Partonic Structure of the Nucleon's Chiral Periphery

Charge and Current Transverse Densities, Energy Momentum Tensor and Orbital Angular Momentum

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QCD Evolution Workshop

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Aim and Context

- Spatial representation of hadrons as relativistic systems
  - GPDs, Transverse Densities
- Universality in large distance dynamics:
  - Chiral symmetry breaking, effective field theory
- Study of chiral periphery of transverse nucleon structure
  - Charge and current densities, EM form factors
  - Matter density and angular momentum, energy momentum tensor and GPDs

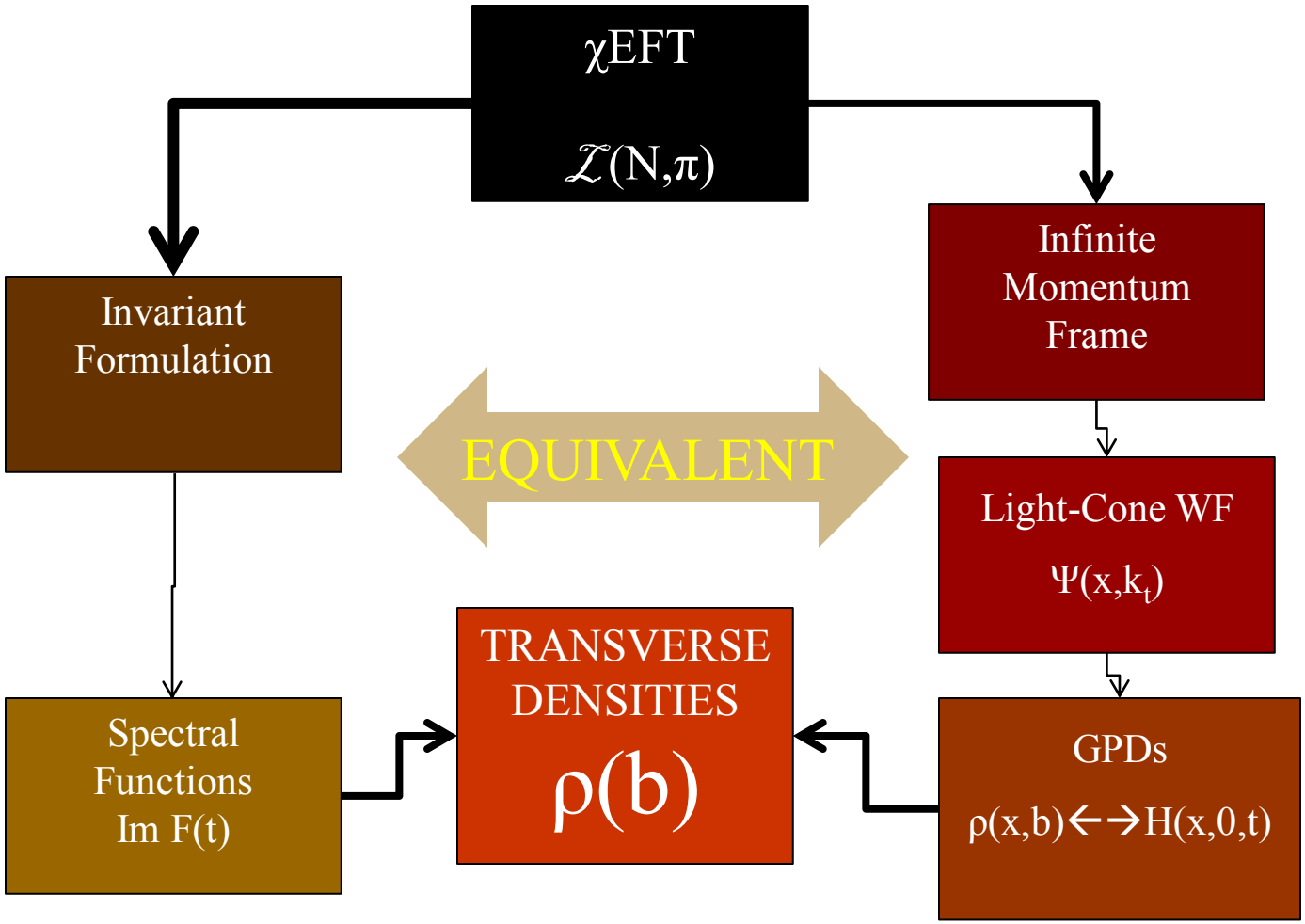
Motivation

- Methodology
  - Reveal spatial structure of χEFT:

    M_π^{-1} vs. short distance contributions
  - Explore different formulations of orbital angular momentum in field theory applied to a πN system
- Practical
  - Calculate model independent chiral components of the nucleon structure
  - Constrain form factors, peripheral GPDs
- Experiment
  - Form factors measurements in the low Q^2 region

    JLab E12-11-106 Q2 ~ 10^{-2} - 10^{-4} GeV^2
  - Connect chiral dynamics with Peripheral Processes in High Energy ep and pp Reactions: EIC, LHC

Methodology



Transverse Charge and Current Densities
Definition

- Fourier Transform in Transverse Momentum

$$\rho(b) \equiv \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot \mathbf{b})} F\left(-\Delta_T^2\right) \Longrightarrow \rho(b) = \int_0^\infty \frac{d\Delta}{2\pi} J_0(\Delta b) F\left(t = -\Delta^2\right)$$

- From Electromagnetic (EM) Form Factors (FF)

$$\left\langle N_2 \middle| J^\mu \middle| N_1 \right\rangle = \overline{U}_2 \left[\gamma^\mu \boldsymbol{F_1}\left(\boldsymbol{\Delta^2}\right) + i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} \boldsymbol{F_2}\left(\boldsymbol{\Delta^2}\right) \right] U_1$$

- From Energy-Momentum Tensor (EMT) FF

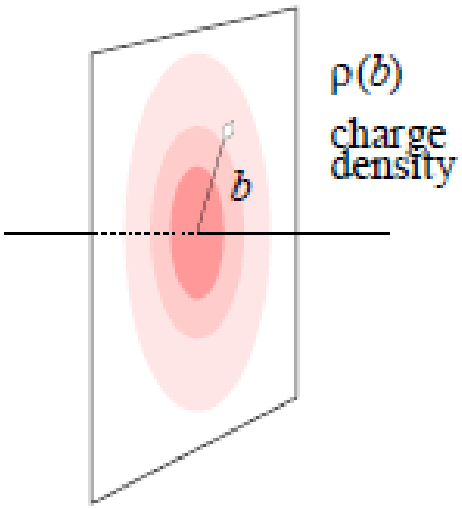
$$\left\langle N_2 \middle| \Theta_{\mu\nu}^{N\pi} \middle| N_1 \right\rangle = \overline{U}_2 \left[\gamma_{(\mu} P_{\nu)} \boldsymbol{A}\left(\boldsymbol{\Delta^2}\right) + P_{(\mu} i\sigma_{\nu)\alpha} \frac{\Delta_\alpha}{2M} \boldsymbol{B}\left(\boldsymbol{\Delta^2}\right) + \left(\frac{\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}}{M} \right) \boldsymbol{C}\left(\boldsymbol{\Delta^2}\right) + M g_{\mu\nu} \tilde{\boldsymbol{C}}\left(\boldsymbol{\Delta^2}\right) \right] U_1$$

Transverse Charge and Current Densities

Definition

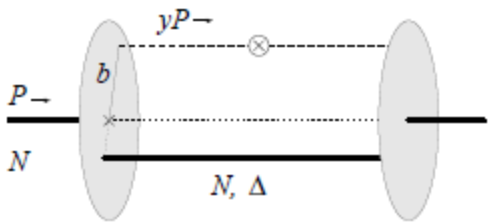
$$\rho(b) \equiv \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot \mathbf{b})} F\left(-\Delta_T^2\right)$$

G.Miller, PRL99(2007)



- Charge distribution in transverse plane:

  Proper densities, Relativistic Systems



Parton current picture

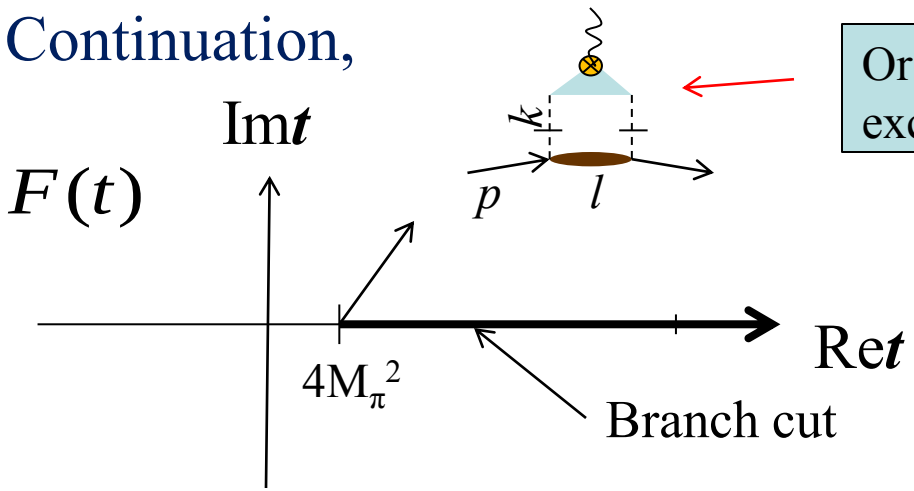
$$\langle N_2 | J^\mu | N_1 \rangle = \langle N_2 | \left[\gamma^\mu \, \mathrm{F}_1(\Delta^2) + i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} \mathrm{F}_2(\Delta^2) \right] | N_1 \rangle$$

$$\langle P, x_{2T}, \sigma_2 | \mathrm{J}^0(0, \mathrm{x}_\mathrm{T}, 0) | P, x_{1T}, \sigma_1 \rangle = \left[2P\delta^{(2)}(x_{2T} - x_{1T}) \right] \delta_{\sigma_1\sigma_2} \, \rho_1(\mathrm{b})$$

$$\langle P, x_{2T}, \sigma_2 | \mathrm{J}^3(0, \mathrm{x}_\mathrm{T}, 0) | P, x_{1T}, \sigma_1 \rangle = [\ldots] \frac{S_{\sigma_1\sigma_2}}{M_N} \left(e_3 \times \frac{\partial}{\partial x} \right) \rho_2(\mathrm{b})$$

