Dynamical
Parton Distribution Functions

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Dynamical
Parton Distribution Functions

The dynamical approach to parton distributions

The longitudinal structure function

The dynamical determination of strange parton distributions

The treatment of heavy quarks: a brief critical review

Weak-gauge and Higgs boson production at hadron colliders

Global QCD analysis

Determination of NP information: **input distributions** $xf(x, Q_0^2)$

for light quarks + gluon: $f = u, d, s, \bar{u}, \bar{d}, \bar{s}$ and g (*no heavy-quark PDFs!*)

Selected **experiment**al information + **parametrizations**

Nucleon DIS structure functions

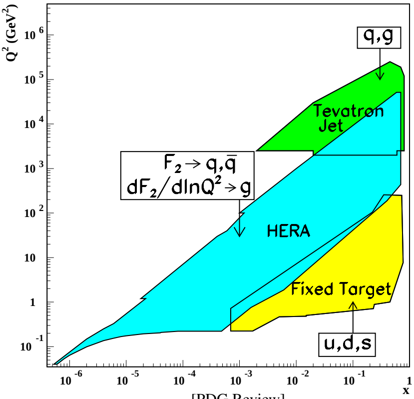
Jets from Tevatron (up to NLO)

Drell-Yan pp + pn (or neutrino DIS) data needed for $\bar{d} \neq \bar{u}$

Not very sensitive to strange PDFs,
$\Rightarrow$ input assumptions $s = \bar{s} (= 0)$
(or asymmetric, discussed later)

Chi-square method:

$$\chi^2(p) \equiv \sum_{i=1}^{N} \left(\frac{\mathrm{data}(i) - \mathrm{theory}(i,p)}{\mathrm{error}(i)} \right)^2$$



[PDG Review]

Estimation of uncertainties

Propagation of **experimental** errors (only!) into the PDFs

Hessian method: quadratic expansion around the global minimum

$$\Delta\chi^2 = \chi^2 - \chi_0^2 \simeq \frac{1}{2}\sum_{i,j=1}^{d} H_{ij}(a_i - a_i^0)(a_j - a_j^0) \leq T^2$$

Tolerance parameter: $T^2 = T_{1\sigma}^2 = \sqrt{2N}/(1.65)^2 \Rightarrow \mathbf{T \simeq 5}$

diagonalization of $H_{ij} \longrightarrow$ (rescaled) eigenvector matrix M_{ij}

"Eigenvector sets": $\quad a_i^{\pm j} = a_i^0 \pm T M_{ij}$

Calculation of a quantity $X \pm \Delta X$:

$$X = X(a^0), \qquad \Delta X = \frac{1}{2}\sum_{j=1}^{d}\sqrt{\left(X(a^{+j}) - X(a^{-j})\right)^2}$$

The dynamical parametrization

Since we are free to (and have to) select an input scale for the RGE:

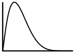
At low-enough Q^2 only "valence" partons would be "resolved"

$\Rightarrow$ *structure at higher Q^2 appears radiatively (i.e. due to QCD dynamics)*

DYNAMICAL:

$Q_0^2 < 1\,\text{GeV}^2$ optimally **determined**

$\mathbf{a} > 0$ *"valence-like"*



"STANDARD":

$Q_0^2 = 2\,\text{GeV}^2$ arbitrarily **fixed**

Unrestricted parameters

$$xf(x, Q_0^2) = N x^{\mathbf{a}} (1-x)^b (1 + A\sqrt{x} + Bx)$$

Positive definite input distributions

QCD **predictions** for $x \lesssim 10^{-2}$

More restrictive, **less uncertainties**

Arbitrary fine tunning ($g < 0!$)

Extrapolations to unmeasured region

Less restrictive, *marginally* smaller χ^2

Physical motivation for the **CC of the DGLAP $\neq$ NP structure** of the nucleon

There are *no extra theoretical assumptions* involved in the dynamical approach with respect to the "standard" one

Brief history of the dynamical PDFs

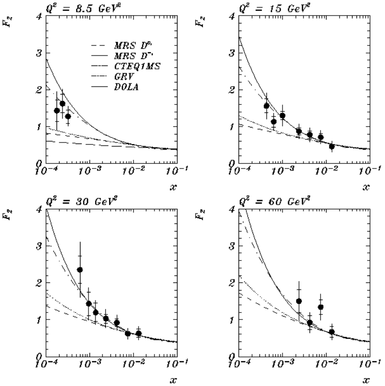
Dynamical assumption [Altarelli, Cabibbo, Maiani, Petronzio 74], [Parisi, Petronzio 76], [Novikov 76], [Glück, Reya 77] in connexion with the *constituent quark model*: only valence quarks

First dynamical determination of parton distributions [Glück, Reya 77]

Used in the 80's: e.g. for the discovery of W and Z bosons (SPS, CERN)

Extended to include **light sea** [Glück, Reya, Vogt 90] and **gluon** [Glück, Reya, Vogt 92] **valence-like input**
$\longrightarrow$ **steep gluon and sea at small-x!!**

Confirmed by first HERA $F_2(x, Q^2)$ data [H1, ZEUS 93]



GRV95 and GRV98 contributed greatly in the 90's and beginning of the 00's

New improved generation (GJR08, JR09):
$\overline{\text{MS}}$ + DIS factorization schemes, NNLO, error analysis, FFNS+VFNS, **new data**

The new generation

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Dynamical Parton Distribution Functions

The links below give access to FORTRAN codes (grids) containing the latest *dynamical* parton distribution functions of the nucleon; some "standard" (following the terminology of the papers) sets are available on request. Both the (old) famous *GRV* parton distributions and these new *(G)JR* sets are also available as part of the LHAPDF library or at HEPDATA. In addition to the references below, details about the *(G)JR* parton distributions (e.g. the parameters of the "error" sets) are given in the Ph.D. thesis of P. Jimenez-Delgado (arXiv:0902.3947).

