Electroweak Processes in the Few Nucleons: the Old and the New

- EM currents in the conventional approach
- EM currents in χEFT up to one loop
- A (sensitive) test case: radiative captures in A=3 and 4 systems
- Nuclear theory at 1%: μ-capture in d and ^{3}H
- Summary and outlook

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References:

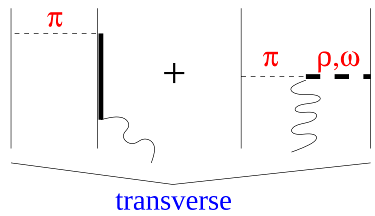
Pastore *et al.* PRC**80**, 034004 (2009); Girlanda *et al.* PRL**105**, 232502 (2010); Marcucci *et al.*, PRC**83**, 014002 (2011)

Conventional approach: EM currents

Marcucci *et al.*, PRC**72**, 014001 (2005)

$$\mathbf{j} = \mathbf{j}^{(1)}$$

$$+\ \mathbf{j}^{(2)}(\mathrm{v}) +$$



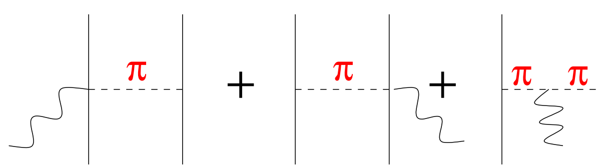
$$+\ \mathbf{j}^{(3)}(\mathrm{V}^{2\pi})$$

- Static part v_0 of v from π-like (PS) and ρ-like (V) exchanges
- Currents from corresponding PS and V exchanges, for example

$$\begin{aligned}\mathbf{j}_{ij}(v_0; PS) &= \mathrm{i}\left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_z \Big[v_{PS}(k_j)\boldsymbol{\sigma}_i \left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right) \\ &+ \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} v_{PS}(k_i)\left(\boldsymbol{\sigma}_i \cdot \mathbf{k}_i\right)\left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right)\Big] + i \rightleftharpoons j\end{aligned}$$

with $v_{PS}(k) = v^{\sigma\tau}(k) - 2\, v^{t\tau}(k)$ projected out from v_0 components

$$\mathbf{j}^{(2)}(\mathrm{v}) \xrightarrow[\text{long range}]{}$$



- Currents from v_p via minimal substitution in i) <u>explicit</u> and ii) <u>implicit</u> p-dependence, the latter from

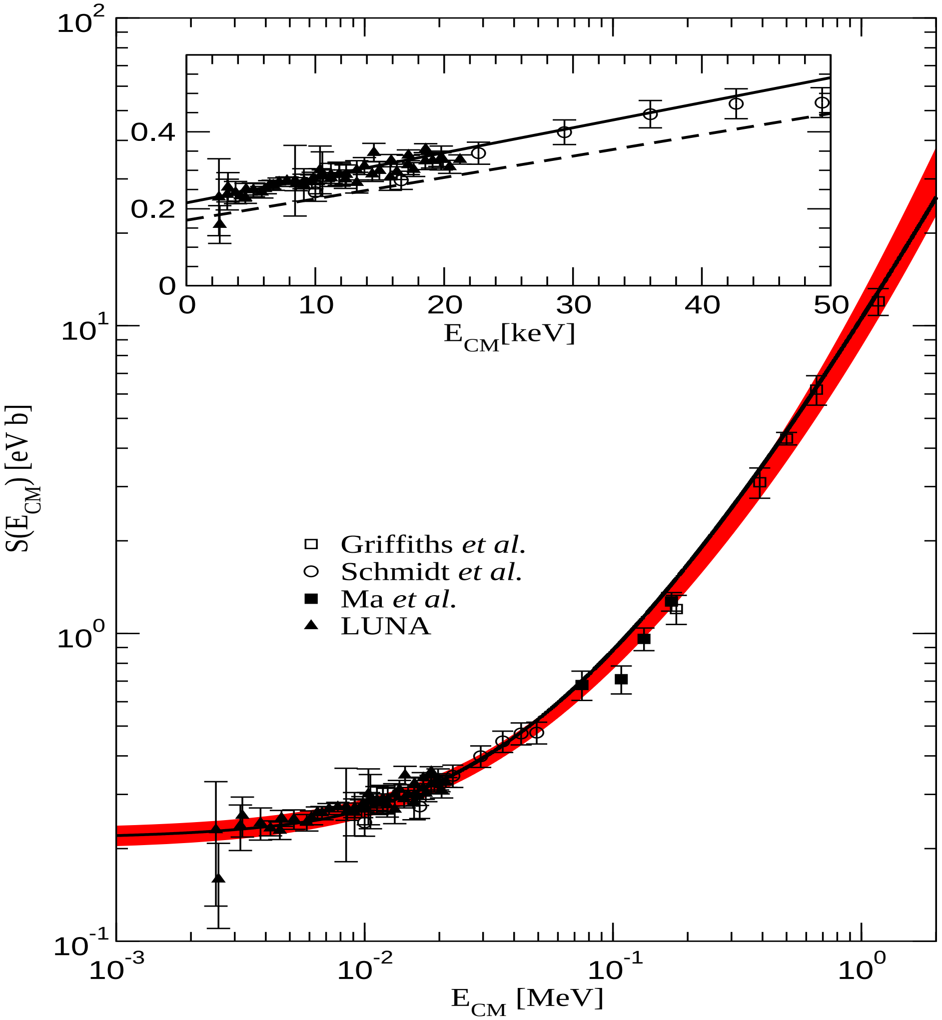
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 + (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, \mathrm{e}^{\mathrm{i}(\mathbf{r}_{ji} \cdot \mathbf{p}_i + \mathbf{r}_{ij} \cdot \mathbf{p}_j)}$$

- Currents are conserved, contain no free parameters, and are consistent with short-range behavior of v and $V^{2\pi}$, but are not unique

Variety of EM observables in A=2–7 nuclei well reproduced, including μ's and $M1$ widths, elastic and inelastic f.f.'s, inclusive response functions, ...