Transverse Charge and Current Densities
Dispersion Representation, Spectral Functions

- Form Factors
  - Analytic Continuation,



Or any t- channel exchange

- Dispersion Relation : For Re*t*<0,

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'-t} \frac{\mathrm{Im}\, F(t'+i0)}{\pi}$$

Transverse Charge and Current Densities

Dispersion Representation, Spectral Functions and Analytic Structure Near Threshold

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'-t} \frac{\operatorname{Im} F(t'+i0)}{\pi}$$

$$\rho(b) = \int_0^{\infty} \frac{d\Delta}{2\pi} J_0(\Delta b) F\left(t = -\Delta^2\right)$$

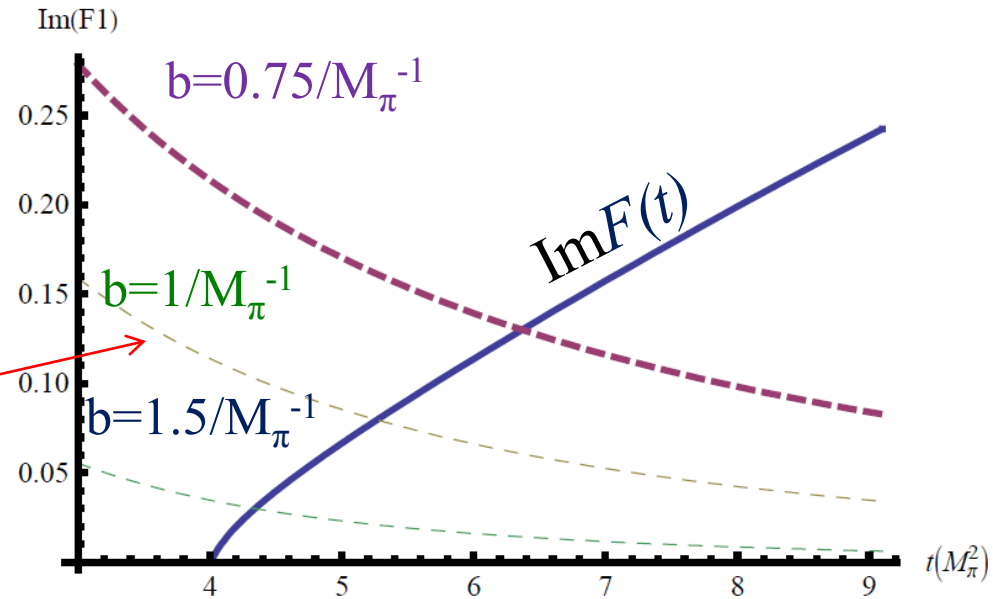
$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0\left(\sqrt{t}b\right) \frac{1}{\pi} \operatorname{Im} F\left(t + i0\right)$$

Transverse Charge and Current Densities

Dispersion Representation, Spectral Functions

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0\left(\sqrt{t}b\right) \frac{1}{\pi} \operatorname{Im} F(t+i0)$$

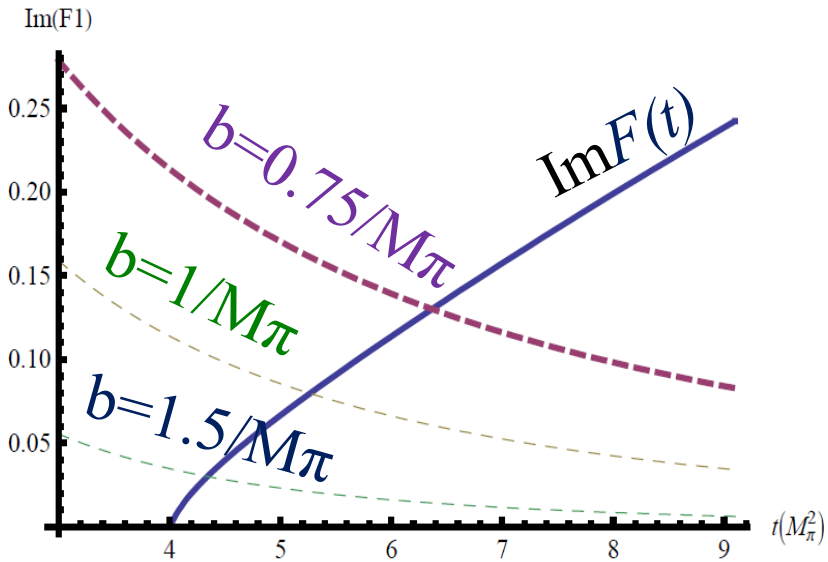
$$K_0\left(\sqrt{t}b\right) \approx \sqrt{\frac{\pi}{2}} \frac{e^{-\sqrt{t}b}}{\left(\sqrt{t}b\right)^{\frac{1}{2}}}$$

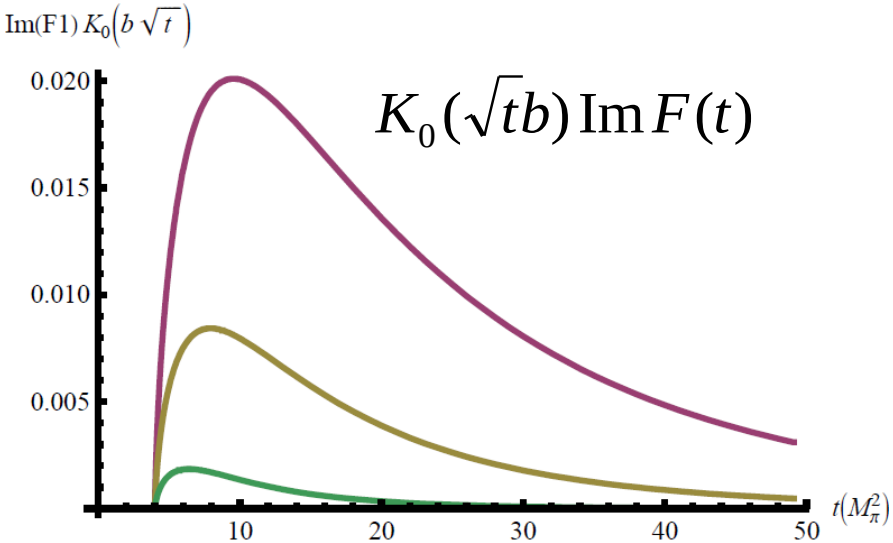


- As *b* grows, *F(t)* is sampled closer to threshold ($t_{thr}=4M_\pi^2$)

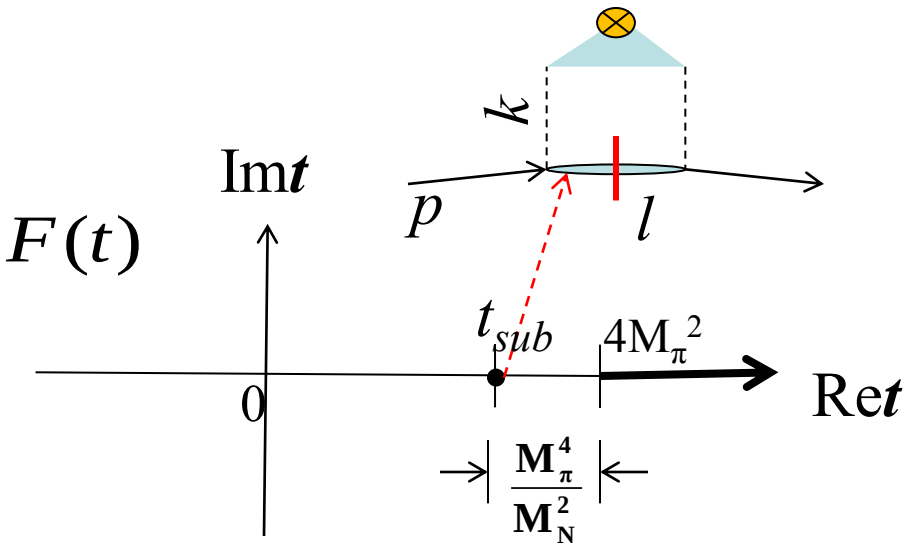
Transverse Charge and Current Densities
Dispersion Representation, Spectral Functions

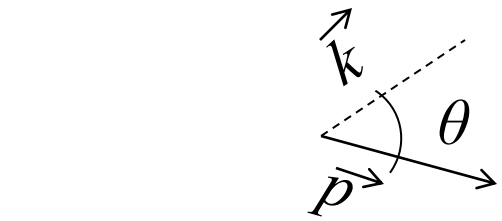
- For large *b*, Transverse densities are dominated by near threshold values of spectral functions

