PAVI '11
Sapienza - Uni. Roma
Sep. 5, 2011

Overview of two-boson exchange in electron-proton scattering

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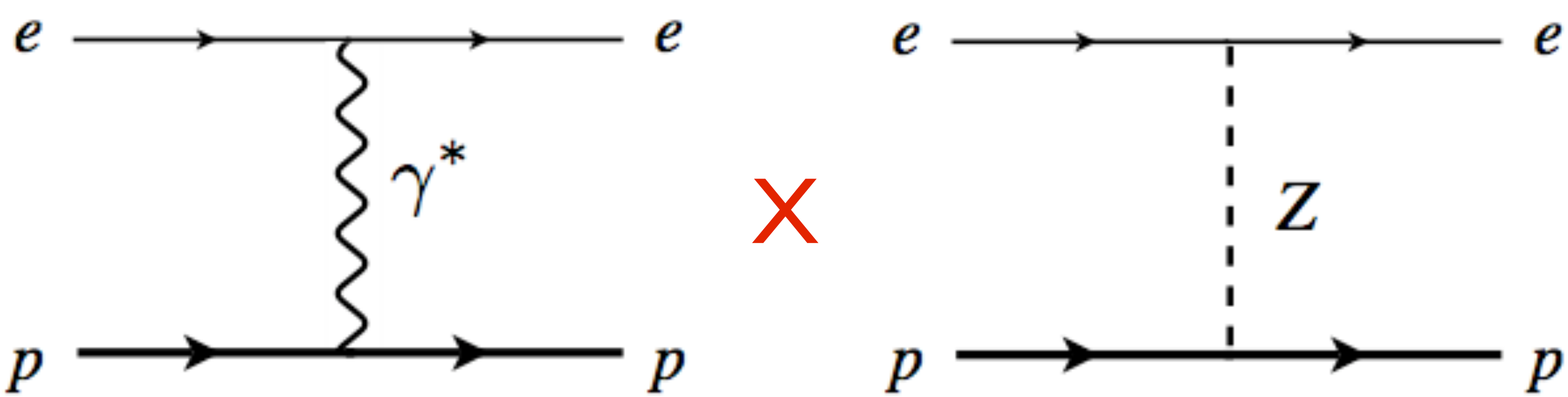
Refs: PNPP (2011) arXiv:1105.0951 (review article)
PRL **107**, 081801 (2011) (γZ correction to Q_W^p)

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents



Born (tree) level

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \rightarrow e\, p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

$\longrightarrow$ measure interference between e.m. and weak currents

vector asymmetry

$$A_V = g_A^e \rho \left[(1 - 4\kappa \sin^2\theta_W) - (\varepsilon G_E^{\gamma p} G_E^{\gamma n} + \tau G_M^{\gamma p} G_M^{\gamma n})/\sigma^{\gamma p}\right]$$

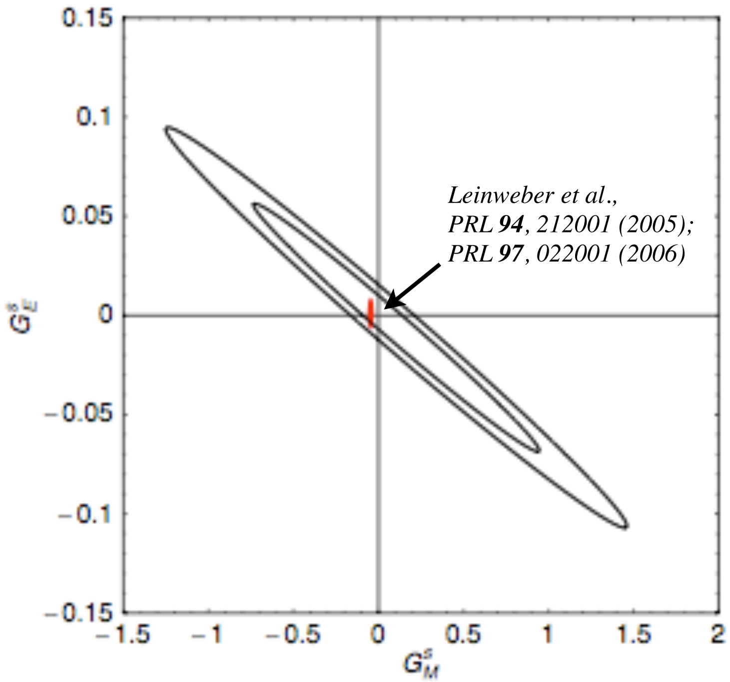
axial vector asymmetry

$$A_A = g_V^e \sqrt{\tau(1+\tau)(1-\varepsilon^2)}\, \widetilde{G}_A^{Zp} G_M^{\gamma p}/\sigma^{\gamma p}$$

strange asymmetry

$$A_s = -g_A^e \rho \left(\varepsilon G_E^{\gamma p} G_E^s + \tau G_M^{\gamma p} G_M^s\right)/\sigma^{\gamma p}$$

Parity-violating e scattering



$$G_E^s = +0.0025 \pm 0.0182$$
$$G_M^s = -0.011 \pm 0.254$$

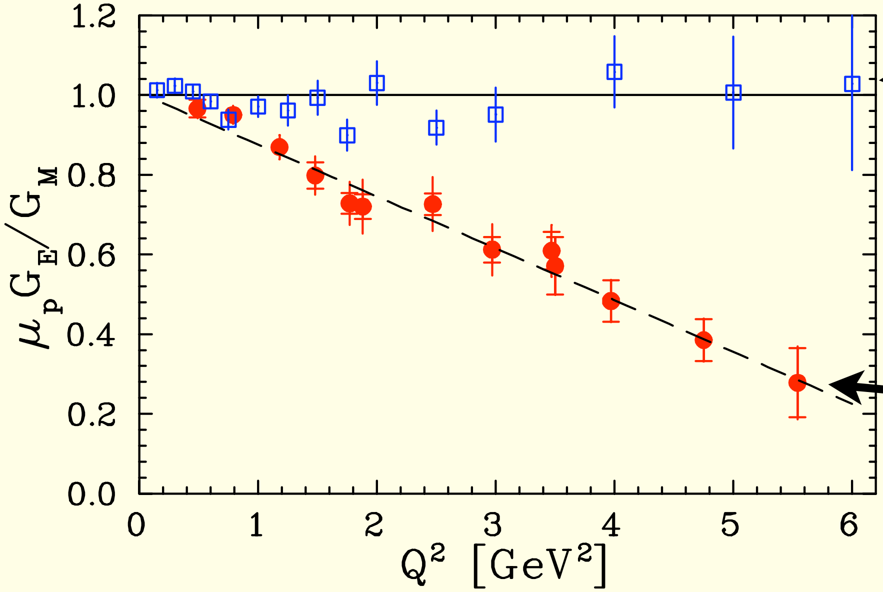
$$Q^2 = 0.1 \text{ GeV}^2$$

Young, Roche, Carlini, Thomas
PRL ***97*** *(2006) 102002*

→ strange form factors <u>small</u>

→ how important are <u>higher order ($e.g.$ γZ) corrections</u>?

Historical background: proton G_E/G_M ratio



Rosenbluth (Longitudinal-Transverse) Separation

Polarization Transfer

LT method

$$\sigma_R = G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

→ G_E from slope in ε plot

→ suppressed at large Q^2

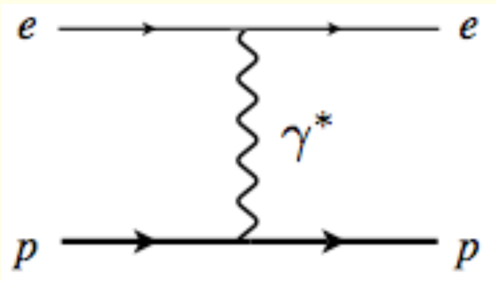
PT method

$$\frac{G_E}{G_M} = -\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\,\frac{P_T}{P_L}$$

→ $P_{T,L}$ recoil proton polarization in $\vec{e}\,p \to e\,\vec{p}$

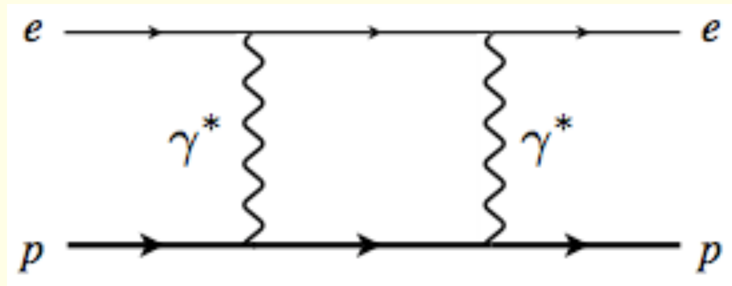
Two-photon exchange corrections

- direct computation of interference between γ and $\gamma\gamma$ exchange diagrams, including effects of *hadron structure*



$\mathcal{M}_0$

X



$\mathcal{M}_{\gamma\gamma}$

$$\delta^{(2\gamma)} = \frac{2\mathcal{R}e\left\{\mathcal{M}_0^\dagger\,\mathcal{M}_{\gamma\gamma}\right\}}{|\mathcal{M}_0|^2}$$

Blunden, WM, Tjon
*PRL **91** (2003) 142304*

$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} \quad + \text{crossed box}$$

$$N(l) = \bar{u}(k')\gamma_\mu(\not{k} - \not{l} + m_e)\gamma_\nu u(k)\ \bar{u}(p')\Gamma^\mu(q-l)(\not{p} + \not{l} + M)\Gamma^\nu(l)u(p)$$

$$D(l) = (l^2 - \lambda^2)((l-q)^2 - \lambda^2)((k-l)^2 - m_e^2)((p+l)^2 - M^2)$$

$\lambda(\rightarrow 0)$ = infrared regulator

Two-photon exchange corrections

- "exact" evaluation of integrals including form factors (Veltman-Passarino functions)

  → *cf.* soft photon approximation (used in most data analyses!) which assumes pole dominance of TPE amplitude & neglects nucleon structure $N(l) \approx N(0)$

