Semi-leptonic weak processes in two-nucleon systems

Impact on neutrino oscillation experiments and astrophysics

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## Collaborators

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<thead>
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<th>Institution</th>
<th>Research Area</th>
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<tbody>
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<td>( \nu d )</td>
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<td>(Numazu Coll. Tech.)</td>
<td>supernova</td>
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**Introduction**

Electroweak processes in few-nucleon systems

⇒ well-established calculational method

\[ \langle \psi_f | H_{ew} | \psi_i \rangle \]

\( |\psi\rangle \) : solution of Schrödinger eq. with high-precision \( NN \) (+ \( NNN \)) potential

\( H_{ew} \) : impulse + meson exchange currents

review : Carlson and Schiavilla, Rev. Mod. Phys. 70, 743841 (1998)

cf. chiral effective field theory
Contribution to neighborhood  (neutrino physics, astrophysics)

* Experiment at Sudbury Neutrino Observatory (SNO)  (part 1)

\[ \nu_e + d \rightarrow e^- + p + p \quad (\nu d \text{ reaction}) \]

* Supernova simulation  (part 2)

\[ \nu d \text{ reaction} \]

\[ p + p \rightarrow d + e^+ + \nu_e \quad (\text{pp-fusion}) \]

\[ N + N \rightarrow N + N + \nu + \bar{\nu} \quad (NN\text{-bremsstrahlung}) \]

* Solar model

\[ \text{pp-fusion} \]
Part 1

**SNO experiment** (neutrino oscillation, solar neutrino problem)

heavy water Cherenkov light detector:

\[
\begin{align*}
\nu_e + d & \rightarrow e^- + p + p \quad \text{(CC)} \\
\nu_x + d & \rightarrow \nu_x + p + n \quad \text{(NC)} \\
\end{align*}
\]

\[x = e, \mu, \tau\]

Solar neutrino fluxes of $\nu_e$ and $\sum_x \nu_x$ are separately measured.

*Theoretical prediction for $\nu d$ reaction is prerequisite!*
Previous work

<table>
<thead>
<tr>
<th>Uncertainty</th>
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<tbody>
<tr>
<td>Ying et al. (IA)</td>
<td>( \sim 10% ) (muon capture exp.)</td>
</tr>
<tr>
<td>Kubodera et al. (IA+EXC)</td>
<td>a few%</td>
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What we do

SN et al., PRC 63, 034617 (2000); NPA 707, 561 (2002)

- Confirm the previous work
- Recent high-precision \( NN \) potential (AV18, CD-Bonn, Nijmegen)
- Exchange axial-vector current tested by tritium \( \beta \)-decay rate
  \( \longrightarrow \) significant reduction of theoretical uncertainty (\( \sim 1\% \))
- Differential cross section
Interaction Hamiltonian

\[ H_{WC}^{CC} = \frac{G'_F V_{ud}}{\sqrt{2}} \int d\mathbf{x} [J_{\chi}^{CC} (\mathbf{x}) L^\lambda (\mathbf{x}) + \text{h. c.}] \quad \text{for CC} \]

\[ H_{WC}^{NC} = \frac{G'_F}{\sqrt{2}} \int d\mathbf{x} [J_{\chi}^{NC} (\mathbf{x}) L^\lambda (\mathbf{x}) + \text{h. c.}] \quad \text{for NC} \]

\[ L^\lambda (\mathbf{x}) = \bar{\psi}_l(\mathbf{x}) \gamma^\lambda (1 - \gamma^5) \psi_\nu(\mathbf{x}) \]
Nuclear Current

\[ J^{CC}_\lambda (x) = V^\pm_\lambda (x) + A^\pm_\lambda (x) \]

\[ J^{NC}_\lambda (x) = V^3_\lambda - 2 \sin^2 \theta_W (V^3_\lambda + V^s_\lambda) + A^3_\lambda \]

\( V(A) \): Vector (Axial) current

\( V^s \): Isoscalar vector current

\( \theta_W \): Weinberg Angle \( \sin^2 \theta_W = 0.23 \)

\[ J_\lambda = (\text{one-body current}) + (\text{two-body exchange current}) \]
Impulse Approximation (IA) Current

\[ < p' | V_\lambda(0) | p > = \bar{u}(p') \left[ f_V \gamma_\lambda + i \frac{f_M}{2M_N} \sigma_{\lambda\rho} q^\rho \right] u(p) \]

\[ < p' | A_\lambda(0) | p > = \bar{u}(p') \left[ f_A \gamma_\lambda \gamma^5 + f_P \gamma^5 q_\lambda \right] u(p) \]

\[ q_\lambda \equiv p'_\lambda - p_\lambda \]

\[ f_M : CVC \quad f_P : PCAC \]

\[ f_A(q^2_\mu) = -g_A \left(1 - \frac{q^2_\mu}{1.04 \text{[GeV}^2\text{]}} \right)^{-2}, \quad g_A = 1.2670 \pm 0.0030 \text{ (PDG)} \]
Exchange axial-vector current

R. Schiavilla et al. PRC 58, 1263 (1998)

- Fit $AN\Delta$ coupling to tritium $\beta$-decay rate
- Rigorous three-body calculation
Why tritium $\beta$ decay?

