TNT Seminar, NCSU
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Anatomy of relativistic pion loop corrections to electromagnetic nucleon properties

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Outline

- *Motivation*: can one understand flavor asymmetries in the nucleon ($e.g.\ \bar{d} - \bar{u}$) from QCD?

  $\longrightarrow$ origin of 5-quark Fock components $|qqq\,\bar{q}q\rangle$ of nucleon

- Effective pion-nucleon interactions

  $\longrightarrow$ pseudovector *vs*. pseudoscalar coupling

- *Example*: self-energy of nucleon dressed by pions

  $\longrightarrow$ equivalence of equal-time and light-front formulations

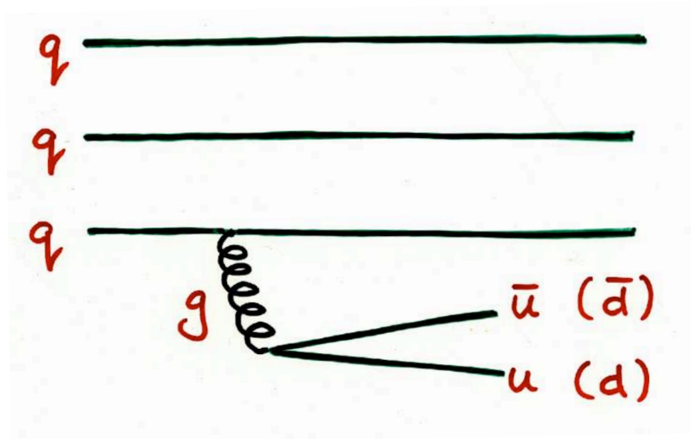
- Vertex corrections

  $\longrightarrow$ light-cone momentum distributions

  $\longrightarrow$ PDF moments: χPT *vs*. "Sullivan" formulations

Flavor asymmetry

- Antiquarks in the proton "sea" produced predominantly by gluon radiation into quark-antiquark pairs, $g \longrightarrow q\bar{q}$



→ since u and d quark masses are similar, expect flavor-symmetric sea, $\bar{d} \approx \bar{u}$

- Experimentally, one finds *large excess* of $\bar{d}$ over $\bar{u}$

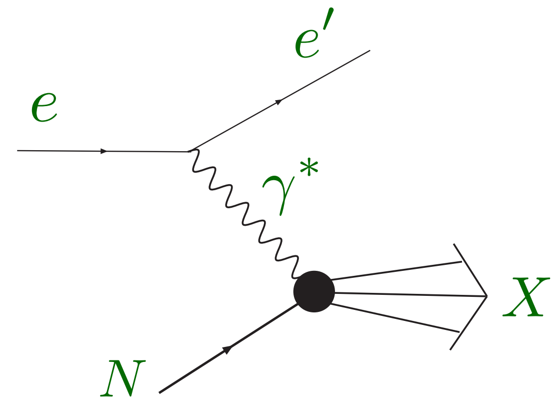
$$\int_0^1 dx\ (\bar{d}(x) - \bar{u}(x)) = 0.118 \pm 0.012$$

*E866 (Fermilab), PRD **64**, 052002 (2001)*

Flavor asymmetry

- Inclusive cross section for $eN \to eX$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$



$$\left.\begin{aligned} \nu &= E - E' \\ Q^2 &= \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\frac{\theta}{2} \end{aligned}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

- F_1 , F_2 structure functions

$\longrightarrow$ contain all information about structure of nucleon

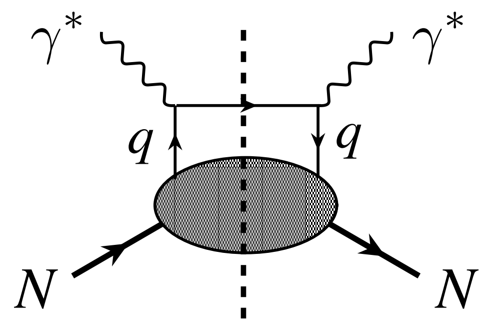
Flavor asymmetry

- In *deep-inelastic* region ($W \gtrsim 2\ \mathrm{GeV},\ Q^2 \gtrsim 1\ \mathrm{GeV}^2$) structure function given by parton distribution functions

$$F_2(x,Q^2) = x \sum_q e_q^2\, q(x,Q^2)$$
$$= \frac{4}{9}x(u+\bar{u}) + \frac{1}{9}x(d+\bar{d}) + \frac{1}{9}x(s+\bar{s}) + \cdots$$

→ $q(x,Q^2)$ = probability to find parton type "q" in nucleon, carrying momentum fraction x

"PDF"

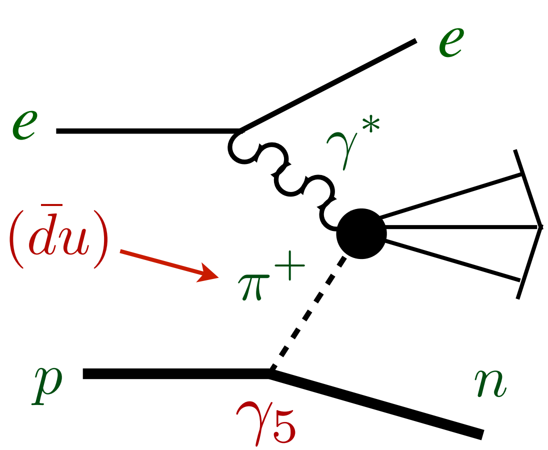


→ at large Q^2, structure functions related by $F_2 = 2xF_1$

Flavor asymmetry

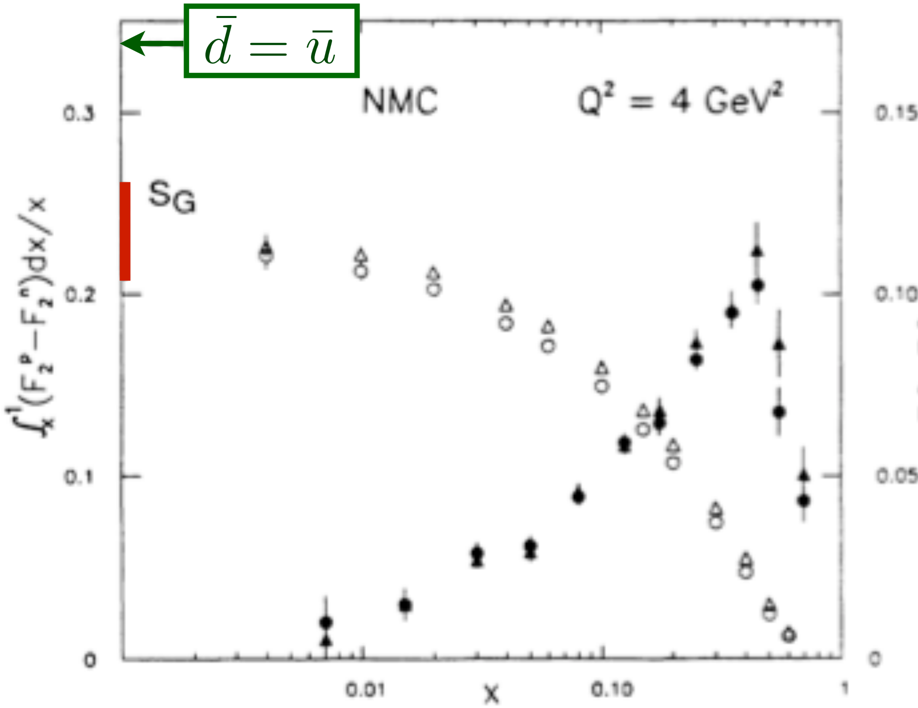
- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