Mo, Tsai (1969)

$$\mathcal{M}_{\gamma\gamma} = e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{N(l)}{D(l)} \quad + \text{crossed box}$$

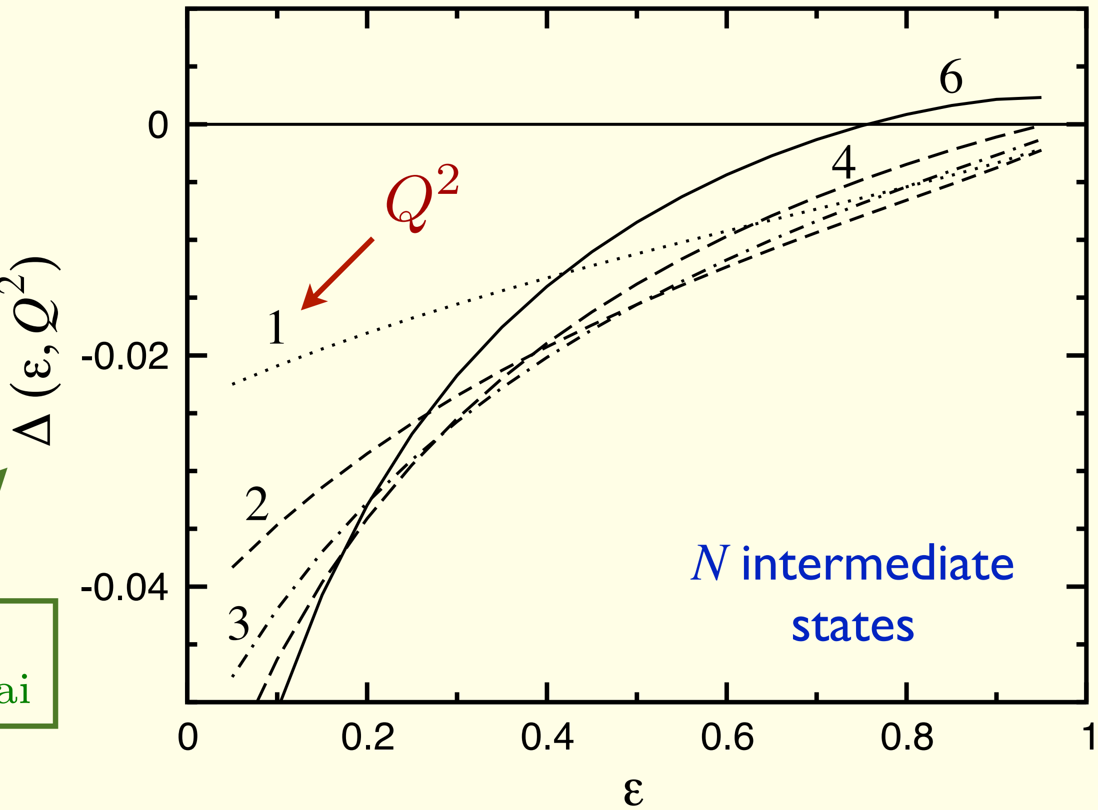
$$N(l) = \bar{u}(k')\gamma_\mu(\not{k} - \not{l} + m_e)\gamma_\nu u(k)\ \bar{u}(p')\Gamma^\mu(q-l)(\not{p} + \not{l} + M)\Gamma^\nu(l)u(p)$$

$$D(l) = (l^2 - \lambda^2)((l-q)^2 - \lambda^2)((k-l)^2 - m_e^2)((p+l)^2 - M^2)$$

$\lambda(\to 0)$ = infrared regulator

Two-photon exchange corrections

- difference between "exact" and Mo-Tsai calculations of TPE



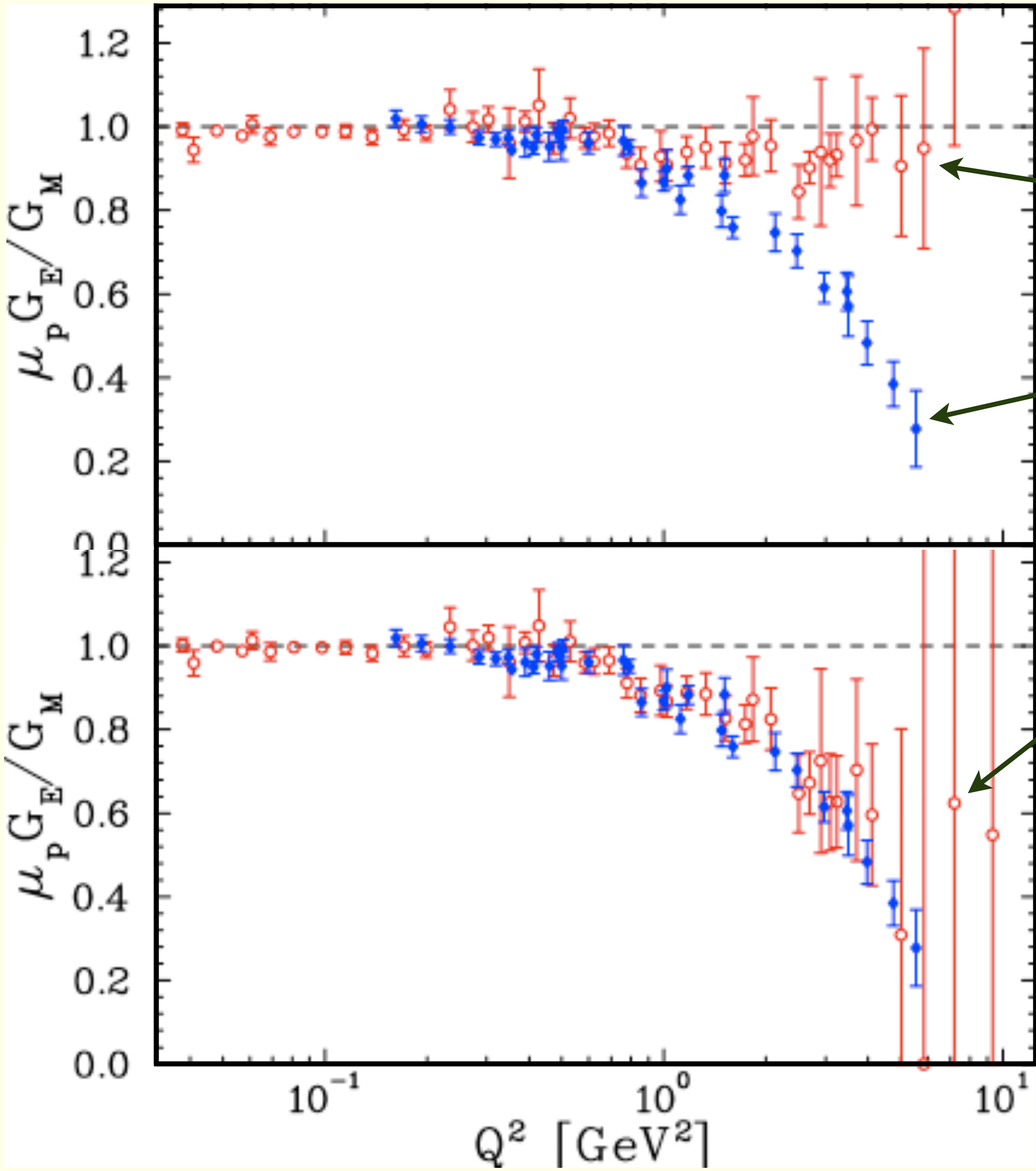
$\delta^{(2\gamma)}_{\rm full} - \delta^{(2\gamma)}_{\rm Mo-Tsai}$

Blunden, WM, Tjon
PRL ***91*** *(2003) 142304;*
PRC ***72*** *(2005) 034612*

$\longrightarrow$ few % magnitude, non-linear in ε, positive slope

$\longrightarrow$ does not depend strongly on vertex form factors

Two-photon exchange corrections



Rosenbluth separation

polarization transfer

with TPE correction

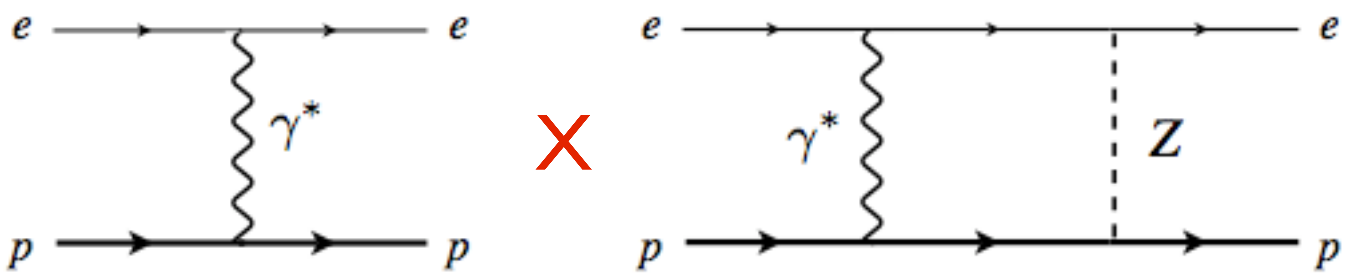
+ higher-mass intermediate states

→ significant effect

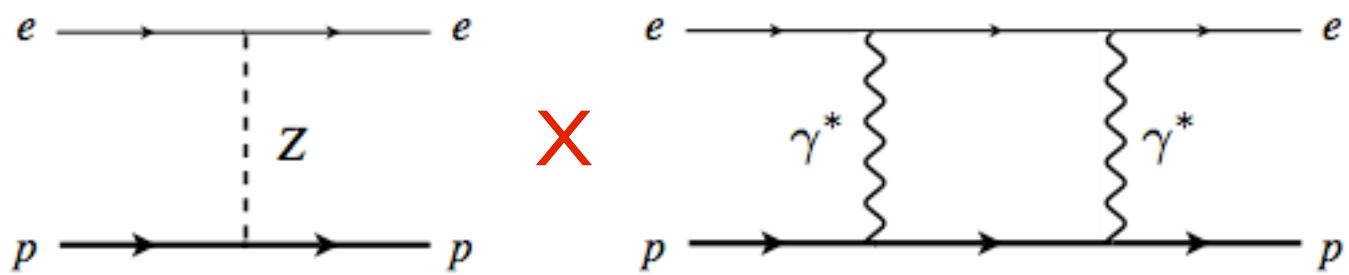
→ resolves discrepancy (within errors)