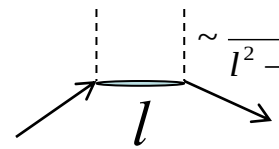




Transverse Charge and Current Densities
Analytic Structure Near Threshold







$$\sim \frac{1}{l^2 - M_N^2 + i0} = -\frac{1}{A(t) - iB(t)\cos\theta}$$

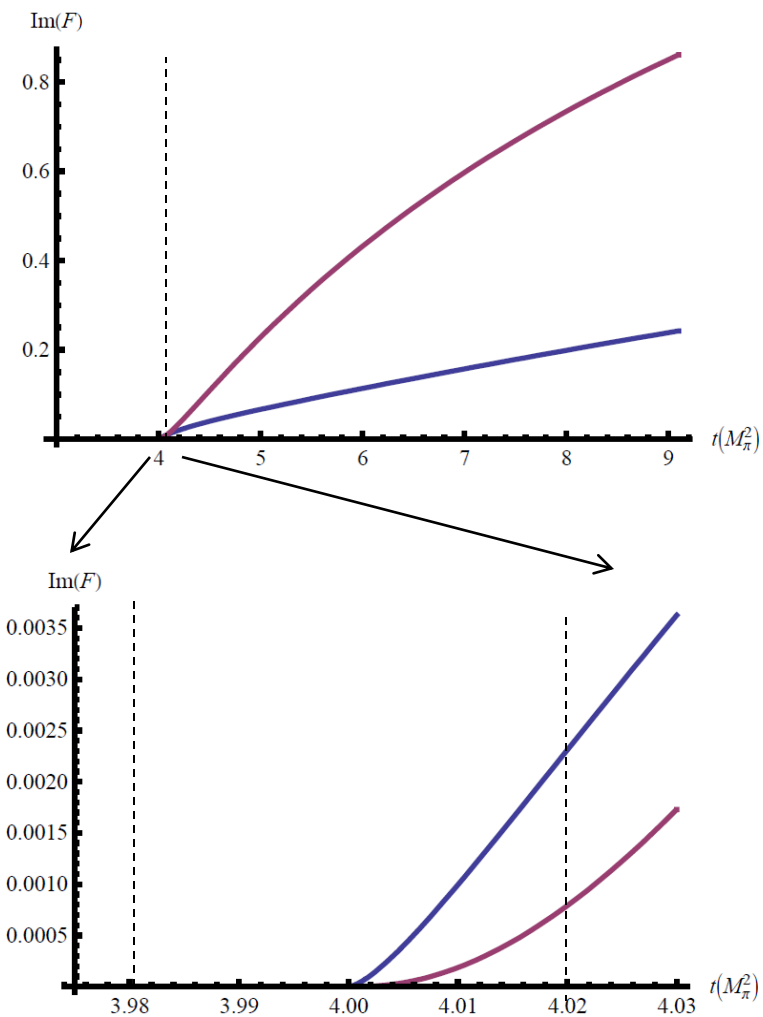
$$\mathrm{Im}\,F = \int_{-1}^{1} \frac{f(t,\cos\theta)}{A(t) - iB(t)\cos\theta} d(\cos\theta)$$

$$A(t_{sub}) = iB(t_{sub}) \Rightarrow t_{sub} = 4M_\pi^2 - \frac{M_\pi^2}{M_N^2}$$

- Sub-threshold singularity
  - End-point singularity
  - Intermediate nucleon on-shell
  - Limits convergence of expansion near threshold
  - Controls large $b(\sim M_N^2 M_\pi^{-3})$ behavior of transverse densities

Transverse Charge and Current Densities
Parametric Regions

- Chiral Region
  - $\Delta b \sim \frac{1}{(\Delta t_\chi)^{1/2}} \sim \frac{1}{M_\pi} \approx 1.5\text{fm}$



- Molecular Region
  - $\Delta b > \frac{M_N^2}{M_\pi^2}\frac{1}{M_\pi} \approx 56\frac{1}{M_\pi}$

$$\approx 90\text{fm}$$

Peripheral Densities from Invariant χPT

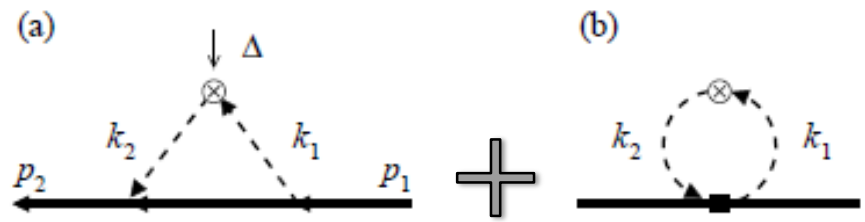
- Chiral EFT Lagrangian
  - Relativistic formulation of pion-nucleon dynamics

$$\mathcal{L}_{int} = -\frac{g_A}{2F_\pi}\overline{\psi}\gamma^\mu\gamma_5\tau^a\psi\partial_\mu\pi^a - \frac{1}{4F_\pi^2}\overline{\psi}\gamma^\mu\tau^a\psi\varepsilon^{abc}\pi^b\partial_\mu\pi^c + \ldots$$

  - Axial Vector coupling and contact terms

- EM current. Leading π contributions to isovector spectral functions:

$$\left\langle N_2 \middle| J^\mu_{\pi N} \middle| N_1 \right\rangle =$$



$$\left\langle N_2 \middle| J^\mu \middle| N_1 \right\rangle = \overline{U}_2\left[\gamma^\mu F_1(\Delta^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2M}F_2(\Delta^2)\right]U_1$$

Peripheral Densities from Invariant χPT

$$F_1 = \frac{2}{F_\pi^2}\left(1-g_A^2\right)I^{(1)} + \frac{8M_N^2 g_A^2}{F_\pi^2} I^{(2)}$$

$$F_2 = \frac{8M_N^2 g_A^2}{F_\pi^2} I^{(3)}$$

$$k \equiv \frac{k_1+k_2}{2}$$

$$P \equiv \frac{p_1+p_2}{2}$$

$$I^{(1)} = i\int \frac{d^4k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2) N^{(1)}$$

$$I^{(2,3)} = i\int \frac{d^4k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2) D_N(l) N^{(2,3)}$$

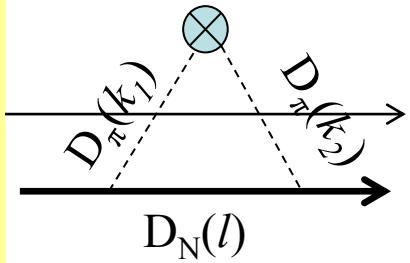
$$N^{(1)} = \frac{1}{3}\left[k^2 - \frac{(k\Delta)^2}{\Delta^2}\right]$$

$$N^{(2)} = \frac{1}{2}\left[k^2 - \frac{(kP)^2}{P^2} - \frac{(k\Delta)^2}{\Delta^2}\right] + \frac{1}{2}\left[-k^2 + 3\frac{(kP)^2}{P^2} + \frac{(k\Delta)^2}{\Delta^2}\right]\frac{M_N^2}{P^2}$$

$$N^{(3)} = -\frac{1}{2}\left[-k^2 + 3\frac{(kP)^2}{P^2} + \frac{(k\Delta)^2}{\Delta^2}\right]\frac{M_N^2}{P^2}$$

- Find Spectral Functions (Im$F(t)$) instead,
  - Cutkosky Rules

    Pion on mass-shell



$$\frac{1}{k_{1,2}^2 - m_\pi^2 + i\varepsilon} \to -2\pi i\,\delta(k_{1,2}^2 - m_\pi^2)$$

$$I(t) = -i\int \frac{d^4k}{(2\pi)^2} D_\pi(k_1) D_\pi(k_2)\Phi(k)$$

$$\frac{1}{\pi}\mathrm{Im}\, I(t) = \frac{k_{cm}^3}{16\pi^2\sqrt{t}}\int_{-1}^{1} d(\cos\theta)\Phi\left(k^0 = 0, |\mathbf{k}| = k_{cm}\right)$$

$$k_{cm} = \sqrt{\frac{t}{4} - M_\pi^2}$$

Peripheral Densities from Invariant χPT

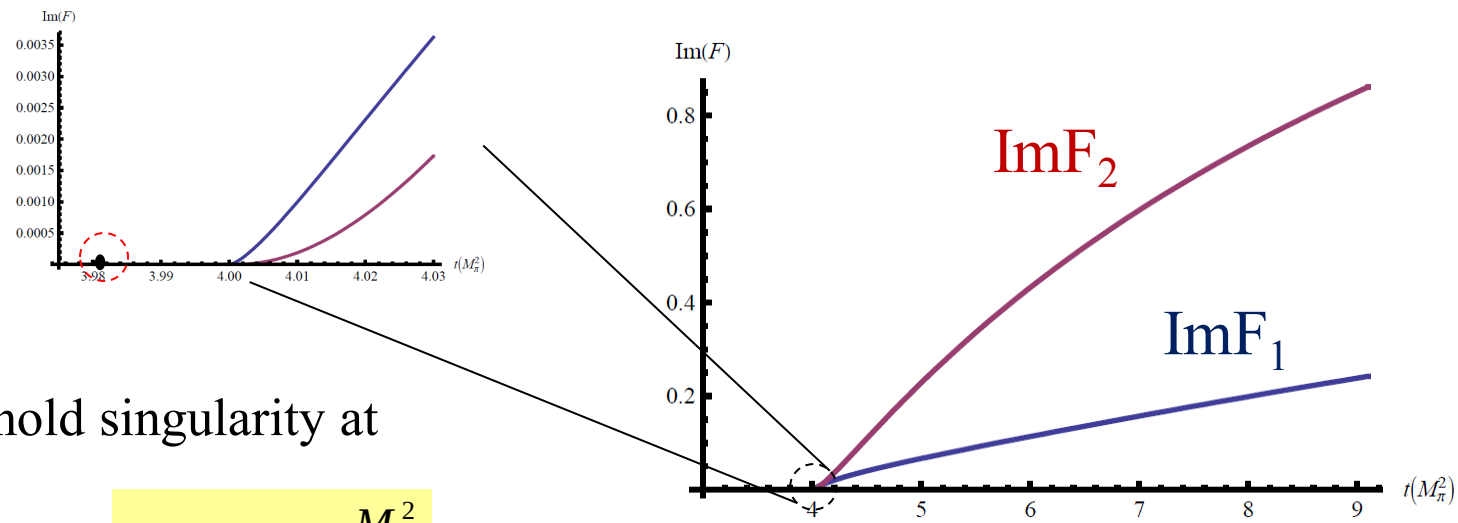
- Spectral Functions

$$\frac{\mathrm{Im}F_1}{\pi} = 2g_{\pi N}^2 \frac{\left(\frac{t}{2}-M_\pi^2\right)^2}{(4\pi)^2\left(P^2\right)^{\frac{5}{2}}\sqrt{t}}\left[-\frac{t}{8}x^2\arctan(x)+\left(M_N^2+\frac{t}{8}\right)\left(x+\arctan(x)\right)\right]+\frac{\left(1-g_A^2\right)\left(t-4M_\pi^2\right)^{\frac{3}{2}}}{6\left(4\pi F_\pi\right)^2}$$

