Electroweak radiative corrections to the weak charge of the proton

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Refs: PRD 82 (2010) 013011 (vector hadron correction)
arXiv:1102.5334 (axial-vector hadron correction)
arXiv:1105.0951 (review article)
Parity-violating $e$ scattering

Left-right polarization asymmetry in $\bar{e} \ p \rightarrow e \ p$ scattering

\[ A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s) \]

→ measure interference between e.m. and weak currents

Born (tree) level
Parity-violating $e$ scattering

**Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e\ p$ scattering**

\[ A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right) (A_V + A_A + A_s) \]

→ **in forward limit measures weak charge of proton $Q_W^p$**

\[ A_{PV} \rightarrow \frac{G_F Q_W^p}{4\sqrt{2}\pi\alpha} t \]

\[ k \quad \gamma^* \downarrow q \quad Z \quad k' \approx k \quad \text{forward limit} \]

\[ t = (k - k')^2 \rightarrow 0 \]

\[ s = (k + p)^2 = M(M + 2E) \]
Proton weak charge

- At tree level, $Q_W^p$ gives weak mixing angle:

$$Q_W^p = 1 - 4 \sin^2 \theta_W$$

- Current best values:

$$\sin^2 \theta_W (M_Z^2) = 0.23116(16)$$
$$\sin^2 \theta_W (0) = 0.23867(13)$$

- Scale dependence from radiative effects

- $Q_W^p$ is a small number – sensitive to higher-order corrections

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\[ Q_W^p = 1 - 4 \sin^2 \theta_W \]
Corrections to proton weak charge

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ e \rightarrow e \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ Z \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ Z \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ Z \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ \gamma^* \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ \gamma^* \]
\[ Z \]
\[ p \rightarrow p \]

\[ e \rightarrow e \]
\[ p \rightarrow p \]
\[ \gamma^* \]
\[ \gamma^* \]
\[ p \rightarrow p \]

\( \theta_{LR} = 90\% \)

If one is interested in an even more precise determination of \( \Delta \theta \), the \( \Lambda \) may be possible at a level of \( 0.01 \) or \( 0.001 \). Constraints from \( \alpha(\gamma) \) can be used as a powerful probe for "new physics" effects.

For small \( \mu \), Møller scattering \( \gamma^* \) and \( \gamma^* \) can be parametrized by the four fermion interaction.

Radiative corrections and sin \( 2 \theta \) corrections parametrized by the four fermion interactions must be included in any detailed study.

Here, we comment on the potential very large. A complete calculation has been carried out for both E158 and future experiments.

For scattering, extremely forward events must be detected. For both E158 and future experiments, it was shown that such effects can be used as a powerful probe for \("new physics\) effects.
Corrections to proton weak charge

- including higher order radiative corrections

\[ Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \]

\[ = 0.0713 \pm 0.0008 \]

\[ \text{Erler et al., PRD 72 (2005) 073003} \]

\( \rightarrow \) WW and ZZ box diagrams dominated by short distances, evaluated perturbatively \( (WW \text{ box gives } \sim 25\% \text{ correction!}) \)

\( \rightarrow \) \( \gamma Z \) box diagram sensitive to long distance physics, has two contributions

\[ \Box_{\gamma Z} = \Box_{\gamma Z}^A + \Box_{\gamma Z}^V \]

vector \( e - \) axial \( h \) (finite at \( E=0 \))

axial \( e - \) vector \( h \) (vanishes at \( E=0 \))
Axial $h$ correction

- **axial** $h$ correction $\square_{\gamma Z}^A$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin (1980s) as sum of two parts:

- ★ low-energy part approximated by *Born* contribution (elastic intermediate state)

- ★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

\[
\square_{\gamma Z}^A = \frac{5\alpha}{2\pi} \left(1 - 4\sin^2\theta_W\right) \left[\ln \frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]
\]

\[\approx 0.0052(5) \quad \text{(short-distance)} \quad \text{long-distance} \approx 3/2 \pm 1\]

Axial $h$ correction

- **axial $h$ correction** $\Box_A^{\gamma Z}$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

- repeat calculation using forward dispersion relations with realistic (structure function) input

\[
\begin{align*}
\hat{k} & \quad \approx \quad \hat{k}' \\
\gamma^* & \downarrow q \\
\gamma^* & \downarrow Z \\
\hat{p} & \quad \approx \quad \hat{p}'
\end{align*}
\]

- **axial $h$ contribution** *antisymmetric* under $E' \leftrightarrow -E'$:

\[
\mathbb{R} e \left( \Box_A^{\gamma Z}(E) \right) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im m \left( \Box_A^{\gamma Z}(E') \right)
\]

- **negative energy part** corresponds to crossed box (crossing symmetry $s \rightarrow u$)
Axial $h$ correction

