Weak charge of the proton: loop corrections to parity-violating electron scattering

Wally Melnitchouk

Jefferson Lab

collaborators: P. Blunden, A. Sibirtsev, A. Thomas, J. Tjon
Outline

- Parity-violating elastic $ep$ scattering (PVES)
  - strange form factors of the proton

- Two-boson exchange corrections
  - $\gamma Z$ box diagrams

- Weak charge of the proton $Q_W^p$
  - dispersive corrections for JLab’s “Qweak” experiment
Parity-violating elastic $ep$ scattering
Parity-violating $e$ scattering

Two linear combinations of $G^{u,d,s}$:

\[ G^{\gamma p} = \frac{2}{3} G^u - \frac{1}{3} G^d - \frac{1}{3} G^s \]
\[ G^{\gamma n} = \frac{2}{3} G^d - \frac{1}{3} G^u - \frac{1}{3} G^s \]

Third combination from PVES:

\[ G^{Zp} = g^u_V G^u + g^d_V G^d + g^s_V G^s \]

\[ g^u_V = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \]
\[ g^{d,s}_V = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \]

Note: PDG definition (factor 1/2 cf. nuclear physics definition)!
Parity-violating $e$ scattering

**Electromagnetic Born amplitude**

$$\mathcal{M}_\gamma = -\frac{e^2}{q^2} j_\gamma^\mu J_{\gamma\mu}$$

$$e = \sqrt{4\pi\alpha}$$

**Weak neutral current Born amplitude**

$$\mathcal{M}_Z = -\frac{g^2}{(4\cos\theta_W)^2} \frac{1}{M_Z^2 - q^2} j_Z^\mu J_{Z\mu}$$

$$\approx -\frac{G_F}{2\sqrt{2}} j_Z^\mu J_{Z\mu}$$

$$g = \frac{e}{\sin^2\theta_W}$$

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_Z^2 \sin^2\theta_W \cos^2\theta_W}$$

\[ q^2 = (k - k')^2 = -t \]
Parity-violating $e$ scattering

- Electroweak lepton currents

\[
j^\mu_\gamma = \bar{u}_e(k') \gamma^\mu u_e(k) \\
j^\mu_Z = \bar{u}_e(k')(g^e_V \gamma^\mu + g^e_A \gamma_5) u_e(k)
\]

\[
g^e_A = -\frac{1}{2}, \quad g^e_V = -\frac{1}{2}(1 - 4 \sin^2 \theta_W)
\]

- Hadronic currents

\[
J^\mu_{\gamma,Z} = \bar{u}_N(p') \Gamma^\mu_{\gamma,Z} u_N(p)
\]

\[
\Gamma^\mu_\gamma = \gamma^\mu F^\gamma_1 + \frac{i \sigma^{\mu\nu} q_\nu}{2M} F^\gamma_2 \\
\Gamma^\mu_Z = \gamma^\mu F^Z_1 + \frac{i \sigma^{\mu\nu} q_\nu}{2M} F^Z_2 + \gamma^\mu \gamma_5 G^Z_A
\]
Parity-violating $e$ scattering

- Born cross section

\[ \frac{d\sigma}{d\Omega} = \left( \frac{\alpha}{4MQ^2} \frac{E'}{E} \right)^2 |M|^2 \]

where total squared amplitude is

\[ |M|^2 = |M_\gamma|^2 + 2 \Re (M_\gamma^* M_Z) + |M_Z|^2 \]

- P-conserving
- P-violating $\gamma Z$ interference
- negligible
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\bar{e} \, p \rightarrow e \, p$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \alpha} \right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

Born (tree) level
Parity-violating $e$ scattering

- **Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e \ p$ scattering**

  \[
  A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \alpha} \right) (A_V + A_A + A_s)
  \]

  measure interference between e.m. and weak currents

  vector asymmetry
  \[
  A_V = g_A^e \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - (\epsilon G_E^\gamma p G_E^\gamma n + \tau G_M^\gamma p G_M^\gamma n) / \sigma^\gamma p \right]
  \]

  axial vector asymmetry
  \[
  A_A = g_V^e \sqrt{\tau (1 + \tau) (1 - \epsilon^2)} \, \tilde{G}_A^Z \gamma p G_M^\gamma p / \sigma^\gamma p
  \]

