^*}(Q^2, M_1^*) \mathcal{D}_{N_1^* \rightarrow N_2^* \pi}(M_1^*, M_2^*) \right|$$

transition
form factor

decay function

■ Partonic description

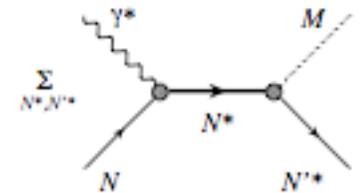


$$\mathcal{N}_N^\pi(x, z) = e_u^2 u^N(x) D_u^\pi(z) + e_d^2 d^N(x) D_d^\pi(z)$$

→ ratios given by quark charges

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = \frac{e_u^2}{e_d^2} = 4$$

■ Hadronic description



→ magnetic interaction operator for $\gamma N \rightarrow N_1^*$

$$\sum_i e_i \sigma_i^+$$

→ pion emission operator for $N_1^* \rightarrow N_2^* \pi^\pm$

$$\sum_i \tau_i^\mp \sigma_{zi}$$

■ Relative probabilities \mathcal{N}_N^π in SU(6) symmetric quark model
(summed over N_1^*)

	N_2^*					sum	spin-averaged
	${}^2\mathbf{8}, 56^+$	${}^4\mathbf{10}, 56^+$	${}^2\mathbf{8}, 70^-$	${}^4\mathbf{8}, 70^-$	${}^2\mathbf{10}, 70^-$		
$\gamma p \rightarrow \pi^+ N_2^*$	100 (100)	32 (-16)	64 (64)	16 (-8)	4 (4)	216 (144)	spin-averaged
$\gamma p \rightarrow \pi^- N_2^*$	0 (0)	24 (-12)	0 (0)	0 (0)	3 (3)	27 (-9)	spin-dependent
$\gamma n \rightarrow \pi^+ N_2^*$	0 (0)	96 (-48)	0 (0)	0 (0)	12 (12)	108 (-36)	
$\gamma n \rightarrow \pi^- N_2^*$	25 (25)	8 (-4)	16 (16)	4 (-2)	1 (1)	54 (36)	

■ π^-/π^+ ratios for p and n targets (summing over N_2^*)

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{8}, \quad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{1}{2}$$

$$\frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{2}, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^-}} = \frac{\mathcal{N}_p^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = 4$$

■ Consistent with parton model in SU(6) limit, $d/u=1/2$

- For *spin-dependent* ratios (e & N longitudinally polarized)

$$\frac{\Delta\mathcal{N}_p^{\pi^-}}{\Delta\mathcal{N}_p^{\pi^+}} = -\frac{1}{16}, \quad \frac{\Delta\mathcal{N}_n^{\pi^-}}{\Delta\mathcal{N}_n^{\pi^+}} = -1$$

$$\frac{\Delta\mathcal{N}_p^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{2}{3}, \quad \frac{\Delta\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^-}} = -\frac{1}{3}$$

$$\frac{\Delta\mathcal{N}_n^{\pi^+}}{\mathcal{N}_n^{\pi^+}} = -\frac{1}{3}, \quad \frac{\Delta\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{2}{3}$$

- Consistent with parton model ratios

$$\Delta u/u = 2/3, \quad \Delta d/d = -1/3, \quad \Delta d/\Delta u = -1/4$$

- Inclusive results recovered by summing over π^+ & π^-

$$\frac{\mathcal{N}_n^{\pi^++\pi^-}}{\mathcal{N}_p^{\pi^++\pi^-}} = \frac{F_1^n}{F_1^p} = \boxed{\frac{2}{3}}$$

$$\frac{\Delta\mathcal{N}_p^{\pi^++\pi^-}}{\mathcal{N}_p^{\pi^++\pi^-}} = \frac{g_1^p}{F_1^p} = \boxed{\frac{5}{9}}, \quad \frac{\Delta\mathcal{N}_n^{\pi^++\pi^-}}{\mathcal{N}_n^{\pi^++\pi^-}} = \frac{g_1^n}{F_1^n} = \boxed{0}$$

■ SU(6) symmetry may be valid at $x \sim 1/3$, but is (badly) broken at large x

■ Color-magnetic interaction

→ suppression of transitions to states with $S=3/2$

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{56}, \quad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{7}{2}$$

→ consistent with $d/u=1/14$ at parton level

■ Scalar diquark dominance

→ suppression of symmetric (λ) component of wfn.

