Excited meson spectroscopy and radiative transitions from LQCD

Christopher Thomas, Jefferson Lab

DAMTP, Cambridge, September 2010

With Jo Dudek, Robert Edwards, Mike Peardon, David Richards and the Hadron Spectrum Collaboration
Outline

• Introduction and motivation
• Excited spectra from LQCD – method outline
• Results – isovector spectra
• Photocouplings – charmonium
• Summary and outlook

PR D79 094504 (2009)
PRL 103 262001 (2009)
PR D82 034508 (2010)
Motivation

Renaissance in excited charmonium spectroscopy
BABAR, Belle, BES, CLEO-c, ...

Upcoming experimental efforts (in charmonium and light meson sector)
GlueX (JLab), BESIII, PANDA, ...
Motivation

Renaissance in excited charmonium spectroscopy

Upcoming experimental efforts (in charmonium and light meson sector)

Exotics (\(J^{PC} = 1^{-+}, 2^{++}, \ldots\))? – can’t just be a \(q\bar{q}\) pair
e.g. hybrids, multi-mesons
Motivation

Renaissance in excited charmonium spectroscopy

Upcoming experimental efforts (in charmonium and light meson sector)

Exotics ($J^{PC} = 1^{-+}, 2^{++}, ...$)? – can’t just be a $q\bar{q}$ pair

Two spin-half fermions: $^{2S+1}L_J$

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$

$J^{PC} = 0^{-+}, 0^{++}, 1^{-}, 1^{++}, 1^{--}, 2^{--}, 2^{++}, 2^{-+}, ...$
Motivation

Renaissance in excited charmonium spectroscopy

Upcoming experimental efforts (in charmonium and light meson sector)

Exotics ($J^{PC} = 1^{-+}, 2^{++}, ...$)? — can’t just be a $q\bar{q}$ pair
  e.g. hybrids, multi-mesons

Photoproduction at GlueX (JLab 12 GeV upgrade)
Motivation

Renaissance in excited charmonium spectroscopy

Upcoming experimental efforts (in charmonium and light meson sector)

Exotics ($J^{PC} = 1^{-+}, 2^{++}, \ldots$)? — can’t just be a $q\bar{q}$ pair
  e.g. hybrids, multi-mesons

Photoproduction at GlueX (JLab 12 GeV upgrade)

Use Lattice QCD to extract excited spectrum...

... and photocouplings (tested in charmonium)
QCD on a Lattice

Discretise on a grid (spacing = a) – regulator

Finite volume $\rightarrow$ finite no. of d.o.f.
QCD on a Lattice

Discretise on a grid (spacing = a) – regulator

Finite volume → finite no. of d.o.f.

Quarks fields on lattice sites

\[ \psi(x) \rightarrow \psi_x \]

Gauge fields on links

\[ A_\mu(x) \rightarrow U_{x,\mu} = e^{-a A_{x,\mu}} \]
QCD on a Lattice

Discretise on a grid (spacing = a) – regulator

Finite volume → finite no. of d.o.f.

Quarks fields on lattice sites

\[ \psi(x) \rightarrow \psi_x \]

Gauge fields on links

\[ A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x,\mu}} \]

Path integral formulation

\[ \int D\psi D\bar{\psi} DU f(\psi, \bar{\psi}, U) e^{iS[\psi, \bar{\psi}, U]} \]
QCD on a Lattice

- Discretise on a grid (spacing = a) – regulator
- Finite volume → finite no. of d.o.f.
- Quarks fields on lattice sites \( \psi(x) \rightarrow \psi_x \)
- Gauge fields on links \( A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x,\mu}} \)
- Path integral formulation
- Euclidean time: \( t \rightarrow it \)
- Do fermion integral analytically then use importance sampling Monte Carlo
Spectroscopy on the lattice

Calculate energies and matrix elements (“overlaps”, Z’s) from correlation functions of meson interpolating fields

\[ C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle \]
Spectroscopy on the lattice

Calculate **energies** and **matrix elements** ("overlaps", Z’s) from correlation functions of meson interpolating fields

\[ C_{ij}(t) = \langle 0 | O_i(t)O_j(0) | 0 \rangle \]

\[
O(t) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \bar{\psi}(x) \Gamma_i \bar{D}_j \bar{D}_k \ldots \psi(x)
\]

(p = 0)

More about operators later...

