Quark spin distributions at large $x$

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Outline

- Why large-$x$ quarks are important

- $x \to 1$ behavior from perturbative QCD
  - role of orbital angular momentum

- Nuclear effects in $^3\text{He}$
  - limitations of “effective polarizations”
Why large $x$?
Most direct connection between quark distributions and models of nucleon structure is via *valence* quarks.

→ most cleanly revealed at $x > 0.4$
Ideal testing ground for nonperturbative & perturbative models of the nucleon

→ e.g. ratio \( d/u \) or \( \Delta d/d \) sensitive to spin-flavor dynamics

**SU(6) proton wave function**

\[
p^\uparrow = -\frac{1}{3}d^\uparrow(uu)_1 - \frac{\sqrt{2}}{3}d^\downarrow(uu)_1
\]

\[+ \frac{\sqrt{2}}{6}u^\uparrow(ud)_1 - \frac{1}{3}u^\downarrow(ud)_1 + \frac{1}{\sqrt{2}}u^\uparrow(ud)_0\]
SU(6) symmetry

\[ u(x) = 2 \, d(x) \text{ for all } x \]

\[ \frac{d}{u} = \frac{1}{2} \quad \frac{F_2^n}{F_2^p} = \frac{2}{3} \]

\[ \frac{\Delta u}{u} = \frac{2}{3} \quad A_1^p = \frac{5}{9} \]

\[ \frac{\Delta d}{d} = -\frac{1}{3} \quad A_1^n = 0 \]

50% \ S=0 (qq)  
50% \ S=1 (qq)

e.g. Close, “An Introduction to Quarks and Partons” (1979)
Scalar diquark dominance

\[ M_\Delta > M \rightarrow (qq)_1 \text{ has larger energy than } (qq)_0 \]

\[ \rightarrow \text{ scalar diquark dominant in } x \rightarrow 1 \text{ limit} \]

\[ \rightarrow \text{ since only } u \text{ quarks couple to scalar diquarks} \]

- \[ \frac{d}{u} \rightarrow 0 \]
- \[ \frac{\Delta u}{u} \rightarrow 1 \]
- \[ \frac{\Delta d}{d} \rightarrow -\frac{1}{3} \]
- \[ \frac{F^n_2}{F^p_2} \rightarrow \frac{1}{4} \]
- \[ A^p_1 \rightarrow 1 \]
- \[ A^n_1 \rightarrow 1 \]

_Feynman, “Photon-Hadron Interactions” (1972)_
_Close, *PLB* 43, 422 (1973)_
Local duality models

→ duality in quark models realized by summing over complete sets of even and odd parity resonances, e.g. $56$ ($L=0$) and $70$ ($L=1$) multiplets of SU(6)

<table>
<thead>
<tr>
<th>representation</th>
<th>$^28[56^+]$</th>
<th>$^410[56^+]$</th>
<th>$^28[70^-]$</th>
<th>$^48[70^-]$</th>
<th>$^210[70^-]$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9\rho^2$</td>
<td>$8\lambda^2$</td>
<td>$9\rho^2$</td>
<td>$0$</td>
<td>$\lambda^2$</td>
<td>$18\rho^2+9\lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3\rho+\lambda)^2/4$</td>
<td>$8\lambda^2$</td>
<td>$(3\rho-\lambda)^2/4$</td>
<td>$4\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2+27\lambda^2)/2$</td>
</tr>
<tr>
<td>$g_1^p$</td>
<td>$9\rho^2$</td>
<td>$-4\lambda^2$</td>
<td>$9\rho^2$</td>
<td>$0$</td>
<td>$\lambda^2$</td>
<td>$18\rho^2-3\lambda^2$</td>
</tr>
<tr>
<td>$g_1^n$</td>
<td>$(3\rho+\lambda)^2/4$</td>
<td>$-4\lambda^2$</td>
<td>$(3\rho-\lambda)^2/4$</td>
<td>$-2\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2-9\lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda$ ($\rho$) = (anti) symmetric component of ground state wave function