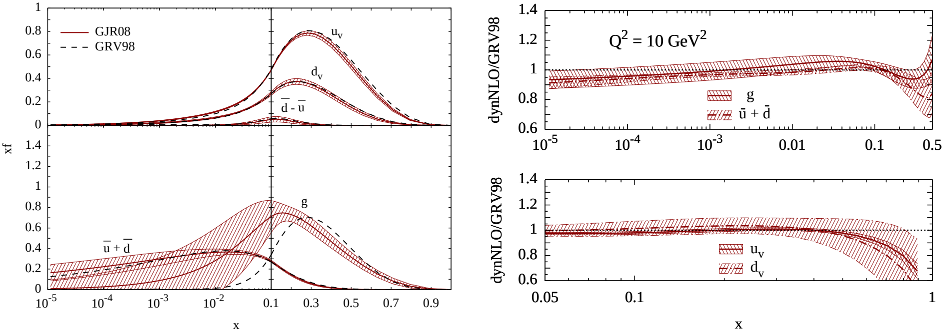
JR09VFNNLO.tar.gz contains the *NNLO-MSbar* (including uncertainties) *dynamical* parton distribution functions of the nucleon generated in the *VFNS* (Variable Flavor Number Scheme) as presented in *Phys. Rev. D80 (2009) 114011* (arXiv:0909.1711).

JR09FFNNLO.tar.gz contains the *NNLO-MSbar* (including uncertainties) *dynamical* parton distribution functions of the nucleon generated in the *FFNS* (Fixed Flavor Number Scheme) as presented in *Phys. Rev. D79 (2009) 074023* (arXiv:0810.4274). Note that the scale dependence of the running coupling constant is governed by a variable number of active flavors as discussed in *Mod. Phys. Lett. A22 (2007) 351* (hep-ph/0608276).

GJR08VFNS.tar.gz contains the *LO* and *NLO-MSbar* (including uncertainties) *dynamical* parton distribution functions of the nucleon generated in the *VFNS* (Variable Flavor Number Scheme) as presented in *Phys. Lett. B664 (2008) 133* (arXiv:0801.3618).

GJR08FFNS.tar.gz contains the *LO*, *NLO-MSbar* (including uncertainties and the light- as well as the fixed-order heavy-quark-contributions to *F2p*) and *NLO-DIS dynamical* parton distribution functions of the nucleon generated in the *FFNS* (Fixed Flavor Number Scheme) as presented in *Eur. Phys. J. C53 (2008) 355* (arXiv:0709.0614). Note that the scale dependence of the running coupling constant is governed by a variable number of active flavors as discussed in *Mod. Phys. Lett. A22 (2007) 351* (hep-ph/0608276).

Comparison with GRV98



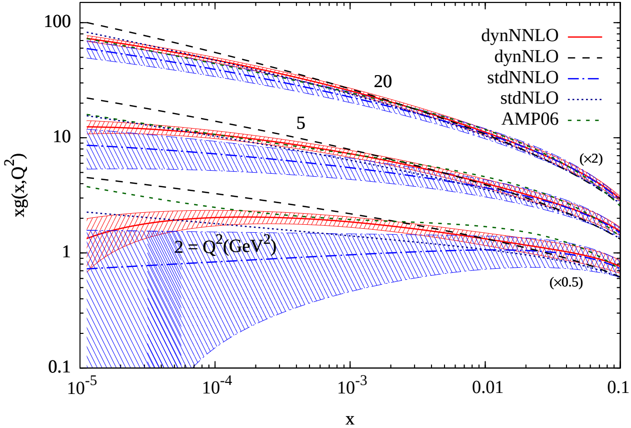
Very **similar** to the previous **dynamical** (input) distributios **GRV98** [up to NLO]

All quark distributions **within error** estimates (note the flat sea)

Similar gluon as well: peaks at slightly different x but within 2σ

Stable after evolution, less than $10-20\%$ of "acceptable" (1σ) difference

Dynamical vs standard: gluon

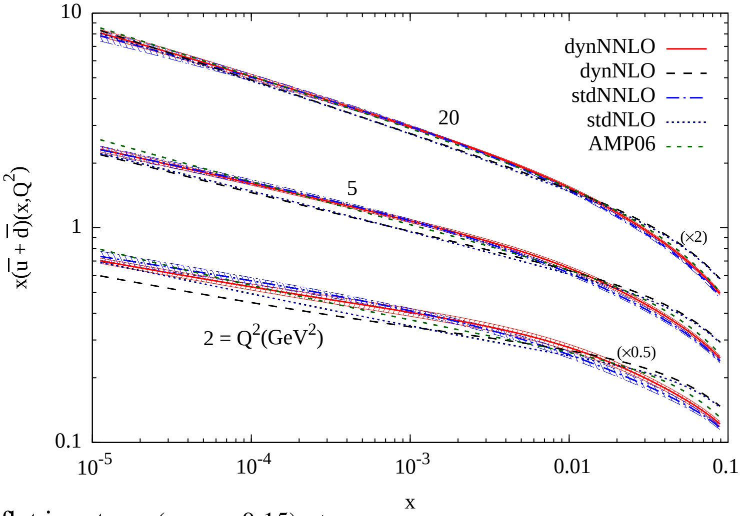


Uncertainties decrease as Q^2 increase: *pQCD evolution*

Valence-like input, i.e., *larger "evolution* distance" ⇒ **less uncertainties**

Q_0^2 also play another role ⇒ standard gluons fall below dynamical

Dynamical vs standard: sea



Rather flat input sea ($a_{\bar{u}+\bar{d}} \simeq 0.15$) $\Rightarrow$

equally increasing down to $x \simeq 10^{-2}$ $\Rightarrow$ **marginally smaller** errors

The strong coupling constant

$\alpha_s(\mu^2)$ and HQ masses are *parameters* which *depend on the theoretical input* (order, scheme, scales, etc.)