‘Distillation’ technology for constructing on lattice PR D80 054506 (2009)
Spectroscopy on the lattice

Calculate energies and matrix elements ("overlaps", Z's) from correlation functions of meson interpolating fields

\[ C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle \]

\[ O(t) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \bar{\psi}(x) \Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \ldots \psi(x) \] \hspace{1cm} (p = 0)

More about operators later...

‘Distillation’ technology for constructing on lattice PR D80 054506 (2009)

\[ Z_i^{(n)} = \langle 0 | O_i | n \rangle \]

\[ C_{ij}(t) = \sum_n e^{-\frac{E_n}{2} t} \langle 0 | O_i(0) | n \rangle \langle n | O_j(0) | 0 \rangle \]
Variational Method

Large basis of operators $\rightarrow$ matrix of correlators

Generalised eigenvector problem:

$$C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

$$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$$
Variational Method

Large basis of operators $\rightarrow$ matrix of correlators

Generalised eigenvector problem:

$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle$

$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

Eigenvalues $\rightarrow$ energies

$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)} \quad (t >> t_0)$
Variational Method

Large basis of operators $\rightarrow$ matrix of correlators

$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0 \rangle$

Generalised eigenvector problem:

$C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

Eigenvalues $\rightarrow$ energies

$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)} \quad (t \gg t_0)$

Eigenvectors $\rightarrow$ optimal linear combination of operators to overlap on to a state

$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$

$Z^{(n)}$ related to eigenvectors

$Z_i^{(n)} = \langle 0|O_i|n \rangle$
Variational Method

Large basis of operators $\rightarrow$ matrix of correlators

Generalised eigenvector problem:

$$C_{ij}(t) = \langle 0| O_i(t) O_j(0) |0 \rangle$$

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

Eigenvalues $\rightarrow$ energies

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)} \quad (t >> t_0)$$

Eigenvectors $\rightarrow$ optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

$$Z^{(n)} \text{ related to eigenvectors}$$

$$Z_i^{(n)} = \langle 0| O_i |n \rangle$$

Var. method uses orthog of eigenvectors; don’t just rely on separating energies
On a lattice, 3D rotation group is broken to Octahedral Group

In continuum:
Infinite number of irreps: $J = 0, 1, 2, 3, 4, ...$
Spin on the lattice

On a lattice, 3D rotation group is broken to Octahedral Group

In continuum:
Infinite number of *irreps*: \( J = 0, 1, 2, 3, 4, \ldots \)

On lattice:
Finite number of *irreps*: \( A_1, A_2, T_1, T_2, E \) (and others for half-integer spin)

<table>
<thead>
<tr>
<th>Irrep</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( T_1 )</th>
<th>( T_2 )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dim</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cont. Spin</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irrep(s)</td>
<td>( A_1 )</td>
<td>( T_1 )</td>
<td>( T_2 + E )</td>
<td>( T_1 + T_2 + A_2 )</td>
<td>( A_1 + T_1 + T_2 + E )</td>
<td>...</td>
</tr>
</tbody>
</table>
Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

$\Gamma \times D \times D \times \ldots$ (up to 3 derivs)

Couple using SU(2) Clebsch Gordans

$\langle 0 | O^{J,M} | J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$ definite $J^{PC}$
Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

$\Gamma \times D \times D \times \ldots$ (up to 3 derivs)

Couple using $\text{SU}(2)$ Clebsch Gordans

$\langle 0| O^{J,M} |J', M' \rangle = Z^{[J]} \delta_{J,J'} \delta_{M,M'}$

definite $J^{PC}$

‘Subduce’ operators into lattice irreps ($J \rightarrow \Lambda$):