→ summing over all resonances in $56^+$ and $70^-$ multiplets

$F_1^n/F_1^p \rightarrow 2/3$, $A_1^p \rightarrow 5/9$, $A_1^n \rightarrow 0$

as in parton model (if $u=2d$)!
Local duality models

→ various ways of breaking SU(6) while respecting duality

- spin-1/2 dominance ($\Delta$ suppression)

$$ A_p^1 = \frac{19 - 23 \sin^2 \theta_s}{19 - 11 \sin^2 \theta_s} \quad A_n^1 = \frac{1 - 2 \sin^2 \theta_s}{1 + \sin^2 \theta_s} $$

- helicity-1/2 dominance

$$ A_p^1 = \frac{7 - 9 \sin^2 \theta_h}{7 - 5 \sin^2 \theta_h} \quad A_n^1 = 1 - 2 \sin^2 \theta_h $$

- antisymmetric wave function ($\rho$) dominance

$$ A_p^1 = \frac{6 - 7 \sin^2 \theta_w}{6 - 3 \sin^2 \theta_w} \quad A_n^1 = \frac{1 - 2 \sin^2 \theta_w}{1 + 2 \sin^2 \theta_w} $$
Local duality models

→ various ways of breaking SU(6) while respecting duality

Local duality models

various ways of breaking $SU(6)$ while respecting duality

$x$ dependence of polarized & unpolarized PDFs correlated

Perturbative QCD
In QCD, “exceptional” $x \rightarrow 1$ configurations of proton wave function generated from “typical” wave function (for which $x_i \sim 1/3$) by exchange of $\geq 2$ hard gluons, with mass $k^2 \sim -\langle k_{\perp}^2 \rangle / (1 - x)$

Since $|k^2|$ is large, coupling at $q$-$g$ vertex is small

→ use lowest-order perturbation theory!

Assume wave function vanishes sufficiently fast as $|k^2| \rightarrow \infty$ and unperturbed wave function dominated by 3-quark Fock component with $SU(2) \times SU(3)$ symmetry
Perturbative QCD

- **If spectator “diquark” spins are anti-aligned** (helicity of struck quark = helicity of proton)
  - can exchange *transverse* or *longitudinal* gluon

- **If spectator “diquark” spins are aligned** (helicity of struck quark ≠ helicity of proton)
  - can exchange *only longitudinal* gluon

- **Coupling of (large-\(k^2\)) longitudinal gluon to (small-\(p^2\)) quark** is suppressed by \((p^2/k^2)^{1/2} \sim (1-x)^{1/2}\) w.r.t. transverse
  - \(q^\downarrow \sim (1-x)^2 q^\uparrow \sim (1-x)^5\)
**Perturbative QCD**

**Phenomenological consequences of** $S_Z = 0$ **qq dominance**

(dominance of helicity-$1/2$ photoproduction cross section)

→ assuming unperturbed SU(6) wave function

\[
\begin{align*}
\frac{d}{u} & \rightarrow \frac{1}{5} & \frac{F_2^n}{F_2^p} & \rightarrow \frac{3}{7} \\
\frac{\Delta u}{u} & \rightarrow 1 & A_1^p & \rightarrow 1 \\
\frac{\Delta d}{d} & \rightarrow 1 & A_1^n & \rightarrow 1
\end{align*}
\]

→ dramatically different predictions for $\Delta d/d$

cf. nonperturbative models

* valid in Abelian & non-Abelian theories
Above results assume quarks in lowest Fock state are in relative $s$-wave.

\[ \text{higher Fock states and nonzero quark OAM will in general introduce additional suppression in } (1-x) \]

**BUT** nonzero OAM can provide logarithmic enhancement of helicity-flip amplitudes!

\[ \text{quark OAM modifies asymptotic behavior of nucleon’s Pauli form factor} \]

\[ F_2(Q^2) \sim \log^2(Q^2/\Lambda^2) \frac{1}{Q^6} \]

consistent with surprising $Q^2$ dependence of proton’s $G_E/G_M$ form factor ratio

*Belitsky, Ji, Yuan
PRL 91, 092003 (2003)*
Role of orbital angular momentum

For $L_z = 1$ Fock state, expand hard scattering amplitude in powers of $k_\perp$ ("collinear expansion")