It is **desirable** that their values come out of the global PDF fits

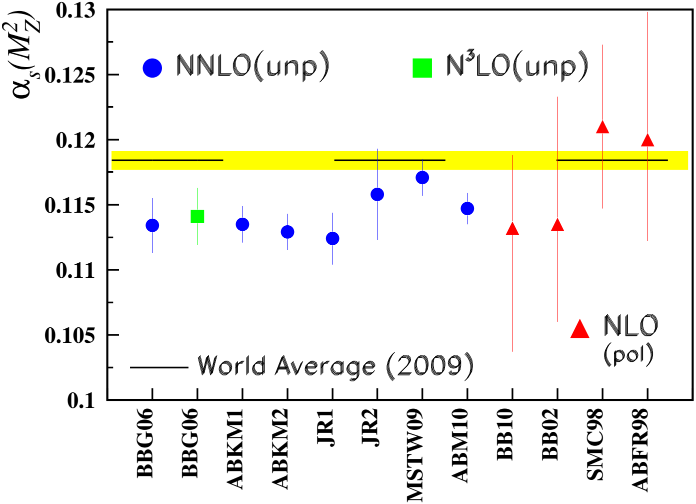
We determine $\alpha_s(M_Z^2)$ **together** together with the distributions:

	dynamical	**"standard"**
NNLO	0.1124 ± 0.0020	0.1158 ± 0.0035
NLO	0.1145 ± 0.0018	0.1178 ± 0.0021
LO	0.1263 ± 0.0015	0.1339 ± 0.0030

Dynamical constraints reduce the uncertainty! (in particular at NNLO)

Dynamical results are smaller: larger "evolution distance" ($Q_0^2 < 1\,\mathrm{GeV}^2$)

Comparing with other determinations



[Blümlein 2010]

DIS data generally prefer lower values than LEP or hadron colliders:

Differences should be interpreted as **uncertainties** (*not* be "removed" by convention!)

DIS reduced cross-section

$$\sigma_r^{NC} \equiv \left(\frac{2\pi\alpha^2}{xyQ^2}Y_+\right)^{-1}\frac{d^2\sigma^{NC}}{dx\,dy} = F_2^{NC} - \frac{y^2}{Y_+}F_L^{NC} \mp \frac{Y_-}{Y_+}xF_3^{NC}$$

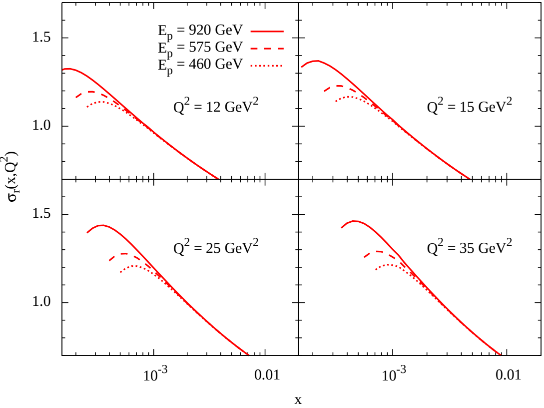
Usually dominated by F_2^γ

$$y = \frac{Q^2}{s}\frac{1}{x}$$

for fixed Q^2 (and s)
F_L relevant with
increasing y (decreasing x)

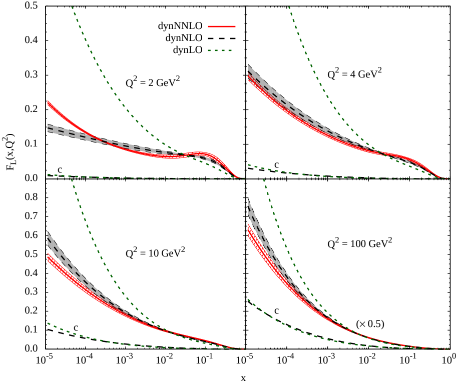
$\rightarrow$ *turnover* at small x

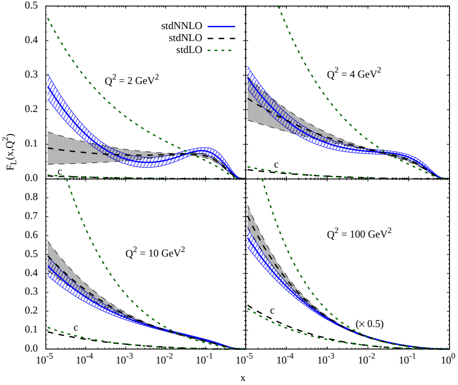
$\Rightarrow$ F_L **positive**



gluon dominated in the small-x region $\Rightarrow$ **positive gluon** (also beyond LO!)

Perturbative stability





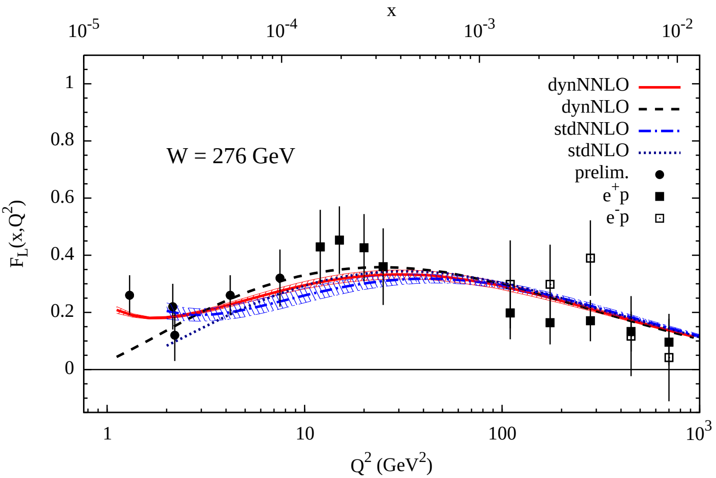
Observed [M(R)ST(W)] instabilities *unphysical*: **artefact** of negative gluons

