Duality: Towards Bridging the Quark-Hadron Chasm

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Outline

- Quark-hadron duality: historical perspective

- Duality and Quantum ChromoDynamics (QCD)
  - twists and moments
  - insights from nonperturbative models

- Implications of duality for quark distributions

- Outlook
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Pentaquark Summary

- Existence or otherwise is a CRUCIAL question in strong interaction physics.
- Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD.
- Jefferson Lab is the ideal facility to definitively answer this question!

strength of nuclear force acting between quarks given by
\[ \alpha_{\text{QCD}} \] (or \[ \alpha_s \])
Existence or otherwise is a CRUCIAL question in strong interaction physics.

Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD.

Jefferson Lab is the ideal facility to definitively answer this question! Nobel Prize (2004) for discovery of “asymptotic freedom”

Physical observables described in terms of quarks and gluons:

"strong" coupling constant $\alpha_s$ small, calculate using perturbation theory.
Operated by the Southeastern Universities Research Association for the U.S. Department of Energy.

Thomas Jefferson National Accelerator Facility

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**Diagram Notes**

- $\alpha_s$ large, cannot describe observables in terms of quarks perturbatively (need nonperturbative methods e.g. lattice QCD)

- Meson & baryon (hadron) degrees of freedom prominent (e.g. chiral effective field theory)
Existence or otherwise is a CRUCIAL question in strong interaction physics.

Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD.

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“quarks”

low energy long distance

high energy short distance
Looking for quarks in hadrons is like looking for the Mafia in Sicily – everybody *knows* they’re there, but it’s hard to find the evidence!

Anonymous
one way of seeing connections...

"QUARKS, NEUTRINOS, MESONS, ALL THOSE DAMN PARTICLES YOU CAN'T SEE. THAT'S WHAT DROVE ME TO DRINK. BUT NOW I CAN SEE THEM."
Quark-hadron duality

Complementarity between *quark* and *hadron* descriptions of observables

\[ \sum_{\text{hadrons}} = \sum_{\text{quarks}} \]

Can use either set of *complete* basis states to describe physical phenomena
Duality in hadron-hadron scattering

\[ p_{\text{Lab}} \Delta \sigma \]

\[(\text{mb GeV})\]

\[\sum R(s) \approx \sum j \alpha_j(t)\]
Duality in electron-hadron scattering

“Bloom-Gilman duality”

\[
\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \; \nu W_2(\nu, Q^2) = \int_1^{\omega_m'} d\omega' \; \nu W_2(\omega')
\]

“hadrons”

“quarks”
**Electron-nucleon scattering**

**Inclusive cross section for** \( eN \rightarrow eX \)

\[
\frac{d^2 \sigma}{d \Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2 \frac{\theta}{2}}{Q^4} \left( 2 \tan^2 \frac{\theta}{2} \frac{F_1}{2M} + \frac{F_2}{\nu} \right)
\]

\[\begin{align*}
\nu &= E - E' \\
Q^2 &= \vec{q}^2 - \nu^2 = 4EE' \sin^2 \frac{\theta}{2}
\end{align*}\]

\[x = \frac{Q^2}{2M\nu}\]

**Bjorken scaling variable**

**\( F_1, F_2 \) “structure functions”**

\[\rightarrow\] contain all information about structure of nucleon

\[\rightarrow\] functions of \( x, Q^2 \) in general
Electron-nucleon scattering

Bjorken variable in terms of $Q^2$ & $W$:

$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$
In deep inelastic region \((W \gtrsim 2\ \text{GeV}, \ Q^2 \gtrsim 1\ \text{GeV}^2)\), structure function given by quark and antiquark ("parton") distributions

\[
F_2(x, Q^2) = x \sum_q e_q^2 \ q(x, Q^2)
\]

\[
= \frac{4}{9}x(u + \bar{u}) + \frac{1}{9}x(d + \bar{d}) + \frac{1}{9}x(s + \bar{s}) + \cdots
\]

\[\rightarrow q(x, Q^2) = \text{probability to find quark type "}q\text{" in nucleon, carrying momentum fraction} \ x\]
In deep inelastic region \((W \gtrsim 2 \text{ GeV}, \ Q^2 \gtrsim 1 \text{ GeV}^2)\), structure function given by quark and antiquark ("parton") distributions

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\]

→ \(q(x, Q^2) = \) probability to find quark type "q" in nucleon, carrying momentum fraction \(x\)

In resonance region \((W \lesssim 2 \text{ GeV})\), or at low \(Q^2 \ (Q^2 \lesssim 1 \text{ GeV}^2)\) can no longer resolve individual quark structure