$$\frac{\mathrm{Im}F_2}{\pi} = g_{\pi N}^2 \frac{\left(\frac{t}{2}-M_\pi^2\right)^2}{(4\pi)^2\left(P^2\right)^{\frac{5}{2}}\sqrt{t}}\left[\left(x^2+3\right)\arctan(x)-3x\right]$$

$$x = \frac{2\sqrt{M_N^2-\frac{t}{4}}\sqrt{\frac{t}{4}-M_\pi^2}}{\frac{t}{2}-M_\pi^2}$$

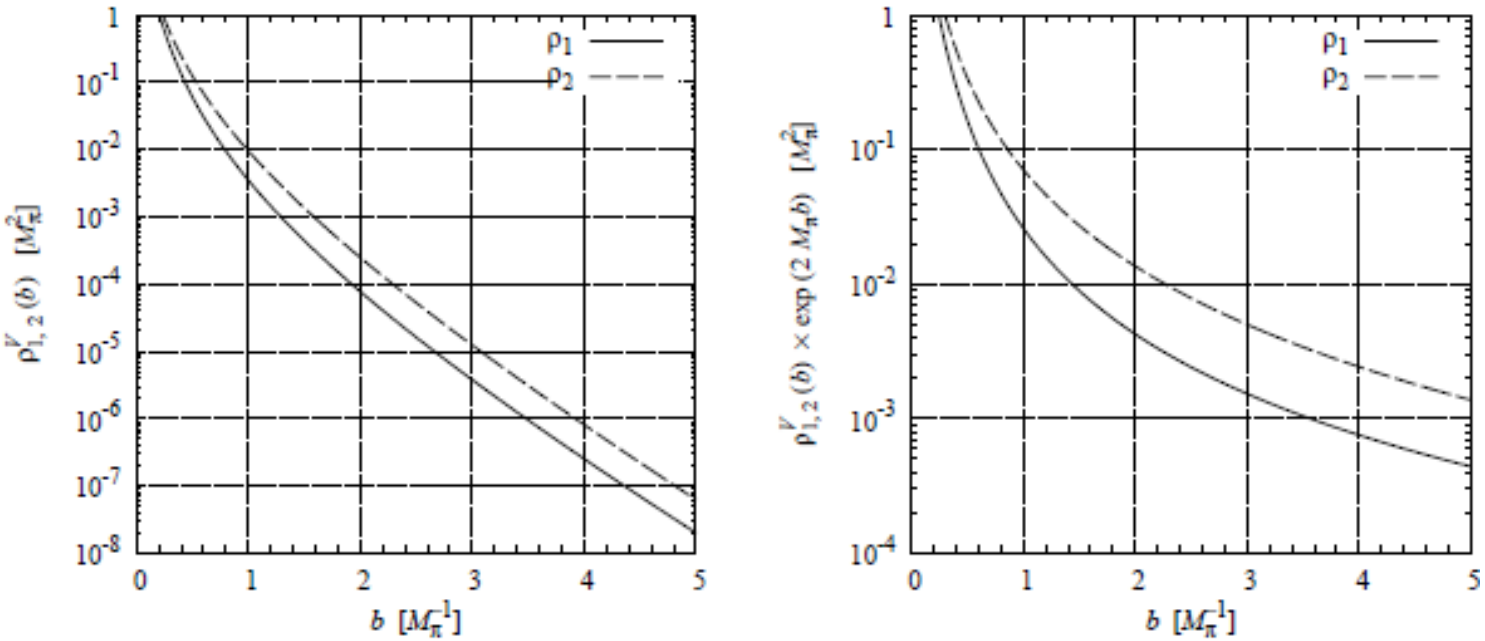
Strikman, Weiss, PRC82 (2010) 042201



- Sub-threshold singularity at $x(t_{sub})=\pm i$

$$\rightarrow \quad t_{sub} = 4M_\pi^2 - \frac{M_\pi^2}{M_N^2}$$

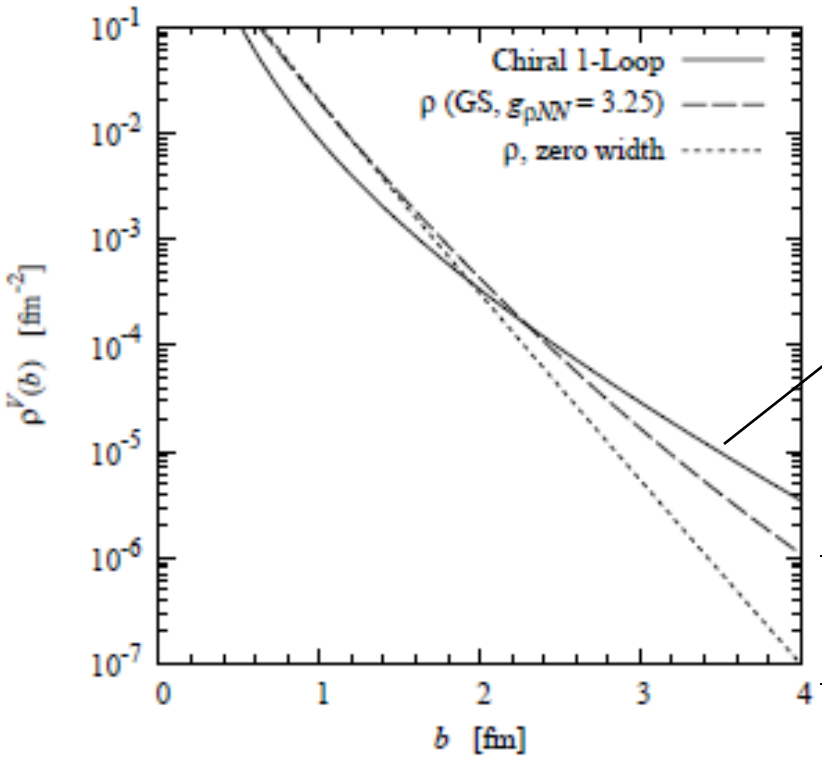
Peripheral Densities from Invariant χPT



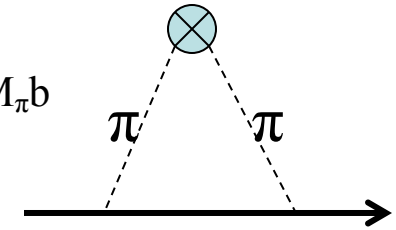
$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0\left(\sqrt{t}b\right) \frac{1}{\pi} \operatorname{Im} F\left(t + i0\right)$$

Peripheral Densities from Invariant χPT
Chiral vs. Non-Chiral

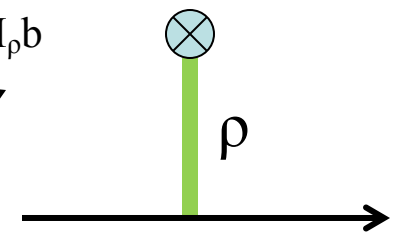
Chiral component dominant only at b >> 2 fm



$\rho(b) \sim e^{-2M_\pi b}$



$\rho(b) \sim e^{-M_\rho b}$



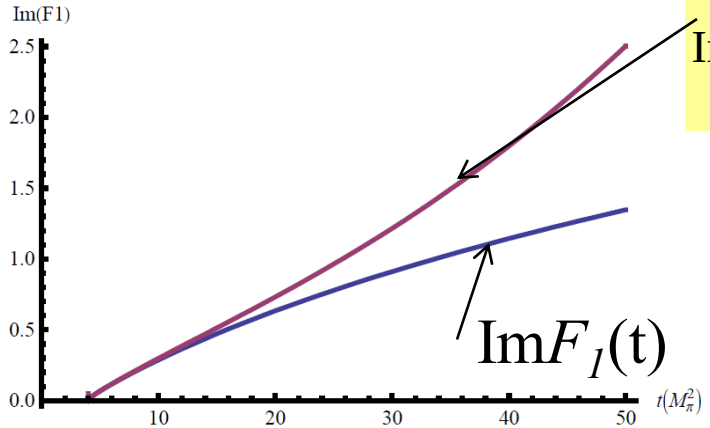
Miller, Strikman, Weiss, PRC84 (2011) 045205

Peripheral Densities from Invariant χPT

Heavy Baryon Expansion

$$\varepsilon \equiv \frac{M_\pi}{M_N}$$

$$\varepsilon << 1, \quad \frac{t}{M_\pi^2} \sim O(\varepsilon^0)$$



$$\mathrm{Im}\,F_1^{HB}(t) = \pi \frac{g_A^2 M_\pi^2}{(4\pi F_\pi)^2}\left[\sum_{i=0}^{\infty} \varepsilon^i C_i\left(\frac{\sqrt{t}}{2M_\pi}\right)\right]$$

$$C_0(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}}\left[\tau^4 - \frac{3}{2}\tau^2 + \frac{1}{2}\right]$$

$$C_1(\tau) = -\frac{\pi}{2\tau^2}\left[3\tau^5 - 3\tau^3 + \frac{1}{2}\tau\right]$$