Both dynamical *and* standard results manifestly **positive** at all orders

Dynamical predictions **stable** already at $Q^2 \gtrsim 2$ GeV^2

Standard differ more but less distinguishable due to the **larger error bands**

Confronting results with data



[H1 2001+2003]

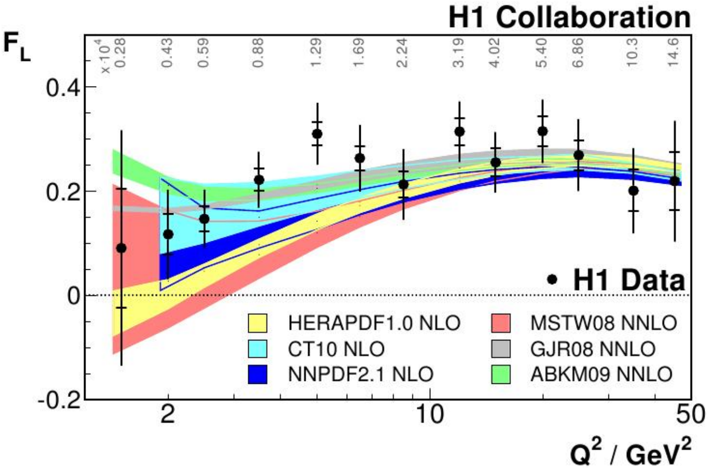
From our paper, preliminary data: [Lobodzinska 2004] [Lastovicka 2004]

Positive and in complete **agreement** with measurements

Dynamical predictions more tightly constrained

Higher-twist effects may contribute for $Q^2 \leq 2$ GeV^2

Confronting results with data



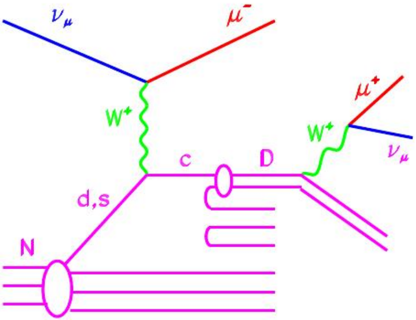
[H1 2010]

Positive and in complete **agreement** with measurements (confirmed!)

Greater precision achieved within the dynamical framework

Other results less precise and even **turning negative** at the lower Q^2 values

Dimuon production



[NuTeV 2001]

Signature: Two muons of different sign

Directly related to **charged current charm production** $\propto s(x,Q^2)$ (FFNS)

Sensitive to differences between s and $\bar{s}$

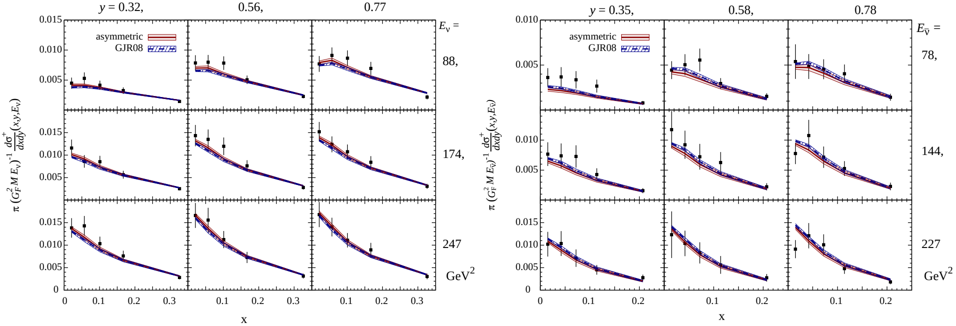
Overall normalization proportional to B_c

$$\frac{d\sigma^+}{dxdy}\left(x,y,E_{\nu(\bar{\nu})}\right) = \frac{G_F^2 M E_{\nu(\bar{\nu})}}{\pi}\, B_c\, \mathscr{A}\left(x,y,E_{\nu(\bar{\nu})}\right)\, \frac{d\sigma^{\nu(\bar{\nu})}}{dxdy}\left(x,y,E_{\nu(\bar{\nu})}\right)$$

Acceptance corrections [Kretzer et al.] at NLO!

Nuclear corrections (iron) using FFNS NLO GRV98 [de Florian et al.]

Fitting the data



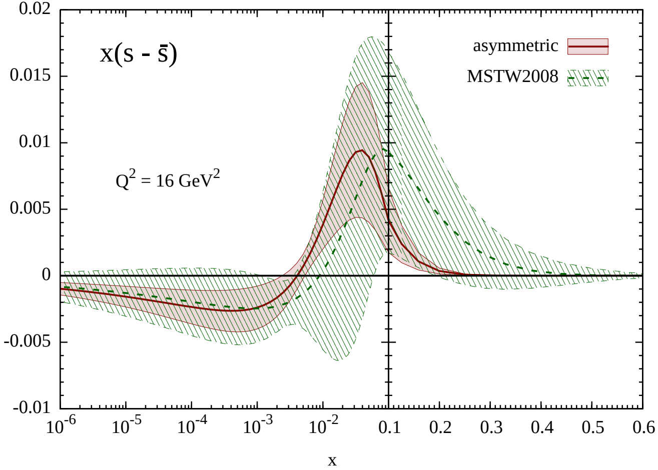
Already **well described by GJR08**: $\chi^2 = 65$ for 90 data points (1σ)

$\Rightarrow$ **radiatively generated strangeness plausible!**: $s(x,Q_0^2) + \bar{s}(x,Q_0^2) = 0$