  strange asymmetry
  \[
  A_s = -g_A^e \rho \left( \epsilon G_E^\gamma p G_E^s + \tau G_M^\gamma p G_M^s \right) / \sigma^\gamma p
  \]
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e \ p$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \alpha} \right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

vector asymmetry

$$A_V = g^e_A \rho \left[ (1 - 4\kappa \sin^2 \theta_W) - (\varepsilon G^\gamma p_E G^\gamma n_E + \tau G^\gamma p_M G^\gamma n_M) / \sigma^\gamma p \right]$$

radiative corrections, including TBE

using isospin to relate weak and e.m. form factors

$$G_{E,M}^Z p = (1 - 4 \sin^2 \theta_W) G_{E,M}^\gamma p - G_{E,M}^\gamma n - G_{E,M}^s$$
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e \ p$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

Axial vector asymmetry

$$A_A = g_V^e \sqrt{\tau(1 + \tau)(1 - \varepsilon^2)} \ \tilde{G}_A^{Zp} G_{\gamma p}^\gamma / \sigma_{\gamma p}$$

Insensitive to axial contribution at forward angles ($\varepsilon \rightarrow 1$)
Parity-violating $e$ scattering

- Left-right polarization asymmetry in $\vec{e} \ p \rightarrow e \ p$ scattering

$$A_{PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = - \left( \frac{G_F Q^2}{4\sqrt{2}\alpha} \right) (A_V + A_A + A_s)$$

→ measure interference between e.m. and weak currents

extract strange electric & magnetic form factors

strange asymmetry

$$A_s = -g_A^e \rho \left( \varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s \right) / \sigma^{\gamma p}$$
Young et al., PRL 97 (2006) 102002

\[ Q^2 \sim 0.04 - 0.5 \text{ GeV}^2 \]
PVES experiments

Armstrong et al., *PRL* 95 (2005) 092001
(forward angle G0)

Androic et al., *PRL* 104 (2010) 012001
(backward angle G0)
PVES global analysis

Young, Roche, Carlini, Thomas
PRL 97 (2006) 102002

Liu, McKeown, Ramsey-Musolf
PRC 76 (2007) 025202

\[ G_E^s = +0.0025 \pm 0.0182 \]
\[ G_M^s = -0.011 \pm 0.254 \]

\[ Q^2 = 0.1 \text{ GeV}^2 \]

\[ G_E^s = -0.008 \pm 0.016 \]
\[ G_M^s = +0.29 \pm 0.21 \]

→ strange form factors small (analyses compatible)

→ how important are higher order (e.g. $\gamma Z$) corrections?
Two-boson exchange corrections
Two-photon exchange corrections

Calculation uses same framework as that for computing two-photon exchange corrections to e.m. form factors

\[ \delta^{(2\gamma)} = \frac{2 \text{Re} \left\{ M_0^\dagger M_{\gamma\gamma} \right\}}{|M_0|^2} \]

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} + \text{crossed box} \]

\[ N(l) = \bar{u}(k')\gamma_\mu (\not{k} - \not{l} + m_e)\gamma_\nu u(k) \bar{u}(p')\Gamma^\mu (q - l)(\not{\phi} + \not{l} + M)\Gamma^\nu (l)u(p) \]

\[ D(l) = (l^2 - \lambda^2)((l - q)^2 - \lambda^2)((k - l)^2 - m_e^2)((p + l)^2 - M^2) \]

\[ \lambda(\to 0) = \text{infrared regulator} \]
Two-photon exchange corrections

“exact” evaluation of integrals including form factors (Veltman-Passarino functions)

\[ D(l) = (l^2 - \lambda^2)((l - q)^2 - \lambda^2)((k - l)^2 - m_e^2)((p + l)^2 - M^2) \]

\[ \lambda(\to 0) = \text{infrared regulator} \]

\[ \mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} + \text{crossed box} \]

\[ N(l) = \bar{u}(k')\gamma_\mu(k - l + m_e)\gamma_\nu u(k) \bar{u}(p')\Gamma^\mu(q - l)(\not{p} + \not{l} + M)\Gamma^\nu(l)u(p) \]

\[ Mo, Tsai (1969) \]

\[ \text{cf. soft photon approximation (used in most data analyses!)} \]

which assumes pole dominance of TPE amplitude & neglects nucleon structure \( N(l) \approx N(0) \)