→ Sullivan process



Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*

$$\boxed{\bar{d} > \bar{u}}$$



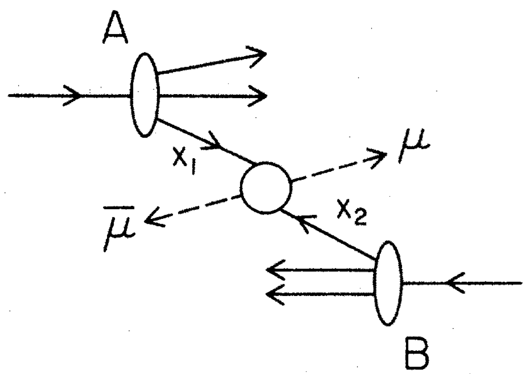
$$\int_0^1 \frac{dx}{x}(F_2^p - F_2^n) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx\,(\bar{d} - \bar{u})$$
$$= 0.235(26)$$

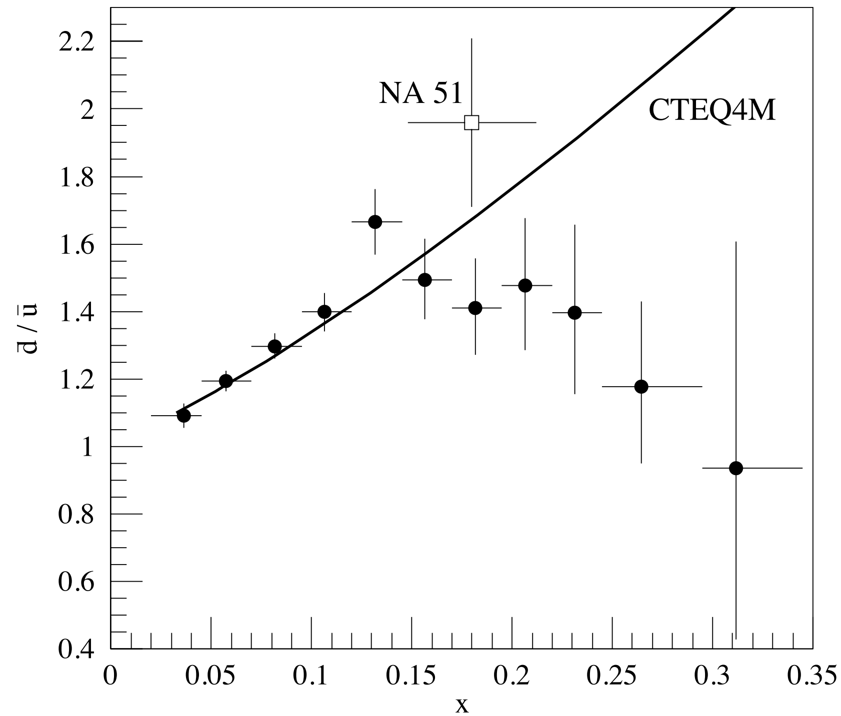
New Muon Collaboration, PRD ***50****, 1 (1994)*

Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

  → Drell-Yan process





E866, PRL ***80****, 3715 (1998)*

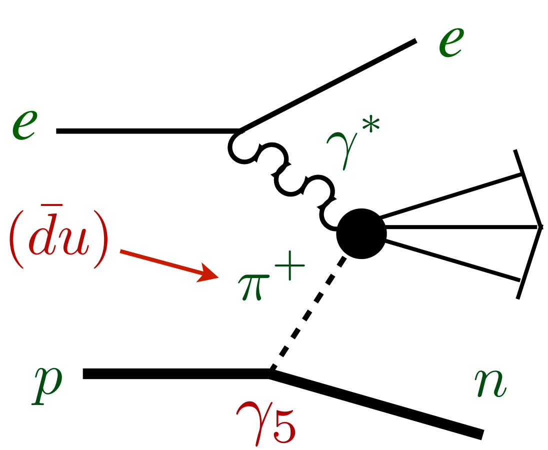
$$\frac{d^2\sigma}{dx_b dx_t} = \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2\left(q(x_b)\bar{q}(x_t) + \bar{q}(x_b)q(x_t)\right)$$

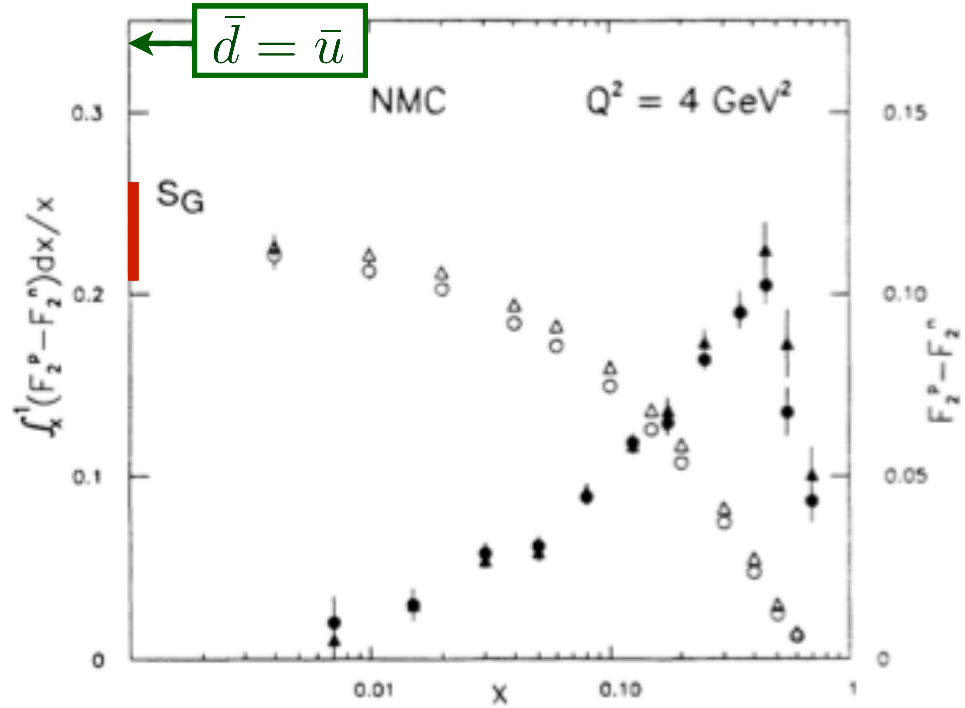
For $x_b \gg x_t$ $$\frac{\sigma^{pd}}{2\sigma^{pp}} \approx \frac{1}{2}\left(1 + \frac{\bar{d}(x_t)}{\bar{u}(x_t)}\right)$$

Flavor asymmetry

- Large flavor asymmetry in proton sea suggests important role of π cloud in high-energy reactions

$\longrightarrow$ Sullivan process





Sullivan, PRD ***5****, 1732 (1972)*
Thomas, PLB ***126****, 97 (1983)*

$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y}\, f_\pi(y)\ \bar{q}^\pi(x/y)$$

$$\boxed{f_\pi(y) = \frac{3g_{\pi NN}^2}{16\pi^2}\, y \int dt \frac{-t\ \mathcal{F}_{\pi NN}^2(t)}{(t - m_\pi^2)^2}}$$

connection to QCD?

Connection with QCD?