<u>current</u> predictions for ${}^2\mathrm{H}(n,\gamma){}^3\mathrm{H}$ and ${}^3\mathrm{He}(n,\gamma){}^4\mathrm{He}$ cross-sections shown later

^{2}H$(p,\gamma)^3$He capture at low energies



Nuclear χEFT approach

Weinberg, PLB**251**, 288 (1990); NPB**363**, 3 (1991); PLB**295**, 114 (1992)

- χEFT exploits the χ-symmetry exhibited by QCD to restrict the form of π interactions with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum Q, and $\mathcal{L}_{\rm eff}$ can be systematically expanded in powers of Q/Λ_χ ($\Lambda_\chi \simeq 1$ GeV)

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

- χEFT allows for a perturbative treatment in terms of a Q–as opposed to a coupling constant–expansion
- The unknown coefficients in this expansion–the LEC's–are fixed by comparison with experimental data
- Nuclear χEFT provides a practical calculational scheme, susceptible (in principle) of systematic improvement

Work in nuclear χEFT: a partial listing

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A very incomplete list:

- NN and NNN potentials:
  - van Kolck *et al.* (1994–96)
  - Kaiser, Weise *et al.* (1997–98)
  - Glöckle, Epelbaum, Meissner *et al.* (1998–2005)
  - Entem and Machleidt (2002–03)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al.* (1996–2009), hybrid studies in A=2–4
  - Meissner *et al.* (2001), Kölling *et al.* (2009–2010)
  - Phillips (2003), deuteron static properties and f.f.'s

Lots of work in pionless EFT too ...

Formalism

- Time-ordered perturbation theory (TOPT):

$$
\begin{aligned}
-\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}}\cdot\mathbf{j} &= \langle N'N' \mid T \mid NN;\gamma\rangle \\
&= \langle N'N' \mid H_1 \sum_{n=1}^{\infty}\left(\frac{1}{E_i - H_0 + i\,\eta} H_1\right)^{n-1} \mid NN;\gamma\rangle
\end{aligned}
$$

- Power counting:

$$
T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots\,, \quad \text{and} \quad T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}
$$

- Irreducible and recoil-corrected reducible contributions retained in T expansion
- A contribution with N interaction vertices and L loops scales as

$$
\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2}\right)}_{H_1\ \text{scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}
$$

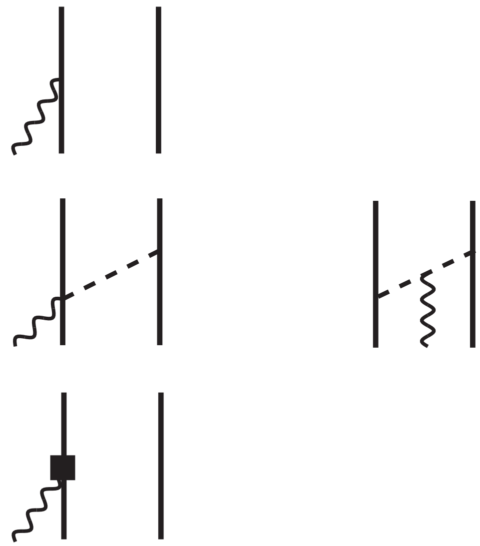
α_i = number of derivatives (momenta) and β_i = number of π's at each vertex

Two-body EM currents in χEFT up to N^{2}LO $(e\,Q^0)$

LO : $\mathbf{eQ^{-2}}$

NLO : $\mathbf{eQ^{-1}}$

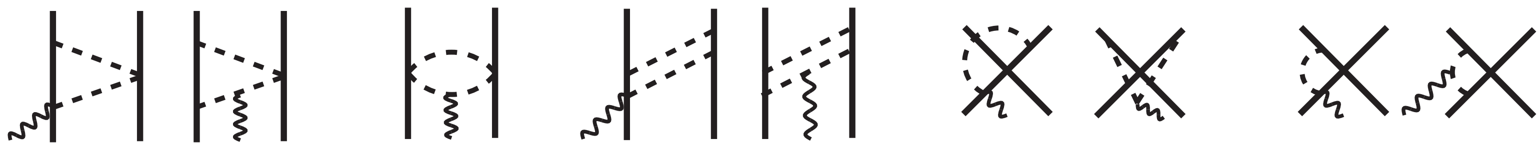
N^{2}LO : $\mathbf{eQ^{0}}$



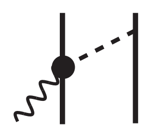
- These depend on the proton and neutron μ's ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$), g_A, and F_π
- One-loop corrections to one-body current are absorbed into μ_N and $\langle r_N^2 \rangle$

N^{3}LO $(e\,Q)$ corrections

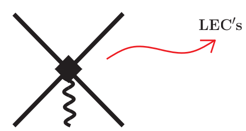
- One-loop corrections:



- Tree-level current with one $e\,Q^2$ vertex from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al.* (1998), involving 3 LEC's ($\sim \gamma N \Delta$ and $\gamma\rho\pi$ currents) :



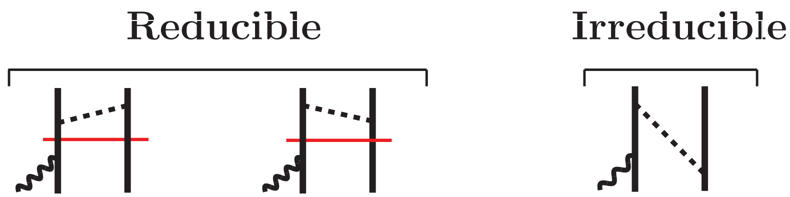
- Contact currents



from i) minimal substitution in the interactions involving $\partial\, N$ (7 LEC's determined from strong-interaction sector) and ii) non-minimal couplings (2 LEC's)

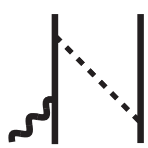
Technical issues I: recoil corrections at N^{2}LO

- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$ 

- Recoil corrections to the reducible contributions obtained by expanding in powers of $(E_i - E_I)/\omega_\pi$ the energy denominators

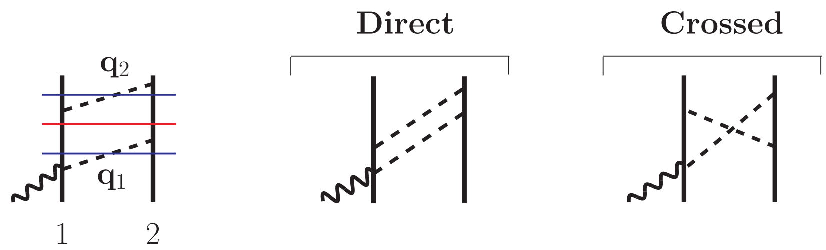
$$= v^\pi \left(1 + \frac{E_i - E_I}{2\,\omega_\pi}\right) \frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{LO}}$$



$$= -\frac{v^\pi}{2\,\omega_\pi}\,\mathbf{j}^{\mathrm{LO}}$$

- Recoil corrections to reducible diagrams cancel irreducible contribution

Technical issues II: recoil corrections at N^{3}LO

$\mathbf{j}^{\mathbf{N^3LO}} =$ 

- Reducible contributions

$$\mathbf{j}_{\rm red} = \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\rm NLO}(\mathbf{q}_1)$$
$$-2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