Mo, Tsai (1969)
Two-photon exchange corrections

- calculation uses same framework as that for computing two-photon exchange corrections to e.m. form factors

\[ \delta_{\text{full}}^{(2\gamma)} - \delta_{\text{Mo-Tsai}}^{(2\gamma)} \]

\[ \Delta(\varepsilon, Q^2) \]

- few % magnitude, non-linear in \( \varepsilon \), positive slope
- does not depend strongly on vertex form factors
Two-photon exchange corrections

Rosenbluth separation

polarization transfer

with TPE correction

→ significant effect

→ resolves discrepancy (within errors)

Arrington, WM, Tjon, PRC 76 (2007) 035205
Two-photon exchange corrections

$\gamma (2\gamma)$ exchange changes sign (invariant) under $e^+ \leftrightarrow e^-$

**Very preliminary Novosibirsk data**

Arrington, Holt et al. (2010)
Two-boson exchange corrections

\[ A_{\text{PV}} = (1 + \delta) A_{\text{PV}}^0 \equiv \left( \frac{1 + \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)}}{1 + \delta_{\gamma(\gamma\gamma)}} \right) A_{\text{PV}}^0 \]

\[
\delta_{\gamma(Z\gamma)} = \frac{2 \text{Re}(M_{\gamma\gamma}^* M_{\gamma Z} + M_{\gamma Z}^* M_{\gamma\gamma})}{2 \text{Re}(M_{Z\gamma}^* M_{\gamma\gamma})}
\]

\[
\delta_{\gamma(\gamma\gamma)} = \frac{2 \text{Re}(M_{\gamma\gamma}^* M_{\gamma\gamma})}{|M_{\gamma\gamma}|^2}
\]

\[
\delta_{Z(\gamma\gamma)} = \frac{2 \text{Re}(M_{Z\gamma}^* M_{\gamma\gamma})}{2 \text{Re}(M_{Z\gamma}^* M_{\gamma\gamma})}
\]

\[
\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}
\]
Two-boson exchange corrections

- nucleon intermediate states

\[ Q^2 = 0.01 \text{ GeV}^2 \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ \delta_N(\epsilon, Q^2) \]

\[ \gamma(\gamma\gamma) \]

\[ Z(\gamma\gamma) \]

\[ \gamma(Z\gamma) \]

\[ Tjon, WM, PRL 100 (2008) 082003 \]
\[ Tjon, Blunden, WM, PRC 79 (2009) 055201 \]

→ cancellation between \( Z(\gamma\gamma) \) and \( \gamma(\gamma\gamma) \) corrections, especially at low \( Q^2 \)

→ dominated by \( \gamma(Z\gamma) \) contribution
Two-boson exchange corrections

\[ \Delta \] intermediate states

\[ Q^2 = 0.01 \text{ GeV}^2 \]

\[ \delta (\varepsilon, Q^2) \]

\[ 0 \leq \varepsilon \leq 1 \]

\[ \delta_N, \delta_\Delta, \delta_N + \delta_\Delta \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ 0 \leq \varepsilon \leq 1 \]

\[ T\text{jon}, WM, PRL 100 (2008) 082003 \]

\[ T\text{jon}, Blunden, WM, PRC 79 (2009) 055201 \]

\[ \Delta \] contribution small, except at very forward angles

(numerators have higher powers of loop momenta)

\[ \Delta \] calculation less reliable for \( \varepsilon \rightarrow 1 \)

(grows faster with \( s \) than nucleon)
Two-boson exchange corrections

\[ \sim 2\text{–}4\% \text{ correction for } Q^2 \sim 0.01\text{–}0.1 \text{ GeV}^2 \]

\[ \rightarrow \text{ stronger } Q^2 \text{ dependence at larger } Q^2 \]