*Arrington, WM, Tjon, PRC **76** (2007) 035205*

Two-boson exchange corrections



"$\gamma(Z\gamma)$"



"$Z(\gamma\gamma)$"

$$A_{\rm PV} = (1+\delta)A^0_{\rm PV} \equiv \left(\frac{1+\delta_{Z(\gamma\gamma)}+\delta_{\gamma(Z\gamma)}}{1+\delta_{\gamma(\gamma\gamma)}}\right) A^0_{\rm PV}$$

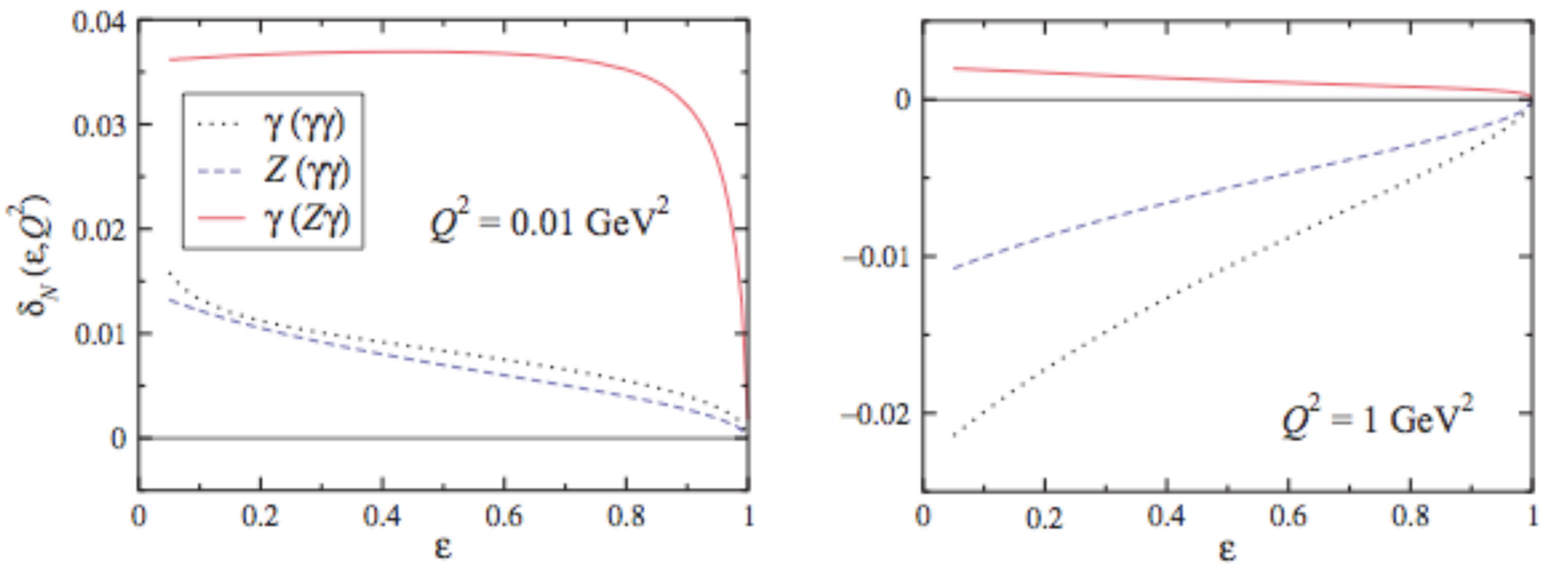
Born asymmetry

$$\delta_{\gamma(Z\gamma)} = \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma Z} + \mathcal{M}^*_\gamma \mathcal{M}_{Z\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_\gamma)} \qquad \delta_{\gamma(\gamma\gamma)} = \frac{2\Re e(\mathcal{M}^*_\gamma \mathcal{M}_{\gamma\gamma})}{|\mathcal{M}_\gamma|^2}$$

$$\delta_{Z(\gamma\gamma)} = \frac{2\Re e(\mathcal{M}^*_Z \mathcal{M}_{\gamma\gamma})}{2\Re e(\mathcal{M}^*_Z \mathcal{M}_\gamma)} \qquad \delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$

Two-boson exchange corrections

- nucleon intermediate states



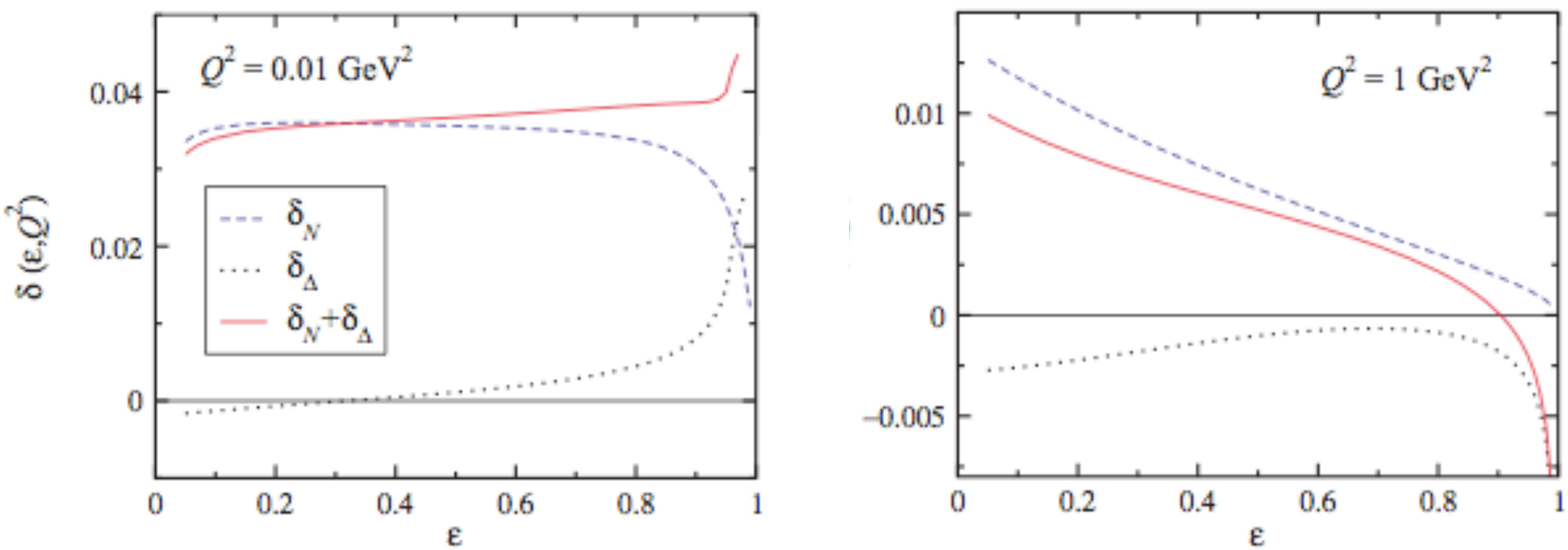
Tjon, WM, PRL ***100*** *(2008) 082003*
Tjon, Blunden, WM, PRC ***79*** *(2009) 055201*

→ cancellation between $Z(\gamma\gamma)$ and $\gamma(\gamma\gamma)$ corrections, especially at low Q^2

→ dominated by $\gamma(Z\gamma)$ contribution

Two-boson exchange corrections

- Δ intermediate states



*Tjon, WM, PRL **100** (2008) 082003*
*Tjon, Blunden, WM, PRC **79** (2009) 055201*

→ Δ contribution small, except at very forward angles
(numerators have higher powers of loop momenta)

→ Δ calculation less reliable for $\varepsilon \to 1$
(grows faster with s than nucleon)

Effect on strange form factors

- include TBE corrections in global analysis

→ *e.g.* Young et al. (preliminary)

$$\boxed{\begin{aligned} G_E^s &= +0.0025 \pm 0.0182 \\ G_M^s &= -0.011 \pm 0.254 \end{aligned}}$$

↓

$$\boxed{\begin{aligned} G_E^s &= +0.0023 \pm 0.0182 \\ G_M^s &= -0.020 \pm 0.254 \end{aligned}}$$

at $Q^2 = 0.1\ \mathrm{GeV}^2$

→ small (absolute) shift in strange form factors from TBE (large relative shift to G_M^s), well within experimental errors

→ global reanalysis (incl. TBE) in progress

Young et al. (2011)

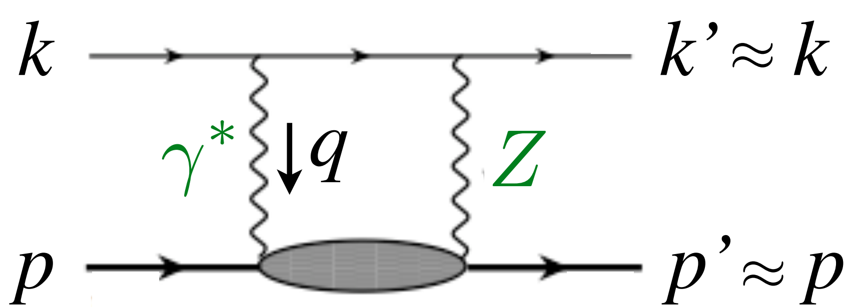
Proton weak charge

- Left-right polarization asymmetry in $\vec{e}\, p \longrightarrow e\ p$ scattering

$$A_{\rm PV} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\left(\frac{G_F Q^2}{4\sqrt{2}\alpha}\right)(A_V + A_A + A_s)$$