- Logarithmic singularities arise when integrating over longitudinal momentum fractions $x_i$ of soft quarks

- Leads to additional $\log^2(1-x)$ enhancement of $q^\downarrow$

$$q^\downarrow \sim (1 - x)^5 \log^2(1 - x)$$

Avakian, Brodsky, Deur, Yuan, PRL 99, 082001 (2007)

(similar contributions to positive helicity $q^\uparrow$ are power-suppressed)
Role of orbital angular momentum

- $k_\perp$-odd transverse momentum dependent (TMD) distributions (vanish after $k_\perp$ integration)
  - arise from interference between $L_z = 0$ and $L_z = 1$ states

- $T$-even TMDs
  - $g_{1T}$ (longitudinally polarized $q$ in a transversely polarized $N$)
  - $h_{1L}$ (transversely polarized $q$ in a longitudinally polarized $N$)

- $T$-odd TMDs
  - $f_{1T}$ (unpolarized $q$ in a transversely polarized $N$ – “Sivers”)
  - $h_{1T}$ (transversely polarized $q$ in an unpolarized $N$ – “Boer-Mulders”)

- Each behaves in $x \to 1$ limit as
  \[ \text{TMD} \sim (1 - x)^4 \]

Brodsky, Yuan
PRD 74, 094018 (2006)
Phenomenological implications

- Power counting rule constraints used in exploratory fit to limited set of inclusive DIS spin structure function data

\[
q^\uparrow = x^\alpha [A (1 - x)^3 + B (1 - x)^4] \\
q^\downarrow = x^\alpha [C (1 - x)^5 + D (1 - x)^6]
\]

Brodsky, Burkardt, Schmidt
NPB 441, 197 (1995)
Phenomenological implications

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\[ q^\downarrow = x^\alpha [C(1 - x)^5 + D(1 - x)^6 + C'(1 - x)^5 \log^2(1 - x)] \]

Additional \( L_z = 1 \) term

LSS’98 (pQCD-inspired)

ABDY’07 (including OAM)

Improved fit for \( \Delta d/d \)

Avakian et al., PRL 99, 082001 (2007)
Phenomenological implications

- Determining $x \rightarrow 1$ behavior experimentally is problematic

- Simple $x^\alpha(1 - x)^\beta$ parametrizations inadequate for describing high-precision data, and global fits typically require more complicated $x$ dependence, e.g.

$$q \sim x^\alpha(1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x)$$

- Recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5$ GeV$^2$)

$$\beta \approx 3.3 \ (\Delta u_V), \ 3.9 \ (\Delta d_V)$$

but with $\gamma, \eta \sim \mathcal{O}(10-100)$

- Challenge to perform constrained global fit to all DIS, SIDIS & $\vec{p}p$ scattering data
Phenomenological implications

Determining $x \to 1$ behavior experimentally is problematic

\[ q \sim x^\alpha (1 - x)^\beta (1 + \gamma \sqrt{x} + \eta x) \]

recent global fits of spin-dependent PDFs find (at $Q^2 \sim 5 \text{ GeV}^2$)

\[ \beta \approx 3.3 \left( \Delta u_V \right), \ 4.1 \left( \Delta d_V \right) \]

but with $\gamma, \eta \sim \mathcal{O}(10-100)$

Challenge to perform constrained global fit to all DIS, SIDIS & $\vec{p} \vec{p}$ scattering data
Phenomenological implications

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- Challenge to perform constrained global fit to all DIS, SIDIS & $\vec{p}\vec{p}$ scattering data
Phenomenological implications

- Challenges for large-\(x\) PDF analysis
  - at fixed \(Q^2\), increasing \(x\) corresponds to decreasing \(W\)
    - eventually run into nucleon resonance region as \(x \to 1\)
  - subleading \(1/Q^2\) corrections (target mass, higher twists)
  - nuclear corrections in extraction of neutron information from nuclear \((\text{deuterium}, ^3\text{He})\) data

- New “JAM” (JLab Angular Momentum) global PDF analysis*
  - dedicated to describing large-\(x\), moderate-\(Q^2\) region
  - preliminary results expected this summer
  - global spin asymmetry / structure function database currently being compiled