(especially at forward angles)
TBE corrections at experimental kinematics

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>$\theta$</th>
<th>Expt.</th>
<th>$\delta_N$</th>
<th>$\delta_\Delta$</th>
<th>$\delta_{N+\Delta}$</th>
<th>$\delta_{\text{FMS}}^{\text{frwd}}$</th>
<th>$\delta_{\text{FMS}}^{\text{bckd}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.099</td>
<td>6.0°</td>
<td>HAPPEX [1]</td>
<td>0.19</td>
<td>-1.20</td>
<td>-1.01</td>
<td>0.45</td>
<td>2.12</td>
</tr>
<tr>
<td>0.477</td>
<td>12.3°</td>
<td>HAPPEX [1]</td>
<td>0.13</td>
<td>-0.44</td>
<td>-0.31</td>
<td>0.16</td>
<td>0.86</td>
</tr>
<tr>
<td>0.077</td>
<td>6.0°</td>
<td>HAPPEX [3]</td>
<td>0.22</td>
<td>-1.04</td>
<td>-0.82</td>
<td>0.52</td>
<td>2.78</td>
</tr>
<tr>
<td>0.1</td>
<td>144.0°</td>
<td>SAMPLE [5]</td>
<td>1.63</td>
<td>-0.09</td>
<td>1.54</td>
<td>0.06</td>
<td>0.33</td>
</tr>
<tr>
<td>0.108</td>
<td>35.37°</td>
<td>PVA4 [7]</td>
<td>1.05</td>
<td>0.78</td>
<td>1.83</td>
<td>0.37</td>
<td>1.98</td>
</tr>
<tr>
<td>0.23</td>
<td>35.31°</td>
<td>PVA4 [7]</td>
<td>0.62</td>
<td>0.34</td>
<td>0.96</td>
<td>0.23</td>
<td>1.22</td>
</tr>
<tr>
<td>0.122</td>
<td>6.68°</td>
<td>G0 [2]</td>
<td>0.18</td>
<td>-1.06</td>
<td>-0.88</td>
<td>0.40</td>
<td>2.13</td>
</tr>
<tr>
<td>0.128</td>
<td>6.84°</td>
<td>G0 [2]</td>
<td>0.18</td>
<td>-1.03</td>
<td>-0.85</td>
<td>0.39</td>
<td>2.07</td>
</tr>
<tr>
<td>0.136</td>
<td>7.06°</td>
<td>G0 [2]</td>
<td>0.18</td>
<td>-0.99</td>
<td>-0.81</td>
<td>0.37</td>
<td>1.99</td>
</tr>
<tr>
<td>0.144</td>
<td>7.27°</td>
<td>G0 [2]</td>
<td>0.17</td>
<td>-0.96</td>
<td>-0.79</td>
<td>0.36</td>
<td>1.92</td>
</tr>
<tr>
<td>0.153</td>
<td>7.5°</td>
<td>G0 [2]</td>
<td>0.17</td>
<td>-0.92</td>
<td>-0.75</td>
<td>0.35</td>
<td>1.85</td>
</tr>
<tr>
<td>0.164</td>
<td>7.77°</td>
<td>G0 [2]</td>
<td>0.17</td>
<td>-0.88</td>
<td>-0.71</td>
<td>0.33</td>
<td>1.77</td>
</tr>
<tr>
<td>0.177</td>
<td>8.09°</td>
<td>G0 [2]</td>
<td>0.16</td>
<td>-0.83</td>
<td>-0.67</td>
<td>0.32</td>
<td>1.69</td>
</tr>
<tr>
<td>0.192</td>
<td>8.43°</td>
<td>G0 [2]</td>
<td>0.16</td>
<td>-0.79</td>
<td>-0.63</td>
<td>0.30</td>
<td>1.60</td>
</tr>
<tr>
<td>0.21</td>
<td>8.84°</td>
<td>G0 [2]</td>
<td>0.16</td>
<td>-0.73</td>
<td>-0.57</td>
<td>0.28</td>
<td>1.51</td>
</tr>
<tr>
<td>0.232</td>
<td>9.31°</td>
<td>G0 [2]</td>
<td>0.16</td>
<td>-0.68</td>
<td>-0.52</td>
<td>0.26</td>
<td>1.41</td>
</tr>
<tr>
<td>0.262</td>
<td>9.92°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.62</td>
<td>-0.47</td>
<td>0.24</td>
<td>1.30</td>
</tr>
<tr>
<td>0.299</td>
<td>10.63°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.55</td>
<td>-0.40</td>
<td>0.22</td>
<td>1.19</td>
</tr>
<tr>
<td>0.344</td>
<td>11.46°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.48</td>
<td>-0.33</td>
<td>0.20</td>
<td>1.07</td>
</tr>
<tr>
<td>0.41</td>
<td>12.59°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.41</td>
<td>-0.26</td>
<td>0.18</td>
<td>0.95</td>
</tr>
<tr>
<td>0.511</td>
<td>14.2°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.32</td>
<td>-0.17</td>
<td>0.15</td>
<td>0.81</td>
</tr>
<tr>
<td>0.631</td>
<td>15.98°</td>
<td>G0 [2]</td>
<td>0.15</td>
<td>-0.26</td>
<td>-0.11</td>
<td>0.13</td>
<td>0.70</td>
</tr>
<tr>
<td>0.788</td>
<td>18.16°</td>
<td>G0 [2]</td>
<td>0.16</td>
<td>-0.23</td>
<td>-0.07</td>
<td>0.11</td>
<td>0.60</td>
</tr>
<tr>
<td>0.922</td>
<td>20.9°</td>
<td>G0 [2]</td>
<td>0.17</td>
<td>-0.22</td>
<td>-0.05</td>
<td>0.10</td>
<td>0.51</td>
</tr>
<tr>
<td>0.23</td>
<td>110.0°</td>
<td>G0 [4]</td>
<td>1.37</td>
<td>-0.10</td>
<td>1.27</td>
<td>0.09</td>
<td>0.47</td>
</tr>
<tr>
<td>0.62</td>
<td>110.0°</td>
<td>G0 [4]</td>
<td>1.10</td>
<td>-0.15</td>
<td>0.95</td>
<td>0.07</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Partial TBE corrections ($\gamma Z$ at $Q^2 = 0$) need to be removed before adding new results.