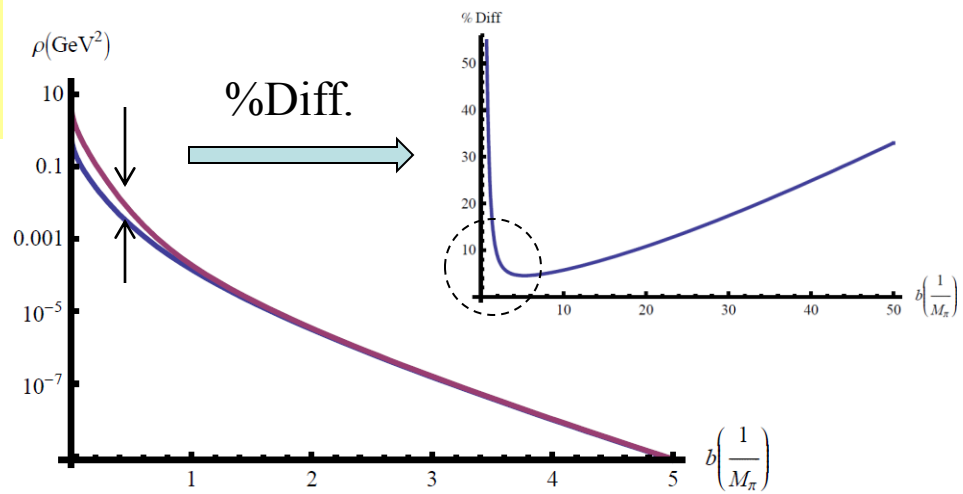
$$C_2(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}}\left[4\tau^6 - 6\tau^4 + \frac{9}{4}\tau^2 - \frac{1}{8}\right]$$

$$\rho_1^{HB}(b) = \frac{1}{2\pi}\frac{4g_A^2 M_\pi^2}{(4\pi F_\pi)^2}\left[\sum_{i=0}^{\infty} \varepsilon^i f_i(2M_\pi b)\right]$$

$$f_0(\beta) = \frac{1}{16}(K_2(\beta))^2 - \frac{1}{8}(K_1(\beta))^2 + \frac{1}{16}(K_0(\beta))^2$$

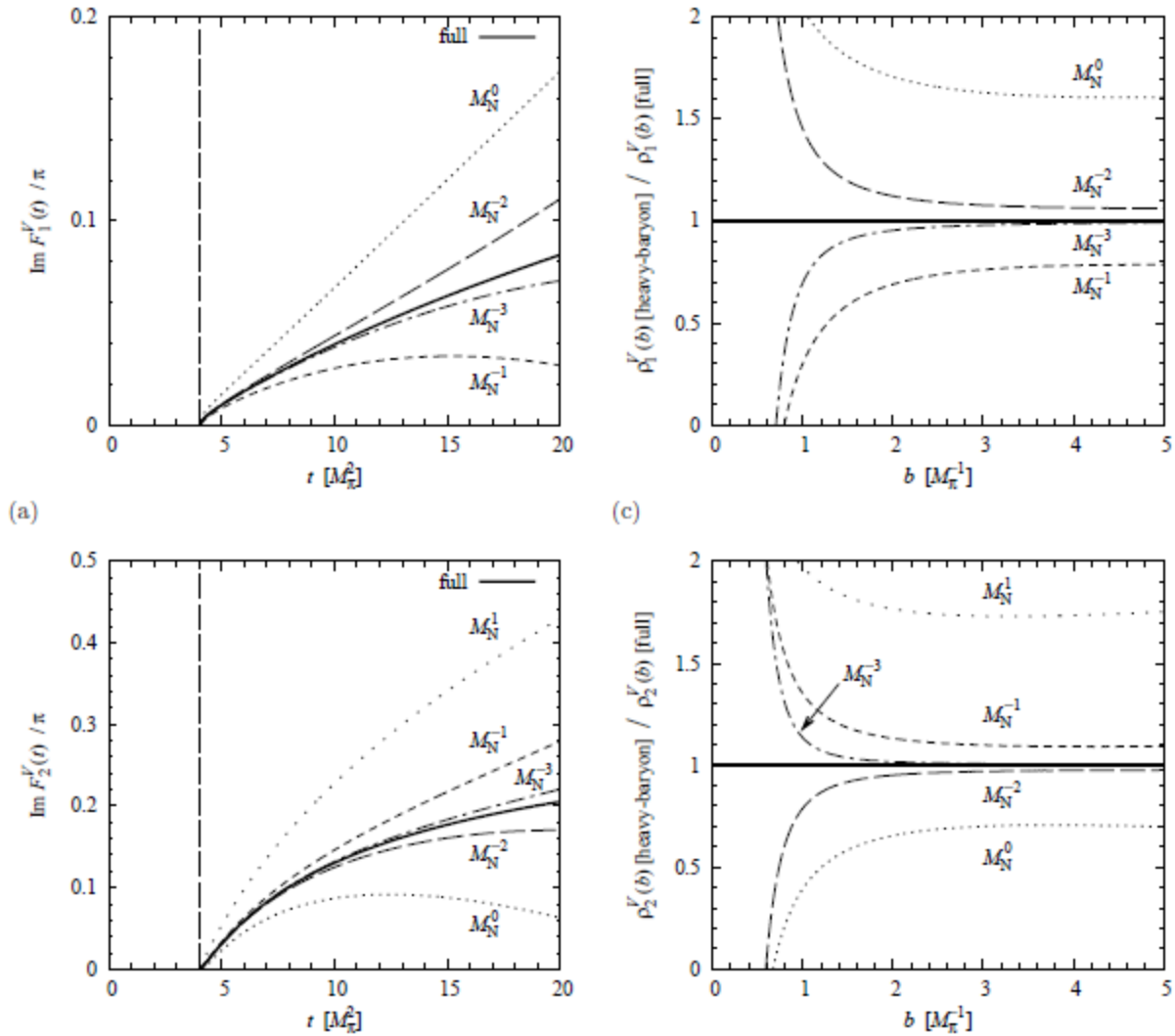
$$f_1(\beta) \approx -\left(\frac{\pi}{2}\right)^{\frac{3}{2}}\left[3\beta^{-5}\Gamma\left(\frac{9}{2},\beta\right) - 3\beta^{-3}\Gamma\left(\frac{5}{2},\beta\right) + \frac{1}{2}\beta^{-1}\Gamma\left(-\frac{1}{2},\beta\right)\right]$$

$$f_2(\beta) = \frac{1}{2}(K_3(\beta))^2$$



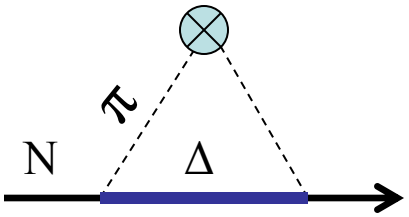
Peripheral Densities from Invariant χPT

Heavy Baryon Expansion



Peripheral Densities from Invariant χPT
Contributions from Δ and Large N_c Limit

- Consistency with QCD in the Large N_c Limit
  - At Large N_c
    - $M_\pi \sim N_c^0$, $M_{N,\Delta} \sim N_c^1$
    - $g_{\pi NN} \sim N_c^{3/2}$, $g_{\pi N\Delta} \sim N_c^{3/2}$
    - **$\rho \sim N_c^0$**



- But in the Large N_c Limit, considering only nucleonic intermediate state χPT contributions to F(t) lead to

$$\rho_{(N\pi)N} \approx A_N N_c + B_N N_c^0$$

- Contributions from Δ-intermediate states remedy this discrepancy ,

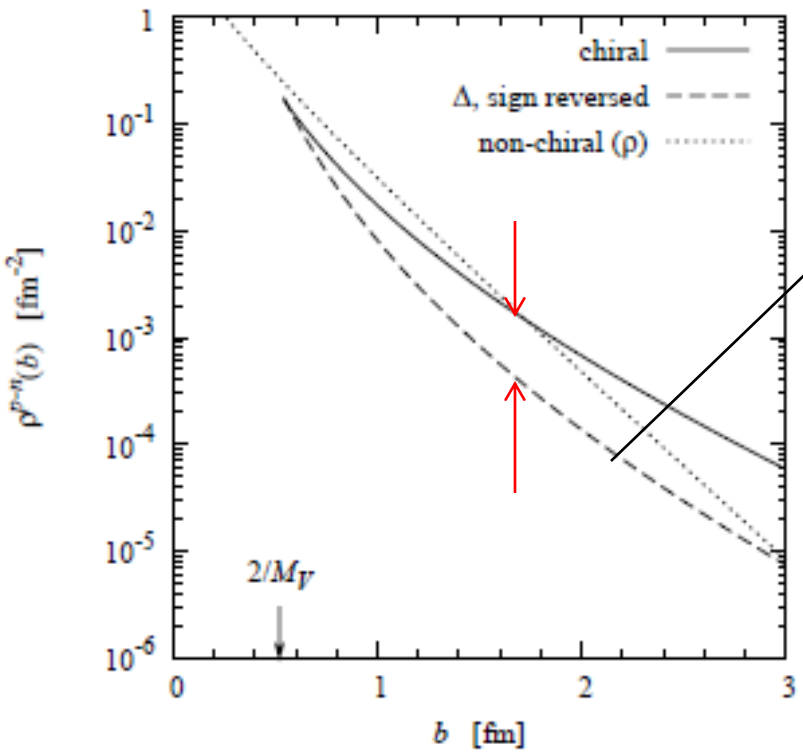
$$\rho_{(\Delta\pi)N} \approx -A_N N_c + B_\Delta N_c^0$$

$$\rho_N = \rho_{(N\pi)N} + \rho_{(\Delta\pi)N}$$

$$\approx (B_N + B_\Delta) N_c^0$$

Peripheral Densities from Invariant χPT

Contributions from Δ and Large N_c Limit



$$\frac{1}{\pi}\mathrm{Im}\,F_1 = \frac{g^2_{\pi N\Delta}A(-2M_N F + AG)}{48\pi^2 M_N^5\sqrt{t}}(x - \arctan x)$$

$$= -\frac{g^2_{\pi N\Delta}(t/2 - M_\pi^2)^2}{18\pi^2 M_N^3\sqrt{t}}x \qquad (\text{large } N_c)$$