* JAM collaboration: P. Jimenez-Delgado, A. Accardi, WM (theory) + Halls A, B, C (expt.)
  http://www.jlab.org/jam
Nuclear corrections to spin structure functions
Nuclear structure functions

- Incoherent scattering from nucleons in nucleus \((x \gg 0)\)
- Expansion in powers of \(p^2/M^2\) & binding energy

\[ x g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{i,j}^N(y, \gamma) x g_j^N(x/y, Q^2) \quad i, j = 1, 2 \]

- Photon “velocity” \(\gamma = |q|/q_0 = \sqrt{1 + 4M^2x^2/Q^2}\)
- Light-cone momentum fraction \(y = \frac{p \cdot q}{P \cdot q} = \frac{p_0 + \gamma p_z}{M}\)

\[ f_{i,j}^N(y, \gamma) = \int \frac{d^3p}{(2\pi)^3} D_{i,j}^N(\varepsilon, p, \gamma) \delta \left( y - 1 - \frac{\varepsilon + \gamma p_z}{M} \right) \]
$^3$He structure functions

For $^3$He, nuclear functions $D_{ij}$ given in terms of components of $^3$He spectral function

\[
D_{11} = \mathcal{F}_1 + \frac{3 - \gamma^2}{6\gamma^2}(3\hat{p}_z^2 - 1)\mathcal{F}_2 + \frac{p_z}{3\gamma}(3\mathcal{F}_1 + 2\mathcal{F}_2) \\
+ \frac{p^2}{M^2} \frac{(3 - \gamma^2)\hat{p}_z^2 - 1 - \gamma^2}{12\gamma^2}(3\mathcal{F}_1 - \mathcal{F}_2) \quad \text{etc.}
\]

where spectral function is defined as

\[
\mathcal{P}(\varepsilon, \vec{p}, \vec{S}) = \frac{1}{2} \left[ \mathcal{F}_0 I + \mathcal{F}_1 \sigma \cdot \vec{S} + \mathcal{F}_2 (\hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}) S_i \sigma_j \right]
\]
\[ D_{11} = \mathcal{F}_1 + \frac{3 - \gamma^2}{6\gamma^2} (3\hat{p}_z^2 - 1)\mathcal{F}_2 + \frac{p_z}{3\gamma} (3\mathcal{F}_1 + 2\mathcal{F}_2) \]

\[ + \frac{p^2}{M^2} \frac{(3 - \gamma^2)\hat{p}_z^2 - 1 - \gamma^2}{12\gamma^2} (3\mathcal{F}_1 - \mathcal{F}_2) \quad \text{etc.} \]

where spectral function is defined as

\[ \mathcal{P}(\varepsilon, p, S) = \frac{1}{2} \left[ \mathcal{F}_0 I + \mathcal{F}_1 \sigma \cdot S + \mathcal{F}_2 (\hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}) S_i \sigma_j \right] \]

spin-averaged \quad spin-dependent
$^3\text{He}$ structure functions

- Proton and neutron contributions differ qualitatively
  
  \[ \mathcal{F}^p_{1,2} = \mathcal{F}^{p\,(\text{cont})}_{1,2}(E, p) + \mathcal{F}^{p(d)}_{1,2}(p) \delta(E + \varepsilon_{^3\text{He}} - \varepsilon_d) \]

  \[ \mathcal{F}^n_{1,2} = \mathcal{F}^{n\,(\text{cont})}_{1,2}(E, p) \]

  deuteron pole contributes
  \[ \sim 60\% \text{ to normalization!} \]

- Normalizations

  \[ \int \frac{d^3p}{(2\pi)^3} \mathcal{F}^{p\,(n)}_0 = 2 \quad (1) \]

  number sum rules

  \[ \int \frac{d^3p}{(2\pi)^3} \mathcal{F}^N_1 = \langle \sigma_z \rangle^N \]

  average $N$ polarization

  \[ \int \frac{d^3p}{(2\pi)^3} \mathcal{F}^N_2 = \frac{9}{2} \langle T_{zi} \sigma_i \rangle^N \]

  tensor polarization
$^3\text{He structure functions}$

- Proton and neutron contributions differ qualitatively

\[
\mathcal{F}_{1,2}^p = \mathcal{F}_{1,2}^{p\text{(cont)}}(E,p) + \mathcal{F}_{1,2}^{p(d)}(p) \delta(E + \varepsilon_{^3\text{He}} - \varepsilon_d)
\]

\[
\mathcal{F}_{1,2}^n = \mathcal{F}_{1,2}^{n\text{(cont)}}(E,p)
\]

- deuteron pole contributes \(~ 60\% \) to normalization!