- Chiral expansion of moments of $f_\pi(y)$

→ model-independent leading nonanalytic (LNA) behavior

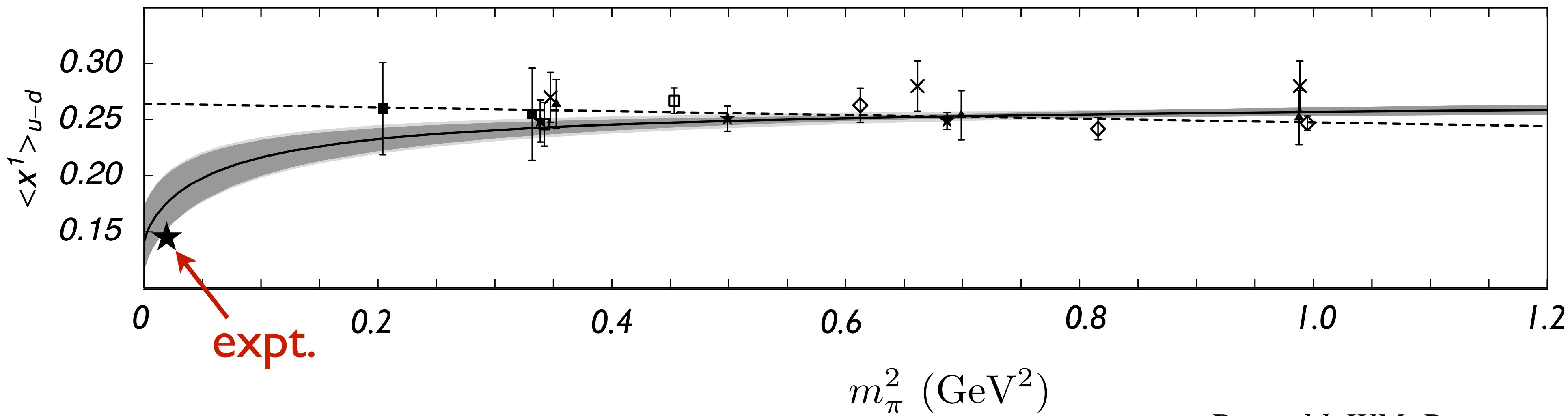
$$\begin{aligned}
\langle x^0 \rangle_{\bar{d}-\bar{u}} &\equiv \int_0^1 dx(\bar{d}-\bar{u}) \\
&= \frac{2}{3}\int_0^1 dy\, f_\pi(y) \\
&= \frac{2g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \;+\; \text{analytic terms}
\end{aligned}$$

Thomas, WM, Steffens
PRL ***85****, 2892 (2000)*

→ can *only* be generated by pion cloud!

Connection with QCD?

- Nonanalytic behavior vital for chiral extrapolation of lattice data



Detmold, WM, Renner et al. PRL ***87****, 172001 (2001)*

→ allows lattice QCD calculations to be reconciled with experiment

Connection with QCD?

- Direct calculation of matrix elements of local twist-2 operators in ChPT disagrees with "Sullivan" result

$$\langle x^n \rangle_{u-d} = a_n \left(1 + \frac{(3g_A^2 + 1)}{(4\pi f_\pi)^2} m_\pi^2 \log(m_\pi^2/\mu^2) \right) + \mathcal{O}(m_\pi^2)$$

Chen, X. Ji, PLB ***523****, 107 (2001)*
Arndt, Savage, NPA ***692****, 429 (2002)*

$\boxed{\textit{cf.}\ 4g_A^2 \text{ in "Sullivan", via moments of } f_\pi(y)}$

→ is there a problem with application of ChPT or "Sullivan process" to DIS?

→ is light-front treatment of pion loops problematic?

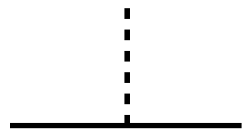
→ investigate relation between *covariant*, *instant-form*, and *light-front* formulations

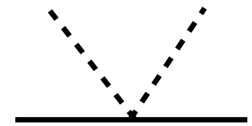
→ consider simple test case: nucleon self-energy

πN Lagrangian

- Chiral (pseudovector) Lagrangian

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\,\bar{\psi}_N \gamma^\mu \gamma_5\, \vec{\tau}\cdot\partial_\mu \vec{\pi}\,\psi_N - \frac{1}{(2f_\pi)^2}\,\bar{\psi}_N \gamma^\mu\, \vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\,\psi_N$$





$g_A = 1.267$

$f_\pi = 93$ MeV

$\longrightarrow$ lowest order approximation of chiral perturbation theory Lagrangian

- Pseudoscalar Lagrangian

$$\mathcal{L}^{\mathrm{PS}}_{\pi N} = -g_{\pi NN}\,\bar{\psi}_N\, i\gamma_5 \vec{\tau}\cdot\vec{\pi}\,\psi_N$$

Self-energy

- From lowest order PV Lagrangian

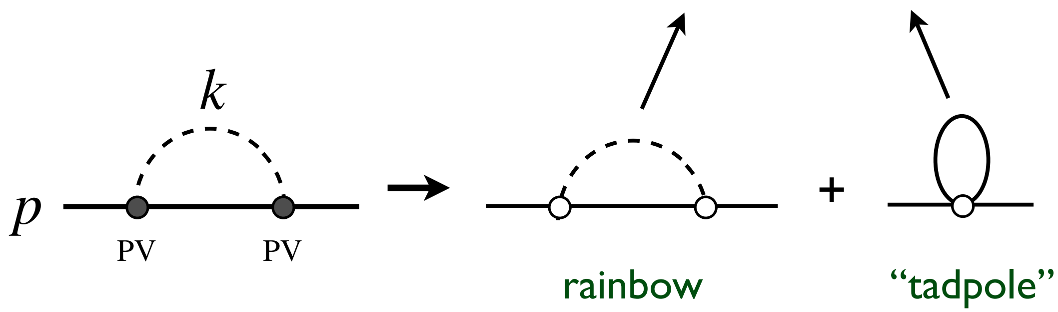
$$\Sigma = i\left(\frac{g_{\pi NN}}{2M}\right)^2 \bar{u}(p) \int \frac{d^4k}{(2\pi)^4} (\not{k}\gamma_5\vec{\tau})\, \frac{i\,(\not{p}-\not{k}+M)}{D_N} (\gamma_5\not{k}\vec{\tau})\, \frac{i}{D_\pi^2}\, u(p)$$

Goldberger-Treiman $\quad \dfrac{g_A}{f_\pi} = \dfrac{g_{\pi NN}}{M}$

$$\begin{aligned} D_\pi &\equiv k^2 - m_\pi^2 + i\varepsilon \\ D_N &\equiv (p-k)^2 - M^2 + i\varepsilon \end{aligned}$$

$\longrightarrow$ rearrange in more transparent "reduced" form

$$\Sigma = -\frac{3ig_A^2}{4f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{2M} \left[4M^2\left(\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N}\right) + \cancel{\frac{2p\cdot k}{D_\pi}}\right]$$



C.-R. Ji, WM, Thomas, PRD ***80****, 054018 (2009)*

Self-energy

- Covariant (dimensional regularization)

$$\int d^{4-2\varepsilon}k \frac{1}{D_\pi D_N} = -i\pi^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \int_0^1 dx \log\frac{(1-x)^2 M^2 + x m_\pi^2}{\mu^2} + \mathcal{O}(\varepsilon)\right)$$

$$\int d^{4-2\varepsilon}k \frac{1}{D_N} = -i\pi^2 M^2 \left(\gamma + \log\pi - \frac{1}{\varepsilon} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\varepsilon)\right)$$