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = 0, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_n^{\pi^-}} = 0, \quad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{4}$$

→ consistent with $d/u=0$ at parton level

■ SU(6) symmetry may be valid at $x \sim 1/3$, but is (badly) broken at large x

■ Helicity conservation

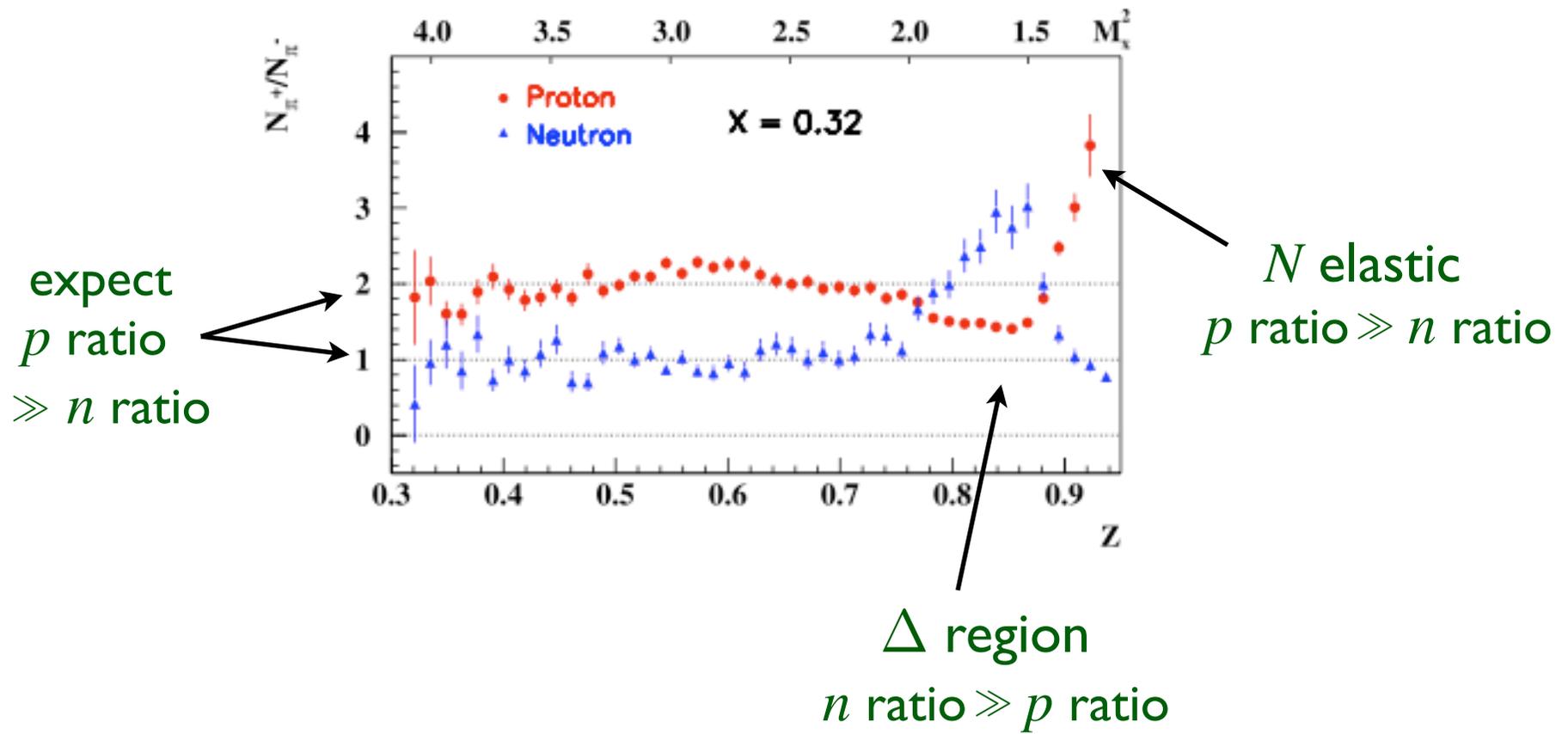
→ suppression of helicity-3/2 amplitude

$$\frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_p^{\pi^+}} = \frac{1}{20}, \quad \frac{\mathcal{N}_n^{\pi^-}}{\mathcal{N}_n^{\pi^+}} = \frac{5}{4}, \quad \frac{\mathcal{N}_n^{\pi^+}}{\mathcal{N}_p^{\pi^+}} = \frac{\mathcal{N}_p^{\pi^-}}{\mathcal{N}_n^{\pi^-}} = \frac{1}{5}$$