$G_0$ (fwd): < 1% (negative)

$G_0$ (bck): ~ 1% (positive)
Effect on strange form factors

- include TBE corrections in global analysis

- e.g. Young et al.

\[
\begin{align*}
G_E^s &= +0.0025 \pm 0.0182 \\
G_M^s &= -0.011 \pm 0.254
\end{align*}
\]

\[
\begin{align*}
G_E^s &= +0.0023 \pm 0.0182 \\
G_M^s &= -0.020 \pm 0.254
\end{align*}
\]

- small (absolute) shift in strange form factors from TBE (large relative shift to \(G_M^s\)), well within experimental errors

\(Q^2 = 0.1 \text{ GeV}^2\)
Extraction of proton’s weak charge

– JLab Qweak experiment –
Correction to proton weak charge

- In **forward** limit $A_{PV}$ measures weak charge of proton $Q_W^p$

$$A_{PV} \rightarrow \frac{G_F Q_W^p}{4\sqrt{2}\pi\alpha} t$$

- At **tree level** $Q_W^p$ gives weak mixing angle

$$Q_W^p = 1 - 4\sin^2\theta_W$$
Correction to proton weak charge

- including higher order radiative corrections

\[
Q_W^p = (1 + \Delta \rho + \Delta_e)(1 - 4 \sin^2 \theta_W(0) + \Delta'_e) \\
+ \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \quad \text{box diagrams}
\]

\[
= 0.0713 \pm 0.0008^* \quad \text{Erler et al., PRD 72 (2005) 073003}
\]

* \(\sin^2 \theta_W(0) = 0.23867(16)\)

- \(WW\) and \(ZZ\) box diagrams dominated by short distances, evaluated perturbatively

- \(\gamma Z\) box diagram sensitive to long distance physics, has two contributions

\[
\Box_{\gamma Z} = \Box^A_{\gamma Z} + \Box^V_{\gamma Z}
\]

- \(\text{vector } e - \text{axial } h\)
  (finite at \(E=0\))

- \(\text{axial } e - \text{vector } h\)
  (vanishes at \(E=0\))
Axial $h$ correction

- **Axial $h$ correction** $\Box^A_{\gamma Z}$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy

→ computed by Marciano & Sirlin as sum of two parts:

- **Low-energy part** approximated by *Born* contribution (elastic intermediate state)

- **High-energy part** (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

\[
\Box^A_{\gamma Z} = \frac{5\alpha}{2\pi} (1 - 4 \sin^2 \theta_W) \left[ \ln \frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda) \right]
\]

\[\approx 0.0048\]

Axial $h$ correction

- **Axial** $h$ correction $\Box^A_{\gamma Z}$ dominant $\gamma Z$ correction in atomic parity violation at very low (zero) energy.