→ in <u>forward</u> limit measures weak charge of proton Q_W^p

$$A_{\rm PV} \;\; \rightarrow \;\; \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha} t$$



forward limit

$t = (k - k')^2 \rightarrow 0$

$s = (k + p)^2$

$\quad = M(M + 2E)$

Proton weak charge

- At tree level Q_W^p gives weak mixing angle

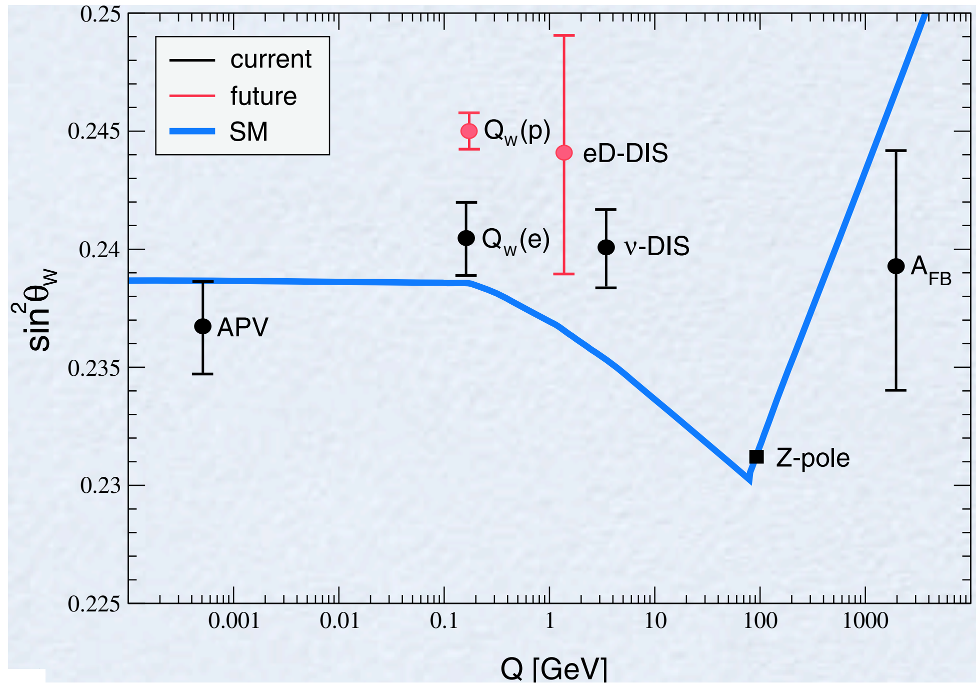
$$Q_W^p = 1 - 4\sin^2\theta_W$$

→ current best values

$$\sin^2\theta_W(M_Z^2) = 0.23116(16)$$

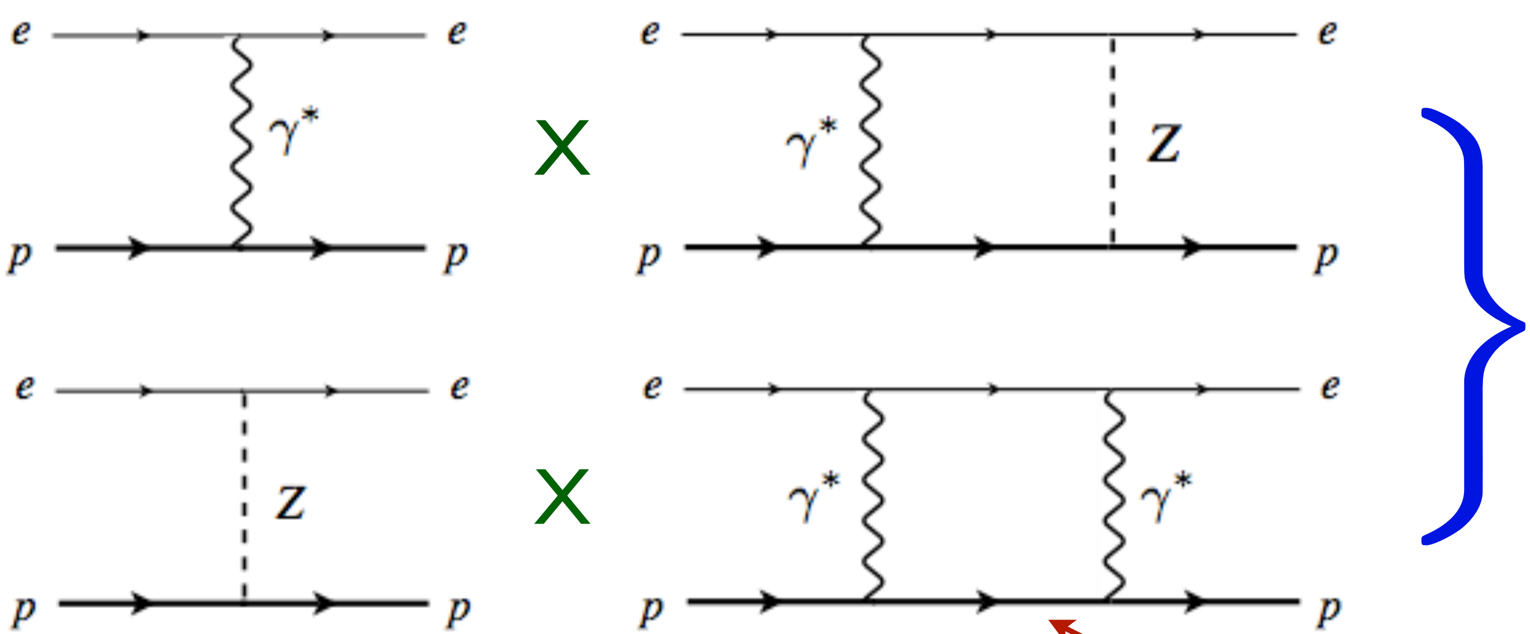
$$\sin^2\theta_W(0) = 0.23867(13)$$

scale dependence from radiative effects



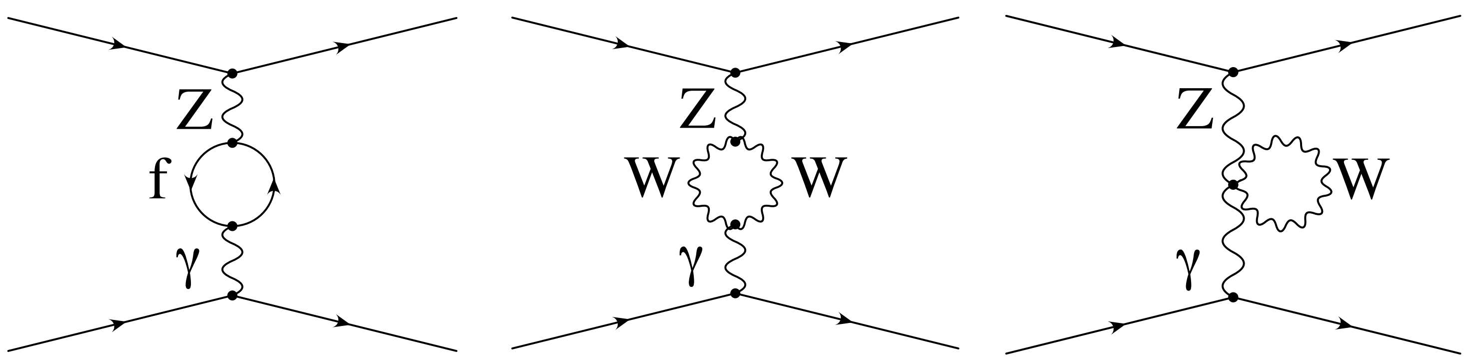
→ Q_W^p small number – sensitive to higher-order corrections

Corrections to proton weak charge



two-boson exchange

two-photon exchange vanishes at $t = 0$



vacuum polarization

+ ...

Corrections to proton weak charge

- including higher order radiative corrections

$$Q_W^p = (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta'_e) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \quad \longleftarrow \text{box diagrams}$$

$$= 0.0713 \pm 0.0008$$

Erler et al., PRD **72** *(2005) 073003*

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively (*WW* box gives ~ 25% correction!)

→ γZ box diagram sensitive to long distance physics, has two contributions $\Box_{\gamma Z} = \Box_{\gamma Z}^A + \Box_{\gamma Z}^V$

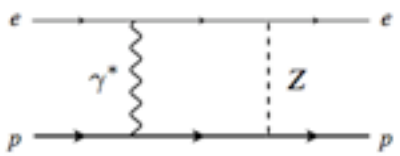
vector e – axial h (finite at E=0)

axial e – vector h (vanishes at E=0)

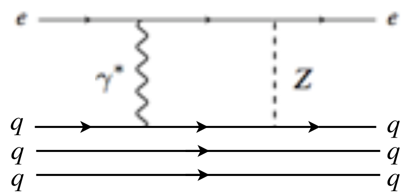
Axial h correction

- axial h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

$\longrightarrow$ computed by Marciano & Sirlin (1980s) as sum of two parts:



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box^A_{\gamma Z} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\underbrace{\ln\frac{M_Z^2}{\Lambda^2}}_{\text{short-distance}} + \underbrace{C_{\gamma Z}(\Lambda)}_{\text{long-distance } \approx 3/2 \pm 1}\right]$$

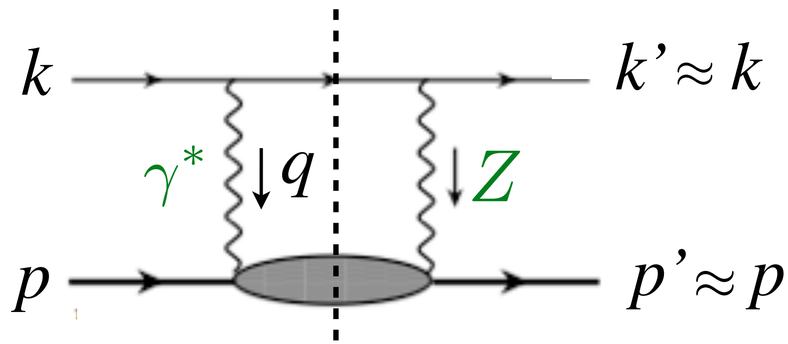
$$\approx \; 0.0052(5)$$

Marciano, Sirlin, PRD ***29*** *(1984) 75; Erler et al., PRD* ***68*** *(2003) 016006*

Axial h correction

- <u>axial</u> h correction $\Box^A_{\gamma Z}$ dominant γZ correction in atomic parity violation at very low (zero) energy

→ repeat calculation using <u>forward dispersion relations</u> with realistic (structure function) input



- axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e\, \Box^A_{\gamma Z}(E) = \frac{2}{\pi} \int_0^\infty dE' \frac{E'}{E'^2 - E^2}\, \Im m\, \Box^A_{\gamma Z}(E')$$

- negative energy part corresponds to crossed box (crossing symmetry $s \to u$)

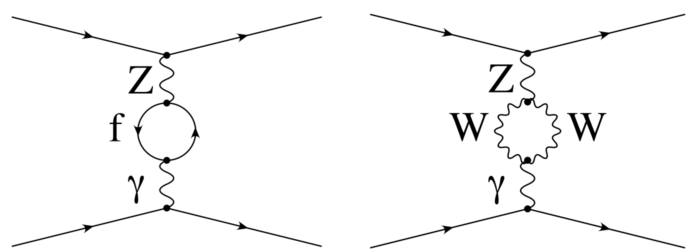
Axial h correction

- imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\mathcal{I}m\,\Box^A_{\gamma Z}(E) = \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\max}} dQ^2\,\frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left(\frac{2ME}{W^2-M^2+Q^2}-\frac{1}{2}\right) F_3^{\gamma Z}$$

with $v_e(Q^2) = 1 - 4\kappa(Q^2)\sin^2\theta_W(Q^2)$

→ scale dependence of v_e, α given by vacuum polarization corrections, *e.g.*



$$\frac{\alpha}{\alpha(Q^2)} = 1 - \Delta\alpha_{\rm lep}(Q^2) - \Delta\alpha^{(5)}_{\rm had}(Q^2)$$

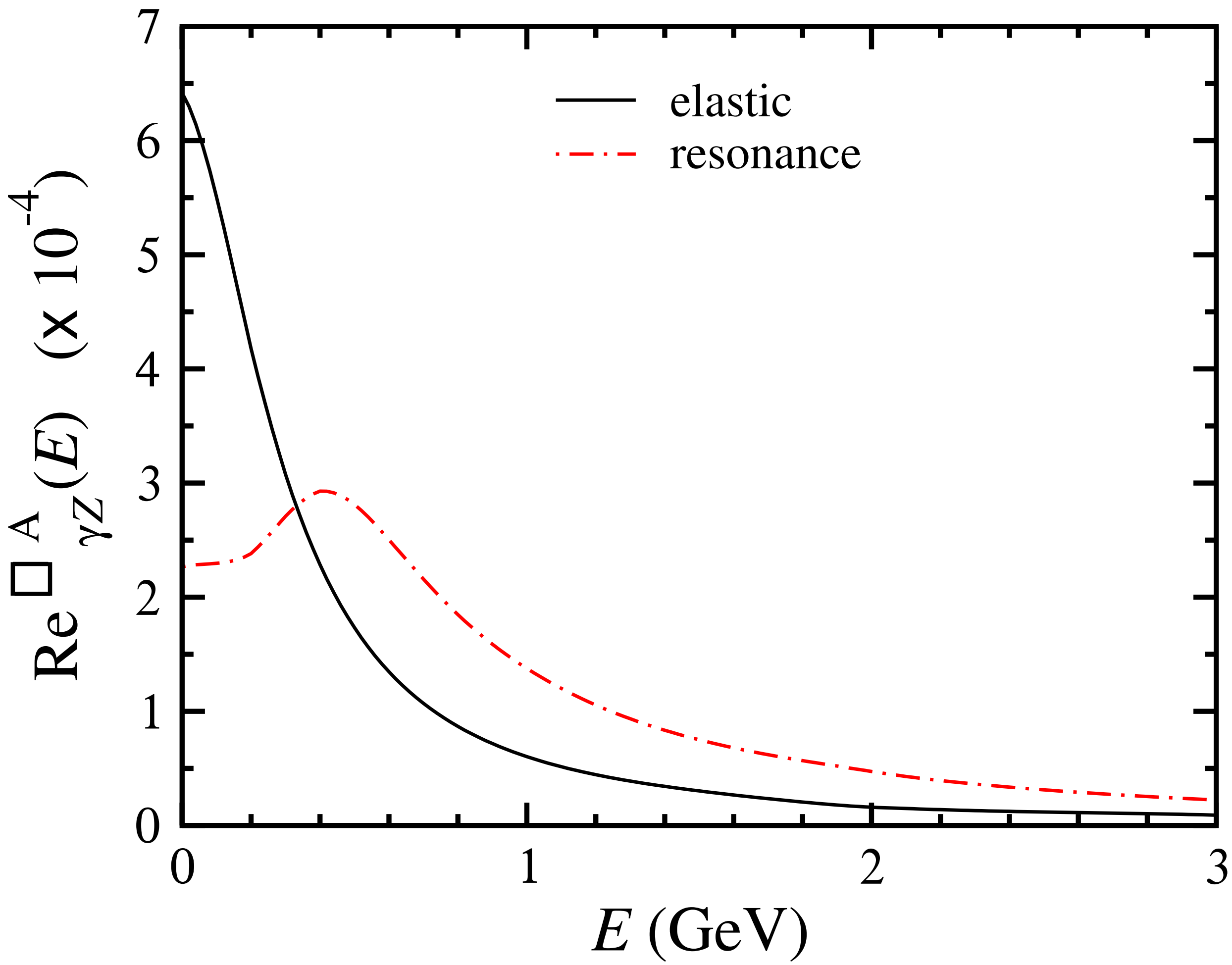
$$\alpha^{-1}(M_Z^2) = 128.94$$

... similarly for weak charges

Axial h correction

- <u>elastic</u> part $F_3^{\gamma Z(\mathrm{el})} = -Q^2\, G_M^p(Q^2)\, G_A^Z(Q^2)\, \delta(W^2 - M^2)$

- <u>resonance</u> part from parametrization of ν scattering data
(includes lowest four spin-1/2 and 3/2 states) *Lalakulich, Paschos (2006)*



Blunden, WM, Thomas
*PRL **107**, 081801 (2011)*

Axial h correction

- <u>DIS</u> part dominated by leading twist PDFs at high W (small x)

$$e.g.\ \text{at LO},\quad F_3^{\gamma Z(\mathrm{DIS})} = \sum_q 2e_q\, g_A^q \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

→ switching order of integration (energy integral analytic!), expand integrand in $1/Q^2$ in DIS region ($Q^2 \gtrsim 1\ \mathrm{GeV}^2$)

$$\mathcal{R}e\,\Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left[M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{\gamma Z(3)}\right]$$

with moments $M_3^{\gamma Z(n)}(Q^2) = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

Axial h correction

- structure function moments

<u>$n = 1$</u> $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\,\Box_{\gamma Z}^{A(\mathrm{DIS})} \approx (1 - 4\hat{s}^2)\,\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1+Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ precisely result from Marciano & Sirlin!

(works because result depends on lowest moment of *valence* PDF, with <u>model-independent normalization</u>!)

<u>$n = 3$</u> $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

$\longrightarrow$ related to x^2 -weighted moment of valence quarks

Axial h correction

- "<u>DIS</u>" region at $Q^2 < 1\ \mathrm{GeV}^2$ does not afford PDF description

→ in absence of data, consider models with general constraints

- ☆ $F_3^{\gamma Z}(x_{\max}, Q^2)$ should not diverge in limit $Q^2 \to 0$
- ☆ $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 = 1\,\mathrm{GeV}^2$

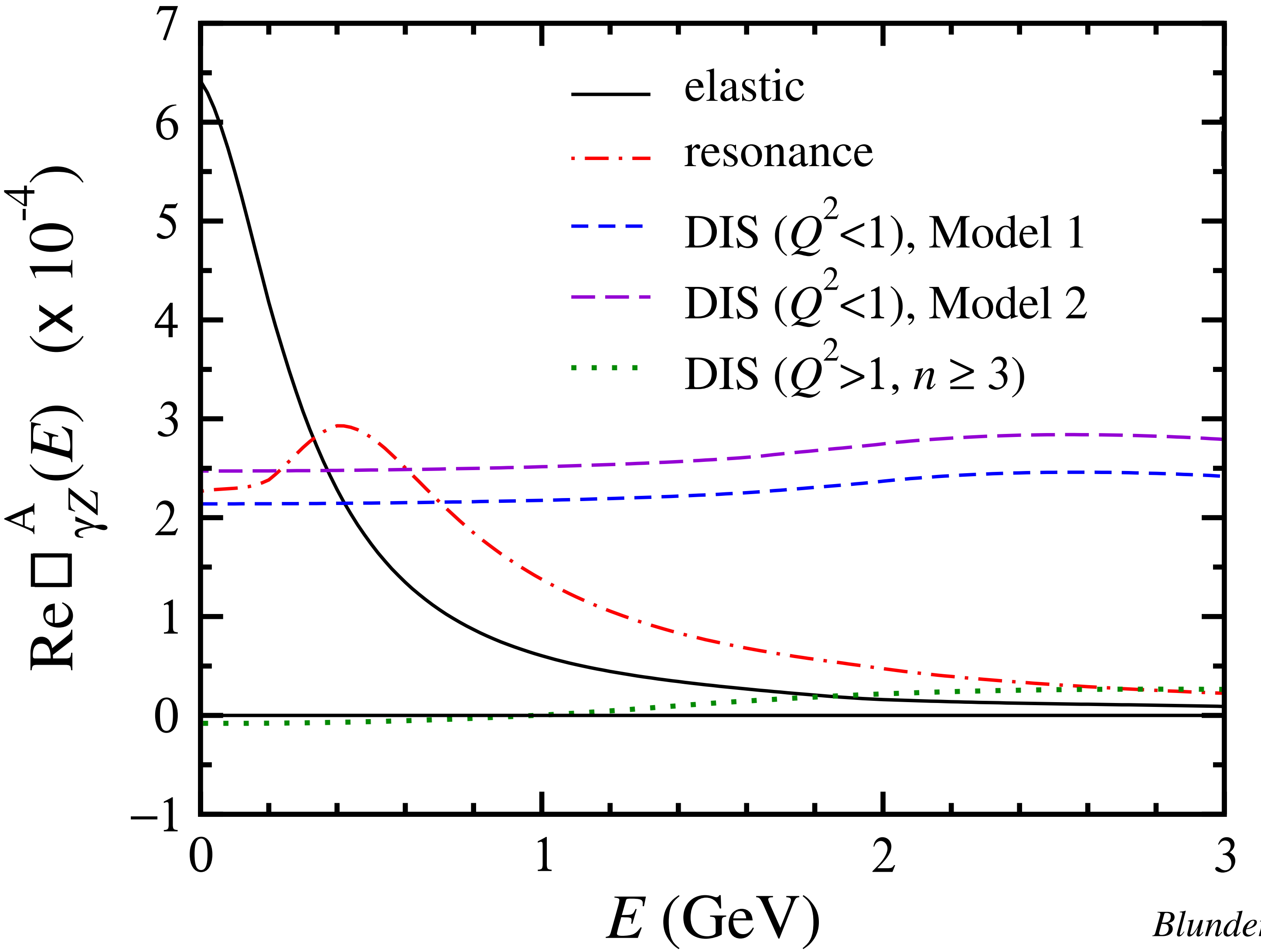
<u>Model 1</u> $$F_3^{\gamma Z}(x, Q^2) = \left(\frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)$$