Introducing an asymmetry χ^2 goes down to 60: $s(x,Q_0^2) - \bar{s}(x,Q_0^2) \neq 0$

Neutrino increases, antineutrino decreases $\Rightarrow$ **“positive” asymmetry**

The strangeness asymmetry



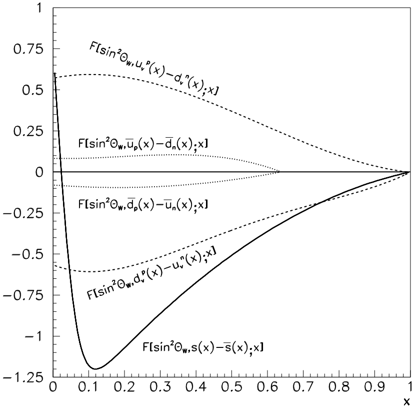
Compatible with previous determinations but smaller uncertainties

Very **small effect** (for most applications): $S^- \equiv \int_0^1 dx\, x(s-\bar{s}) = 0.0008 \pm 0.0005$

Important for dedicated experiments (e.g. NuTeV anomaly)

The NuTeV anomaly

Experimental methods(functionals): $\Delta s_W^2 = \int_0^1 F[s_W^2, \delta \overset{(-)}{q}; x] x \delta \overset{(-)}{q}(x, Q^2)\, dx$



Total shift: $\Delta s_W^2|_{\text{total}} =$

$= \Delta s_W^2|_{\text{QED}} + \Delta s_W^2|_{\text{NP}} + \Delta s_W^2|_{\text{strange}}$

Isospin–symmetry violating PDFs:

NP mass effects: $\Delta s_W^2|_{\text{NP}}$ [Londergan et al.]
radiative QED effects: $\Delta s_W^2|_{\text{QED}}$

Strange asymmetric PDFs: $\Delta s_W^2|_{\text{strange}}$

All effects combined remove the "anomaly" (within SM)!

Using $R^- \equiv \frac{\sigma_{\text{NC}}^{\nu N} - \sigma_{\text{NC}}^{\bar{\nu} N}}{\sigma_{\text{CC}}^{\nu N} - \sigma_{\text{CC}}^{\bar{\nu} N}} = R_{\text{PW}}^- + \delta R_I^- + \delta R_s^-$ overestimates the corrections ($\approx 20\%$–40%)

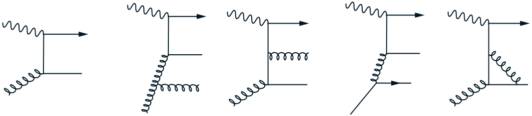
FOPT heavy-quark contributions

Experiment: **No** (< 1%) **intrinsic** heavy-quark content in the nucleon

HQ generated in hard collisions, *not collinearlly*, short "lifetime" ($\neq$ parton)

$\Rightarrow$ **Final state** $\equiv$ **extrinsic** heavy-quark content; **fully massive calculations**

$\Rightarrow$**FOPT** initiated by gluons and light (u, d, s) quarks $\equiv$ **FFNS** (in this context)



$$F^h_{k=2,L}(x,Q^2,m^2) = \frac{\alpha_s(\mu^2)Q^2}{4\pi^2 m^2} \sum_{f=g,q,\bar{q}} \int_{z_{th}}^{1} dz\, f(z,\mu^2)\, \hat{F}^h_{k=2,L}(\tfrac{x}{z},\mu^2,Q^2,m^2), \quad z_{th} = x(1+\tfrac{4m^2}{Q^2})$$

Gluon dominated (starts at LO), therefore "small-x"-dominated:

about 80% originates in the region $z_{th} \le z \le 3z_{th}$ [A.Vogt 96]

$\Rightarrow$ *threshold region is always important* (irrespective of Q^2)

Massive coefficient functions

Theoretical status:

The (inclusive) coefficient functions are known at LO [Witten 75, Glück and Reya 79] and NLO [Laenen,Riemersma,Smith,van Neerven 93]:

$$\hat{F}^{h}_{k=2,L}(\tfrac{x}{z},\mu^2,Q^2,m^2) = e_h^2\delta_{fg}\,c^{(0)}_{k,g} + 4\pi\alpha_s(\mu^2)\left[e_h^2\left(c^{(1)}_{k,f}+\bar{c}^{(1)}_{k,f}\ln\frac{\mu^2}{m^2}\right)+e_f^2\left(d^{(1)}_{k,f}+\bar{d}^{(1)}_{k,f}\ln\frac{\mu^2}{m^2}\right)\right]$$

There is a fully exclusive NLO calculation [Harris and Smith 95]: **HVQDIS**,
in which all the experimental analysis at HERA is based

At NNLO only the asymptotic ($Q^2 \gg m^2$) coefficients [Bierenbaun,Blümlein,Klein 09] exist:

There is no complete NNLO (massive) calculation of HQ contributions in DIS

Some *approximations* can be made using small-x [Catani, Cialfoni, Hautmann 91] and threshold [Laenen and Moch 99] resummations

The coefficient functions contain potentially large $\ln\frac{\mu^2}{m^2}$'s (**not mass divergences**):

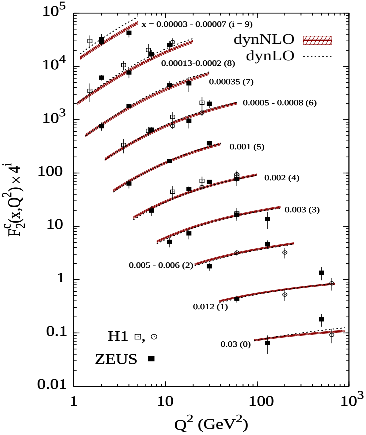
Are these terms dangerous or is the FFNS stable for DIS phenomenology?