→ repeat calculation using forward dispersion relations with realistic (structure function) input.

\[ \begin{align*}
  k & \rightarrow_{\gamma^*} q \rightarrow \gamma Z \\
  p & \rightarrow \approx p
\end{align*} \]

★ **Axial** $h$ contribution *antisymmetric* under $E' \leftrightarrow -E'$:

\[ \Re \Box^A_{\gamma Z}(E') = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2} \Im \Box^A_{\gamma Z}(E') \]
★ Imaginary part can only grow as $\log E' / E'$.
Axial $h$ correction

imaginary part given by interference $F_{3}^{\gamma Z}$ structure function

\[
\text{Im} \mathbf{A}_{\gamma Z}^{A}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_{\pi}^2}^{s} dW^2 \int_{0}^{Q_{\text{max}}^2} dQ^2 \frac{dQ^2}{1 + Q^2/M_Z^2} \times \frac{g_{V}^{e}}{2g_{A}^{e}} \left( \frac{4M E}{W^2 - M^2 + Q^2} - 1 \right) F_{3}^{\gamma Z}
\]

with $g_{A}^{e} = -\frac{1}{2}$, $g_{V}^{e} = -\frac{1}{2}(1 - 4\hat{s}^2)$

$F_{3}^{\gamma Z}$ structure function

- elastic part given by $G_{M}^{P} G_{A}^{Z}$
- resonance part from parametrization of $\nu$ scattering data (Lalakulich-Paschos)
- DIS part dominated by leading twist PDFs at small $x$ (MSTW, CTEQ, Alekhin)
Axial $h$ correction

- change integration variable $W^2 \rightarrow x$ and switch order of integration

$$\text{Im} \, A_{\gamma Z}^\square = (1 - 4s^2) \frac{\alpha}{2ME} \int_0^{2ME} \frac{dQ^2}{1 + Q^2/M_Z^2} \int_{x_{\text{min}}}^1 \frac{dx}{x} \left(1 - \frac{y}{2}\right) F_{3\gamma Z}$$

where $y = (W^2 - M^2 + Q^2)/2ME$

→ in DIS region ($Q^2 \gtrsim 1 \text{ GeV}^2$), expand integrand in $1/Q^2$

$$\text{Re} \, A_{\gamma Z}^{\text{DIS}} = (1 - 4s^2) \frac{3\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2(1 + Q^2/M_Z^2)}$$

$$\times \left[ M_{3\gamma Z}^{\gamma Z(1)} - \frac{2M^2}{9Q^4} (5E^2 - 3Q^2) M_{3\gamma Z}^{\gamma Z(3)} \right]$$

with moments $M_{3\gamma Z}^{\gamma Z(n)}(Q^2) = \int_0^1 dx \, x^{n-1} \, F_{3\gamma Z}(x, Q^2)$
Axial $h$ correction

- **structure function moments**

\[ n = 1 \quad M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) \]

\[ \rightarrow \gamma Z \text{ analog of Gross-Llewellyn Smith sum rule} \]

\[ \Re \epsilon^{A \gamma Z(\text{DIS})} \approx (1 - 4\hat{s}^2) \frac{5\alpha}{2\pi} \int_{Q_0^2}^{\infty} \frac{dQ^2}{Q^2(1+Q^2/M_Z^2)} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} \right) \]

\[ \rightarrow \text{precisely result from Marciano & Sirlin!} \]

(works because result depends on lowest moment of valence PDF, with model-independent normalization!)

\[ n = 3 \quad M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3} \left( 2 \langle x \rangle_{uv} + \langle x \rangle_{dv} \right) \left( 1 + \frac{5\alpha_s(Q^2)}{12\pi} \right) \]

\[ \rightarrow \text{related to momentum carried by valence quarks} \]
Axial $h$ correction

$Q_{\text{weak}} E = 1.165 \text{ GeV}$

$\rightarrow$ dominated by DIS contribution (weak $E$ dependence)

Blunden, WM, Thomas (2011)
Axial $h$ correction

$\rightarrow$ correction at $E = 0$

$$\Re e \Box^A_{\gamma Z} = 0.0009 + 0.0003 + 0.0037 = 0.0050$$

$\uparrow$ elastic $\uparrow$ resonance $\uparrow$ DIS

$\rightarrow$ correction at $E = 1.165 \text{ GeV}$ (Qweak)

$$\Re e \Box^A_{\gamma Z} = 0.00006 + 0.0002 + 0.0037 = 0.0039$$

$c f$. MS value: $0.0048$ (~1% shift in $Q^p_W$)
Vector $h$ correction

- vector $h$ correction $\Box_{\gamma Z}^V$ vanishes at $E = 0$, but experiment has $E \sim 1$ GeV – what is energy dependence?