$\langle 0| O^{[J]}_{\Lambda,\lambda} |J', M \rangle = S^{J,M}_{\Lambda,\lambda} Z^{[J]} \delta_{J,J'}$

Up to 26 ops in $\Lambda^{PC}$ channel

Given continuum op $\rightarrow$ same $Z$ in each $\Lambda$
(ignoring lattice mixing)

e.g. $O^{[2]} \rightarrow T_2$ and $O^{[2]} \rightarrow E$

21
Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

\[ \Gamma \times D \times D \times ... \quad \text{(up to 3 derivs)} \]

Couple using SU(2) Clebsch Gordans

\[ \langle 0 | O^{J, M} | J', M' \rangle = Z^{[J]} \delta_{J, J'} \delta_{M, M'} \]

definite \( J^{PC} \)

‘Subduce’ operators into lattice irreps (\( J \rightarrow \Lambda \)):

\[ \langle 0 | O^{[J]}_{\Lambda, \lambda} | J', M \rangle = S^{J, M}_{\Lambda, \lambda} Z^{[J]} \delta_{J, J'} \]

Up to 26 ops in \( \Lambda^{PC} \) channel

Given continuum op \( \rightarrow \) same \( Z \) in each \( \Lambda \)
(ignoring lattice mixing)

(1) Look for ‘large’ overlaps with \( O^{[J]}_{\Lambda, \lambda} \)

(2) Compare \( Z \)’s of same cont. op. subduced to different irreps

\[ O^{[J]}_{\Lambda, \lambda} = \sum_{M} S^{J, M}_{\Lambda, \lambda} O^{J, M} \]
Spin and operator construction

Construct operators which only overlap on to one spin in the continuum limit

\[ \Gamma \times D \times D \times \ldots \]

Clebsch-Gordan coefficients

\[ \langle 0 | \mathcal{O}^{J,M}_{j} | J' \rangle \]

\[ \langle 0 | \mathcal{O}_{\Lambda_{j}}^{J} | J' \rangle \]

‘Subduce’ operators into lattice irreps

\[ \sum_{M} S^{J,M}_{\Lambda_{j},\Lambda} \mathcal{O}^{J,M} \]

Given continuum op \( \rightarrow \) same \( Z \) in each \( \Lambda \) channel (ignoring lattice mixing)

(1) Look for ‘large’ overlaps

(2) Compare \( Z \)'s of same cont. op. subduced to different irreps

\[ C_{ij}/\sqrt{C_{ii}C_{jj}} \]
Calculation details

• Dynamical calculation. Clover fermions

• Anisotropic \( (a_s/a_t = 3.5) \), \( a_s \sim 0.12 \) fm, \( a_t^{-1} \sim 5.6 \) GeV

• Two volumes: \( 16^3 \) (\( L_s \approx 2.0 \) fm) and \( 20^3 \) (\( L_s \approx 2.4 \) fm)

Lattice details in: PR D78 054501, PR D79 034502
Calculation details

• Dynamical calculation. Clover fermions

• Anisotropic ($a_s/a_t = 3.5$), $a_s \sim 0.12$ fm, $a_t^{-1} \sim 5.6$ GeV

• Two volumes: $16^3$ ($L_s \approx 2.0$ fm) and $20^3$ ($L_s \approx 2.4$ fm)

Lattice details in: PR D78 054501, PR D79 034502

• Only connected diagrams – Isovectors ($I=1$) and kaons

• As an example: three degenerate ‘light’ quarks ($N_f = 3$, $M_\pi \approx 700$ MeV)

• Also ($N_f = 2+1$) $M_\pi \approx 520, 440, 400$ MeV

SU(3) sym

~ 500 cfgs x 9 t-sources

Method details and results: PRL 103 262001 (2009), PR D82 034508 (2010)
$N_f = 3$ isovectors