$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0$$

<u>Model 2</u> $F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all W^2

$$F_3^{\gamma Z} \text{ finite as } Q^2 \to 0$$

Axial h correction



Blunden, WM, Thomas
*PRL **107**, 081801 (2011)*

→ dominated by $n = 1$ DIS moment: $\underline{32.8 \times 10^{-4}}$
(weak E dependence)

Axial h correction

→ correction at $\underline{E = 0}$

$$\Re e\,\Box^{A}_{\gamma Z} \;=\; \underbrace{0.00064}_{\text{elastic}} \;+\; \underbrace{0.00023}_{\text{resonance}} \;+\; \underbrace{0.00350}_{\text{DIS}} \;\to\; \underline{0.0044(4)}$$

→ correction at $\underline{E = 1.165\ \mathrm{GeV}}$ (Qweak)

$$\Re e\,\Box^{A}_{\gamma Z} \;=\; 0.00005 \;+\; 0.00011 \;+\; 0.00352 \;=\; \underline{0.0037(4)}$$

cf. MS value: $\underline{0.0052(5)}$ (~1% shift in Q_W^p)

→ shifts Q_W^p from $\underline{0.0713(8)} \longrightarrow \underline{0.0705(8)}$

Vector h correction

- vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but experiment has $E \sim 1\ \mathrm{GeV}$ – what is energy dependence?

→ forward dispersion relation

★ $\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$

★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$
$$\times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

*Gorchtein, Horowitz, PRL **102** (2009) 091806*
(note: factor 2 missing in original formula)

Vector h correction

→ $F_{1,2}^{\gamma Z}$ structure functions

- parton model for DIS region $F_2^{\gamma Z} = 2x \sum_q e_q \, g_V^q \, (q + \bar{q}) = 2x F_1^{\gamma Z}$
  - → $F_2^{\gamma Z} \approx F_2^{\gamma}$ good approximation at *low x*
  - → provides upper limit at *large x* $(F_2^{\gamma Z} \lesssim F_2^{\gamma})$

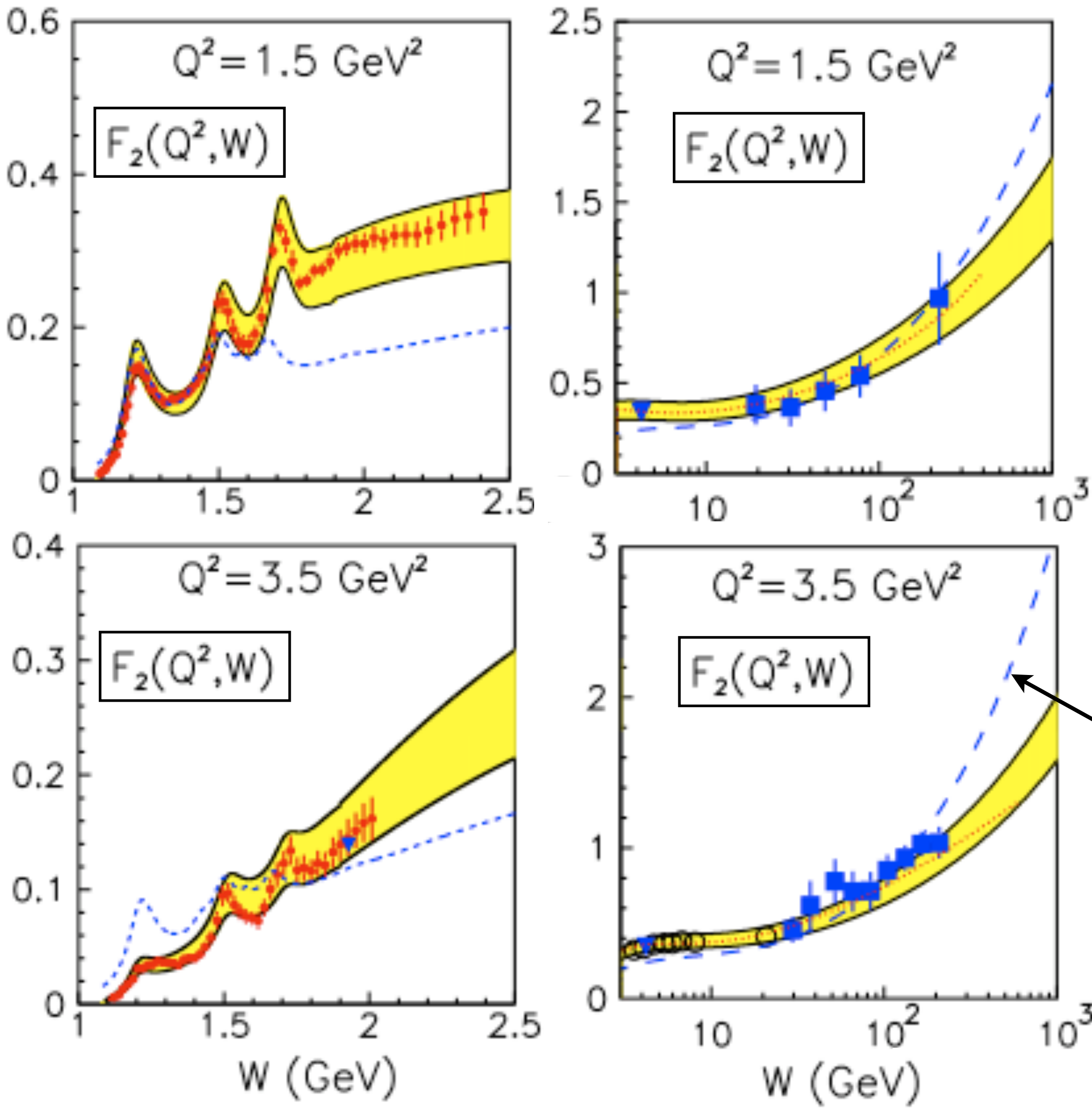
- in resonance region use phenomenological input for F_2, empirical (SLAC) fit for R
  - → for transitions to $I = 3/2$ states (*e.g.* Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$
  - → for transitions to $I = 1/2$ states, SU(6) wave functions predict Z & γ transition couplings equal to a few %

Vector h correction

→ compare structure function input with data

low W

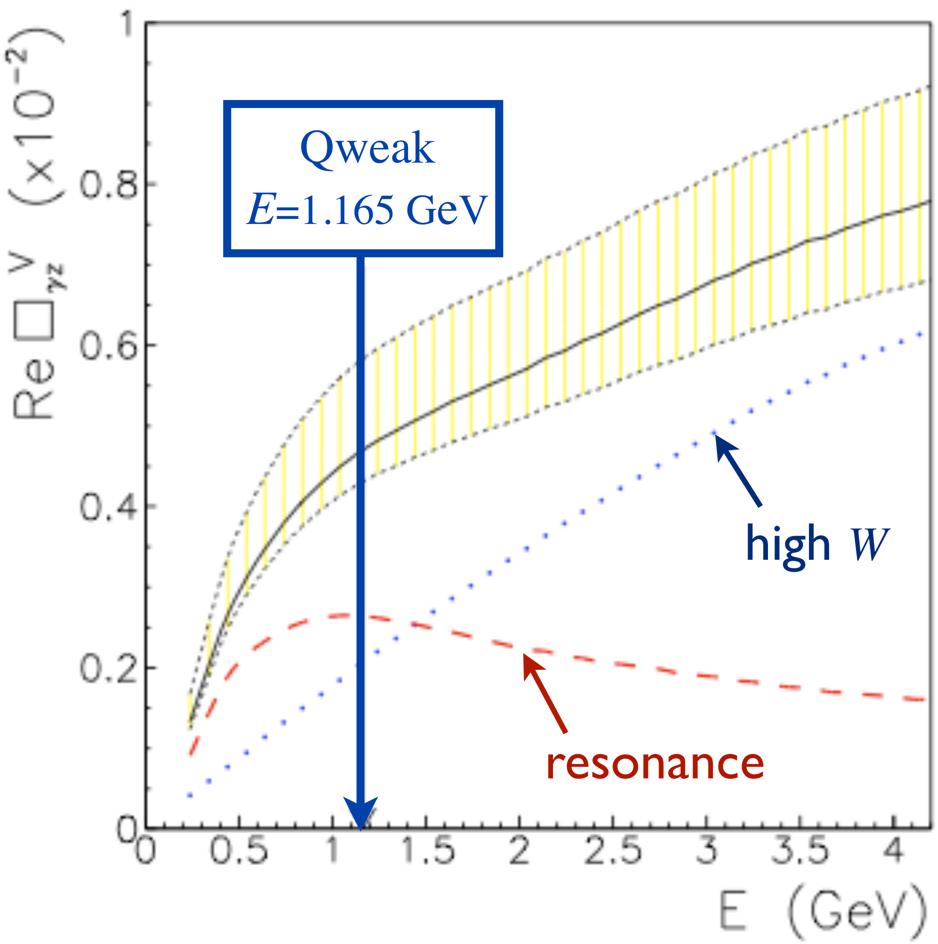
high W



GVMD model
(used as input by
Gorchtein & Horowitz)