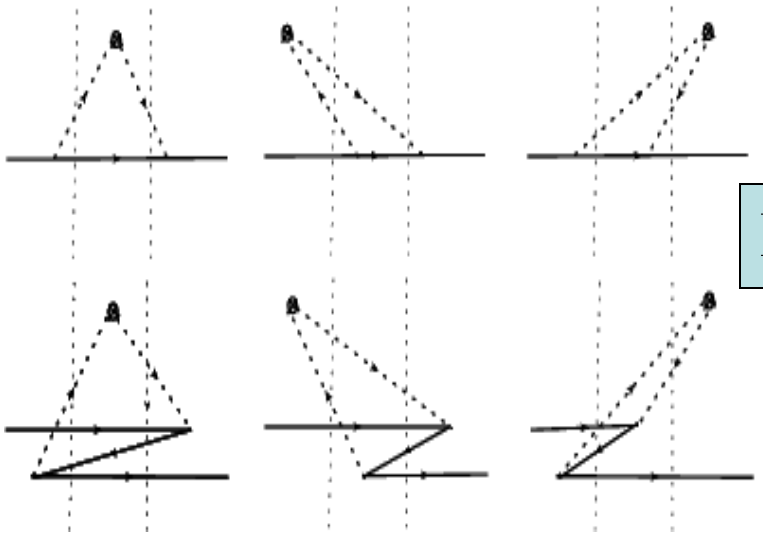
- $F_{N\pi\Delta}$ comparable to $F_{N\pi N}$ at b< 2fm
- Cancelation of leading Nc component

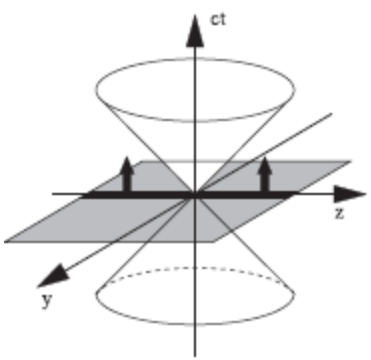
Peripheral Densities in Light-Front χPT

- Develop a partonic formulation of chiral dynamics
  - Charge and current of pions in the chiral periphery
  - Orbital angular momentum decomposition of chiral π-N LCWF
- Demonstrate equivalence with invariant formalism
- Connect to GPD formalism
  - Compute model–independent χGPDs

Peripheral Densities in Light-Front χPT

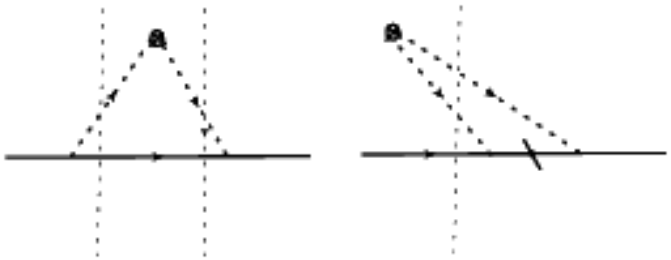
Equal time x_o

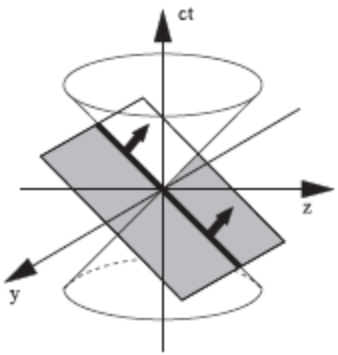


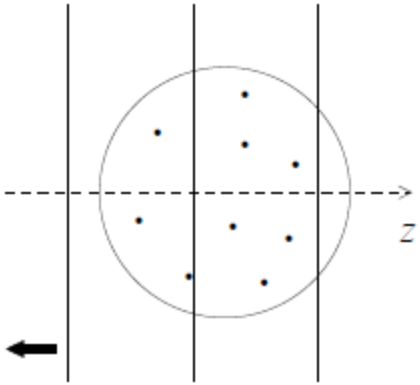


IMF

Equal LF time x+

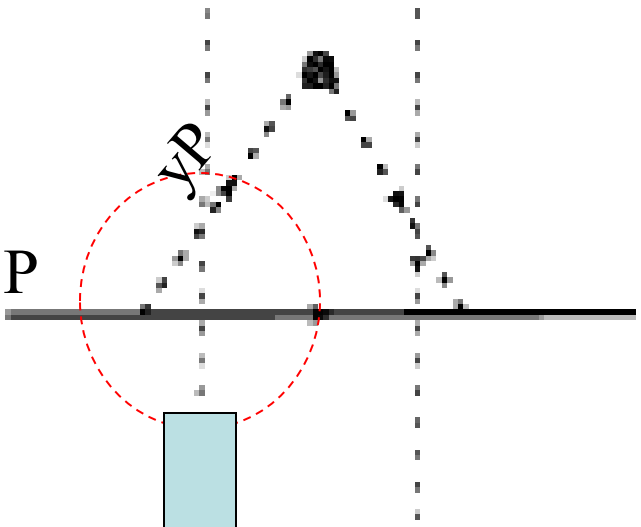


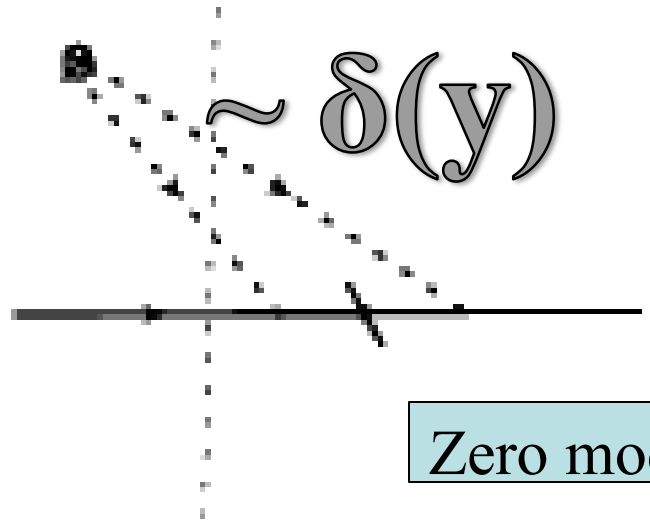




$x^+ = x^0 + x^3 = \text{const.}$

Peripheral Densities in Light-Front χPT





Zero mode

$$\Psi_{\pi N}(y, k_T, \lambda, \lambda') = -\left[\frac{y(1-y)}{k_\perp^2 + \overline{M}_\pi^2(y)}\right]\overline{U}_{\lambda'} i\Gamma_{\pi NN} U_\lambda \phi_\pi$$

Light-Cone Wave-function

Peripheral Densities in Light-Front χPT

- Form factors from the infinite momentum frame

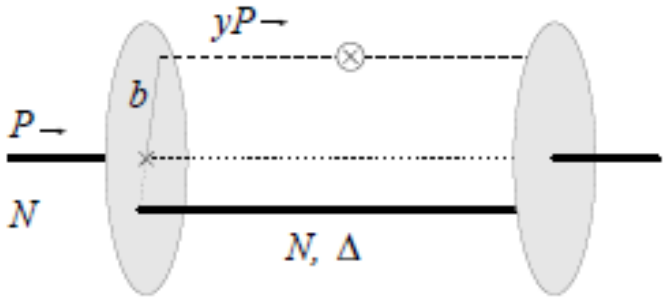
$$F_1 = \left\langle P_2,+\left|\frac{J^3}{2P}\right|P_1,+\right\rangle$$

$$-\frac{\Delta_L}{2M}F_2 = \left\langle P_2,+\left|\frac{J^3}{2P}\right|P_1,-\right\rangle$$

$$F_1 = 2\int\frac{d^2k_\perp}{(2\pi)^2}\frac{dy}{y(1-y)}\sum_{\lambda'}\Psi_{\pi N}^\dagger(y,k'_T,+,\lambda')\Psi_{\pi N}(y,k_T,+,\lambda')+\delta(y)(1-g_A^2)\mathbf{ct},$$

$$-\frac{\Delta_L}{2M}F_2 = 2\int\frac{d^2k_\perp}{(2\pi)^2}\frac{dy}{y(1-y)}\sum_{\lambda'}\Psi_{\pi N}^\dagger(y,k'_T,+,\lambda')\Psi_{\pi N}(y,k_T,-,\lambda')$$

- LCWF from $N\pi N$ pseudo scalar coupling



$$\Psi_{\pi N}(y,k_T,\lambda,\lambda') = -\frac{ig_A}{2F_\pi}\left[\frac{y(1-y)}{k_\perp^2+\overline{M}_\pi^2(y)}\right]\overline{U}_{\lambda'}\gamma_5 U_\lambda$$