$16^3 \, (\sim 2 \, \text{fm})$ and $20^3 \, (2.4 \, \text{fm})$
$N_f = 3$ isovectors

$16^3$ (≈ 2 fm) and $20^3$ (2.4 fm)
First $J = 4$ mesons in LQCD

$N_f = 3$ isovectors

$16^3 \text{ (} \sim 2 \text{ fm)} \text{ and } 20^3 \text{ (} 2.4 \text{ fm)}$

Exotics
Z values

\[ \langle 0 \mid \mathcal{O}^{[J]}_{\Lambda, \lambda} \mid J', M \rangle = S^{J, M}_{\Lambda, \lambda} Z^{[J]} \delta_{J, J'} \]

J = continuum spin of op
\[
\langle 0 | \mathcal{O}_{\Lambda',\lambda}^{[J]} | J', M \rangle = S_{\Lambda',\lambda}^{J, M} Z^{[J]} \delta_{J,J'}
\]

J = continuum spin of op
\[ \langle 0 | \mathcal{O}^{[J]}_{\Lambda, \lambda} | J', M \rangle = S^{J, M}_{\Lambda, \lambda} Z^{[J]} \delta_{J, J'} \]

J = continuum spin of op
Vector Hybrid?

\[ \langle 0 | \mathcal{O}^{[J]}_{\Lambda, \lambda} | J', M \rangle = S^{J, M}_{\Lambda, \lambda} Z^{[J]} \delta_{J, J'} \]

\( J = \) continuum spin of op

Z values

\( m/m_0 \)

\( 1^3D_1 \)

\( 2^3S_1 \)

\( 1^3S_1 \)

\( A_1^{--} \)

\( T_1^{--} \)

\( T_2^{--} \)

\( E^{--} \)

\( A_2^{--} \)

Normalised \( T_1^{--} Z's \)

This operator \( \sim [D_i, D_j] \)
This operator \( \sim [D_i, D_j] \)

\[
\langle 0 | \mathcal{O}_{L,L}^{[J]} | J', M \rangle = S_{L,L}^{J,M} Z^{[J]} \delta_{J,J'}
\]

\( J = \) continuum spin of op

Z values

Vector Hybrid?
This operator $\sim [D_i, D_j]$
Given continuum op → same \( Z \) for each subduced irrep

\[
\langle 0 | O^{[J]}_{\Lambda, \lambda} | J', M \rangle = S^{J,M}_{\Lambda, \lambda} Z^{[J]} \delta_{J,J'}
\]
Z values – spin 3

\[ \langle 0 | O^{[J]}_{\Lambda, \lambda} | J', M \rangle = S^J_M Z^J_{\delta J,J'} \]

Given continuum op \( \rightarrow \) same Z for each subduced irrep

\[ m_{A_2}/m_\Omega = 1.210(5) \quad m_{A_2}/m_\Omega = 1.626(16) \]
\[ m_{T_1}/m_\Omega = 1.207(5) \quad m_{T_1}/m_\Omega = 1.648(23) \]
\[ m_{T_2}/m_\Omega = 1.204(4) \quad m_{T_2}/m_\Omega = 1.626(8) \]
Z values – spin 4

\[ \langle 0 | \mathcal{O}_\Lambda^{[J]} | J', M \rangle = S_{\Lambda, J}^{J', M} Z^{[J]} \delta_{J, J'} \]

Given continuum op \( \rightarrow \) same Z for each subduced irrep
$N_f = 3$ isovectors

$16^3$ ($\sim 2$ fm) and $20^3$ (2.4 fm)
$N_f = 3$ isovectors

$\begin{align*}
16^3 (\sim 2 \text{ fm}) \text{ and } 20^3 (2.4 \text{ fm})
\end{align*}$
First J = 4 mesons in LQCD