→ see gross features of hadron (complex, multi-parton effects)
Duality in electron-hadron scattering

average over (strongly $Q^2$ dependent) resonances

$\approx Q^2$ independent scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

Niculescu et al., PRL 85, 1182 (2000)
Duality in electron-hadron scattering

\[ \Delta \]

\[ S_{11} \]

also exists locally in individual resonance regions
Duality in QCD era

- Operator product expansion

  expand moments of structure functions in powers of $1/Q^2$

\[ M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) \]

\[ = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]
Duality in QCD era

- **Operator product expansion**

  - expand *moments* of structure functions in powers of $1/Q^2$

  \[
  M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) \\
  = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots
  \]

  - matrix elements of operators with specific “twist” $\tau$

  \[
  \tau = \text{dimension} - \text{spin}
  \]

  - $\tau = 2$
  - $\tau > 2$
Duality in QCD era

\[ \tau = 2 \]

single quark scattering

e.g. \( \bar{\psi} \gamma_\mu \psi \)

\[ \tau > 2 \]

qq and qg correlations

e.g. \( \bar{\psi} \gamma_\mu \psi \bar{\psi} \gamma_\nu \psi \)

or \( \bar{\psi} \tilde{G}_{\mu\nu} \gamma^\nu \psi \)
Duality in QCD era

- **Operator product expansion**
  - expand *moments* of structure functions in powers of $1/Q^2$

  \[
  M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots
  \]

- If moment $\approx$ independent of $Q^2$
  - higher twist terms $A_n^{(\tau>2)}$ small

- Duality $\leftrightarrow$ suppression of higher twists

*de Rujula, Georgi, Politzer*  
*Ann. Phys.** 103, 315 (1975)*
Truncated moments

Seldom have sufficient data to form complete moments

$\Rightarrow$ usually require $x \to 0$ and $x \to 1$ extrapolations
Truncated moments

- Seldom have sufficient data to form complete moments
  - usually require $x \to 0$ and $x \to 1$ extrapolations

- *Truncated* moments allow study of restricted regions in $x$ (or $W$) within pQCD in well-defined, systematic way

\[
\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \ x^{n-2} \ F_2(x, Q^2)
\]
Truncated moments

- Seldom have sufficient data to form complete moments
  - usually require \( x \to 0 \) and \( x \to 1 \) extrapolations

- Truncated moments allow study of restricted regions in \( x \) (or \( W \)) within pQCD in well-defined, systematic way

\[
\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx \; x^{n-2} \; F_2(x, Q^2)
\]

- Obey DGLAP-like evolution equations, similar to PDFs

\[
\frac{d\overline{M}_n(\Delta x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \left( P_{(n)}' \otimes \overline{M}_n \right)(\Delta x, Q^2)
\]

\[ P_{(n)}'(z, \alpha_s) = z^n \; P_{NS,S}(z, \alpha_s) \]

truncated splitting function

Forte, Magnea, PLB 448, 295 (1999)
Kotlorz, Kotlorz, PLB 644, 284 (2007)
Truncated moments

- Follow evolution of specific resonance (region) with $Q^2$ in pQCD framework

how much of this region is leading twist?
Analysis of JLab $F_2^p$ resonance region data

Psaker, WM, Christy, Keppel
PRC 78, 025206 (2008)
Analysis of JLab $F_2^p$ resonance region data

higher twists $< 10\text{–}15\%$ for $Q^2 > 1\text{ GeV}^2$
Resonances & twists

- Total higher twist "small" at scales $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$

- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
  
  $\rightarrow$ nontrivial interference between resonances
Resonances & twists

- Total higher twist "small" at scales $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$

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- Can we understand this dynamically, at quark level?
  $\rightarrow$ is duality an accident?

- Can we use resonance region data to learn about leading twist structure functions?
  $\rightarrow$ expanded data set has potentially significant implications for global PDF studies
Consider simple quark model with spin-flavor symmetric wave function

**Form factors**

\[ d\sigma \sim \left( \sum_i e_i \right)^2 \]

**Coherent scattering from quarks**

**Structure functions**

\[ d\sigma \sim \sum_i e_i^2 \]

**Incoherent scattering from quarks**
Consider simple quark model with spin-flavor symmetric wave function

form factors
- \textit{coherent} scattering from quarks \hspace{1cm} \begin{align*}
    d\sigma & \sim \left( \sum_i e_i \right)^2
\end{align*}

structure functions
- \textit{incoherent} scattering from quarks \hspace{1cm} \begin{align*}
    d\sigma & \sim \sum_i e_i^2
\end{align*}

For duality to work, these must be equal
- \textit{how can square of a sum become sum of squares?}
Accidental cancellations of charges?