→ gives well-known m_π^3 LNA behavior (from $1/D_\pi D_N$ term)

$$\Sigma_{\rm cov}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

- Equal time (rest frame)

$$\int d^4k \frac{1}{D_\pi D_N} = \int d^3k \int_{-\infty}^{\infty} dk_0 \frac{1}{(-2)(\omega_k - i\varepsilon)} \left(\frac{1}{k_0 - \omega_k + i\varepsilon} - \frac{1}{k_0 + \omega_k - i\varepsilon} \right)$$
$$\times \frac{1}{2(E' - i\varepsilon)} \left(\frac{1}{k_0 - E + E' - i\varepsilon} - \frac{1}{k_0 - E - E' + i\varepsilon} \right)$$

$$\omega_k = \sqrt{\mathbf{k}^2 + m_\pi^2}\ , \qquad E' = \sqrt{\mathbf{k}^2 + M^2}$$

$\longrightarrow$ four time-orderings

$$\Sigma_{\mathrm{ET}}^{(+-)}, \quad \Sigma_{\mathrm{ET}}^{(-+)}, \quad \Sigma_{\mathrm{ET}}^{(++)}, \quad \Sigma_{\mathrm{ET}}^{(--)}$$

positive energy

"Z-graph"

= 0

$$\Sigma_{\mathrm{ET}}^{(+-)\mathrm{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(m_\pi^3 + \frac{3}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$
$$\Sigma_{\mathrm{ET}}^{(-+)\mathrm{LNA}} = -\frac{3g_A^2}{32\pi f_\pi^2} \left(- \frac{1}{4\pi} \frac{m_\pi^4}{M} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right)$$

Self-energy

- Equal time (infinite momentum frame)

$$\Sigma_{\rm IMF}^{(+-)} = -\frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E-E'-\omega_k)}$$

$$= \frac{3g_A^2 M}{32\pi^2 f_\pi^2}\int_0^1 dy \int_0^{\Lambda^2} dk_\perp^2 \, \frac{m_\pi^2}{k_\perp^2 + M^2(1-y)^2 + m_\pi^2 y}$$

$$p_z \equiv P \to \infty$$

$$y = p_z'/p_z$$

$$\Sigma_{\rm IMF}^{(-+)} = \frac{3g_A^2 M}{16\pi^3 f_\pi^2}\int_{-\infty}^{\infty} dy \int d^2k_\perp \frac{P}{2E'}\frac{1}{2\omega_k}\frac{m_\pi^2}{(E+E'+\omega_k)} \;=\; \mathcal{O}(1/P^2)$$

→ nonanalytic behavior same as in rest frame

$$\Sigma_{\rm IMF}^{\rm LNA} \;=\; \Sigma_{\rm IMF}^{(+-)\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

Self-energy

Light-front

$$\int dk^+ dk^- d^2k_\perp \frac{1}{D_\pi D_N} = \frac{1}{p^+}\int_{-\infty}^{\infty} \frac{dx}{x(x-1)} d^2k_\perp \int dk^- \left(k^- - \frac{k_\perp^2 + m_\pi^2}{xp^+} + \frac{i\varepsilon}{xp^+}\right)^{-1}$$
$$\times \left(k^- - \frac{M^2}{p^+} - \frac{k_\perp^2 + M^2}{(x-1)p^+} + \frac{i\varepsilon}{(x-1)p^+}\right)^{-1}$$

$$= 2\pi^2 i \int_0^1 dx\ dk_\perp^2\ \frac{1}{k_\perp^2 + (1-x)m_\pi^2 + x^2 M^2}$$

$x = k^+/p^+$

→ identical nonanalytic results as covariant & instant form

$$\Sigma_{\rm LF}^{\rm LNA} = -\frac{3g_A^2}{32\pi f_\pi^2}\left(m_\pi^3 + \frac{1}{2\pi}\frac{m_\pi^4}{M}\log m_\pi^2 + \mathcal{O}(m_\pi^5)\right)$$

*C.-R. Ji, WM, Thomas, PRD **80**, 054018 (2009)*

Self-energy

- **Light-front**

→ $1/D_N$ "tadpole" term has k^- pole that depends on k^+ and moves to infinity as $k^+ \to 0$
("treacherous" in LF dynamics)

→ use LF cylindrical coordinates $k^+ = r\cos\phi,\ k^- = r\sin\phi$

$$\int d^4k \frac{1}{D_N} = \frac{1}{2}\int d^2k_\perp \int \frac{dk^+}{k^+}\int dk^- \left(k^- - \frac{k_\perp^2 + M^2}{k^+} + \frac{i\varepsilon}{k^+}\right)^{-1}$$

$$= -2\pi \int d^2k_\perp \left[\int_0^{r_0} dr \frac{r}{\sqrt{r_0^4 - r^4}} + i \lim_{R\to\infty}\int_{r_0}^{R} dr \frac{r}{\sqrt{r^4 - r_0^4}}\right]$$

$$= \frac{1}{2}\int d^2k_\perp \lim_{R\to\infty}\left(-\pi^2 + 2\pi i\ \log\frac{r_0^2}{R^2} + \mathcal{O}(1/R^4)\right)$$

relevant also for $1/D_\pi$ term

$$r_0 = \sqrt{2(k_\perp^2 + M^2)}$$

contains $\log(k_\perp^2 + M^2)$ term as required

Self-energy

- Pseudoscalar interaction

$$\Sigma^{\mathrm{PS}} = i g_{\pi NN}^2 \; \overline{u}(p) \int \frac{d^4k}{(2\pi)^4} \, (\gamma_5 \vec{\tau}) \, \frac{i\,(\not{p} - \not{k} + M)}{D_N} (\gamma_5 \vec{\tau}) \frac{i}{D_\pi^2} \, u(p)$$

$$= -\frac{3 i g_A^2 M}{2 f_\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{m_\pi^2}{D_\pi D_N} + \frac{1}{D_N} - \frac{1}{D_\pi} \right]$$

→ contains additional ("treacherous") pion "tadpole" term

→ similar evaluation as for $1/D_N$ term

$$\Sigma_{\mathrm{LNA}}^{\mathrm{PS}} = \frac{3 g_A^2}{32\pi f_\pi^2} \left(\frac{M}{\pi} m_\pi^2 \log m_\pi^2 - m_\pi^3 - \frac{m_\pi^4}{2\pi M^2} \log \frac{m_\pi^2}{M^2} + \mathcal{O}(m_\pi^5) \right)$$

additional *lower order* term in PS theory!

Self-energy

- Alberg & Miller claim on light-front $\Sigma^{\rm PS} = \Sigma^{\rm PV}$
  – "form factor removes $k^+ = 0$ contribution" *PRL* **108**, *172001 (2012)*

- In practice, AM drop "treacherous" $k^+ = 0$ (end-point) term

$$\Sigma^{\rm PS} = \Sigma^{\rm PV} + \Sigma^{\rm PS}_{\rm end-pt}$$

  after which PS result happens to coincide with PV

→ but, *even with* form factors, end-point term is non-zero

$$\Sigma^{\rm PS}_{\rm end-pt} = \frac{3g_A^2 M}{16\pi^2 f_\pi^2} \int_0^\infty dt \frac{\sqrt{t}\, F^2(m_\pi^2, -t)}{\sqrt{t + m_\pi^2}} \quad \xrightarrow{\rm LNA} \quad \frac{3g_A^2}{32\pi f_\pi^2} \frac{M}{\pi} m_\pi^2 \log m_\pi^2$$