$N_f = 3$ isovectors

$16^3$ ($\sim 2$ fm) and $20^3$ (2.4 fm)
First $J = 4$ mesons in LQCD

$N_f = 3$ isovectors

$16^3$ (≈2 fm) and $20^3$ (2.4 fm)
First J = 4 mesons in LQCD

S-wave

P-wave

Exotics

$N_f = 3$ isovectors

$16^3$ (≈2 fm) and $20^3$ (2.4 fm)
First J = 4 mesons in LQCD

$N_f = 3$ isovectors

$16^3$ ($\sim 2$ fm) and $20^3$ (2.4 fm)
First J = 4 mesons in LQCD

- S-wave
- D-wave
- P-wave
- F-wave
- Exotics

$N_f = 3$ isovectors

$16^3 (\sim 2 \text{ fm})$ and $20^3 (2.4 \text{ fm})$
First \( J = 4 \) mesons in LQCD

-\( S \)-wave
-\( D \)-wave
-\( P \)-wave
-\( F \)-wave

\( N_f = 3 \) isovectors

\( 16^3 \) (\( \sim 2 \) fm) and \( 20^3 \) (2.4 fm)
First J = 4 mesons in LQCD

\[ N_f = 3 \text{ isovectors} \]

D + G-waves

S-wave

D-wave

P-wave

F-wave

Exotics

Vector hybrid?

Large overlap with op \( \sim [D_i, D_j] \)

\[ 16^3 (\sim 2 \text{ fm}) \text{ and } 20^3 (2.4 \text{ fm}) \]
Lower pion masses

\[
\frac{a_t m_H \cdot m_{\Omega}}{a_t m_{\Omega}}
\]

\( M_\pi \approx 400 \text{ MeV} \quad 440 \text{ MeV} \quad 520 \text{ MeV} \quad 700 \text{ MeV} \)

\[
l_\Omega \equiv \frac{9 (a_t m_\pi)^2}{4 (a_t m_\Omega)^2}
\]
Lower pion masses

\[ l_{\Omega} = \frac{9 (a_t m_\pi)^2}{4 (a_t m_{\Omega})^2} \]

Method still works
$l_\Omega \equiv \frac{9 (a_t m_\pi)^2}{4 (a_t m_\Omega)^2}$
Exotics summary
Exotics summary

- Anisotropic lattices (small $a_t$)
- Large basis of ops
- High statistics

In range accessible to GlueX
Multi-particle states?

Finite box → discrete allowed momenta → discrete spectrum of multiparticle states
Multi-particle states?

Finite box
→ discrete allowed momenta
→ discrete spectrum of multiparticle states

Expect two-meson states above $2m_\pi$

$2m_\pi \sim 0.85 \, m_\Omega$

Where are they?
Charmonium

“Hydrogen atom” of meson spectroscopy

Potential models, effective field theories, QCD sum rules, ...

New and improved measurements at BABAR, Belle, BES, CLEO-c
Charmonium

“Hydrogen atom” of meson spectroscopy

Potential models, effective field theories, QCD sum rules, ...

New and improved measurements at BABAR, Belle, BES, CLEO-c

New resonances not easily described by quark model

Theoretical speculation: hybrids, multiquark/molecular mesons, ...

As yet, no exotic $J^{PC}$ observed ($1^{-+}$, $0^{+-}$, $2^{+-}$)
Charmonium radiative transitions

Below DD threshold radiative transitions have significant BRs
Charmonium radiative transitions

Below DD threshold radiative transitions have significant BRs

Meson – Photon coupling
Charmonium radiative transitions

Below DD threshold radiative transitions have significant BRs

Meson – Photon coupling

Exotic 1^{++}?
Photocouplings

Charmonium (quenched) – testing method

\[ C_{ij}(t_f, t, t_i) = \langle 0 | O_i(t_f) \bar{\psi}(t) \gamma^\mu \psi(t) O_j(t_i) | 0 \rangle \]
Photocouplings