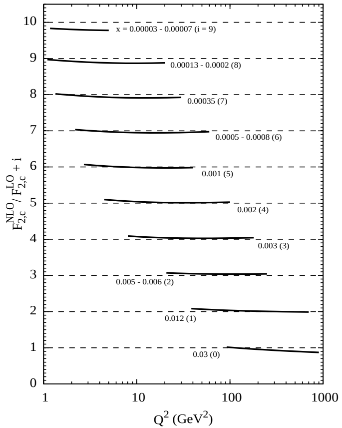
Perturbative stability of the FFNS

It should be clear since (again):

all the experimental analysis at HERA is based on the FFNS (HVQDIS)

Nevertheless we can have a look at the (semi)inclusive calculation (used in the fits):





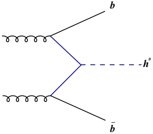
FFNS gets trough *all* "stability tests"!!

No need to resum *supposedly* "large logarithms"... why are there other schemes then?

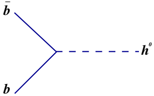
Effective heavy-quark PDFs: VFNS

The only *drawback* of the FFNS is the calculational difficulty, for instance:

FFNS: $g\,g \to b\,\bar{b}\,H$



VFNS: $b\,\bar{b} \to H$



We can construct *effective* heavy-quark PDFs from the the asymptotic limit of the massive calculation [Buza, Matiouine, Smith, van Neerven 98]:

$$H^{Q^2 \gg m^2}\left(\frac{Q^2}{\mu^2}, \frac{\mu^2}{m^2}\right) = A\left(\frac{\mu^2}{m^2}\right) \otimes C\left(\frac{Q^2}{\mu^2}\right) \quad \Rightarrow \quad f_j^{VFNS} = \sum_k A_{jk} \otimes f_k^{FFNS}$$

A's=massive OME's, process independent $\Rightarrow$ **preserves universality!!**
C's=light-parton coefficient functions

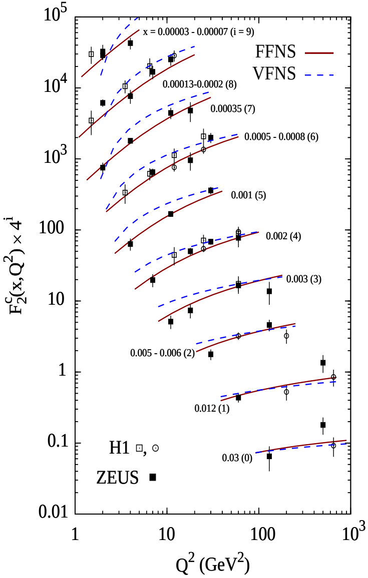
In practice: **massless evolution** with increasing n_f at unphysical "thresholds" $\mu^2 \simeq m^2$

This resums (RGE) the $\ln \frac{\mu^2}{m^2}$'s of the final-state contributions $\neq$ intrinsic HQs

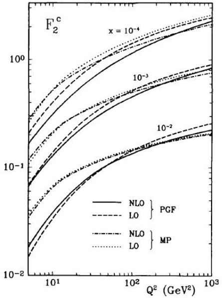
The effective VFNS HQ-PDFs are *assumed* to be correct asymptotically but:

Is this scheme relevant for DIS phenomenology?

Relevance of the VFNS



Even though $W^2 \gg m_c^2$ for *all* data:
At small-x (small Q^2): **gross overestimate**
It does better at larger x (larger Q^2)



[Glück,Reya, Stratmann 94]

It never reduces to the exact (FFNS) result
(not even at very large Q^2):
dropped terms ($\propto \frac{m^2}{Q^2}$) *are always relevant*

The VFNS should not be used for global analyses!! (this is well-known since a long time)

Relevance of the VFNS

VFNS reliable for large invariant mass of the produced system: $W_{th}^2 \gg m^2$

$\Rightarrow$ *non-relativistic* ($\beta = \frac{|\vec{p}|}{E} \lesssim 0.9$) *threshold effects supressed* [Glück,Reya,PJD 08]

For charm production in neutral-current DIS: $\frac{W_{\rm th}}{m_c} = 2 \Rightarrow$ VFNS fails

For the previous example, Higgs-boson production in $b\bar{b}$ fusion:

$$\frac{W_{\rm th}}{m_b} = \frac{2m_b + m_H}{m_b} \simeq \frac{m_H}{m_b} \gg 1 \Rightarrow \text{VFNS should work}$$

Note that *we can* generate VFNS PDFs from our FFNS PDFs (3-flavor input)

Input determined using DIS data and the FFNS!!

This combines the virtues of both **FFNS + VFNS** schemes

Typical scheme-choice uncertainties? Example, *W* production at *LHC*:

$$\sigma^{\rm NLO}(pp \to W^+ + W^- + X) = \begin{cases} 186.5 \pm 4.9_{\rm pdf} \, {}^{+4.8}_{-5.5} \big|_{\rm scale} \text{ nb} & \text{(VFNS)} \\ 192.7 \pm 4.7_{\rm pdf} \, {}^{+3.8}_{-4.8} \big|_{\rm scale} \text{ nb} & \text{(FFNS)} \end{cases}$$

VFNS sufficiently accurate ($\approx 10\%$) for LHC and Tevatron energies.