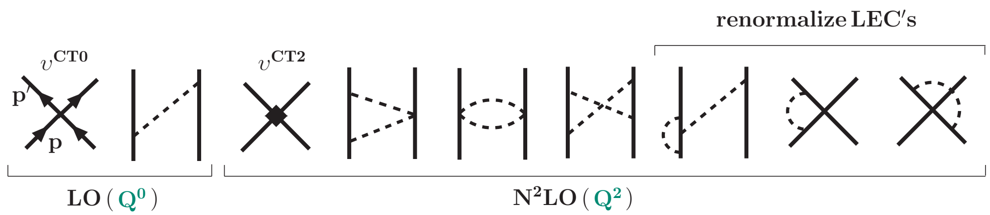
- Irreducible contributions

$$\mathbf{j}_{\rm irr} = 2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$+\ 2\int \frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_1), V_{\pi NN}(2,\mathbf{q}_2)]_-\ V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Partial cancellations between recoil corrections to reducible diagrams and irreducible contributions

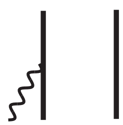
Nuclear χEFT (at Q^2 and $e\,Q$)

NN potential:

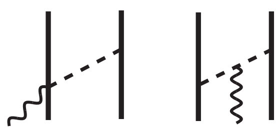


and accompanying set of conserved EM currents:

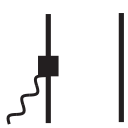
LO : eQ^{-2}



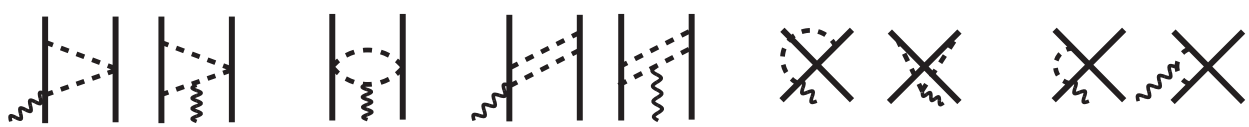
NLO : eQ^{-1}



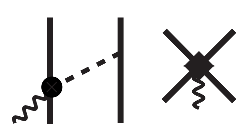
N^{2}LO : eQ^{0}



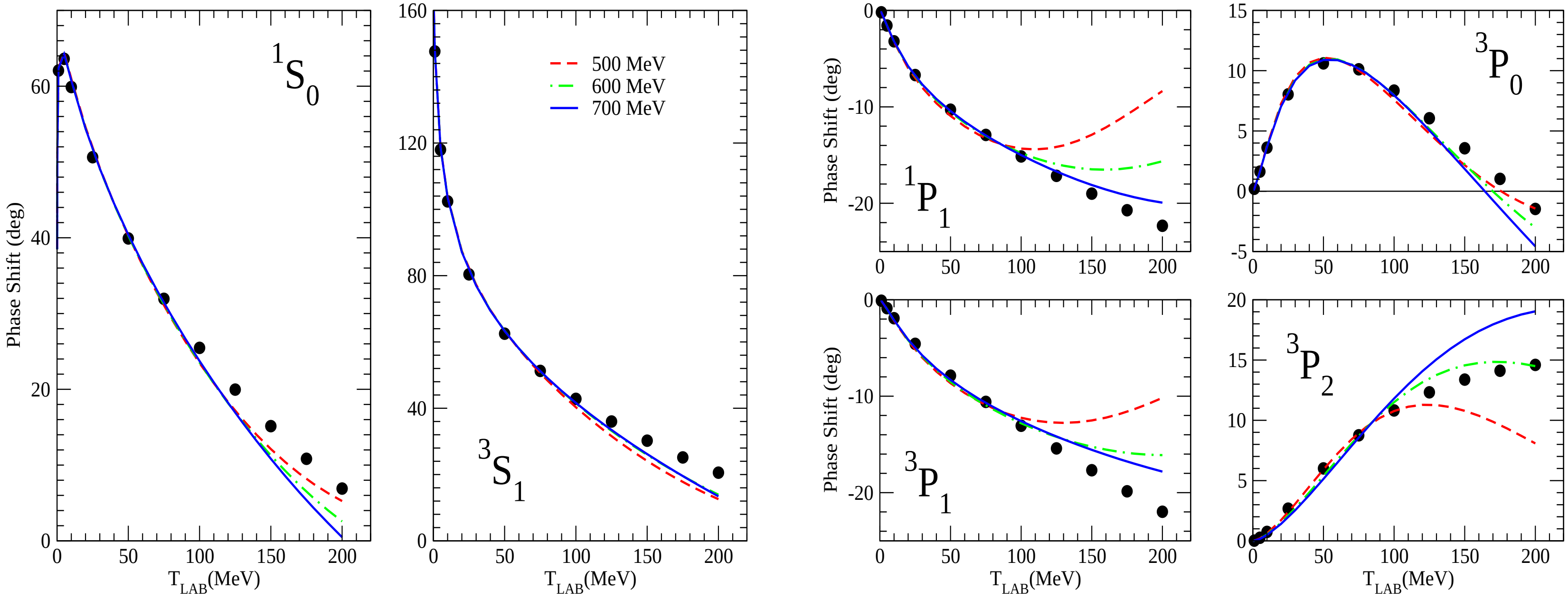
N^{3}LO : eQ



unknown LEC's →

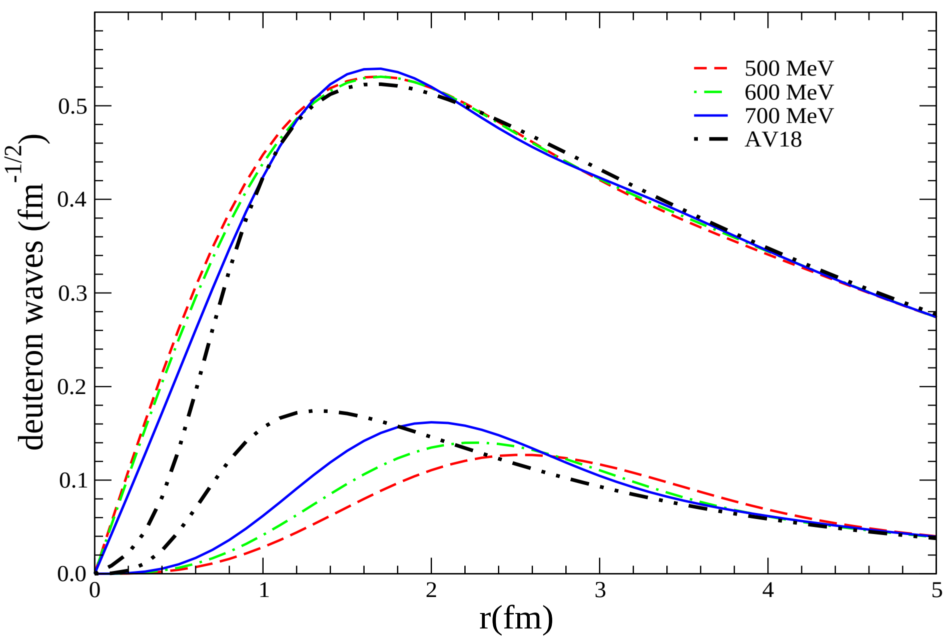


Fits to np phases up to $T_{\rm LAB} = 100$ MeV



LS-equation regulator $\sim \exp(-Q^4/\Lambda^4)$ with Λ=500, 600, and 700 MeV (cutting off momenta $Q \gtrsim 3$–$4\ m_\pi$)

Deuteron properties



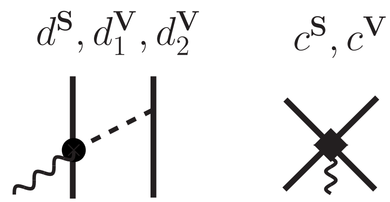
	Λ (MeV)			
	500	600	700	Expt
B_d (MeV)	2.2244	2.2246	2.2245	2.224575(9)
η_d	0.0267	0.0260	0.0264	0.0256(4)
r_d (fm)	1.943	1.947	1.951	1.9734(44)
μ_d (μ_N)	0.860	0.858	0.853	0.8574382329(92)
Q_d (fm^2)	0.275	0.272	0.279	0.2859(3)
P_D (%)	3.44	3.87	4.77	

Previous (and contemporary) work on χEFT currents

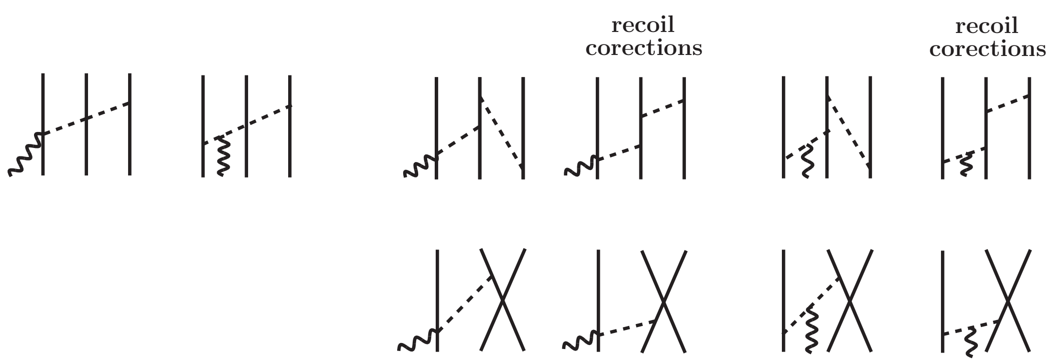
- Expressions for two-body currents (and potential, of course) at one loop in agreement with those of Kölling *et al.* (2009) derived via TOPT and the unitary transformation method
- Park *et al.* (1996) use covariant perturbation theory, but obtain different isospin structure for these loop currents: differences in treatment of box diagrams

EM observables at N^{3}LO

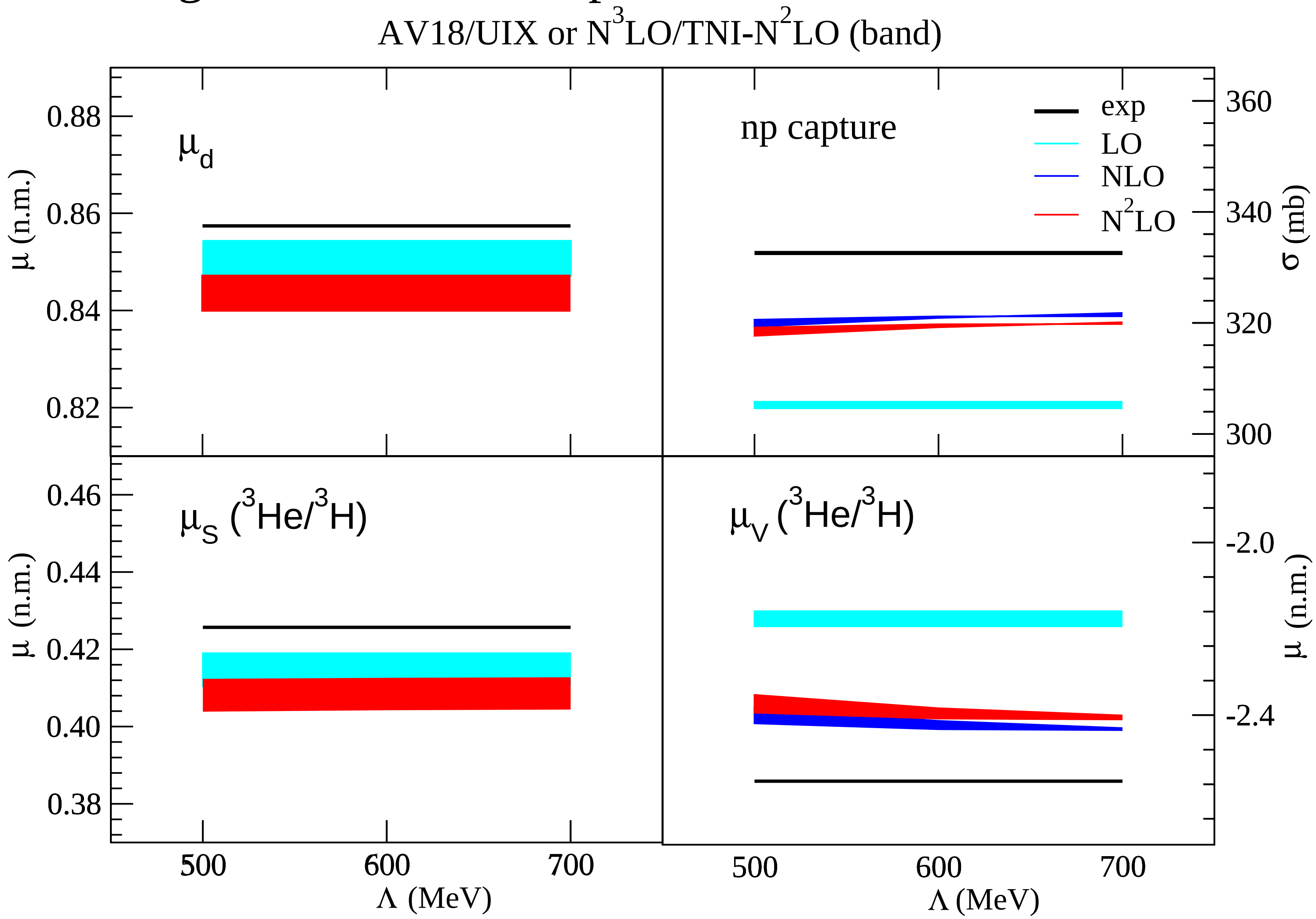
- Pion loop corrections and (minimal) contact terms known
- Five LEC's: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon



- $d_2^V/d_1^V = 1/4$ assuming Δ-resonance saturation
- Three-body currents at N^{3}LO vanish:



Fixing LEC's in EM Properties of A=2 and A=3 Nuclei



Fitted LEC values

- LEC's—in units of Λ—corresponding to Λ = 500–700 MeV for AV18/UIX (N3LO/N2LO)
- Isoscalar d^S (c^S) and isovector d_1^V (c^V) associated with higher-order $\gamma\pi N$ (contact) currents

Λ	$\Lambda^2 d^S \times 10^2$	$\Lambda^4 c^S$	$\Lambda^2 d_1^V$	$\Lambda^4 c^V$
500	–8.85 (–0.225)	–3.18 (–2.38)	5.18 (5.82)	–11.3 (–11.4)
600	–2.90 (9.20)	–7.10 (–5.30)	6.55 (6.85)	–12.9 (–23.3)
700	6.64 (20.4)	–13.2 (–9.83)	8.24 (8.27)	–1.70 (–46.2)

The nd and $n\,^3$He radiative captures

- Suppressed $M1$ processes:

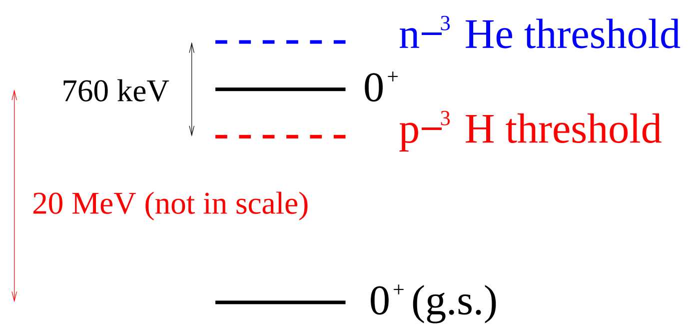
	$\sigma_{\rm exp}$(mb)
^{1}H$(n,\gamma)^2$H	334.2(5)
^{2}H$(n,\gamma)^3$H	0.508(15)
^{3}He$(n,\gamma)^4$He	0.055(3)

- The ^{3}H and ^{4}He bound states are approximate eigenstates of the one-body $M1$ operator, *e.g.* $\hat{\mu}({\rm IA})\,|^3{\rm H}\rangle \simeq \mu_p\,|^3{\rm H}\rangle$ and $\langle nd \mid \hat{\mu}({\rm IA})\,|^3{\rm H}\rangle \simeq 0$ by orthogonality

- A=3 and 4 radiative (and weak) captures very sensitive to i) small components in the w.f.'s and ii) many-body terms in the electro(weak) currents (80-90% of cross section!)

Wave functions: recent progress

- 3 and 4 bound-state w.f.'s and 2+1 continuum routine by now
- Challenges with 3+1 continuum:
  1. Coupled-channel nature of scattering problem: n-^{3}He and p-^{3}H channels both open
  2. Peculiarities of ^{4}He spectrum (see below): hard to obtain numerically converged solutions



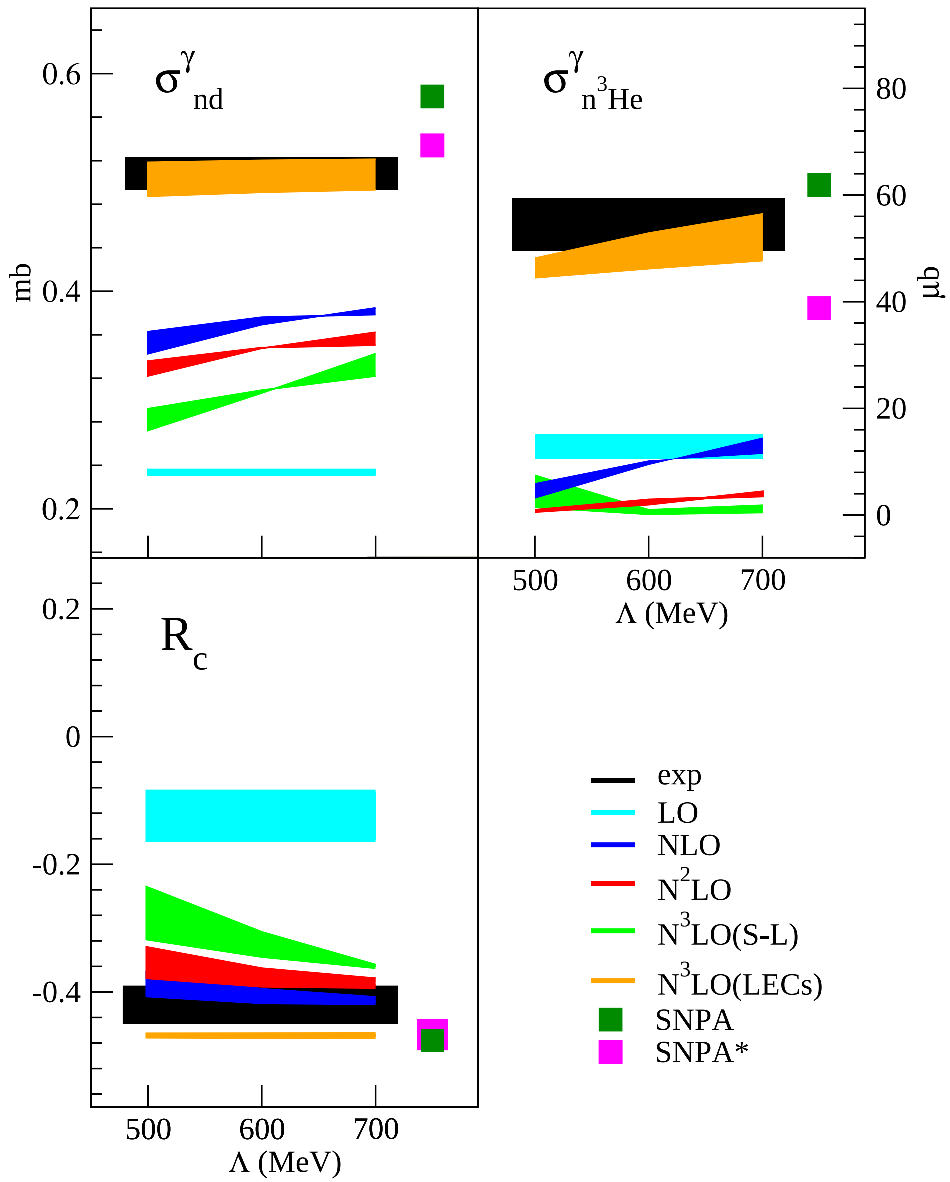
- Major effort by several groups*: both singlet and triplet n-^{3}He scattering lengths in good agreement with data