**cat’s ears diagram** (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i e_j \sim \left( \sum_i e_i \right)^2 - \sum_i e_i^2$$

- coherent
- incoherent
Accidental cancellations of charges?

**cat’s ears diagram** *(4-fermion higher twist \( \sim 1/Q^2 \))*

\[
\alpha \sum_{i \neq j} e_i e_j \sim \left( \sum_i e_i \right)^2 - \sum_i e_i^2
\]

\[\uparrow\text{coherent}\quad \uparrow\text{incoherent}\]

**proton**

\[
HT \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0!
\]

**neutron**

\[
HT \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0
\]

\[\rightarrow \text{duality in proton a } \textit{coincidence!}\]

\[\rightarrow \text{should } \textit{not} \text{ hold for neutron}\]

Brodsky hep-ph/0006310
Dynamical cancellations?

→ e.g. for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

\[ F(\nu, q^2) \sim \sum_n |G_{0,n}(q^2)|^2 \delta(E_n - E_0 - \nu) \]

→ charge operator \( \sum_i e_i \exp(iq \cdot r_i) \) excites

  * odd partial waves with strength \( \propto (e_1 - e_2)^2 \)
  * even partial waves with strength \( \propto (e_1 + e_2)^2 \)
Dynamical cancellations?

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odd partial waves with strength \( \propto (e_1 - e_2)^2 \)

→ resulting structure function

\[ F(\nu, q^2) \sim \sum_n \{(e_1 + e_2)^2 G_{0,2n}^2 + (e_1 - e_2)^2 G_{0,2n+1}^2 \} \]

→ if states degenerate, cross terms \( \sim e_1 e_2 \) cancel when averaged over nearby even and odd parity states
Dynamical cancellations?

→ duality is realized by summing over at least one complete set of even and odd parity resonances *

*Close, Isgur, PLB 509, 81 (2001)*

→ in NR Quark Model, even & odd parity states generalize to 56 (\(L=0\)) and 70 (\(L=1\)) multiplets of spin-flavor SU(6)

- assume magnetic coupling of photon to quarks (better approximation at high \(Q^2\))
- in this limit Callan-Gross relation valid \(F_2 = 2x F_1\)
- structure function given by squared sum of transition FFs

\[
F_1(\nu, q^2) \sim \sum_R |F_{N \to R}(q^2)|^2 \delta(E_R - E_N - \nu)
\]

* realized in many models: ‘t Hooft model, large \(N_c\), RQM, ... see WM et al., Phys. Rep. 406, 127 (2005)
Dynamical cancellations?

→ duality is realized by summing over at least one complete set of even and odd parity resonances

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→ in NR Quark Model, even & odd parity states generalize to 56 ($L=0$) and 70 ($L=1$) multiplets of spin-flavor SU(6)

<table>
<thead>
<tr>
<th>representation</th>
<th>$^2$8[56$^+$]</th>
<th>$^4$10[56$^+$]</th>
<th>$^2$8[70$^-$]</th>
<th>$^4$8[70$^-$]</th>
<th>$^2$10[70$^-$]</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9\rho^2$</td>
<td>$8\lambda^2$</td>
<td>$9\rho^2$</td>
<td>0</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 + 9\lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$8\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$4\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 + 27\lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda \ (\rho)$ = (anti) symmetric component of ground state wfn.

SU(6) limit $\lambda = \rho$

$\longrightarrow$ relative strengths of $N \rightarrow N^*$ transitions:

<table>
<thead>
<tr>
<th>SU(6) :</th>
<th>$[56, 0^+]^2 8$</th>
<th>$[56, 0^+]^4 10$</th>
<th>$[70, 1^-]^2 8$</th>
<th>$[70, 1^-]^4 8$</th>
<th>$[70, 1^-]^2 10$</th>
<th>$\text{total}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>18</td>
</tr>
</tbody>
</table>

$\longrightarrow$ summing over all resonances in $56^+$ and $70^-$ multiplets

$$\frac{F_1^n}{F_1^p} = \frac{18}{27} = \frac{2}{3}$$

$\longrightarrow$ at the quark level, $n/p$ ratio is

$$\frac{F_1^n}{F_1^p} = \frac{4d + u}{d + 4u} = \frac{6}{9} = \frac{2}{3} \quad \text{if} \quad u = 2d$$
Comparison with data

- **Proton** data expected to *overestimate* DIS function in 2nd and 3rd resonance regions (odd parity states)

Data exceeds DIS function

\[ \int \frac{F_2^p(\text{data})}{F_2^p(\text{ALEKHIN})} \, dx \]

- **1st**
- **2nd**
- **3rd**
- **global**

\( Q^2 (\text{GeV}^2) \)