General-mass VFNS's

Phenomenological *models* for HQ contributions in *inclusive* DIS; idea:

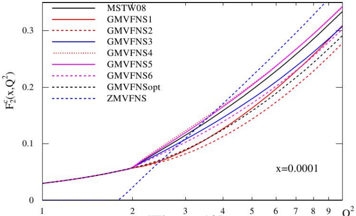
Interpolation between FFNS and VFNS

Constructed (as the VFNS) over the FFNS: **no new information** (e.g. no complete NNLO)

Often goes something like: $F^{GM} \equiv F^{FF} + F^{ZM} - F^{\text{model}}$ with:

$F^{\text{mod}} \to F^{\text{FF}}$ for $Q^2 \gg m^2$

In the intermediate region there is a lot of "freedom" $F^{\text{mod}} \to F^{\text{ZM}}$ for $Q^2 \simeq m^2$



[Thorne 10]

... and correspondingly many implementations:

ACOT [Aivazis,Collins,Olness,Tung] + variations

BMSN [Buza,Matiounine,Smith,van Neerven]

CSN [Chuvakin,Smith,van Neerven]

RT [Roberts,Thorne] + variations

FONNL [Forte,Laenen,Nason,Rojo]

(Although some of them are known "not to work" properly)

Scheme choices affect the PDFs and in turn the predictions for physical observables

Plausibility of the GM-VFNS's

In GM-VFNS's, DIS mass dependences are somewhat absorbed into the PDFs:

Process-dependent PDFs? **What happened with universality?**

GM-VFNS's are *unnecesary* for HERA (FFNS) and for Tevatron or LHC (FF+VF):

What is the advantage of interpolation models?

My opinion (other authors differ):

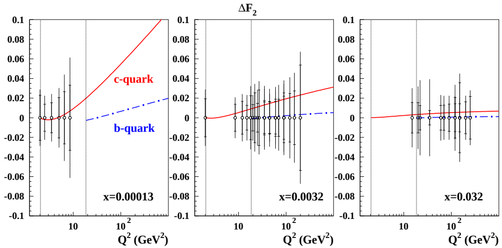
We should not try to model HQ contributions, but stick to schemes unequivocally defined on solid theoretical grounds

Anyway, arguing about which is the best scheme is rather superfluous:

How do the differences compare with the experimental errors?

We can compare $F^{h,FFNS} - F^{h,BMSN}$ with ΔF_2^{tot} [Alekhin,Blümlein,Klein,Moch 10]

$\Rightarrow$ generally effects are smaller (but in some regions in the limit)



Present HQ treatments in DIS

Traditionally the most "popular choice" for global fits was the **VFNS**, with the exception of the GRV group, which used the FFNS *already* for GRV95

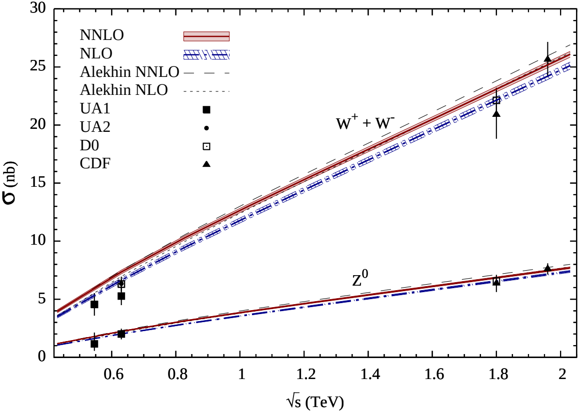
This changed after the release of CTEQ6.5 (2007), where a GM-VFNS was used and the effects of HQ masses in the predictions at hadron colliders were "re-discovered": **today their importance is generally recognized**

Current choices of the (main) PDF groups are:

- CTEQ: ACOT-like
- MSTW and HERAPDF: TR-like
- NNPDF: VFNS (switching to FONLL)
- ABKM: both FFNS and BMSN
- (G)JR: FFNS and VFNS (generated from the FFNS)

The experimental analyses use the FFNS (exclusive calculations needed)

Weak gauge boson production rates

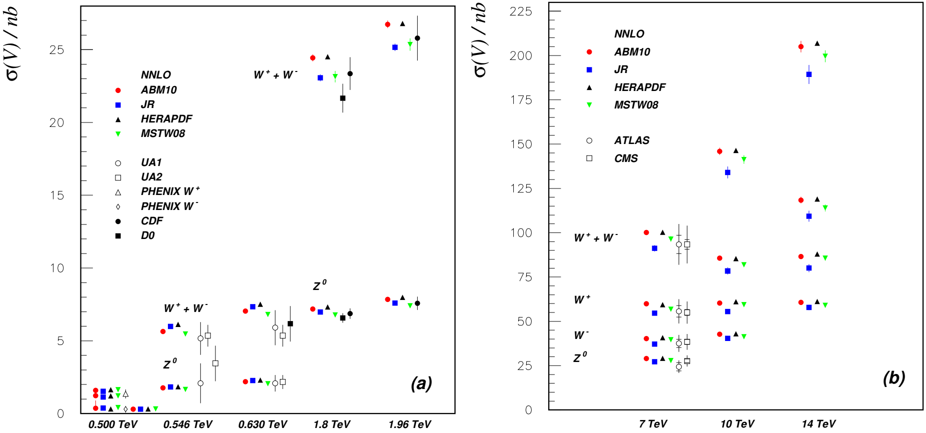


NNLO typically larger but stable; scale uncertainty greatly (%4) reduced

Our NNLO expectations for LHC (≈ 3% accuracy):

$$\sigma(pp \to W^+ + W^- + X) = 190.2 \pm 5.6_{\text{pdf}} \; {}^{+1.6}_{-1.2}|_{\text{scale}} \text{ nb}$$
$$\sigma(pp \to Z^0 + X) = 55.7 \pm 1.5_{\text{pdf}} \; {}^{+0.6}_{-0.3}|_{\text{scale}} \text{ nb}$$