- imaginary part given by interference $F_{3}^{\gamma\pi}$ structure function

\[
\mathcal{I}m \left[ \Box^{A}_{\gamma Z}(E) \right] = \frac{1}{(2ME)^{2}} \int_{M^{2}}^{s} dW^{2} \int_{0}^{Q_{\text{max}}^{2}} dQ^{2} \frac{\nu_{e}(Q^{2}) \alpha(Q^{2})}{1 + Q^{2}/M_{Z}^{2}} \\
\times \left( \frac{2ME}{W^{2} - M^{2} + Q^{2}} - \frac{1}{2} \right) F_{3}^{\gamma\pi}
\]

with $\nu_{e}(Q^{2}) = 1 - 4\kappa(Q^{2}) \sin^{2} \theta_{W}(Q^{2})$

→ scale dependence of $\nu_{e}, \alpha$ given by vacuum polarization corrections, e.g.

\[
\frac{\alpha}{\alpha(Q^{2})} = 1 - \Delta \alpha_{\text{lep}}(Q^{2}) - \Delta \alpha_{\text{had}}^{(5)}(Q^{2})
\]

$\alpha^{-1}(M_{Z}^{2}) = 128.94$

... similarly for weak charges
Axial $h$ correction

- **elastic part**  \[ F_3^{\gamma Z^{(el)}} = -Q^2 G^p_M(Q^2) G^Z_A(Q^2) \delta(W^2 - M^2) \]

- **resonance part** from parametrization of $\nu$ scattering data
  (includes lowest four spin-1/2 and 3/2 states)  
  \[ \text{Lalakulich, Paschos (2006)} \]
**Axial $h$ correction**

- **DIS** part dominated by leading twist PDFs at high $W$ (small $x$)

\[ e.g. \text{ at LO, } F_3^{\gamma Z(DIS)} = \sum_q 2e_q g_A^q (q(x, Q^2) - \bar{q}(x, Q^2)) \]

\[ \Rightarrow \text{ switching order of integration (energy integral analytic!), expand integrand in } 1/Q^2 \text{ in DIS region } (Q^2 \gtrsim 1 \text{ GeV}^2) \]

\[ \Re e \square_{\gamma Z}^{A(DIS)}(E) = \frac{3}{2\pi} \int_{Q_0^2}^{\infty} dQ^2 \frac{v_e(Q^2) \alpha(Q^2)}{1 + Q^2/M_Z^2} \]

\[ \times \left[ M_3^{\gamma Z(1)} - \frac{2M_Z^2}{9Q^4} (5E^2 - 3Q^2) M_3^{\gamma Z(3)} \right] \]

with moments

\[ M_3^{\gamma Z(n)}(Q^2) = \int_0^1 dx x^{n-1} F_3^{\gamma Z}(x, Q^2) \]
Axial $h$ correction

- structure function moments

\[ n = 1 \quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) \]

\[ \rightarrow \gamma Z \text{ analog of Gross-Llewellyn Smith sum rule} \]

\[ \text{Re} \begin{bmatrix} A^{(\text{DIS})} \end{bmatrix}_{\gamma Z} \approx (1 - 4\hat{s}^2) \frac{5\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2(1+Q^2/M_Z^2)} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) \]

\[ \rightarrow \text{precisely result from Marciano & Sirlin!} \]

(works because result depends on lowest moment of \textit{valence} PDF, with \textit{model-independent} normalization!)

\[ n = 3 \quad M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3} \left( 2\langle x^2 \rangle_u + \langle x^2 \rangle_d \right) \left( 1 + \frac{5\alpha_s(Q^2)}{12\pi} \right) \]

\[ \rightarrow \text{related to } x^2 \text{-weighted moment of valence quarks} \]
Axial $h$ correction

“DIS” region at $Q^2 < 1 \text{ GeV}^2$ does not afford PDF description

→ in absence of data, consider models with general constraints

★ $F_3^{\gamma Z}(x_{\text{max}}, Q^2)$ should not diverge in limit $Q^2 \to 0$

★ $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 = 1 \text{ GeV}^2$

Model 1

\[ F_3^{\gamma Z}(x, Q^2) = \left( \frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2} \right) F_3^{\gamma Z}(x, Q_0^2) \]

\[ F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0 \]

Model 2

$F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all $W^2$

\[ F_3^{\gamma Z} \text{ finite as } Q^2 \to 0 \]
Axial $h$ correction

$\text{Re} \frac{A}{\gamma Z} (E) \times 10^{-4}$

- elastic
- resonance
- DIS ($Q^2 < 1$), Model 1
- DIS ($Q^2 < 1$), Model 2
- DIS ($Q^2 > 1, n \geq 3$)

$E$ (GeV)

Blunden, WM, Thomas (2011)

→ dominated by $n = 1$ DIS moment: $32.8 \times 10^{-4}$
(weak $E$ dependence)
Axial $h$ correction

→ correction at $E = 0$

$$\Re e \Box_A^{\gamma Z} = 0.00064 + 0.00023 + 0.00350 \rightarrow 0.0044(4)$$

↑ elastic  ↑ resonance  ↑ DIS

→ correction at $E = 1.165 \text{ GeV}$ (Qweak)

$$\Re e \Box_A^{\gamma Z} = 0.00005 + 0.00011 + 0.00352 = 0.0037(4)$$

\textit{cf. MS value: 0.0052(5) (~1\% shift in $Q_W^p$)}

→ shifts $Q_W^p$ from $0.0713(8) \rightarrow 0.0705(8)$
Vector $h$ correction

- vector $h$ correction $\square_{\gamma Z}^V$ vanishes at $E = 0$, but experiment has $E \sim 1$ GeV – what is energy dependence?