*Deltuva and Fonseca (2007); Lazauskas (2009); Viviani *et al.* (2010)

Triplet scattering length a_1 (fm)

Method	AV18	AV18/UIX
HH	$3.56 - i\,0.0077$	$3.39 - i\,0.0059$
RGM	$3.45 - i\,0.0066$	$3.31 - i\,0.0051$
FY	$3.43 - i\,0.0082$	$3.23 - i\,0.0054$
AGS	$3.51 - i\,0.0074$	
R-matrix	$3.29 \quad - i\,0.0012$	
EXP	$3.28(5) - i\,0.001(2)$	
EXP	$3.36(1)$	
EXP	$3.48(2)$	

Singlet scattering length a_0 (harder to calculate!) also in good agreement with experiment



σ^γ nd
σ^γ n^3He
R c
mb
μb
0.6
0.4
0.2
0
-0.2
-0.4
80
60
40
20
500
600
700
Λ (MeV)
exp
LO
NLO
N^2LO
N^3LO(S-L)
N^3LO(LECs)
SNPA
SNPA*

n-d radiative capture cross section* in μb: $\sigma_{\rm nd}^{\rm EXP} = 508(15)\ \mu$b

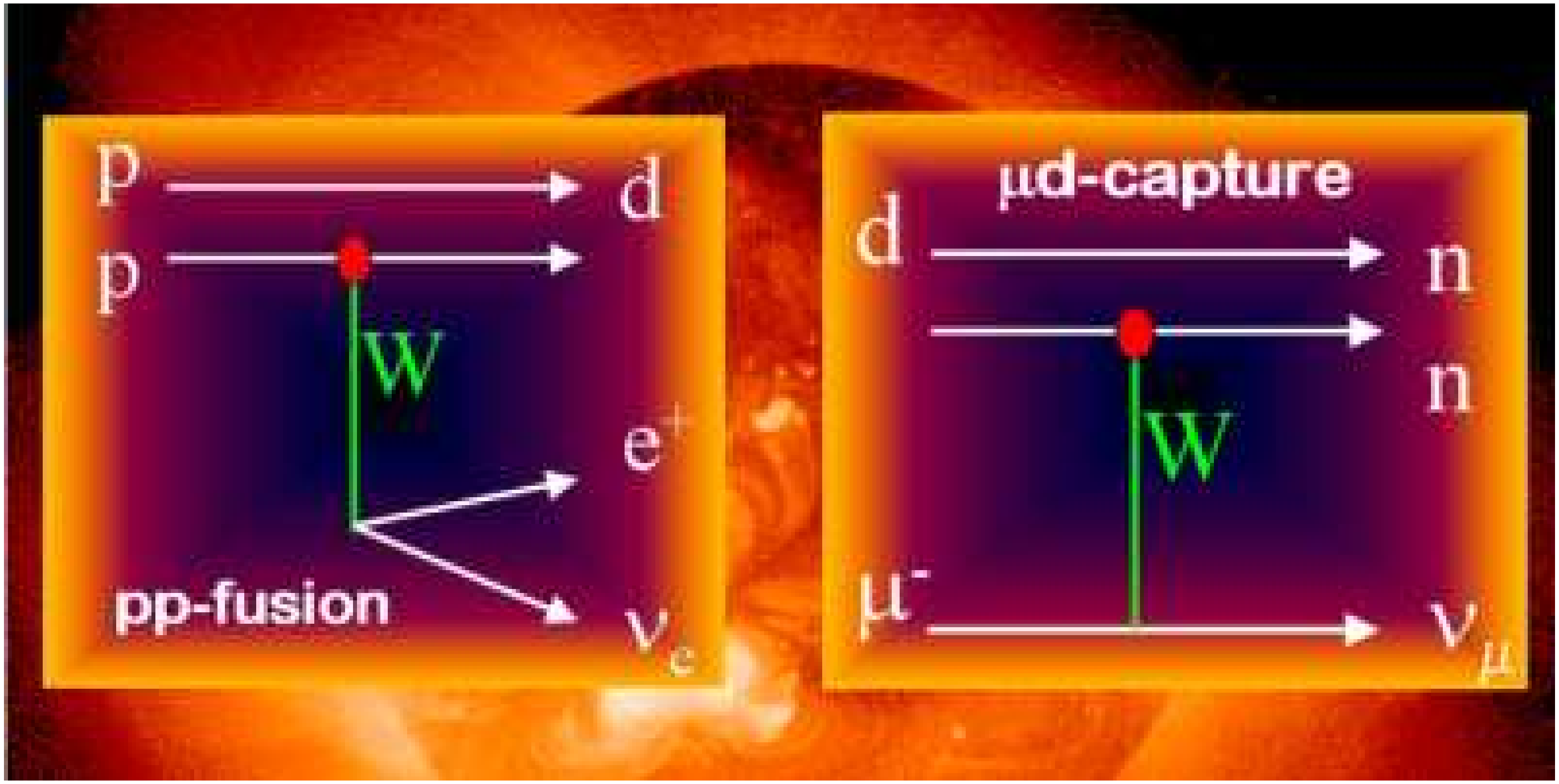
Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	231	343	322	272	487
600	231	369	348	306	491
700	231	385	362	343	493

n-^{3}He radiative capture cross section* in μb: $\sigma_{\rm n\,^3He}^{\rm EXP} = 55(4)\ \mu$b