$\nu d$: Gamow-Teller ($^3S_1 \rightarrow ^1S_0$) $\Rightarrow A_{EXC}$ is main correction

$^3H$: Fermi ($^1S_0 \rightarrow ^1S_0$) & Gamow-Teller

Schiavilla et al., PRC58,1263(1998)
Results

• $d \rightarrow ^1 S_0$ dominance in low-energy region
• confirmation of the past work
\( A_{\text{EXC}} \) contribution

\[ \xi \equiv \frac{\sigma(\text{IA} + A_{\text{EXC}}) - \sigma(\text{IA})}{\sigma(\text{IA})} \]

- 2\% contribution of \( A_{\text{EXC}} \)
- 0.2\% model dependence on \( A_{\text{EXC}} \) (insensitive to detailed structure)
Comparison with EFT results

- **EFT** (Ando et al., PLB 555, 49 (2003))

<table>
<thead>
<tr>
<th>$E_\nu$ (MeV)</th>
<th>$\nu_e d \rightarrow e^- p p$</th>
<th>$\nu d \rightarrow \nu p n$</th>
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</thead>
<tbody>
<tr>
<td>5</td>
<td>1.003</td>
<td>1.004</td>
</tr>
<tr>
<td>10</td>
<td>1.001</td>
<td>1.003</td>
</tr>
<tr>
<td>20</td>
<td>0.998</td>
<td>1.001</td>
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- **KSW**-counting scheme (Butler et al., PRC 63, 035501 (2001))

  1% level agreement if $L_{1,A}$ is appropriately chosen
Angular & energy distributions of electron(neutron) in CC(NC) ($E_\nu$=5MeV)
Strong evidence for neutrino oscillation

Long-standing solar neutrino problem resolved

Theoretical $\nu d$ cross sections played essential role
Summary for Part 1: $\nu d$ reactions

* confirmation of existing work

* update
  
  – high-precision $NN$-potential (AV18, CD-Bonn, Nijmegen)
  
  – exchange axial-vector current tested by tritium $\beta$-decay rate

* differential cross sections

* theoretical uncertainty: $\delta\sigma_{\nu d} \lesssim 1\%$ ($A_{EXC}$, $NN$-model, etc.)

* good agreement with EFT results ($< 1\%$)
Part 2

Neutrino-deuteron reaction as heating mechanism in Supernova

SN et al., PRC 80, 035802 (2009)

In most simulations, supernova doesn’t explode!

⇒ extra assistance needed for re-accelerating shock-wave

* neutrino absorption on nucleon (main)

* neutrino scattering or absorption on nuclei (extra agent)
Heating through neutrino-nucleus scattering

* $^4\text{He}, ^{56}\text{Fe}$  
  Haxton, PRL 60, 1999 (1988)  
  $\Rightarrow$ small effect on supernova dynamics

* $^3\text{He}, ^3\text{H}$  
  O'Connor et al. PRC 75, 055803 (2007)  
  more effective heating than $^4\text{He}$

  Arcones et al. PRC 78, 015806 (2008)  
  $\bar{\nu}$ spectrum can be changed

* deuteron ?  
  can be abundant in supernova, $\sigma_{\nu d} \gg \sigma_{\nu ^3\text{He}}, \sigma_{\nu ^3\text{H}}$
Abundance of light elements in supernova

Nuclear statistical equilibrium is assumed
Quantities of interest for supernova

Thermal average of energy transfer cross section

\[
< \sigma \omega >_{T_\nu} = \int dE_\nu f(T_\nu, E_\nu) \sigma \omega(E_\nu)
\]

Fermi-Dirac distribution for the neutrino

\[
f(T_\nu, E_\nu) = \frac{N}{T_\nu^3} \frac{E_\nu^2}{e^{E_\nu/T_\nu} + 1}
\]

Energy transfer cross section for CC (absorption)

\[
\sigma \omega(E_\nu) = \int dE'_l \frac{d\sigma}{dE'_l} E_\nu
\]

for NC (scattering)

\[
\sigma \omega(E_\nu) = \int dE'_\nu \frac{d\sigma}{dE'_\nu} (E_\nu - E'_\nu)
\]

Only neutrino is treated separately, others are regarded as matter.
Results

Neutrino-deuteron cross sections

\( \sigma \left[ 10^{-42} \text{cm}^2 \right] \)

\( E_\nu \left[ \text{MeV} \right] \)

\( \nu \text{ CC} \)

\( \nu \text{ NC} \)

\( (\nu \text{ N CC}) \)

\[ \sigma(\nu d \text{ CC}) \sim \sigma(\nu N \text{ CC})/3 \quad \text{at } E_\nu = 10 \text{ MeV} \]

\[ \sigma(\nu d \text{ CC}) \sim \sigma(\nu N \text{ CC})/2 \quad \text{at } E_\nu = 50 \text{ MeV} \]

\[ \sigma(\text{elastic } \nu d) \text{ is very small} \]
$E_\nu$-dependence of energy transfer cross section

\[ \sigma \omega (E_\nu) \times f(T, E_\nu) [10^{-42} \text{cm}^2\text{MeV}] \]