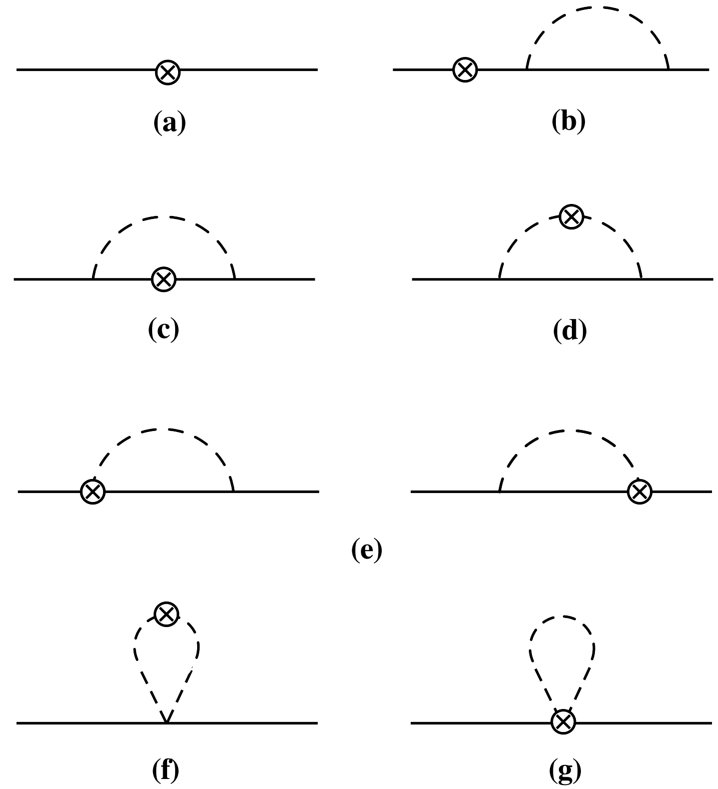
Ji, WM, Thomas, arXiv:1206.3671

→ *ansatz* does not work for other quantities
e.g. vertex renormalization

Vertex corrections

- Pion cloud corrections to electromagnetic N coupling

  → N rainbow **(c)**, π rainbow **(d)**, Kroll-Ruderman **(e)**, π tadpole **(f)**, N tadpole **(g)**



- Vertex renormalization

$$(Z_1^{-1} - 1)\, \bar{u}(p)\, \gamma^\mu\, u(p) = \bar{u}(p)\, \Lambda^\mu\, u(p)$$

→ taking "+" components: $Z_1^{-1} - 1 \approx 1 - Z_1 = \dfrac{M}{p^+}\, \bar{u}(p)\, \Lambda^+\, u(p)$

→ $e.g.$ for N rainbow contribution,

$$\Lambda_\mu^N = -\frac{\partial \hat{\Sigma}}{\partial p^\mu}$$

Vertex corrections

- Define light-cone momentum distributions $f_i(y)$

$$1 - Z_1^i = \int dy\, f_i(y)$$

for isovector ($p-n$) distribution

where
$$f_\pi(y) = 4f^{(\mathrm{on})}(y) + 4f^{(\delta)}(y)$$
$$f_N(y) = -f^{(\mathrm{on})}(y) - f^{(\mathrm{off})}(y) + f^{(\delta)}(y)$$
$$f_{\mathrm{KR}}(y) = 4f^{(\mathrm{off})}(y) - 8f^{(\delta)}(y)$$
$$f_{\pi(\mathrm{tad})}(y) = -f_{N(\mathrm{tad})}(y) \;=\; 2f^{(\mathrm{tad})}(y)$$

with components
$$f^{(\mathrm{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2}\int dk_\perp^2\, \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]^2}$$
$$f^{(\mathrm{off})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2}\int dk_\perp^2\, \frac{y}{k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2}$$
$$f^{(\delta)}(y) = \frac{g_A^2}{4(4\pi f_\pi)^2}\int dk_\perp^2\, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right)\delta(y)$$
$$f^{(\mathrm{tad})}(y) = -\frac{1}{(4\pi f_\pi)^2}\int dk_\perp^2\, \log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right)\delta(y)$$

Burkardt, Hendricks, Ji, WM, Thomas, PRD ***87****, 056009 (2013)*

- Pion distribution $f_\pi(y)$ contains *on-shell* contribution $f^{(\mathrm{on})}(y)$ equivalent to PS ("Sullivan") result

- Nucleon distribution $f_N(y)$ contains in addition new *off-shell* contribution $f^{(\mathrm{off})}(y)$

- Both contain singular $\delta(y)$ components $f^{(\delta)}(y)$, which are present only in PV theory

- Kroll-Ruderman term $f_{\mathrm{KR}}(y)$ needed for gauge invariance $(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{KR}})$

- Nucleon and pion tadpole terms equal & opposite

$$(1 - Z_1^{\pi\,(\mathrm{tad})}) + (1 - Z_1^{N\,(\mathrm{tad})}) = 0$$

Nonanalytic behavior of vertex renormalization factors

$$1 - Z_1^N \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \pi \frac{m_\pi^3}{M} - \frac{2m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^\pi \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ m_\pi^2 \log m_\pi^2 - \frac{5\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{\text{KR}} \xrightarrow{\text{NA}} \frac{3g_A^2}{4(4\pi f_\pi)^2} \left\{ + \frac{2\pi}{3} \frac{m_\pi^3}{M} - \frac{m_\pi^4}{3M^2} \log m_\pi^2 + \mathcal{O}(m_\pi^5) \right\}$$

$$1 - Z_1^{N\,(\text{tad})} \xrightarrow{\text{NA}} -\frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$1 - Z_1^{\pi\,(\text{tad})} \xrightarrow{\text{NA}} \frac{1}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

→ cancellation of $m_\pi^2 \log m_\pi^2$ terms in KR contribution

→ demonstration of gauge invariance condition
(in fact, to *all* orders!)

Nonanalytic behavior of vertex renormalization factors

	$1/D_\pi D_N^2$	$1/D_\pi^2 D_N$	$1/D_\pi D_N$	$1/D_\pi$ or $1/D_\pi^2$	sum (PV)	sum (PS)
$1 - Z_1^N$	g_A^2 *	0	$-\frac{1}{2}g_A^2$	$\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^\pi$	0	g_A^2 *	0	$-\frac{1}{4}g_A^2$	$\frac{3}{4}g_A^2$	g_A^2
$1 - Z_1^{\rm KR}$	0	0	$-\frac{1}{2}g_A^2$	$\frac{1}{2}g_A^2$	0	0
$1 - Z_1^{N\,\rm tad}$	0	0	0	$-1/2$	$-1/2$	0
$1 - Z_1^{\pi\,\rm tad}$	0	0	0	$1/2$	$1/2$	0