Λ	LO	NLO	N^{2}LO	N^{3}LO(L)	N^{3}LO
500	15.2	5.95	0.91	1.36	48.3
600	15.2	10.2	2.87	0.04	53.0
700	15.2	11.5	3.56	0.38	56.6

*N3LO/N2LO potentials and HH wave functions

μ-Capture



From http://www.npl.illinois.edu/exp/musun/

Single-nucleon weak current

$$
\begin{aligned}
\langle n|\overline{d}\gamma_\mu(1-\gamma_5)u|p\rangle &= \overline{u}_n\Big(F_1\,\gamma_\mu + \frac{i}{2m}F_2\,\sigma_{\mu\nu}q^\nu \\
&\quad -G_A\,\gamma_\mu\gamma_5 - \frac{1}{m_\mu}\,G_{PS}\,\gamma_5 q_\mu\Big)u_p
\end{aligned}
$$

- Additional scalar and pseudotensor f.f.'s, associated with second-class currents, possible (discussed later ...)
- $F_1(q^2)$ and $F_2(q^2)$ related to EM f.f.'s via CVC: well known
- $G_A(q^2) = g_A/(1+q^2/\Lambda_A^2)^2$: g_A known from neutron β-decay and $\Lambda_A \simeq 1$ GeV from π-electroproduction and $p(\nu_\mu, \mu^+)n$ data
- $G_{PS}(q^2)$ poorly known: PCAC and χPT predict

$$G_{PS}(q^2) = \frac{2m_\mu g_{\pi pn} F_\pi}{m_\pi^2 - q^2} - \frac{1}{3} g_A m_\mu m r_A^2$$

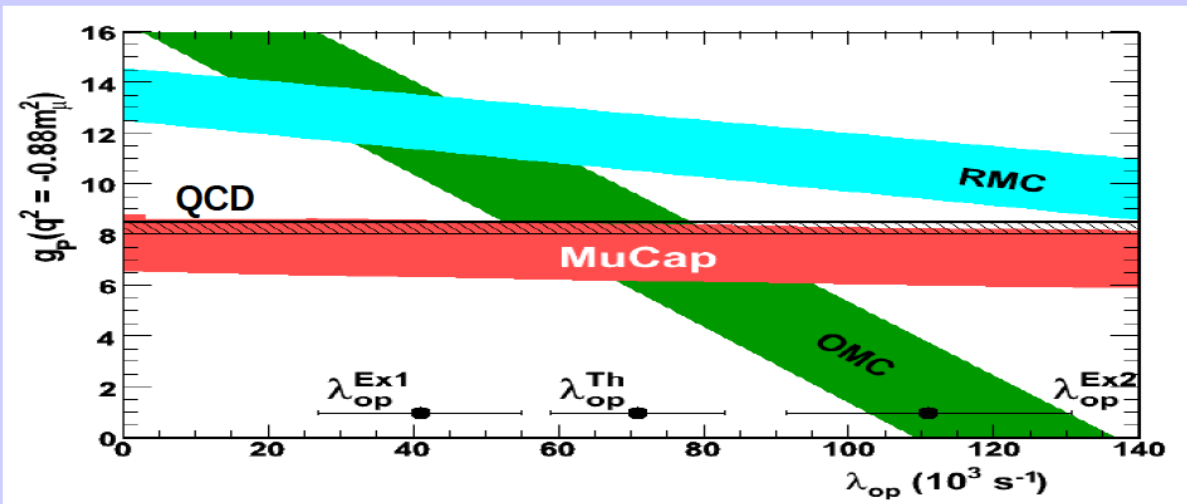
$G_{PS}(q_0^2) = 8.2 \pm 0.2$ at $q_0^2 = -0.88\, m_\mu^2$ relevant for $p(\mu^-, \nu_\mu)n$

Experimental situation I: $\mu^- + p$

MuCap Results

2005: Λ_S = 725.0 ± 13.7(stat) ±10.7(syst) s^{-1}

g_p (q^2 = -0.88m^2_μ) = 7.3±1.1



goal for 2006/2007 dataset is Λ_s to $\pm 5 s^{-1}$

From Gorringe's talk at Elba XI (2010).

Experimental situation II: $\mu^- + d$

Two hyperfine states: 1/2 and 3/2 $\Rightarrow \Gamma^D$ and Γ^Q
From theory: $\Gamma^D \simeq 400\ \mathrm{s}^{-1}$ and $\Gamma^Q \simeq 10\ \mathrm{s}^{-1} \Rightarrow$ only Γ^D

- Wang *et al.*, PR **139**, B1528 (1965): $\Gamma^D = 365(96)\ \mathrm{s}^{-1}$
- Bertini *et al.*, PRD **8**, 3774 (1973): $\Gamma^D = 445(60)\ \mathrm{s}^{-1}$
- Bardin *et al.*, NPA **453**, 591 (1986): $\Gamma^D = 470(29)\ \mathrm{s}^{-1}$
- Cargnelli *et al.*, Workshop on fundamental μ physics, Los Alamos, 1986, LA10714C: $\Gamma^D = 409(40)\ \mathrm{s}^{-1}$
- MuSun Collaboration: result to come!

Experimental situation III: $\mu^- + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \nu_\mu$

Total capture rate Γ_0:

- Folomkin *et al.*, PL **3**, 229 (1963): Γ_0=1410(140) s^{-1}
- Auerbach *et al.*, PR **138**, B127 (1967): Γ_0=1505(46) s^{-1}
- Clay *et al.*, PR **140**, B587 (1965): Γ_0=1465(67) s^{-1}
- Ackerbauer *et al.*, PLB **417**, 224 (1998): Γ_0=1496(4) s^{-1}

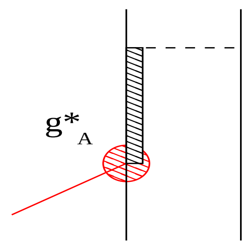
Angular correlation A_v:

- Souder *et al.*, NIMA **402**, 311 (1998): A_v=0.63 $\pm$ 0.09 (stat.)$^{+0.11}_{-0.14}$ (syst.)