Vector h correction

→ total $\Box^V_{\gamma Z}$ correction



$$\Re e\, \Box^V_{\gamma Z} = 0.0047^{+0.0011}_{-0.0004}$$

or $6.6^{+1.5}_{-0.6}\,\%$ of uncorrected Q^p_W

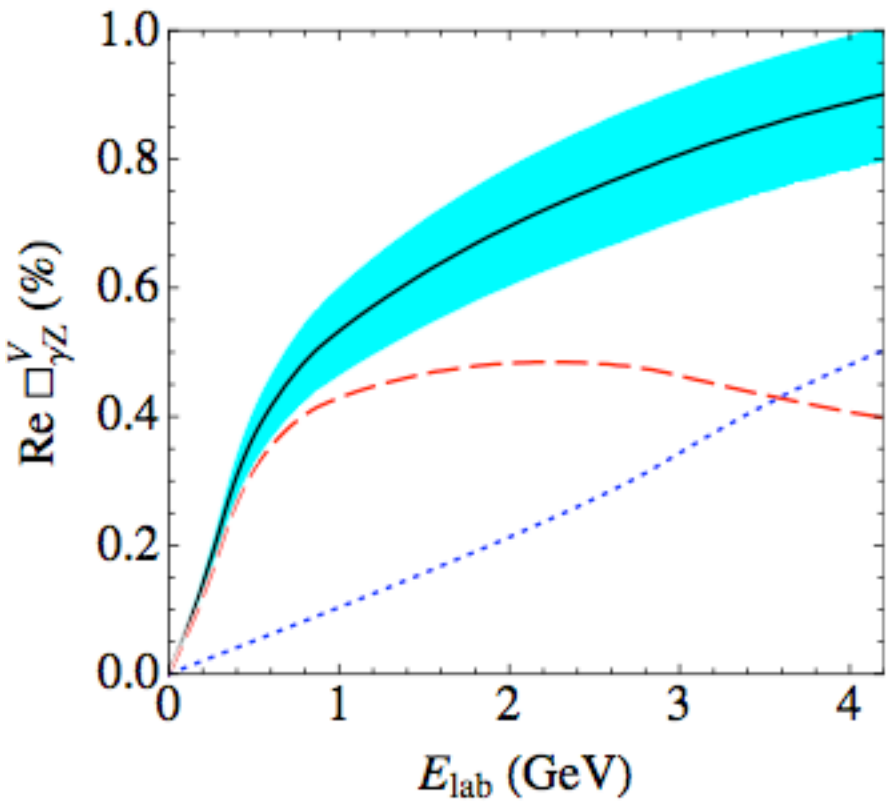
Sibirtsev, Blunden, WM, Thomas
PRD ***82*** *(2010) 013011*

Other vector h calculations

→ total $\Box^V_{\gamma Z}$ correction

$$\Re e\ \Box^V_{\gamma Z} = 0.0057 \pm 0.0009$$

→ compatible with SBMT within errors



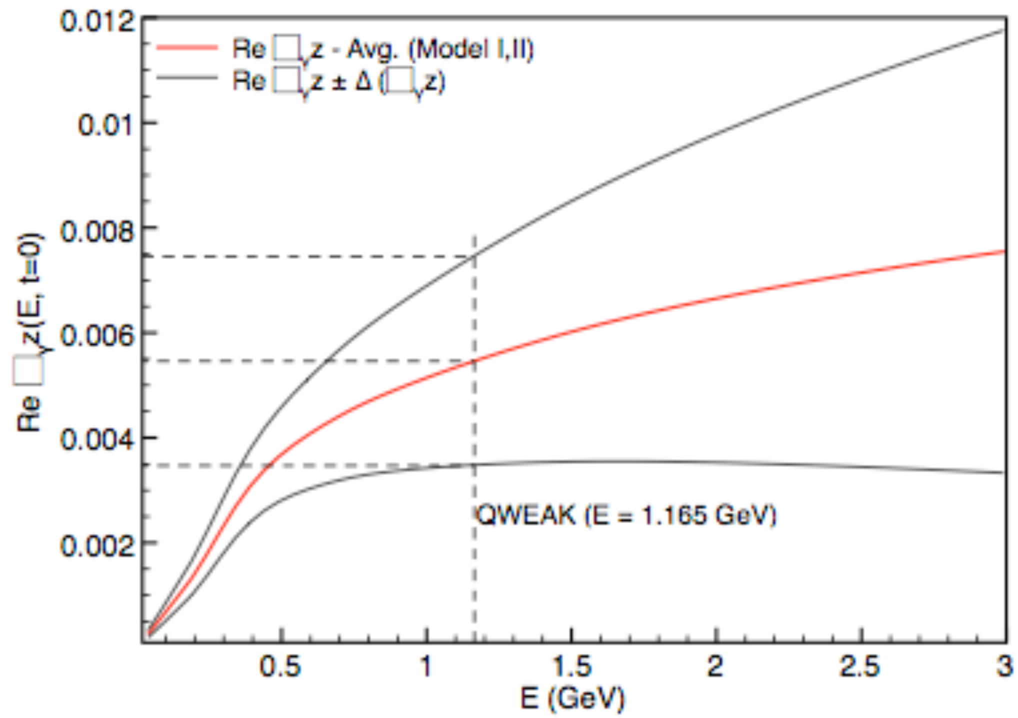
Rislow, Carlson, PRD ***83****, 113007 (2011)*