Charmonium (quenched) – testing method

\[ C_{ij}(t_f, t, t_i) = \langle 0 | O_i(t_f) \bar{\psi}(t) \gamma^\mu \psi(t) O_j(t_i) | 0 \rangle \]

Conventional vector – pseudoscalar transition

\[ J/\psi \to \eta_c \gamma \]
\[ \Gamma \sim 2.5 \text{ keV} \]

\[ \psi' \to \eta_c \gamma \]
\[ \Gamma \sim 0.4 \text{ keV} \]

Magnetic dipole (\(M_1\)) transition – suppressed (in quark model spin flip \(\sim 1/m_c\))
Photocouplings

\[ \Gamma(Y \rightarrow \eta_c \gamma) = 42(18) \text{ keV} \]

Much larger than other 1^- \rightarrow 0^+ M_1 transitions

\[ \Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV} \]

Spectrum analysis suggests a vector hybrid (spin-singlet)

c.f. flux tube model

30 – 60 keV
Photocouplings

Much larger than other $1^- \rightarrow 0^+ M_1$ transitions

\[ \Gamma(Y \rightarrow \eta_c \gamma) = 42(18) \text{ keV} \]

\[ \Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV} \]

Spectrum analysis suggests a vector hybrid (spin-singlet)

c.f. flux tube model

30 – 60 keV

- Usually $M_1 \rightarrow \text{spin flip (e.g. } ^3S_1 \rightarrow ^1S_0) \rightarrow 1/m_c$ suppression
- Spin-singlet hybrid $\rightarrow$ extra gluonic degrees of freedom
  $\rightarrow M_1$ transition without spin flip $\rightarrow$ not suppressed
Exotic meson photocoupling

\[ \eta_{c1}^{(1-\pm)} \rightarrow J/\psi \gamma \]

\[ \Gamma (\eta_{c1} \rightarrow J/\psi \gamma) = 115(16) \text{ keV} \]
Exotic meson photocoupling

Same scale as many measured conventional charmonium transitions

BUT very large for an $M_1$ transition

$\Gamma(\eta_c \rightarrow J/\psi \gamma) = 115(16)$ keV

$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2$ keV

Suggests a spin-triplet hybrid
Exotic meson photocoupling

- Same scale as many measured conventional charmonium transitions
- BUT very large for an $M_1$ transition
  $\Gamma(J/\psi \to \eta_c \gamma) \sim 2$ keV
- Suggests a spin-triplet hybrid

- Usually $M_1 \to$ spin flip (e.g. $^3S_1 \to ^1S_0$) $\to \frac{1}{m_c}$ suppression
- Spin-triplet hybrid $\to$ extra gluonic degrees of freedom
  $\to M_1$ transition without spin flip $\to$ not suppressed
Exotic meson photocoupling

- Same scale as many measured conventional charmonium transitions
- BUT very large for an $M_1$ transition
  \[ \Gamma(\eta_c \rightarrow J/\psi \gamma) = 115(16) \text{ keV} \]
- Suggests a spin-triplet hybrid

- Usually $M_1 \rightarrow$ spin flip (e.g. $^3S_1 \rightarrow ^1S_0$) $\rightarrow 1/m_c$ suppression
- Spin-triplet hybrid $\rightarrow$ extra gluonic degrees of freedom
  $\rightarrow M_1$ transition without spin flip $\rightarrow$ not suppressed
More charmonium results

Tensor – Vector transitions  \[ \chi_{c2}, \chi'_{c2}, \chi''_{c2} \rightarrow J/\psi\gamma \]
Identify \( 1^3P_2, 1^3F_2, 2^3P_2 \) tensors from hierarchy of multipoles \( E_1, M_2, E_3 \)

Vector – Psuedoscalar  \[ J/\psi, \psi', \psi'' \rightarrow \eta_c\gamma \]
Scalar – Vector  \[ \chi_{c0} \rightarrow J/\psi\gamma \quad \psi', \psi'' \rightarrow \chi_{c0}\gamma \]
Axial – Vector  \[ \chi_{c1}, \chi'_{c1} \rightarrow J/\psi\gamma \]