→ forward dispersion relation

★ $\Re \square_{\gamma Z}^V(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E' r^2 - E^2} \Im m \square_{\gamma Z}^V(E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector $h$ contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m \square_{\gamma Z}^V(E) = \frac{\alpha}{(s - M^2)^2} \int_{W^2_\pi}^s dW^2 \int_0^{Q_{\max}^2} dQ^2 \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left( F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s (Q_{\max}^2 - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

factor 2 larger than GH; confirmed by Rislow & Carlson, arXiv:1011:2397 [hep-ph]

Gorchtein, Horowitz, PRL 102 (2009) 091806
Vector $h$ correction

$\rightarrow F_{1,2}^{\gamma Z}$ structure functions

★ parton model for DIS region

\[ F_2^{\gamma Z} = 2x \sum_1^q e_q g_V^q (q + \bar{q}) = 2x F_1^{\gamma Z} \]

$\rightarrow F_2^{\gamma Z} \approx F_2^\gamma$ good approximation at low $x$

$\rightarrow$ provides upper limit at large $x$ $(F_2^{\gamma Z} \lessapprox F_2^\gamma)$

★ in resonance region use phenomenological input for $F_2$, empirical (SLAC) fit for $R$

$\rightarrow$ for transitions to $I = 3/2$ states (e.g. $\Delta$), CVC and isospin symmetry give

\[ F_i^{\gamma Z} = (1 + Q_W^p) F_i^\gamma \]

$\rightarrow$ for transitions to $I = 1/2$ states, SU(6) wave functions predict $Z$ & $\gamma$ transition couplings equal to a few %
Vector $h$ correction

→ compare structure function input with data

low $W$  

![Graphs showing $F_2(Q^2, W)$ at $Q^2 = 1.5$ GeV$^2$ and $Q^2 = 3.5$ GeV$^2$ for low and high $W$.]

high $W$

GVMD model (used as input by Gorchtein & Horowitz)
Vector $h$ correction

$\Re e \Box_{\gamma Z}^V = 0.0047^{+0.0011}_{-0.0004}$

or $6.6^{+1.5}_{-0.6} \%$ of uncorrected $Q_W^p$

Sibirtsev, Blunden, WM, Thomas
PRD 82 (2010) 013011
Qweak

\[
\mathcal{R} \delta_{\gamma Z} = \mathcal{R} \mathbb{V}_{\gamma Z} / Q_{W}^p \approx 6\%
\]

mostly from high-\( W \)
(“Regge”) contribution

→ our formula for \( s m \mathbb{V}_{\gamma Z} \) factor 2 larger*
(“nuclear physics” vs. “particle physics” conventions for weak charges in structure function definitions?)

→ GH omit factor \((1-x)\) in definition of \( F_{1,2} \)
(\( \sim 30\% \) enhancement)

→ GH use \( Q_{W}^p \sim 0.05 \) cf. \( \sim 0.07 \)
(\( \sim 40\% \) enhancement)

→ numerical agreement for \( \delta_{\gamma Z}^V \) coincidental (?)

* confirmed by Rislow/Carlson arXiv:1011.2397
Combined vector and axial $h$ correction

$$Q_W^p = 0.0713 \rightarrow 0.0705$$
(at $E=0$)

At $E=1.165$ GeV
$E$-dependent correction is $+0.0040$
Combined vector and axial $h$ correction

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q^p_W = \pm 0.003$

$\Delta \sin^2 \theta_W \approx 0.0013$

* 4% measurement of $Q^p_W$

Bentz et al., PLB 693 (2010) 462
Extrapolation from $t = -0.03$ GeV$^2$ to $t = 0$

→ phenomenological ansatz

\[ \square \gamma Z(E, t) = \square \gamma Z(0, 0) \frac{e^{-B|t|/2}}{F_{1\gamma p}(t)} \]

with $B = (7 \pm 1)$ GeV$^{-2}$ from forward Compton scattering

*Gorchtein, Horowitz, PRL 102 (2009) 091806*

→ $\sim 2\%$ reduction of $\square \gamma Z$
Summary

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\%$, cf. PDG
  
  $\implies$ would significantly shift extracted weak angle
  
  $\implies$ better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$
  
  (e.g. in PVDIS at JLab)

- New formulation in terms of moments of structure functions
  
  $\implies$ places on firm footing earlier derivation of Marciano/Sirlin in “free quark model”
  
  $\implies$ may have some effect on atomic PV predictions (e.g. Cs, Fr)
The End