- In impact parameter space

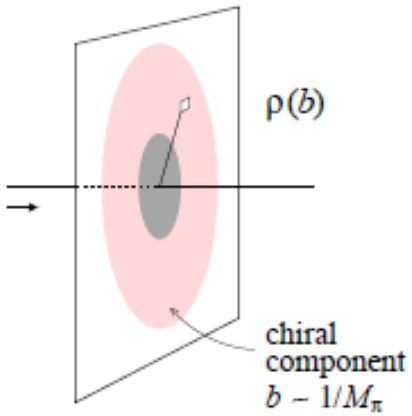
$$\Psi_{\pi N}(y,b) = \int\frac{d^2k_T}{(2\pi)^2}e^{-i(k_T\mathbf{b})}\Psi_{\pi N}(y,k_T)$$

Peripheral Densities in Light-Front χPT

$$\rho(b) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i(\Delta_T \cdot b)} F\left(-\Delta_T^2\right)$$

•Transverse densities from form factors

• Charge and current densities from pion-nucleon light-cone wave functions with



$$\Psi_{\pi N}^{++}(y,\mathbf{b}) = \frac{ig_A}{F_\pi}\frac{y^2\sqrt{1-y}}{2\pi} M_N^2 K_0\left(\overline{M}_\pi \mathbf{b}\right)$$

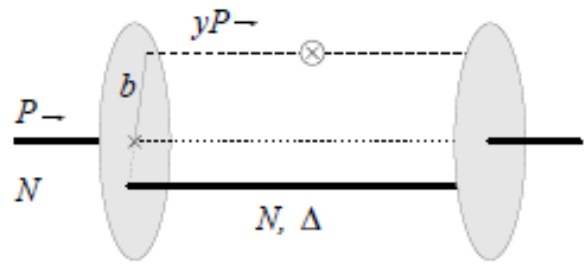
$$\Psi_{\pi N}^{+-}(y,\mathbf{b}) = \frac{g_A}{F_\pi}\frac{y\sqrt{1-y}}{2\pi} M_N \overline{M}_\pi K_1\left(\overline{M}_\pi \mathbf{b}\right) e^{-i\theta}$$

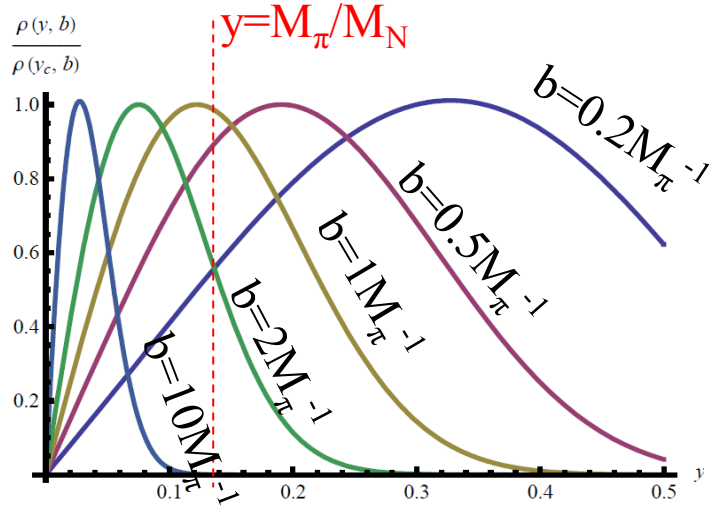
$$\rho_1(b) = 2\sum_{\lambda'}\int \frac{dy}{2\pi}\left(\frac{\left|\Psi_{+\lambda'}(y,b')\right|^2}{y(1-y)^3} + (1-g_A^2)\delta(y)\text{C.T.}\right)$$

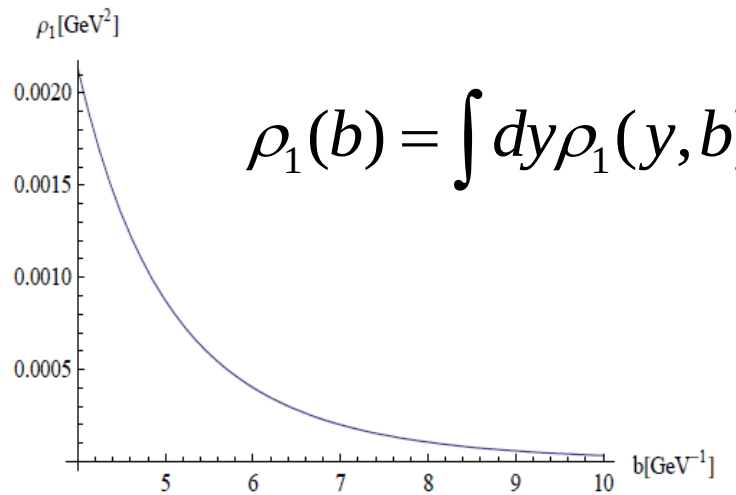
$$\frac{\partial}{\partial b_R}\rho_2(b) = 2iM\sum_{\lambda'}\int \frac{dy}{2\pi}\frac{\Psi_{+,\lambda'}(y,b')\Psi_{-,\lambda'}(y,b')}{y(1-y)^3}$$

Peripheral Densities in Light-Front χPT

$$\rho_1(y,b) \equiv 2\sum_{\lambda\lambda'}\left(\frac{|\Psi_{\lambda\lambda'}(y,b')|^2}{y(1-y)}\right)$$







$$\rho_1(b) = \int dy \rho_1(y,b)$$

- Slower pions at larger impact parameter

$$\rho_1(b) = \int \frac{dy}{2\pi}\frac{2g^2}{(2\pi)^2}\frac{yM_N^2}{(1-y)^2}\left[y^2K_0^2(\overline{M}_\pi b') + \left(\frac{\overline{M}_\pi}{M_N}\right)^2 K_1^2(\overline{M}_\pi b')\right]$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

- Energy-Momentum tensor form factors (calculable in χPT)

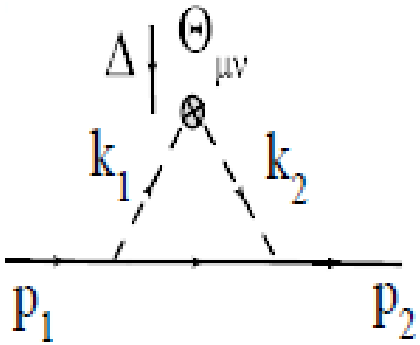
$$\langle N'|\Theta^{N\pi}_{\mu\nu}|N\rangle = \bar{u}_{p'}\Big[\gamma_{(\mu}P_{\nu)}A(\Delta^2) + P_{(\mu}i\sigma_{\nu)\alpha}\frac{\Delta_\alpha}{2M}B(\Delta^2) + \left(\frac{\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu}}{M}\right)C(\Delta^2) + Mg_{\mu\nu}\tilde{C}(\Delta^2)\Big]u_p$$

- Angular momentum of a pion-nucleon system,

$$J_{N\pi} = \frac{1}{2}(A(0) + B(0))$$

- Transverse densities ρ_A, ρ_B from form factors A and B, and A+B.

$$\rho_{A+B}(b) = \frac{1}{2\pi}\int_{thr}^{\infty} dt K_0\left(\sqrt{tb}\right)\frac{1}{\pi}\mathrm{Im}[A(t+i0) + B(t+i0)]$$



$$\Theta_{\mu\nu} = tr\left(\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}g_{\mu\nu}(\partial_\sigma\pi)^2\right)$$

- Calculated leading chiral contribution to spectral functions (Cutkosky Rules). No contact term diagrams!

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

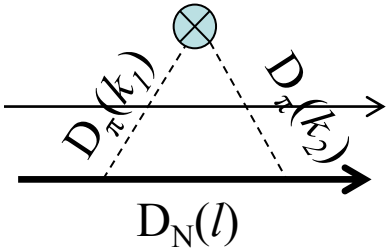
- EMT form factors

$$A(\Delta^2) = 6\frac{g_A^2}{F_\pi^2}M_N^2\left(2J_1^{(3)} + \frac{M_N^2}{P^2}J_3^{(3)}\right)$$
$$B(\Delta^2) = -6\frac{g_A^2}{F_\pi^2}\frac{M_N^4}{P^2}J_3^{(3)}.$$

- Invariant integrals

$$J_1^{(1)} = \int -i\frac{d^4k}{(2\pi)^4}D_\pi(k_1)D_\pi(k_2)D_N(l)\frac{Pk}{P^2}$$
$$J_1^{(3)} = \int -i\frac{d^4k}{(2\pi)^4}D_\pi(k_1)D_\pi(k_2)D_N(l)\frac{1}{2}\frac{Pk}{P^2}\left(k^2 - \frac{(\Delta k)^2}{\Delta^2} - \frac{(Pk)^2}{P^2}\right)$$

- Spectral functions (from Cutkosky rules)