* also in PS

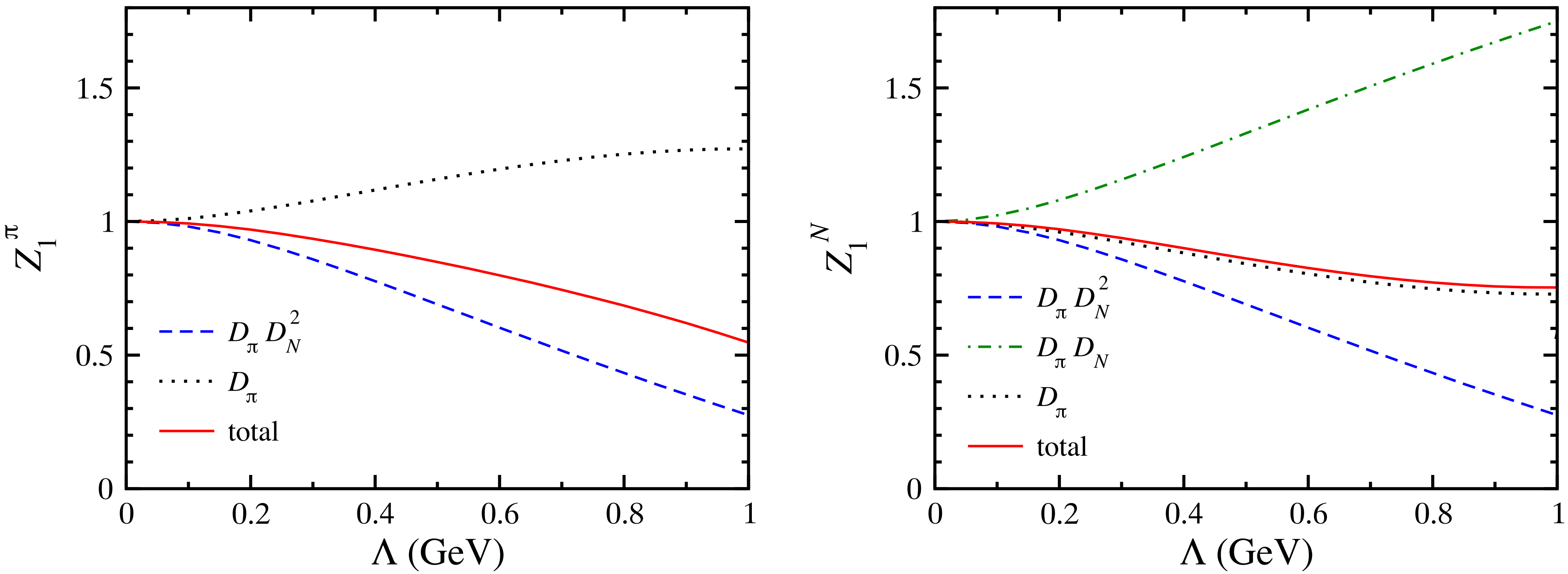
in units of $\dfrac{1}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$

→ origin of ChPT *vs*. Sullivan process difference clear!

$$\left(1 - Z_1^{N\,({\rm PV})}\right)_{\rm LNA} = \frac{3}{4}\left(1 - Z_1^{N\,({\rm PS})}\right)_{\rm LNA}$$

→ no problem with application of light-front to pion loops in DIS (if implemented correctly)

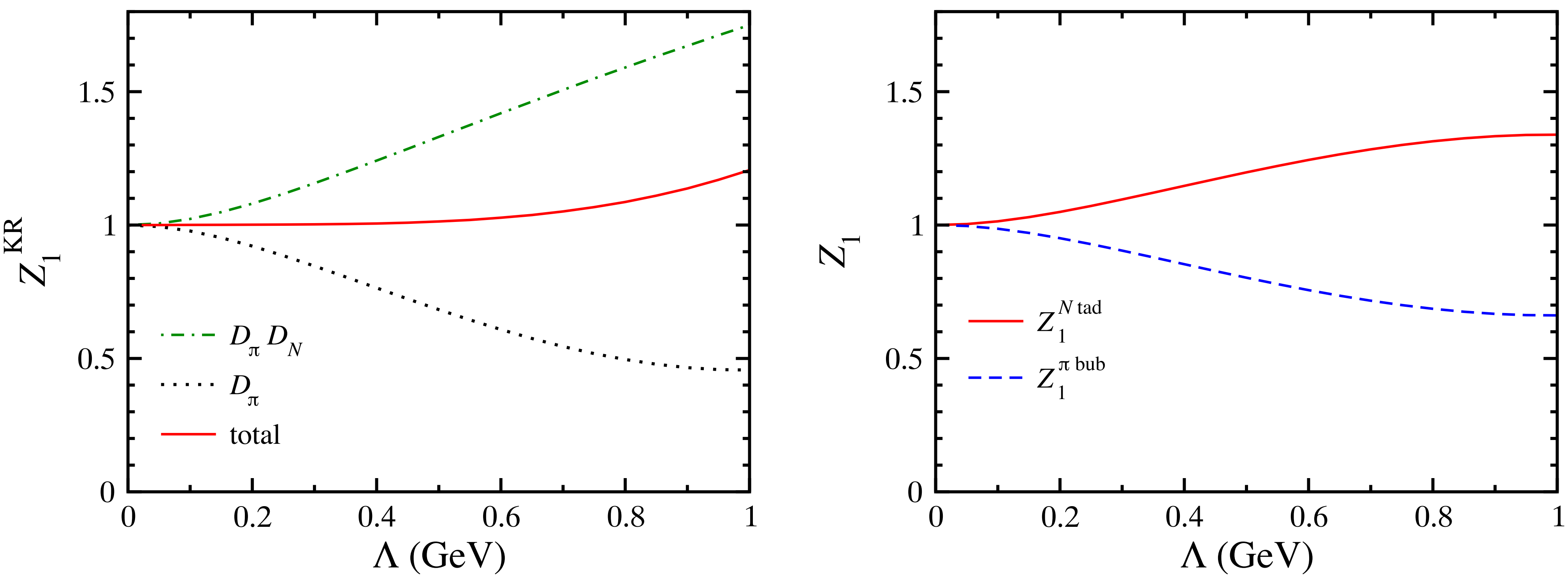
Pion & nucleon rainbow contributions



→ δ-function part reduces on-shell pion contribution

→ almost complete cancellation between on-shell & off-shell parts of nucleon contribution