* Main contribution is from $E_\nu = 20$ (60) MeV for $T_\nu = 5$ (10) MeV

* High energy tail of $\sigma \omega \times f$ is appreciable
Thermal average of energy transfer cross section

\[ \langle \sigma \omega \rangle / 2A \times 10^{-42} \text{cm}^2 \text{MeV} \]

\[ T_\nu [\text{MeV}] \]

\[ 0 \ 2 \ 4 \ 6 \ 8 \ 10 \]

\[ \langle \sigma \omega \rangle / 2A \times 10^{-42} \text{cm}^2 \text{MeV} \]

\[ T_\nu [\text{MeV}] \]

\[ 0 \ 2 \ 4 \ 6 \ 8 \ 10 \]

\[ \ast \ < \sigma \omega > \text{ for the deuteron is much larger than those of } ^3\text{H}, ^3\text{He}, ^4\text{He} \]

\[ \ast \ \text{Small binding energy} \Rightarrow \text{rapid increase of} < \sigma \omega > \text{ at low } T_\nu \]

\[ \ast \ < \sigma \omega >_{\nu_e d} / < \sigma \omega >_{\nu_e N} \sim 0.44 \text{ at } T_{\nu_e} = 5\text{MeV} \]

\[ \ast \ < \sigma \omega >_{\nu_\mu d} / < \sigma \omega >_{\nu_e N} \sim 0.25 \text{ at } T_{\nu_e} = 5\text{MeV} \text{ and } T_{\nu_\mu} = 10\text{MeV} \]
Neutrino emissivity from deuteron (in progress)

Emission of neutrino in supernova

- cooling of matter (99% of total cooling)

- flux and spectrum of neutrino (SN1987A)

- neutrino heating

Abundance of light elements on surface of protoneutron star

⇒ Careful consideration of $\nu$-emission from deuteron (and other light nuclei)
\( \nu \)-emission previously considered

* \( p + e^- \rightarrow n + \nu_e \)
* \( n + e^+ \rightarrow p + \bar{\nu}_e \)
* \( n + n \rightarrow p + n + e^- + \bar{\nu}_e \)  \hspace{1cm} \text{cooling of neutron star}
* \( p + p \rightarrow p + n + e^+ + \nu_e \)
* \( N + N \rightarrow N + N + \nu + \bar{\nu} \)  \hspace{1cm} \text{dominant source of} \ \nu_\mu, \nu_\tau \)
\[ \nu \text{-emission previously considered} \quad \text{Other, possibly significant processes} \]

\[
\begin{align*}
* \quad & p + e^- \rightarrow n + \nu_e \\
* \quad & n + e^+ \rightarrow p + \bar{\nu}_e \\
* \quad & n + n \rightarrow p + n + e^- + \bar{\nu}_e \\
* \quad & p + p \rightarrow p + n + e^+ + \nu_e \\
* \quad & N + N \rightarrow N + N + \nu + \bar{\nu} \\
\end{align*}
\]

Previous calculation of bremsstrahlung: IA, Born Approx. \( \Rightarrow \) Full calculation
Emissivity $Q$

for, e.g., $N_1 + N_2 \rightarrow N'_1 + N'_2 + \nu + \bar{\nu}$

$$Q = \int \frac{d\mathbf{p}_{N_1}}{(2\pi)^3} \frac{d\mathbf{p}_{N_2}}{(2\pi)^3} \frac{d\mathbf{p}_{N'_1}}{(2\pi)^3} \frac{d\mathbf{p}_{N'_2}}{(2\pi)^3} \frac{d\mathbf{p}_\nu}{(2\pi)^3} \frac{d\mathbf{p}_{\bar{\nu}}}{(2\pi)^3}$$

$$\times (2\pi)^4 \delta^{(4)}(p_f - p_i) \sum_{spin} |M|^2 F_{N_1} F_{N_2} (1 - F_{N'_1})(1 - F_{N'_2})$$

$F_N$ : nucleon distribution function
Total cross sections for $pp \rightarrow pne^+\nu_e$, $pp \rightarrow de^+\nu_e$

* Much larger cross section for $pp \rightarrow de^+\nu_e$

* $A_{EXC}$ increases $\sigma$ by 5, 20, 30% at $T_{pp} = 10, 50, 100$ MeV
Summary for Part 2: $\nu$ heating and emissivity in supernova

Abundance of light elements in supernova

⇒ Careful consideration of $\nu$-emission and absorption on the light elements

Deuteron can play an important role!

- $\nu$-heating much more effective than A=3,4 nuclei
- $\sigma(NN \rightarrow d\nu\bar{\nu})$ (emissivity) much larger than $\sigma(NN \rightarrow NN\nu\bar{\nu})$

⇒ Supernova simulation with mixture of light elements and $\nu$-nucleus interactions