→ consistent with $d/u=1/5$ at parton level

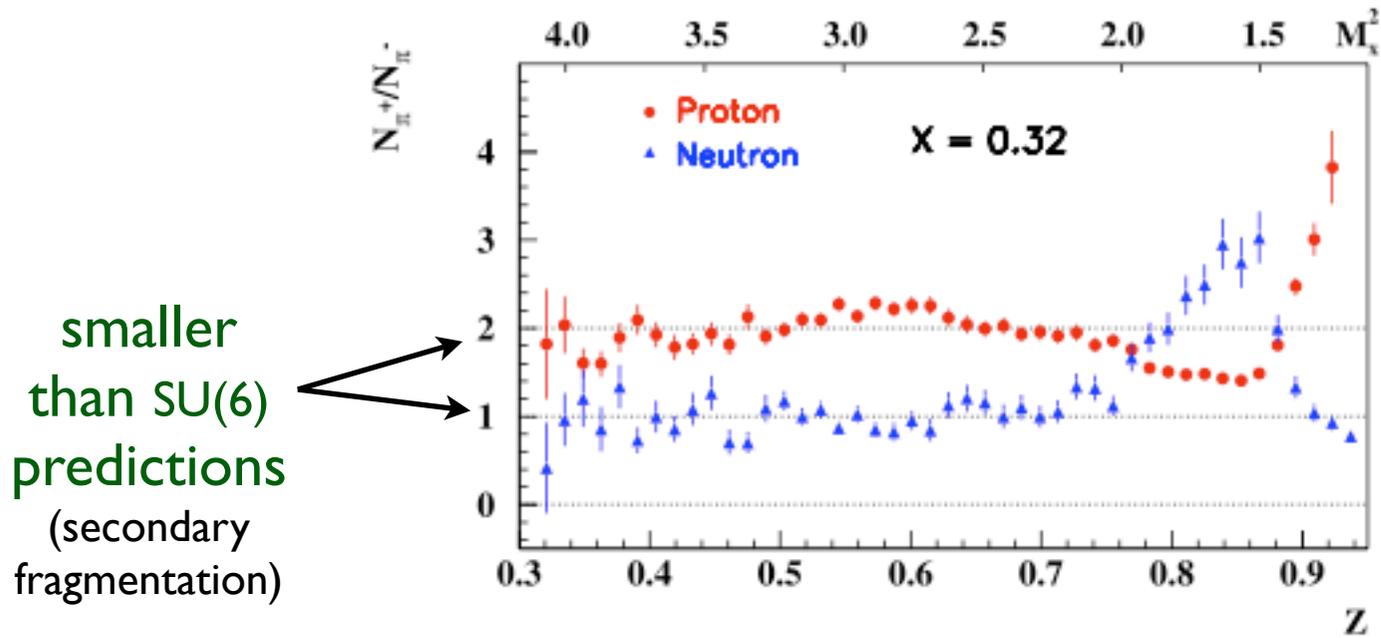
➡ All three scenarios consistent with duality!

Comparison with data (JLab Hall C)



	N_2^*				sum	
	$^28, 56^+$	$^410, 56^+$	$^28, 70^-$	$^48, 70^-$		
$\gamma p \rightarrow \pi^+ N_2^*$	100 (100)	32 (-16)	64 (64)	16 (-8)	4 (4)	216 (144)
$\gamma p \rightarrow \pi^- N_2^*$	0 (0)	24 (-12)	0 (0)	0 (0)	3 (3)	27 (-9)
$\gamma n \rightarrow \pi^+ N_2^*$	0 (0)	96 (-48)	0 (0)	0 (0)	12 (12)	108 (-36)
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■ Comparison with data (JLab Hall C)



■ More quantitative comparison requires secondary fragmentation

$$\frac{\mathcal{N}_d^{\pi^+}}{\mathcal{N}_d^{\pi^-}} = \frac{4 + R}{4R + 1}$$

$$R \equiv \bar{D}/D$$

$$D_d^{\pi^+} = D_u^{\pi^-}$$

“unfavored”

$$D_u^{\pi^+} = D_d^{\pi^-}$$

“favored”

$$z \rightarrow 1$$

Summary

- Remarkable confirmation of quark-hadron duality in *proton* structure functions
 - duality violating higher twists $\sim 10\%$ in few-GeV range
- Truncated moments
 - firm foundation for study of *local* duality in QCD
- Extraction of *neutron* structure function
 - confirmation of local duality at 15-20% level
 - evidence that duality is *not* due to accidental cancellations
- Duality predicted in semi-inclusive pion production
 - quantitative comparison with data requires modeling secondary fragmentation

The End