NNLO benchmarks for W and Z

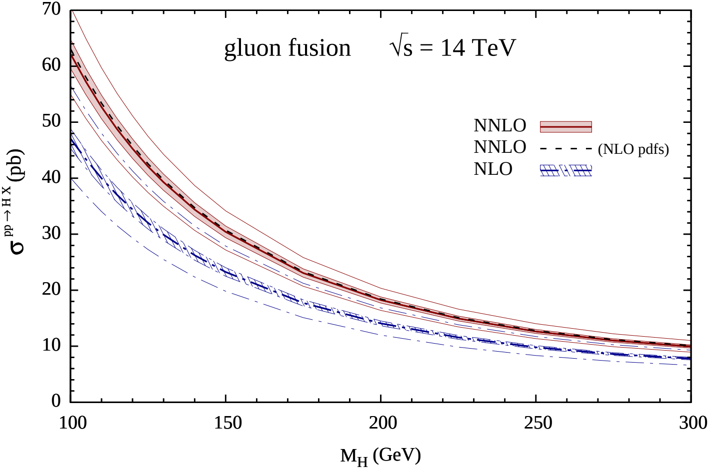


Results from different groups **agree within experimental** uncertainty

Considering results from different groups **accuracy better than** $\approx 10\%$ at LHC

A first inventigation points to *differences in light-sea* distributions

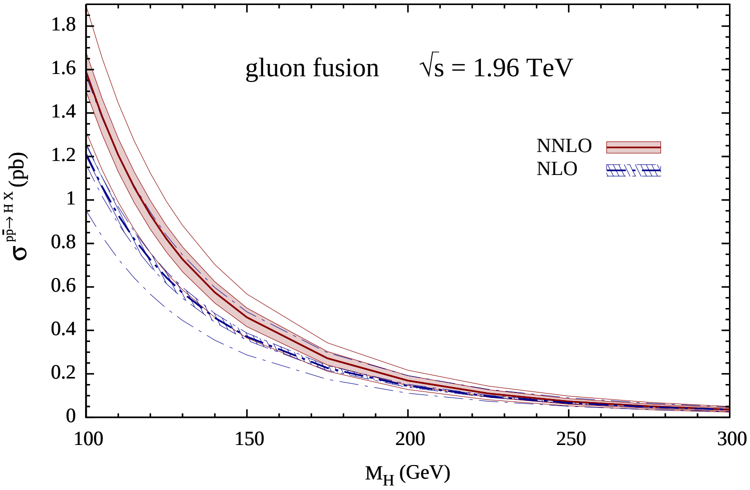
Higgs boson production at LHC



NNLO rather (20%) larger than NLO but *total* uncertainty bands overlap

Not *very* dependent on PDF details. *Our* total errors at NNLO less than 10%

Higgs boson production at Tevatron



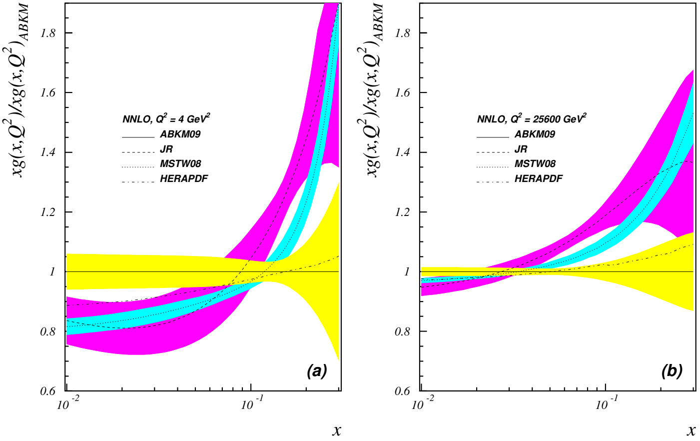
Qualitatively similar features than at LHC but larger uncertainty bands

Briefly speaking uncertainties double at Tevatron (and also double at NLO)

Comparison of parton luminosities

Dominant gluon fusion $\propto \alpha_s^2$, and **quadratic in the gluon** (anticorrelated)

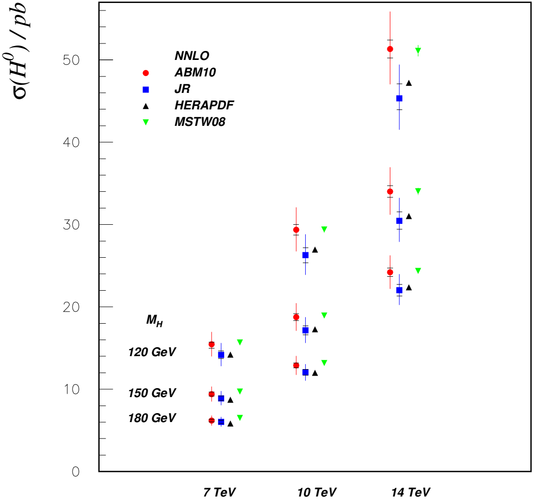
Obtained $\alpha_s(M_Z^2)$ about 4(3)% smaller for JR(ABKM09) than for MSTW08



Differences of about 5% for $<x>=\sqrt{x_1 x_2}=\frac{M_H}{\sqrt{S}}\approx 10^{-2}$ relevant for LHC

(For Tevatron 10-20% at $<x>\approx 10^{-1}$)

NNLO benchmarks for Higgs production



Differences due to $\alpha_s(M_Z^2)$ and gluon distributions, largely **understood**

Considering the different NNLO results $\approx 10-20\%$ **accuracy at LHC**

Summary and conclusions

Dynamical LO and NLO PDFs **updated**: Compatible with **GRV98**

Analyses **extended**: new data, NNLO, errors ...

Dynamical approach: more **predictive** and **smaller uncertainties**

Positive distributions and cross-sections (F_L) in **agreement with all data**

Strangeness asymmetry **precisely** determined: small and positive

FFNS reliable: **no need for heavy-quark distributions!**

Effective (VFNS) "heavy"-quark distributions **practical** for hadron colliders

Total accuracy at LHC: $\approx 10\%$ **for gauge-boson production rates**

$\approx 10-20\%$ **for Higgs production**