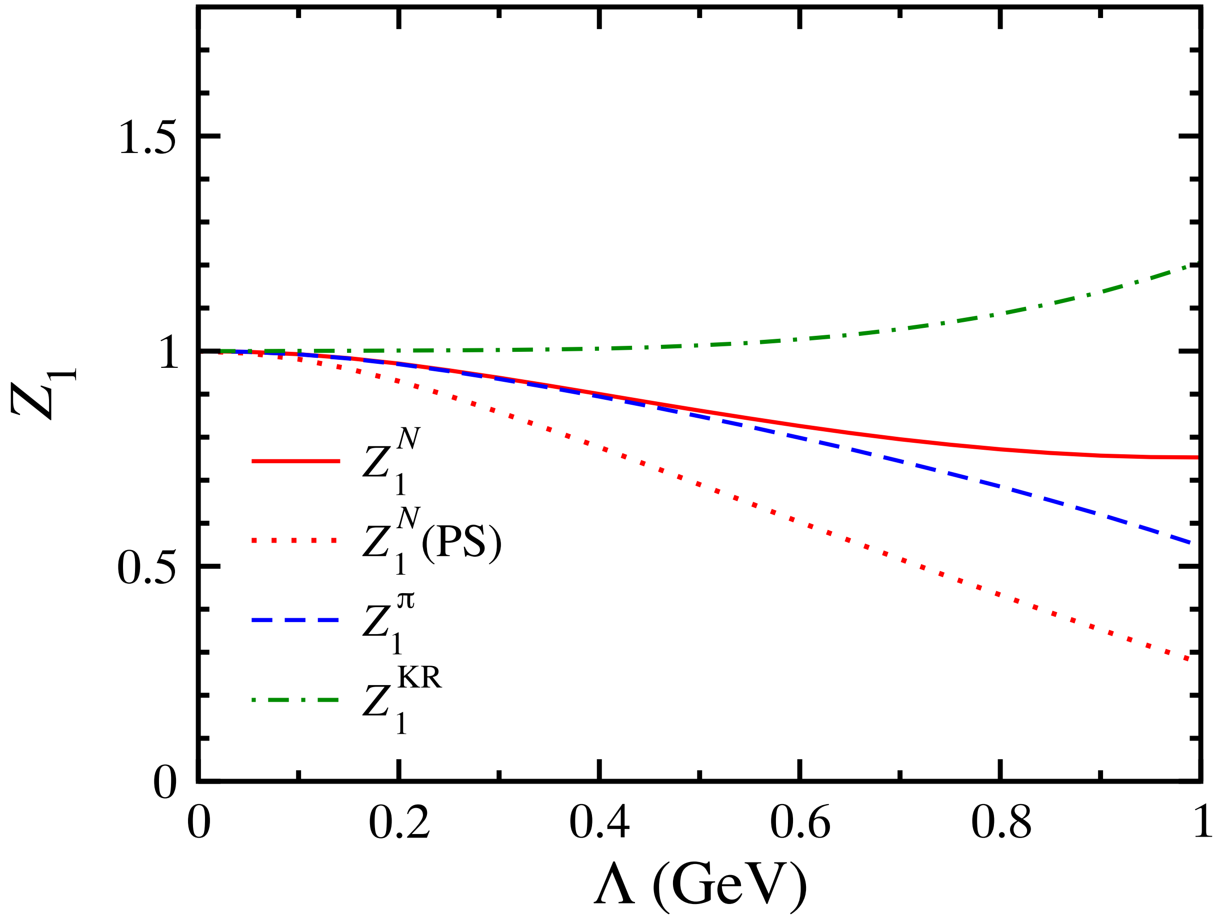
Kroll-Ruderman & tadpoles



→ strong cancellation between off-shell & δ-function parts of KR

→ pion & nucleon tadpoles cancel exactly

Comparison of all contributions to vertex renormalization



→ important differences between PV & PS results (from off-shell & δ-function contributions)

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\mathrm{KR}})$$

Moments of PDFs

- PDF moments related to nucleon matrix elements of local twist-2 operators

$$\langle N | \widehat{\mathcal{O}}_q^{\mu_1 \cdots \mu_n} | N \rangle \; = \; 2 \, \langle x^{n-1} \rangle_q \, p^{\{\mu_1} \cdots p^{\mu_n\}}$$

$\longrightarrow$ n-th moment of (spin-averaged) PDF $q(x)$

$$\langle x^{n-1} \rangle_q \; = \; \int_0^1 dx \, x^{n-1} \left(q(x) + (-1)^n \bar{q}(x) \right)$$

$\longrightarrow$ operator is $\widehat{\mathcal{O}}_q^{\mu_1 \cdots \mu_n} \; = \; \bar{\psi} \gamma^{\{\mu_1} \, iD^{\mu_2} \cdots iD^{\mu_n\}} \psi \; - \; \mathrm{traces}$

- Lowest (n=1) moment $\langle x^0 \rangle_q \equiv \mathcal{M}_N + \mathcal{M}_\pi$ given by vertex renormalization factors $\sim 1 - Z_1^i$

Moments of PDFs

- For couplings involving nucleons

$$\mathcal{M}_N^{(p)} = Z_2 + (1 - Z_1^N) + (1 - Z_1^{N\,(\mathrm{tad})})$$
$$\mathcal{M}_N^{(n)} = 2(1 - Z_1^N) - (1 - Z_1^{N\,(\mathrm{tad})})$$

$\longrightarrow$ wave function renormalization

$$1 - Z_2 = (1 - Z_1^p) + (1 - Z_1^n) \equiv 3(1 - Z_1^N)$$

- For couplings involving only pions

$$\mathcal{M}_\pi^{(p)} = 2(1 - Z_1^\pi) + 2(1 - Z_1^{\mathrm{WT}}) + (1 - Z_1^{\pi\,(\mathrm{tad})})$$
$$\mathcal{M}_\pi^{(n)} = -2(1 - Z_1^\pi) - 2(1 - Z_1^{\mathrm{WT}}) - (1 - Z_1^{\pi\,(\mathrm{tad})})$$

Nonanalytic behavior

$$\mathcal{M}_N^{(p)} \xrightarrow{\text{LNA}} 1 - \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(p)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_N^{(n)} \xrightarrow{\text{LNA}} \frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2 \qquad \mathcal{M}_\pi^{(n)} \xrightarrow{\text{LNA}} -\frac{(3g_A^2+1)}{2(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$\longrightarrow$ no pion corrections to isosclar moments

$\longrightarrow$ isovector correction agrees with ChPT calculation

$$\mathcal{M}_N^{(p-n)} \xrightarrow{\text{LNA}} 1 - \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

$$\mathcal{M}_\pi^{(p-n)} \xrightarrow{\text{LNA}} \frac{\left(4g_A^2 + [1-g_A^2]\right)}{(4\pi f_\pi)^2}\, m_\pi^2 \log m_\pi^2$$

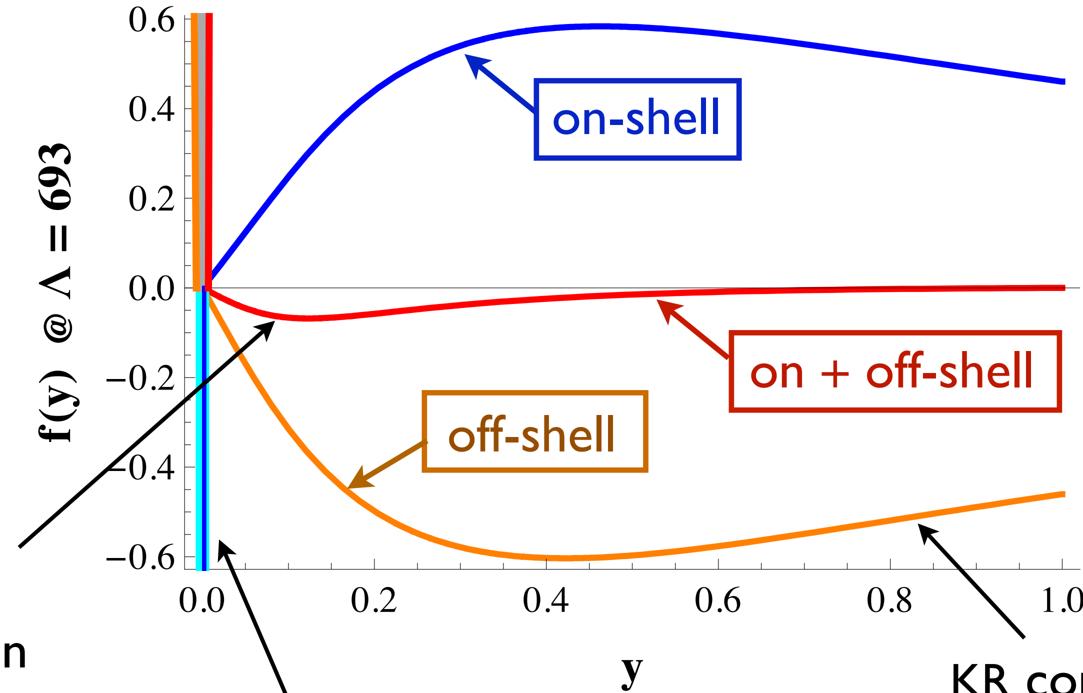
PS (“on-shell”) contribution

δ-function contribution

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

$\longrightarrow$ for transverse momentum cut-off $F(k_\perp) = \Theta(k_\perp^2 - \Lambda^2)$