\( W < 2 \text{ GeV} \)

Malace et al., PRC 80, 035207 (2009)

→ duality violation for proton \( \lesssim 10\% \), integrated over \( x \)
Comparison with data

- **Duality in neutron** not tested because of absence of free neutron targets

- **New extraction method** (using iterative procedure for solving integral convolution equations) has allowed first determination of $F_2^n$ in resonance region & test of neutron duality

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*Kahn, WM, Kulagin*
*PRC 79, 035205 (2009)*

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*Malace, Kahn, WM, Keppel*
*PRL 104, 102001 (2010)*
Comparison with data

- Neutron data expected to lie below DIS function in 2nd region

→ "theory": fit to $W > 2$ GeV data
  Alekhin et al., 0908.2762 [hep-ph]

→ locally, violations of duality in resonance regions $< 15\% - 20\%$
  (largest in $\Delta$ region)

→ globally, violations $< 10\%$

Malace, Kahn, WM, Keppel
PRL 104, 102001 (2010)

duality is not accidental, but a general feature of resonance-scaling transition!
Comparison with data

- Neutron data expected to lie below DIS function in 2nd region

→ “theory”: fit to $W > 2$ GeV data
  
  \textbf{Alekhin et al., 0908.2762 [hep-ph]}

→ \textit{locally}, violations of duality in resonance regions < 15–20% (largest in $\Delta$ region)

→ \textit{globally}, violations < 10%

\textbf{Malace, Kahn, WM, Keppel}

\textbf{PRL 104, 102001 (2010)}

\rightarrow use resonance region data to learn about \textit{leading twist} structure functions?
CTEQ6X global PDF analysis

- New global QCD (next-to-leading order) analysis of expanded set of $p$ and $d$ data, including large-$x$, low-$Q^2$ region
  \[ \rightarrow \] joint JLab-CTEQ theory/experiment collaboration (with Hampton, FSU, FNAL, Duke)

- Systematically study effects of $Q^2$ & $W$ cuts
  \[ \rightarrow \] as low as $Q \sim m_c$ and $W \sim 1.7$ GeV

- Include large-$x$ corrections
  \[ \rightarrow \] TMCs & higher twists $F_2(x, Q^2) = F_2^{LT}(x, Q^2)(1 + C(x)/Q^2)$
  \[ \rightarrow \] realistic nuclear effects in deuteron (binding + off-shell)
  (most analyses assume no nuclear corrections)
CTEQ6X – kinematic cuts

- **cut0**: $Q^2 > 4 \text{ GeV}^2$, \ $W^2 > 12.25 \text{ GeV}^2$
- **cut1**: $Q^2 > 3 \text{ GeV}^2$, \ $W^2 > 8 \text{ GeV}^2$
- **cut2**: $Q^2 > 2 \text{ GeV}^2$, \ $W^2 > 4 \text{ GeV}^2$
- **cut3**: $Q^2 > m_c^2$, \ $W^2 > 3 \text{ GeV}^2$

**factor 2 increase in DIS data from cut0 → cut3**
Systematically reduce $Q^2$ and $W$ cuts, including TMC, HT & nuclear corrections

$\rightarrow$ stable with respect to cut reduction

$\rightarrow$ $d$ quark suppressed by $\sim 50\%$ for $x > 0.5$

(driven by nuclear corrections)

CTEQ6X – $1/Q^2$ corrections

→ important interplay between TMCs and higher twist: HT alone cannot accommodate full $Q^2$ dependence

→ stable leading twist when both TMCs and HTs included
CTEQ6X – final PDF results

\[ \frac{u}{u_{\text{CTEQ6.1}}} \]

\[ \frac{d}{d_{\text{CTEQ6.1}}} \]

\[ Q^2 = 10 \text{ GeV}^2 \]

→ full fits favors smaller \( d/u \) ratio

(CTEQ6.1 had no nuclear or TMC/HT corrections)
CTEQ6X - final PDF results

- full fits favors smaller d/u ratio
  
  (CTEQ6.1 had no nuclear or TMC/HT corrections)

- up to 40-60% reduced errors with weaker cuts extending into resonance region

Summary

- Remarkable confirmation of quark-hadron duality in proton and neutron structure functions
  - duality-violating higher twists \( \sim 10-15\% \) in few-GeV range

- Confirmation of duality in neutron suggests origin in dynamical cancellations of higher twists
  - duality not due to accidental cancellations of quark charges

- Practical application of duality
  - use resonance region data to constrain leading twist PDFs
  - stable fits at low \( Q^2 \) and large \( x \) with significantly reduced uncertainties
The End