Nuclear weak currents

Two-body weak currents:

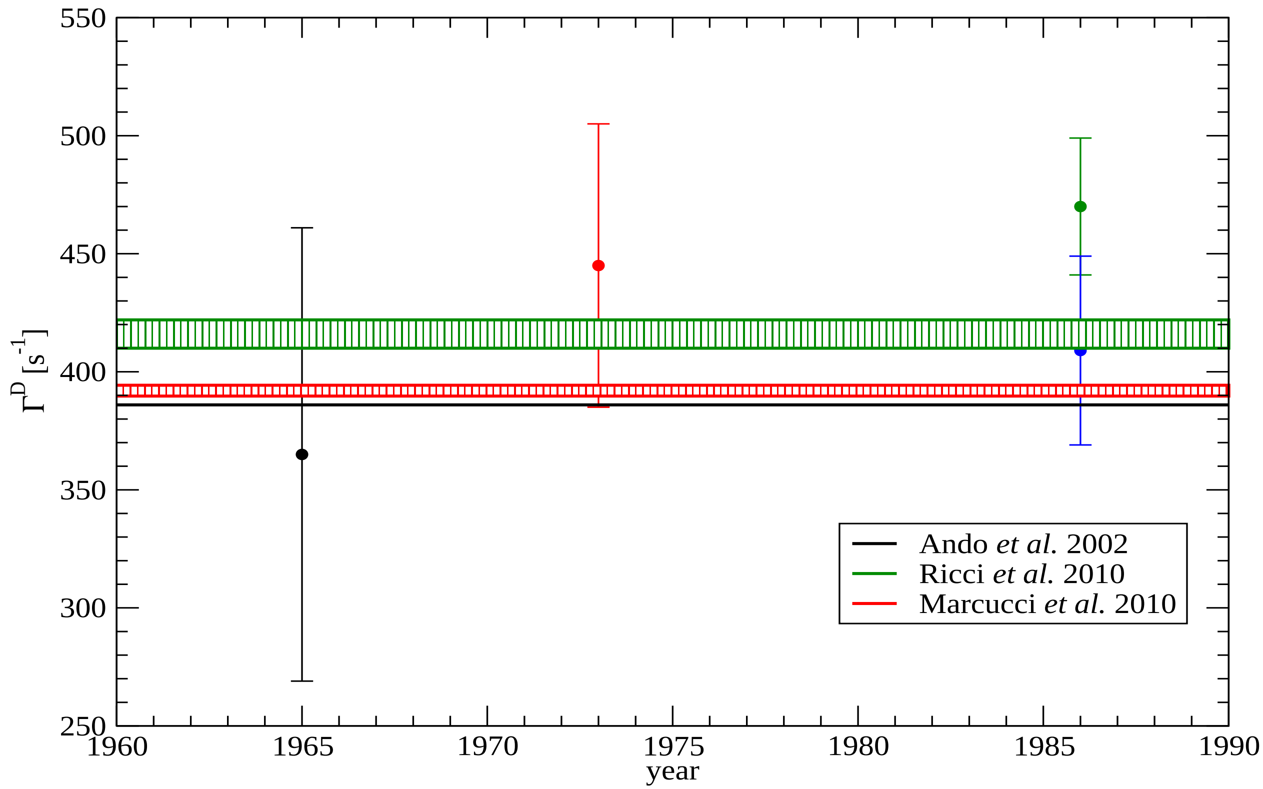
- Vector currents from isovector components of $\mathbf{j}_\gamma$ (CVC)
- In SNPA the leading contribution in the axial currents is from Δ-excitation, additional π- and ρ-meson contributions turn out be tiny



- Axial currents in χEFT at N^{3}LO, derived in Park *et al.* (2003), depend on a single LEC d_R

Common strategy: fix g_A^* in SNPA and $d_R(\Lambda)$ in χEFT by fitting the GT m.e. in ^{3}H β-decay

SNPA and χEFT predictions I: $\Gamma_0(\mu^- +^2\mathrm{H})$



SNPA and χEFT predictions II: $\Gamma_0(\mu^- +^3 \text{He})$

SNPA(AV18/UIX)	Γ_0 s^{-1}
g_A=1.2654(42)	1486(8)
g_A=1.2695(29)	1486(5)
χEFT*(AV18/UIX)	Γ_0
Λ = 500 MeV	1487(8)
Λ = 600 MeV	1488(9)
Λ = 800 MeV	1488(8)
χEFT(N3LO/N2LO; Λ=600 MeV)	1480(9)

- Theory (G_{PS} from χPT): Γ_0= 1484(13) s^{-1}
- With radiative corrections from Czarnecki, Marciano, and Sirlin (2007) $\Gamma_0 \Rightarrow$ 1494(13) s^{-1} vs. Γ_0(exp)=1496(4) s^{-1}

$\Gamma_0(\mu^- +^3 \text{He})$ and second class currents

Standard model allows additional weak f.f.'s

$$\langle n | J_\mu^{\text{second class}} | p \rangle = \overline{u}_n \Big(F_S \, q_\mu - \frac{i}{2m} G_T \, \sigma_{\mu\nu} \, \gamma_5 \, q^\nu \Big) u_p$$

Constraints on F_S and G_T from μ-capture on ^{3}He—analysis by Gazit (2008) but consistent with present predictions:

$$F_S = -0.005 \pm 0.68 \qquad\qquad G_T/G_A = -0.1 \pm 0.68$$

- Limits on F_S tighter than from a survey of $0^+ \to 0^+$ β-decays:

$$F_S = -0.01 \pm 0.27 \qquad \text{Severijns } et\ al.\ (2006)$$

- QCD sum rule estimate for tensor f.f.:

$$G_T/G_A = -0.0152(53) \qquad \text{Shiomi (1996)}$$

Summary and outlook

- Nuclear theory in reasonable agreement with data for suppressed processes
- In some instances, such as μ-capture, it provides predictions with $\lesssim 1\%$ accuracy: extract information on nucleon properties
- Current efforts in χEFT aimed at:
  1. Completing an independent derivation of the parity-violating (PV) potential at N^{2}LO (Q), and an analysis of PV effects in A=2, 3, and 4 systems
  2. EM structure of light nuclei: $d(e,e')pn$ at threshold, charge and magnetic form factors, ...
  3. Including Δ d.o.f. explicitely in nuclear potentials and currents (to improve convergence)