large cancelations of on- and off-shell terms in N rainbow distribution

δ-function contributions

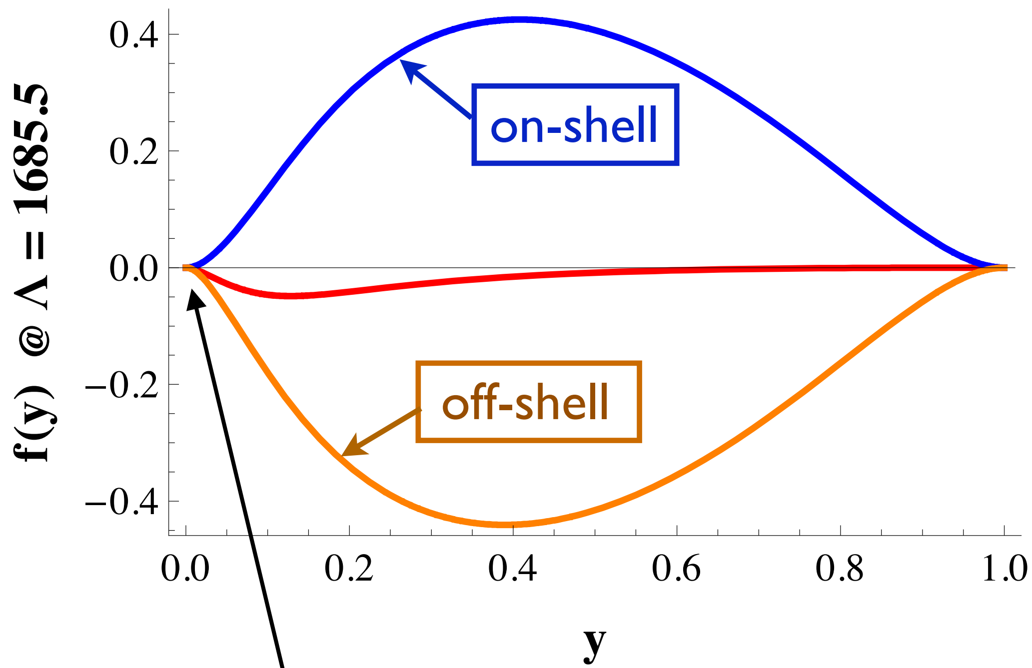
KR contribution < 0 because of off-shell term

Hendricks, Ji, WM, Thomas (2013)

Pion distribution functions

- Using phenomenological form factors, compute functions $f_i(y)$ numerically

→ s-dependent (dipole) form factor $s_{\pi N} = \dfrac{k_\perp^2 + m_\pi^2}{y} + \dfrac{k_\perp^2 + M^2}{1-y}$



suppresses contributions
at $y = 0$ and $y = 1$
– no tadpoles!

Hendricks, Ji, WM, Thomas (2013)

Pion distribution functions

- Alternatively, avoid form factors by using dimensional regularization to compute distribution functions

  $\longrightarrow$ scheme & scale dependence explicit

- Performing $k_\perp$ integration in $d = 2 - 2\epsilon$ dimensions,

$$f^{(\text{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \left\{ \frac{y^3 M^2}{y^2 M^2 + (1-y) m_\pi^2} + y \left(\frac{1}{\epsilon} + \Gamma'(1) - 1 \right) \right.$$
$$\left. - y \ln \left[\frac{y^2 M^2 + (1-y) m_\pi^2}{4\pi \mu^2} \right] \right\}$$

$\longrightarrow$ in $\widetilde{\text{MS}}$ scheme absorb term $1/\epsilon + \Gamma'(1) - 1 + \ln 4\pi$ into (infinite) counter-term

$\longrightarrow$ scale μ set to $\sim M$

Ji, WM et al.
in preparation (2013)

Pion distribution functions

- Renormalized distribution functions in $\widetilde{\mathrm{MS}}$ scheme

$$f^{(\mathrm{on})}_{\widetilde{\mathrm{MS}}}(y) = -\frac{g_A^2 M^2 y}{(4\pi f_\pi)^2}\left\{1 + \frac{(1-y)m_\pi^2}{y^2M^2+(1-y)m_\pi^2} + \ln\left[\frac{y^2M^2+(1-y)m_\pi^2}{\mu^2}\right]\right\}$$

$$f^{(\mathrm{off})}_{\widetilde{\mathrm{MS}}}(y) = \frac{g_A^2 M^2 y}{(4\pi f_\pi)^2}\left\{1 + \ln\left[\frac{y^2M^2+(1-y)m_\pi^2}{\mu^2}\right]\right\}$$

$$f^{(\delta)}_{\widetilde{\mathrm{MS}}}(y) = -\frac{g_A^2}{4(4\pi f_\pi)^2}\delta(y)\ \ln\frac{m_\pi^2}{\mu^2}$$

$\longrightarrow$ compute "model-independently" in terms of single parameter μ !

$\longrightarrow$ numerical analysis, relating to flavor asymmetry, in progress...

Ji, WM et al.
in preparation (2013)

Summary

- Equivalence demonstrated between self-energy in equal-time, covariant, and light-front formalisms

$$\Sigma^{\rm LNA}_{\rm cov} = \Sigma^{(+-)\rm LNA}_{\rm ET} + \Sigma^{(-+)\rm LNA}_{\rm ET} = \Sigma^{(+-)\rm LNA}_{\rm IMF} = \Sigma^{\rm LNA}_{\rm LF}$$

  $\longrightarrow$ non-trivial due to end-point singularities

  $\longrightarrow$ PV and PS results clearly differ

- Vertex corrections computed to all orders in m_π in relativistic framework

$$(1 - Z_1^N) = (1 - Z_1^\pi) + (1 - Z_1^{\rm KR})$$

  $\longrightarrow$ difference between PDF moments in ChPT (PV) $\&$ "Sullivan" process (PS)

  $\longrightarrow$ model-independent constraints on LC distributions $f_i(y)$

  $\longrightarrow$ impact on $\bar{d} - \bar{u}$ data analysis in progress

Exploring Hadron Structure with Tagged Structure Functions

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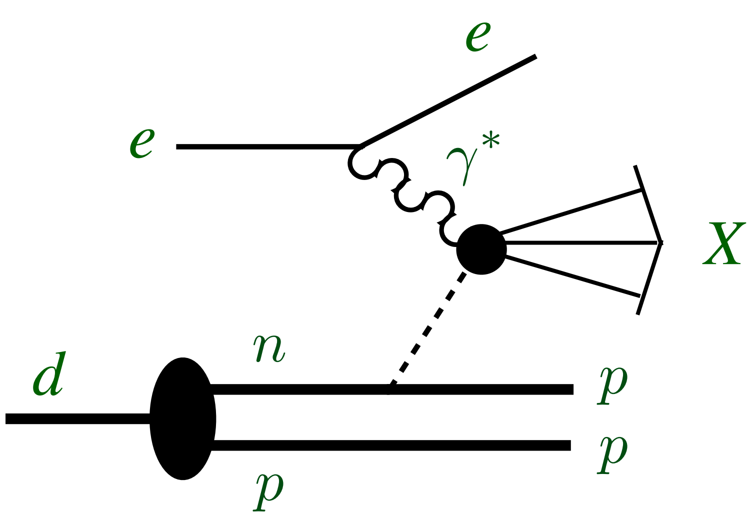

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EHS-TSF

Exploring Hadron Structure with Tagged Structure Functions
January 16-18, 2014
Thomas Jefferson National Accelerator Facility
Newport News, VA

Circular

New experimental techniques are being developed to create effective neutron, pion, and other tagged targets from nucleon and nuclear targets and beams. The effective meson targets in particular open the opportunity to uniquely probe the structure and composition of the nucleon sea. The goal of the Workshop is to first review the status of our current understanding of the mesonic and sea components of nucleons, followed by discussions of key and potential experiments at various existing and future facilities, and finally to address outstanding unresolved theoretical issues in this subject.

The success and open issues of various theoretical approaches, including those based on lattice QCD, light-front methods, chiral symmetry, nonperturbative models and perturbative QCD, will be discussed at this Workshop. The meeting will also aim to identify new experiments which could effectively test these approaches, using in particular novel tagged beam and target configurations, in addition to other techniques at existing or future facilities.

Organizing Committee

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