→ forward dispersion relation

★ $\Re \Box_{\gamma Z}^V (E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2} \Im m \Box_{\gamma Z}^V (E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector $h$ contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m \Box_{\gamma Z}^V (E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^s dW^2 \int_0^{Q_{\text{max}}^2} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left( F_{1\gamma Z}^2 + F_{2\gamma Z}^2 \frac{s \left( Q_{\text{max}}^2 - Q^2 \right)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

factor 2 larger than GH; confirmed by Rislow & Carlson, arXiv:1011.2397 [hep-ph]

Gorchtein, Horowitz, PRL 102 (2009) 091806
Vector $h$ correction

$F^{\gamma Z}_{1,2}$ structure functions

- Parton model for \textbf{DIS} region: $F^{\gamma Z}_{2} = 2x \sum_q e_q g^q_V (q + \bar{q}) = 2x F^{\gamma Z}_1$

- $F^{\gamma Z}_{2} \approx F^{\gamma}_2$ good approximation at low $x$

- Provides upper limit at large $x$ ($F^{\gamma Z}_{2} \lesssim F^{\gamma}_2$)

- In \textbf{resonance} region use phenomenological input for $F_2$, empirical (SLAC) fit for $R$

- For transitions to $I = 3/2$ states (e.g. $\Delta$), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_{SW}^p) F_i^{\gamma}$

- For transitions to $I = 1/2$ states, SU(6) wave functions predict $Z$ & $\gamma$ transition couplings equal to a few %
Vector $h$ correction

→ compare structure function input with data

GVMD model
(used as input by Gorchtein & Horowitz)
Vector $h$ correction

total $\Box_{\gamma Z}^{V}$ correction:

$$\Re \Box_{\gamma Z}^{V} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6} \%$ of uncorrected $Q_{W}^{p}$

$Q_{W}^{p} = 0.0713 \rightarrow 0.0760$

Sibirtsev, Blunden, WM, Thomas
PRD 82 (2010) 013011
Vector $h$ correction

\[ \text{total } \Box_{\gamma Z}^{V} \text{ correction:} \]

\[ \Re \Box_{\gamma Z}^{V} = 0.0057 \pm 0.0009 \]

\[ \rightarrow \text{ compatible with SBMT within errors} \]

\[ \Re \delta_{\gamma Z} = \Re \frac{\Box_{\gamma Z}}{Q_W^p} \approx 6\% \]

mostly from high-\( W \)
(“Regge”) contribution

→ our formula for \( \Sigma m \Box_{\gamma Z} \) factor 2 larger*
("nuclear physics" vs. "particle physics" conventions for weak charges in structure function definitions?)

→ GH omit factor \( (1-x) \) in definition of \( F_{1,2} \)
(\( \sim 30\% \) enhancement)

→ GH use \( Q_W^p \sim 0.05 \) cf. \( \sim 0.07 \)
(\( \sim 40\% \) enhancement)

→ numerical agreement for \( \delta_{\gamma Z}^V \) coincidental (?)

* confirmed by Rislow/Carlson
arXiv:1011.2397
Combined vector and axial $h$ correction

$$Q^p_W = 0.0713 \rightarrow \approx 0.076$$

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q^p_W = \pm 0.003$

* 4% measurement of $Q^p_W$

Bentz et al., PLB 693 (2010) 462
Summary

- Two-boson exchange corrections play minor role in strange form factor extraction
  - cf. significant role of TPE in Rosenbluth extraction of $G_E^p$

- Dramatic effect of $\gamma(\gamma Z)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\%$, cf. PDG
  - would significantly shift extracted weak angle
  - better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$
    (e.g. in PVDIS at JLab)

- New formulation in terms of moments of structure functions
  - places on firm footing earlier derivation of Marciano/Sirlin in “free quark model”
The End