- Nucleon polarizations

\[
\langle \sigma_z \rangle^p = -\frac{2}{3}(P_D - P_{S'}) \approx (-0.04) - (-0.06)
\]

\[
\langle \sigma_z \rangle^n = P_S - \frac{1}{3}(P_D - P_{S'}) \approx 0.86 - 0.89
\]
Smearing functions

Kulagin, WM, PRC 78, 065203 (2008)

→ effectively more smearing for larger $x$ or lower $Q^2$
Smearing functions

\[
\begin{align*}
\text{(a)} & \quad f^{\gamma}_{11}(\gamma, \psi) \\
\text{(b)} & \quad f^{\gamma}_{12}(\gamma, \psi) \\
\text{(c)} & \quad f^{\gamma}_{21}(\gamma, \psi) \\
\text{(d)} & \quad f^{\gamma}_{22}(\gamma, \psi)
\end{align*}
\]

\( \gamma = 1, 1.5, 2 \)

\[Kulagin, WM, PRC 78, 065203 (2008)\]

\[\rightarrow \text{diagonal smearing functions} \quad \Rightarrow \text{off-diagonal}\]
Smearing functions

Kulagin, WM, PRC 78, 065203 (2008)

→ proton smearing functions ⇐ neutron
Nuclear effects in $^3$He

significant smearing, especially in resonance region
Nuclear effects in $^3$He

[Graphs showing](#) nuclear wave function model dependence ($\text{KPSV}^1$, $\text{SS}^2$) not significant

1 Kievsky, Pace, Salme, Viviani, *PRC* **56**, 64 (1997)

Nuclear effects in $^{3}\text{He}$

**Effective polarizations**

\[
\begin{align*}
    f_{ii}^N(y, \gamma) & \rightarrow \langle \sigma_z \rangle^N \delta(y - 1) & \text{(zero width)} \\
    f_{i \neq j}^N(y, \gamma) & \rightarrow 0 & \text{(no off-diagonal)}
\end{align*}
\]

\[\rightarrow\text{ assumes nuclear corrections independent of } x \text{ and } Q^2\]

\[
\begin{align*}
    g_1^{^3\text{He}} & \rightarrow \langle \sigma_z \rangle^p g_1^p + \langle \sigma_z \rangle^n g_1^n \\
    g_2^{^3\text{He}} & \rightarrow \langle \sigma_z \rangle^p g_2^p + \langle \sigma_z \rangle^n g_2^n
\end{align*}
\]
Nuclear effects in $^3\text{He}$

significant differences between “effective polarizations” and full results, especially at low $W$
At large $x$, correct treatment of nuclear corrections essential for extraction of free-$n$ information from $^3$He

$\rightarrow$ difficult to observe $\log^2(1-x)$ enhancement of $q^\downarrow$
predicted from $L_Z = 1$ component of wave function

Avakian et al., PRL 99, 082001 (2007)
Summary

- New JLab 12 GeV measurements of $A_1^{3\text{He}}$ will provide vital information on $\Delta d/d$ at $x \to 1$
  - test applicability of pQCD vs. nonperturbative models, and role of OAM

- Nuclear effects in $^3\text{He}$ important at large $x$
  - "effective polarization" method insufficient for $x \gtrsim 0.6$, and especially low $W$
    (could distort information extracted on $\Delta d/d$)

- New "JAM" global analysis of spin-dependent PDFs dedicated to large-$x$, moderate-$Q^2$ region
  - initial focus on helicity PDFs; later expand scope to TMDs (first results soon)