Dudek, Edwards & CT, PR \textbf{D79} 094504 (2009)
Summary and Outlook

Summary

- Our first results on light mesons – technology and method work
- **Spin identification** is possible using operator overlaps
- **First spin 4 meson** extracted and confidently identified on lattice
- **Exotics** (and non-exotic **hybrids**)
- Isovectors and kaons
Summary and Outlook

Summary

• Our first results on light mesons – technology and method work
• Spin identification is possible using operator overlaps
• First spin 4 meson extracted and confidently identified on lattice
• Exotics (and non-exotic hybrids)
• Isovectors and kaons

Outlook – ongoing work

• Multi-meson operators – resonance physics
• Disconnected diagrams – isoscalars and multi-mesons
• Baryons
• Photocouplings
• Lighter pion masses and larger volumes
Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) sym breaking

\[
\begin{array}{cccc}
M_\pi / \text{MeV} & 700 & 520 & 440 & 400 \\
M_K / M_\pi & 1 & 1.2 & 1.3 & 1.4 \\
\end{array}
\]

c.f. physical
$M_K / M_\pi = 3.5$
Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) sym breaking

- $M_\pi / \text{MeV}$: 700, 520, 440, 400
- $M_K / M_\pi$: 1, 1.2, 1.3, 1.4
  - c.f. physical $M_K / M_\pi = 3.5$

No longer is C-parity a good quantum number for kaons (or a gen. of C-parity)

Combine $J^{P^+}$ and $J^{P^-}$ operators

Physically, axial kaons [$K_{1}(1270)$, $K_{1}(1400)$] are a mixture
Suggested mixing angle $\approx 45^\circ$ (combination of exp and models)

But...
Kaons

Lower the light quark mass ($N_f = 2+1$) — SU(3) symmetry breaking

$M_{\pi} \approx 400 \text{ MeV}$
$M_K / M_{\pi} \approx 1.4$

No longer is $C$-parity a good quantum number for kaons (or a general of $C$-parity)

c.f. physical $M_K / M_{\pi} = 3.5$

Combine $J^P^+$ and $J^P^-$ operators

Physically, axial kaons [$K_1^{(1270)}, K_1^{(1400)}$] are a mixture

Suggested mixing angle $45^\circ$ (combination of explicit and models)

But...
Kaons – Operator Overlaps

16³
$M_\pi \approx 520 \text{ MeV}$
$M_K/M_\pi \approx 1.2$

16³
$M_\pi \approx 400 \text{ MeV}$
$M_K/M_\pi \approx 1.4$
Kaons – Operator Overlaps

16^3
M_π ≈ 520 MeV
M_K/M_π ≈ 1.2

16^3
M_π ≈ 400 MeV
M_K/M_π ≈ 1.4

J^{++}
Kaons – Operator Overlaps

$16^3$

$M_\pi \approx 520 \text{ MeV}$

$M_K / M_\pi \approx 1.2$

$16^3$

$M_\pi \approx 400 \text{ MeV}$

$M_K / M_\pi \approx 1.4$

$J^{++}$

$J^{+-}$

(1)

(2)

(3)

(4)

(5)
Kaons - spectrum

$M_{\pi} \approx 400 \text{ MeV}$

$M_K / M_{\pi} \approx 1.4$

$16^3 (\sim 2 \text{ fm})$ and $20^3 (2.4 \text{ fm})$
Kaons – Various pion masses

\[ K^*(1^-) \]

\[ M_\pi \approx 400 \text{ MeV} \quad 440 \text{ MeV} \quad 520 \text{ MeV} \quad 700 \text{ MeV} \]

\[ l_\Omega = \frac{9 (a_t m_\pi)^2}{4 (a_t m_\Omega)^2} \]
Kaons – Various pion masses

Will things change as we lower pion mass further?

Is the physical axial kaon mixing angle determination correct or too model dependent?