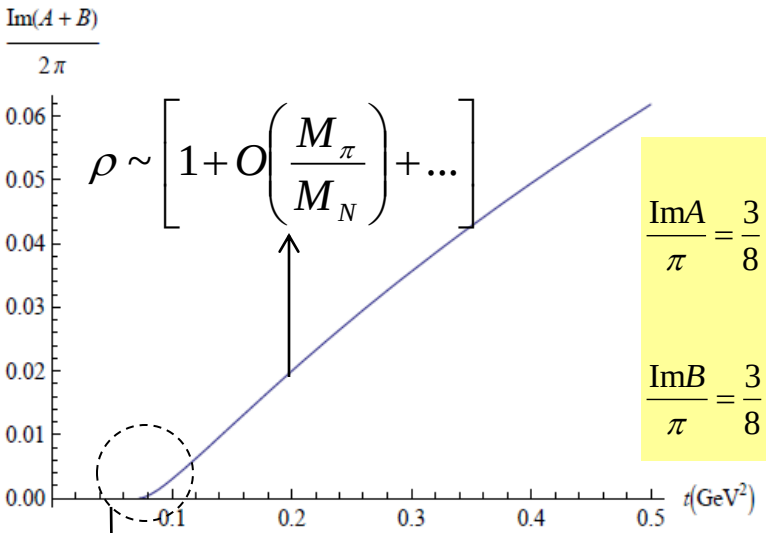
$$\frac{1}{k_{1,2}^2 - m_\pi^2 + i\varepsilon} \to -2\pi i\delta(k_{1,2}^2 - m_\pi^2)$$

$$\frac{1}{\pi}\mathrm{Im}J_1^{(3)} = \frac{k_{cm}^4}{16\pi^2}\frac{1}{\sqrt{t}\sqrt{P^2}}\frac{U^3}{V^4}\left(\frac{2}{3}x^3 + x - (x^2+1)\arctan(x)\right)$$
$$\frac{1}{\pi}\mathrm{Im}J_3^{(3)} = \frac{k_{cm}^4}{16\pi^2}\frac{1}{\sqrt{t}\sqrt{P^2}}\frac{U^3}{V^4}\left(-\frac{4}{3}x^3 - 5x + (3x^2+5)\arctan(x)\right)$$

$$x = \frac{2\sqrt{M_N^2 - \frac{t}{4}}\sqrt{\frac{t}{4} - M_\pi^2}}{\frac{t}{2} - M_\pi^2}$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

- Asymptotic behavior of ρ controled by ImF at threshold and near threshold



$$\rho \sim \left[1 + O\left(\frac{M_\pi}{M_N}\right) + \ldots\right]$$

$$\rho \sim \left[1 + O\left(\frac{M_N^2}{M_\pi^3 b}\right) + \ldots\right]$$

$$\frac{\mathrm{Im}A}{\pi} = \frac{3}{8} g^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^3}{(4\pi)^2 \sqrt{P^2}^5 \sqrt{t}} \left[\frac{4}{3}\left(1 - \frac{M_N^2}{P^2}\right)x^3 + 2x - \left(\left(2 - 3\frac{M_N^2}{P^2}\right)x^2 - 3\right)\arctan(x)\right]$$

$$\frac{\mathrm{Im}B}{\pi} = \frac{3}{8} g^2 \frac{\left(\frac{t}{2} - M_\pi^2\right)^3}{(4\pi)^2 \sqrt{P^2}^5 \sqrt{t}} \left[\frac{4}{3}x^3 + 5x - \left(3x^2 + 5\right)\arctan(x)\right]$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum

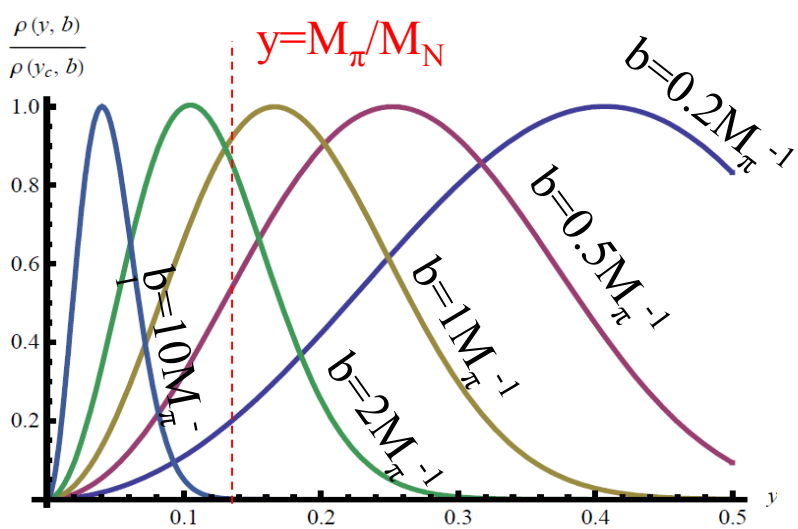
- EMT form factors in IMF
- Corresponding Transverse densities as overlap of LC- wave functions
- Matter distributions in impact parameter space:
  - Moments of parton distributions

$$A = \left\langle P_2,+ \left| \frac{\Theta^{33}}{2P^2} \right| P_1,+ \right\rangle$$

$$-\frac{\Delta_L}{2M} B = \left\langle P_2,+ \left| \frac{\Theta^{33}}{2P^2} \right| P_1,- \right\rangle$$

$$\rho_A(b) = \frac{3}{2} \sum_{\lambda'} \int \frac{dy}{2\pi} y \left[\frac{\left|\Psi_{\lambda\lambda'}(y,b)\right|^2}{y(1-y)^3} \right]$$

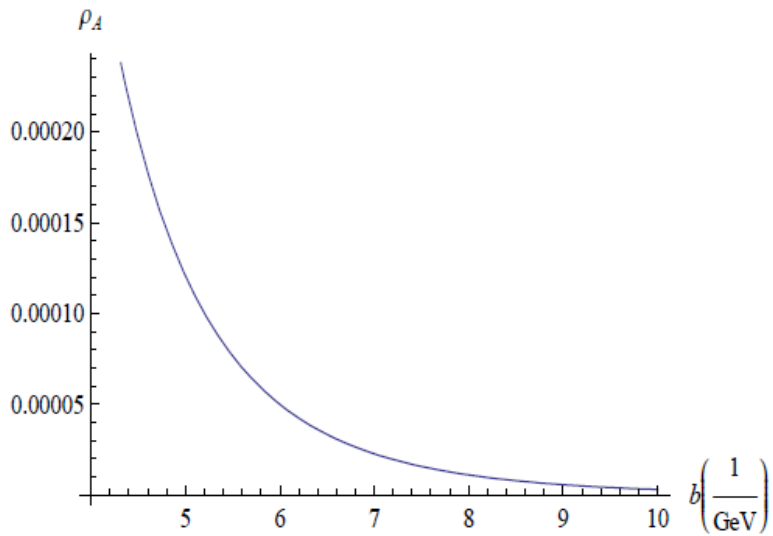
$$\frac{\partial}{\partial b_R} \rho_B(b) = \frac{3}{2} iM \sum_{\lambda'} \int \frac{dy}{2\pi} y \left[\frac{\Psi_{+\lambda'}(y,b)\Psi_{-\lambda'}(y,b)}{y(1-y)^3} \right]$$

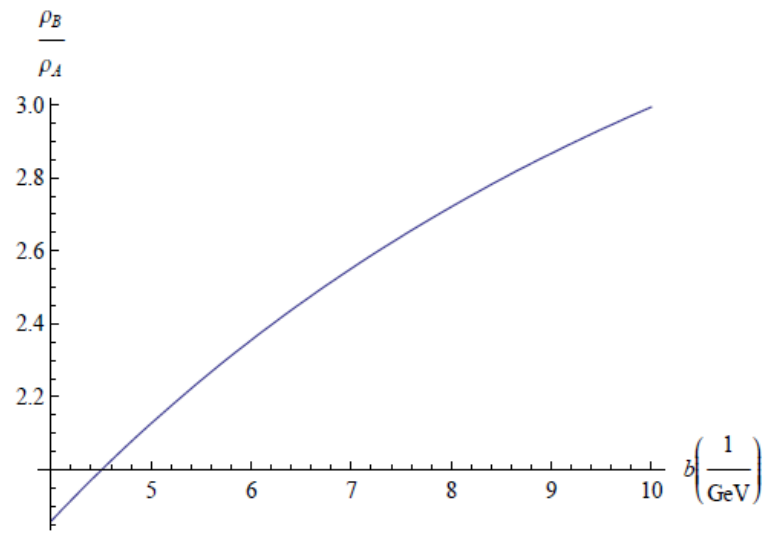


$$\rho_A(y,b) \equiv \frac{3}{2} y \sum_{\lambda\lambda'} \left(\frac{\left|\Psi_{\lambda\lambda'}(y,b')\right|^2}{y(1-y)^3} \right)$$

$$\rho_A(y,b) = 3y\rho_1(y,b)/4$$

Energy-Momentum Tensor: Matter density and Orbital Angular Momentum





$$\rho_A(b) = \frac{3}{2}\int \frac{dy}{2\pi}\frac{g^2}{(2\pi)^2}\frac{y^2 M_N^2}{(1-y)^2}\left[y^2 K_0^2\left(\overline{M}_\pi b'\right) + \left(\frac{\overline{M}_\pi}{M_N}\right)^2 K_1^2\left(\overline{M}_\pi b'\right)\right]$$

$$\rho_B(b) = \frac{3}{2}\int \frac{dy}{2\pi}\frac{g^2}{(2\pi)^2}\frac{y^2 M_N^2}{(1-y)}\left[2yK_0^2\left(\overline{M}_\pi b'\right)\right]$$

Summary

- Explored Nucleonic Structure in a setting that guarantees a model independent analysis of the dynamics governed by χ EFT.
- Derived EM and EMT transverse peripheral densities from corresponding FF
- From spectral functions (invariant formalism)
  - Distinguish parametrical regions (chiral and molecular scales)
  - Accuracy of the Heavy Baryon expansion
  - Consistency with the QCD Large Nc limit (add Δ-contribution)
- Light–front χPT (IMF)
  - Equivalent to invariant formalism
  - Calculated transverse densities from LC-Wave Functions (Connection to GPD formalism)

Outlook

- Understand origin of contact term (higher mass states, nucleon compositeness)
- Use the chiral pion –nucleon system as a toy model for exploring the nature of orbital angular momentum OAM and other operators in field theory (moments of GPDs, Axial form factors)
- Test use of πN-LCWF in experimental studies at Low and High energies

Peripheral exclusive processes in e-N scattering at EIC

[Strikman, Weiss Phys.Rev. D69 (2004) 054012; EIC White Paper 2012]