Other vector h calculations

→ total $\Box_{\gamma Z}^{V}$ correction

$$\Re e\ \Box_{\gamma Z}^{V} = 0.0054 \pm 0.0020$$

→ central value consistent with SBMT and RC, but error 2 x as large



Gorchtein, Horowitz, Ramsey-Musolf
PRC ***84****, 015502 (2011)*

Other vector h calculations

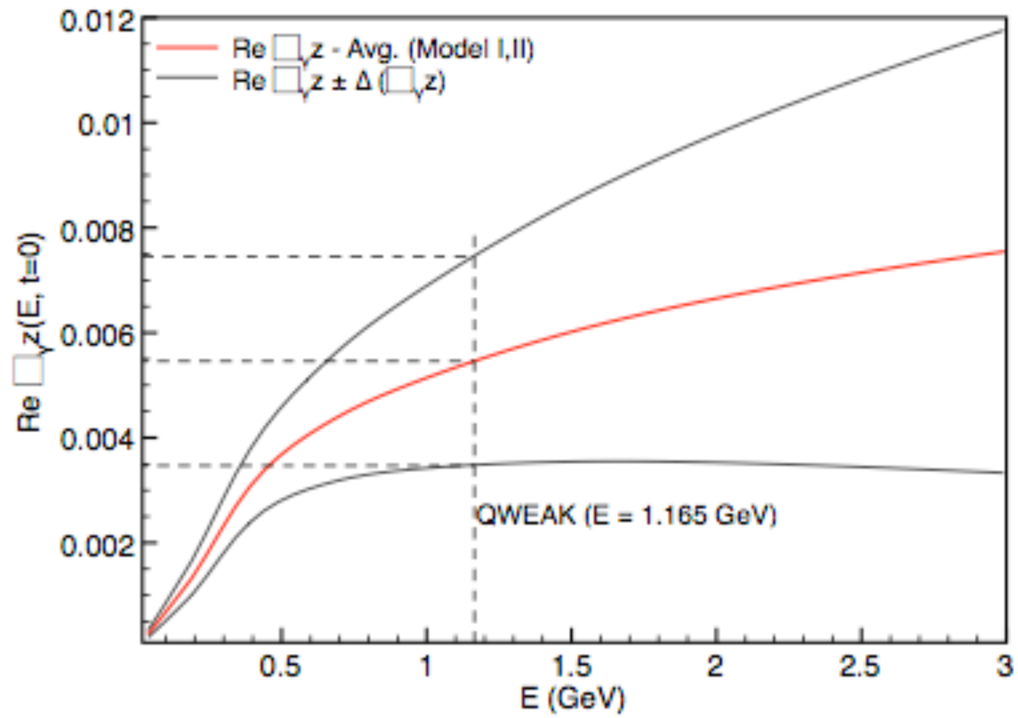
→ total $\Box^V_{\gamma Z}$ correction

$$\Re e\ \Box^V_{\gamma Z} = 0.0054 \pm 0.0020$$

→ central value consistent with SBMT and RC, but error 2 x as large

→ consistent estimate of uncertainty needed

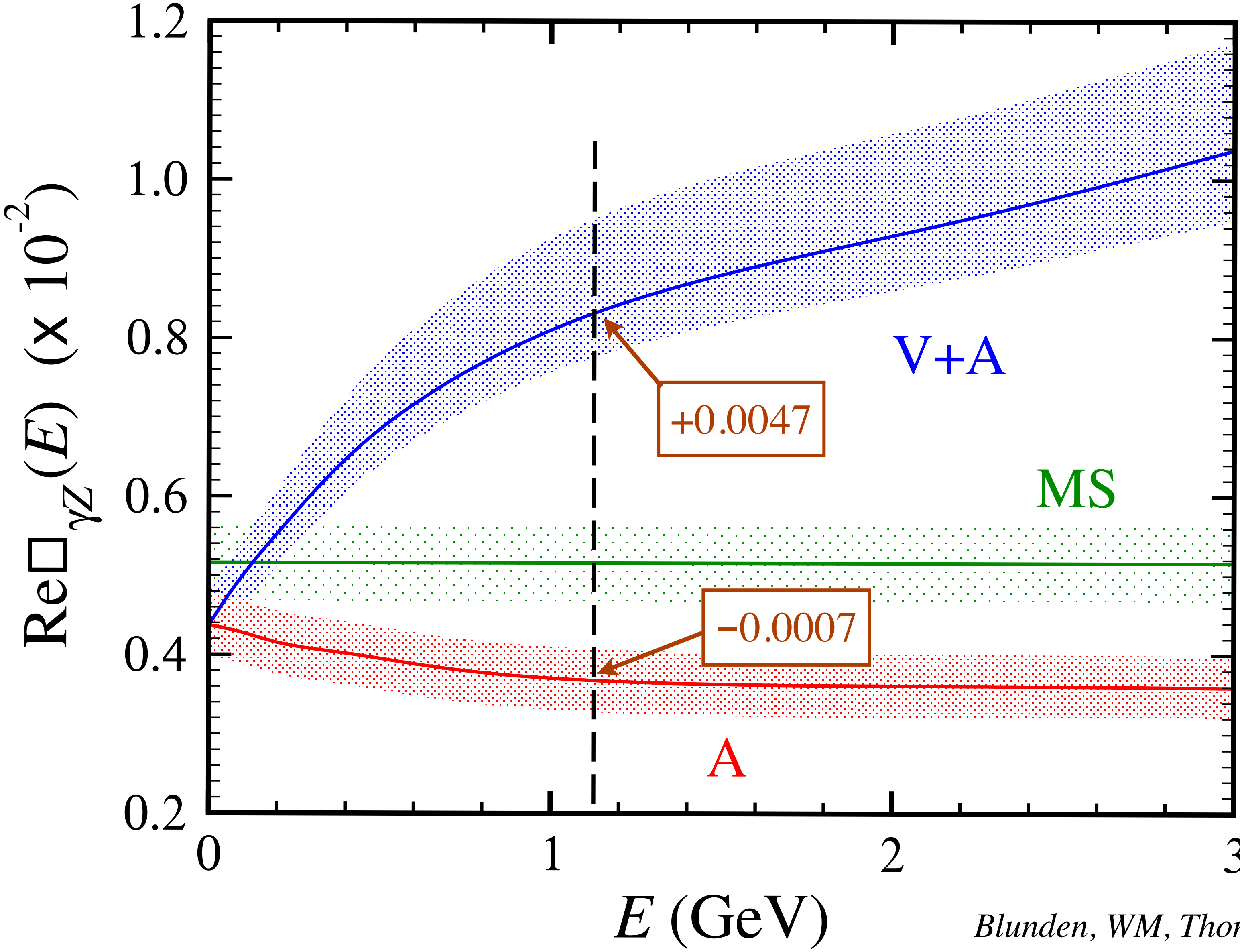
→ data on $F^{\gamma Z}_{1,2}$ at low W and Q^2 vital !



Gorchtein, Horowitz, Ramsey-Musolf
*PRC **84**, 015502 (2011)*

Combined vector and axial h correction

$$\boxed{Q_W^p = 0.0713 \rightarrow 0.0705}$$ (at E=0)



At E=1.165 GeV, E-dependent correction is +0.0040

*Blunden, WM, Thomas, PRL **107**, 081801 (2011)*

t dependence

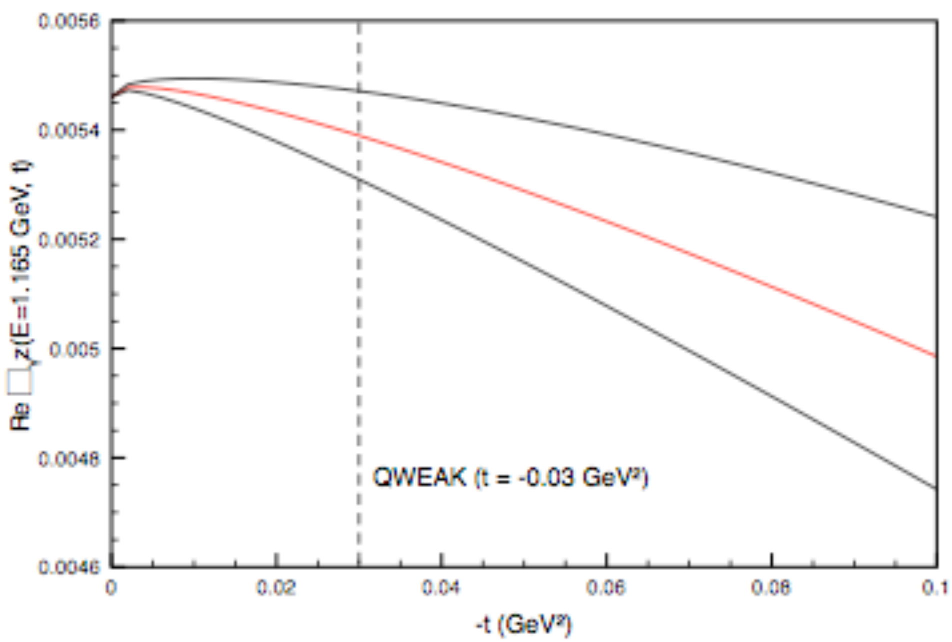
- Extrapolation from $t = -0.03\ \mathrm{GeV}^2$ to $t = 0$

  $\longrightarrow$ phenomenological *ansatz*

$$\Box_{\gamma Z}(E,t) = \Box_{\gamma Z}(0,0)\ \frac{e^{-B|t|/2}}{F_1^{\gamma p}(t)}$$

with $B = (7 \pm 1)\,\mathrm{GeV}^{-2}$ from forward Compton scattering

Gorchtein, Horowitz, PRL ***102*** *(2009) 091806*

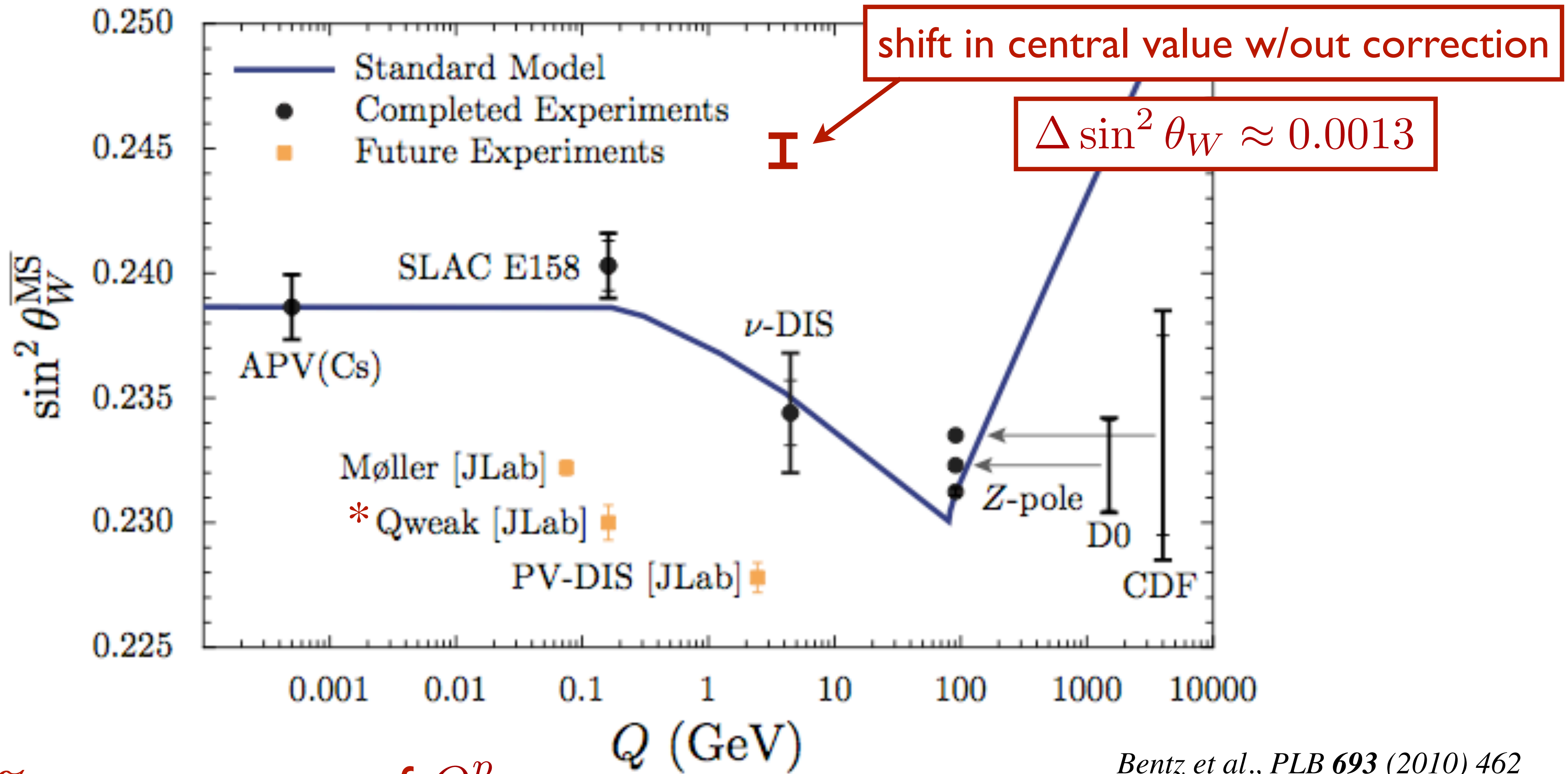


$\longrightarrow$ $\sim 2\%$ reduction of $\Box_{\gamma Z}$

$\longrightarrow$ more work needed for quantitative extrapolation

Combined vector and axial h correction

→ significant shift in central value, errors within projected experimental uncertainty $\Delta Q_W^p = \pm 0.003$



* 4% measurement of Q_W^p

*Bentz et al., PLB **693** (2010) 462*

Summary

- Two-boson exchange corrections likely play minor role in *strange form factor* extraction
  - $\longrightarrow$ *cf*. significant role of TPE in Rosenbluth extraction of G_E^p

- Dramatic effect of $\gamma(Z\gamma)$ corrections at forward angles on proton weak charge, $\Delta Q_W^p \sim 6\%$, *cf*. PDG
  - $\longrightarrow$ would significantly shift extracted weak angle
  - $\longrightarrow$ better constraints from direct measurement of $F_{1,2,3}^{\gamma Z}$ (*e.g*. in PVDIS at JLab)

- New formulation in terms of *moments* of structure functions
  - $\longrightarrow$ places on firm footing earlier derivation of Marciano/Sirlin in "free quark model"
  - $\longrightarrow$ may affect atomic PV calculations ($e.g.$ Cs, Fr)

The End