working with Graham Ross 1974-1984
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Duality & Unitarisation
Footnote in Physics

KNO Scaling
TESTS OF GEOMETRICAL SCALING AND GENERALIZATIONS *

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the prescription

\[
\frac{d\sigma/dt(s, t)}{d\sigma/dt(s, 0)} = f\left(\frac{\sigma_t^2 t}{\sigma_{el}}\right),
\]

with imaginary non-flip amplitudes and \( f \) some universal function, proposed independently as a generalization of GS by Pennington and Ross [12]. For small \( t \) this prescription is almost trivial, since it is well known that \( d\sigma/dt \) is universally exponential here [13]; the interest lies at larger \( t \)-values.

\[ \mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b} \bar{q} \left( i \gamma_\mu D^\mu - m_q \right) q - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} \]
DIS, Renormalization Group & pQCD

\[ q^2 = -Q^2 \]
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\[ \alpha(p_1^2, p_2^2, p_3^2) \]
ROADS TO FREEDOM

\[ \alpha(p_1^2, p_2^2, p_3^2) \]
What can asymptotic freedom say about $e^+e^- \rightarrow \text{hadrons}$?
Where does pQCD apply?

\[ s = q^2 \]
Where does pQCD apply?

$s = q^2$
Where does \( pQCD \) apply?

\[ s = q^2 \]

deep Euclidean  

de Rujula, Georgi
Where does pQCD apply?

Re $s$

Im $s$

depth Euclidean
de Rujula, Georgi

$s = q^2$
\[ R(s) \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

\[ s = W^2 \]
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$$s = W^2$$
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\[ R(s) = N_c \sum_f e_f^2 \]
$R(s)$

$S_F(p) = \frac{\mathcal{F}(p)}{\phi - \mathcal{M}(p)}$
\[ S_F(p) = \frac{\mathcal{F}(p)}{\phi - \mathcal{M}(p)} \]
Where does pQCD apply?

deep Euclidean

s = q^2
Where does pQCD apply?
Where does pQCD apply?

Im \( s \)

Re \( s \)

s = q^2

deep Euclidean
Adler $\mathcal{D}$ - function

\[\Pi(s) \xrightarrow{s} \frac{1}{12\pi^3} \mathcal{D}(s) \equiv s \frac{\partial}{\partial s} \Pi(s)\]

\[\mathcal{D}\left(\frac{s}{\mu^2}, \alpha(\mu^2)\right) = \mathcal{D}(1, \alpha(s)) = \sum_{c,f} e_f^2 \left[ 1 + \frac{\alpha(s)}{\pi} + \mathcal{O}(\alpha^2) \right]\]
running coupling

\[ \alpha(s) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}} \]

\[ \beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f \]

\[ \mu^2 < 0 \]
running coupling

\[ \alpha(s) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}} \]

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timelike \ s = q^2

\[ \ln \left( \frac{s}{-\mu^2} \right) + i \pi \]

\[ s > 0 \]
\[ \mu^2 < 0 \]
running coupling

\[ \alpha(s) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{4\pi} \alpha(\mu^2) \ln \frac{s}{\mu^2}} \]

\[ \beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f \]

\( s > 0 \)

\( \mu^2 < 0 \)
\[ S_F^{-1}(p, m) \bigg|_{p^2=m^2} = \not{\Phi} - m \quad \text{defining mass at pole} \]

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta (g(\mu^2), m/\mu) \frac{\partial}{\partial g} + \sum_i \gamma_i (g(\mu^2), m/\mu) \right] D = 0
\]

\[
D \left( \frac{s}{\mu^2}, \frac{m^2}{\mu^2}, \alpha(\mu^2, m^2) \right) = D \left( 1, \frac{m^2}{s}, \alpha(s, m^2) \right)
\]

\[
\frac{1}{\alpha(s, m^2)} = \frac{1}{\alpha(\mu^2, m^2)} + \frac{1}{4\pi} \left[ 11 \ln \frac{s}{\mu^2} - \frac{2}{3} \sum_j \int_{\mu^2}^s \frac{dz}{z} F_1 \left( \frac{m_j^2}{z} \right) \right]
\]

\[
F_1(x) = 1 - 6x + \frac{12x^2}{\sqrt{1 + 4x}} \ln \left[ \frac{\sqrt{1 + 4x + 1}}{\sqrt{1 + 4x - 1}} \right]
\]
$S_F^{-1}(p, m(\mu^2)) \bigg|_{p^2=\mu^2} = \rho - m(\mu^2)$
defining mass at renorm. pt

$m^2(s) \simeq m^2(\mu^2) \left( \frac{\alpha(s)}{\alpha(\mu^2)} \right)^{d_m}$
$S + i \Delta$
\[ \mathcal{R}(s, \Delta) = \frac{1}{2i} \left[ \Pi(s + i\Delta) - \Pi(s - \Delta) \right] \]

\[ = \frac{\Delta}{\pi} \int_{4m_{\pi}^2}^{\infty} ds' \frac{R(s')}{(s' - s)^2 + \Delta^2} \]

Poggio, Quinn, Weinberg
ANALYTIC CONTINUATION
\[ \alpha \left( \frac{Q^2}{\Lambda^2} \right) \text{ in timelike region } Q^2 < 0 \]

\[ A \alpha(s) + B \pi^2 \alpha^3(s) + \ldots \rightarrow A \alpha(s) \]
AMBIGUITIES FROM QUARK MASSES IN THE RENORMALIZATION GROUP *

H. David POLITZER

Renormalizability ensures that any consistent prescription will lead to the same physical predictions, whether the $\beta$ functions are the same or not. More precisely, any discrepancies between two calculations carried out to a given order must be yet higher order in the coupling constant. One may still ask, in the spirit of Moorhouse, Pennington and Ross [4], whether one particular prescription is better than others in the following practical sense: if we compute to lowest order and ignore yet higher orders, may one prescription be closer to the complete theory than another? That is to ask: can choice of a particular prescription minimize the numerical coefficient of $g^2$ in the next correction? Typically the answer is yes, but it is impossible to prove without actually computing that next correction. However, for the bulk of phenomenological applications, the use of the light quark-gluon vertex to define $g$ seems a likely candidate because it is precisely that vertex which occurs in lowest-order amplitudes and is subsequently renormalized by higher orders.

References

QCD Sum Rules

\[ \int ds \ \omega(s) \Pi(s) = 0 \]

current correlator

|s| = s₀
QCD Sum Rules

\[ \Pi(s) \]

\[ \langle qq \rangle_0, \langle \alpha GG \rangle_0, \ldots \]
\[ 2i \int_0^{s_0} ds \, \omega(s) \, \text{Im} \, \Pi(s) = -\oint ds \, \omega(s) \, \Pi(s) \]
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