N* Resonances and Duality in Deep-Inelastic Scattering

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Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

\[ \sum_{\text{hadrons}} = \sum_{\text{quarks}} \]

→ can use either set of *complete* basis states to describe physical phenomena
In practice, at finite energy typically have access only to **limited** set of basis states.
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Question is not why duality exists, but how it arises where it exists, and how we can make use of it.
Duality in hadron-hadron scattering

\[ p_{\text{Lab}} \Delta \sigma \quad (\text{mb GeV}) \]

\[ \begin{array}{c}
\sum_j \sigma^\pi^+ p - \sigma^\pi^- p \\
R(s) = \sum_j \alpha_j(t)
\end{array} \]

Igi (1962)
Dolen, Horn, Schmidt (1968)

“s-t channel duality”
Duality in electron-nucleon scattering

“Bloom-Gilman duality”

\[ \frac{2M}{Q^2} \int_0^{\nu_m} d\nu \, \nu W_2(\nu, Q^2) = \int_{\omega'_m}^{\omega'_1} d\omega' \, \nu W_2(\omega') \]

“hadrons”

“quarks”

finite-energy sum rules

Bloom, Gilman
PRL 85, 1185 (1970)

\[ \omega' = \frac{1}{x} + \frac{M^2}{Q^2} \]
Duality in electron-nucleon scattering

average over
(strongly $Q^2$ dependent)
resonances
$\approx Q^2$ independent
scaling function

“Nachtmann” scaling variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

Niculescu et al., PRL 85, 1182 (2000)
Duality in electron-nucleon scattering

→ also exists locally in individual resonance regions
Duality in electron-nucleon scattering

- **In deep-inelastic region** \((W \gtrsim 2 \text{ GeV}, \, Q^2 \gtrsim 1 \text{ GeV}^2)\) 
  structure function given by parton distributions 
  \[
  F_2(x, Q^2) = x \sum_q e_q^2 q(x, Q^2)
  \]

- **In resonance region** \((W \lesssim 2 \text{ GeV}), \text{ or at low } Q^2 (Q^2 \lesssim 1 \text{ GeV}^2)\) 
  can no longer resolve individual quark structure

- Resonance and DIS regions intimately connected
  \[\rightarrow\] resonances an *integral* part of scaling structure function
  *e.g.* in large-\(N_c\) limit, spectrum of zero-width resonances is “maximally dual” to quark-level (smooth) structure function
How to build up a scaling structure function from $\gamma^*NN^*$ transitions?

Earliest attempts predate QCD

- e.g. harmonic oscillator spectrum $M_n^2 = (n + 1)\Lambda^2$
  including states with spin $= 1/2, \ldots, n+1/2$
  ($n$ even: $I = 1/2$, $n$ odd: $I = 3/2$)

  $M_n^2 = (n + 1)\Lambda^2$

- at large $Q^2$ magnetic coupling dominates

  $G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2/M_n^2)^2}$

  $r^2 \approx 1.41$

- in Bjorken limit, $\sum_n \rightarrow \int dz$, $z \equiv M_n^2/Q^2$

  $F_2 \sim (\omega' - 1)^{1/2}(\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$

- scaling function of $\omega' = \omega + M^2/Q^2$ ($\omega = 1/x$)
How to build up a scaling structure function from $\gamma^*NN^*$ transitions?

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- e.g. harmonic oscillator spectrum \( M_n^2 = (n + 1)\Lambda^2 \)
  including states with spin = 1/2, ..., \( n+1/2 \)
  (\( n \) even: \( I = 1/2 \), \( n \) odd: \( I = 3/2 \))

- in \( \Gamma_n \to 0 \) limit
  \[ F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2) \frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4} \]

  cf. Drell-Yan-West relation
  \[ G(Q^2) \sim \left( \frac{1}{Q^2} \right)^m \iff F_2(x) \sim (1 - x)^{2m-1} \]

- similar behavior found in many models
  
  - Einhorn, PRD 14, 3451 (1976) ('t Hooft model)
  - Greenberg, PRD 47, 331 (1993) (NR scalar quarks in HO potential)
  - Pace, Salme, Lev, PRC 57, 2655 (1995) (relativistic HO with spin)
  - Isgur et al., PRD 64, 054005 (2001) (transition to scaling)
How to build up a scaling structure function from $\gamma^*NN^*$ transitions?

More recent phenomenological analyses at finite $Q^2$

- additional constraints from threshold behavior at $q \to 0$
- and asymptotic behavior at $Q^2 \to \infty$

Davidovsky, Struminsky, 

\[
\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M\nu \left[ |G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2 \right] \delta(W^2 - M_R^2)
\]

- 21 isospin-1/2 & 3/2 resonances (with mass < 2 GeV)

\[
|G_+^R(Q^2)|^2 = |G_+^R(0)|^2 \left( \frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2} \right)^{\gamma_1} \left( \frac{\Lambda^2}{Q^2 + \Lambda^2} \right)^{m_+} \quad m_{+,0,-} = 3, 4, 5
\]

\[
|G_0^R(Q^2)|^2 = C^2 \left( \frac{Q^2}{Q^2 + \Lambda''^2} \right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left( \frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2} \right)^{\gamma_2} \left( \frac{\Lambda^2}{Q^2 + \Lambda^2} \right)^{m_0}
\]

- in $x \to 1$ limit,

\[
F_2(x) \sim (1 - x)^{m_+}
\]
How to build up a scaling structure function from $\gamma^* NN^*$ transitions?

More recent phenomenological analyses at finite $Q^2$

Valence-like structure of dual function suggests “two-component duality”:

- **Valence** (Reggeon exchange) dual to **resonances** $F_2^{(\text{val})} \sim x^{0.5}$
- **Sea** (Pomeron exchange) dual to **background** $F_2^{(\text{sea})} \sim x^{-0.08}$


Duality and QCD

Operator product expansion

expand moments of structure functions
in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific “twist” $\tau$

$\tau = \text{dimension} - \text{spin}$

$x \to 1 \iff W \to M$

$\tau = 2$

$\tau > 2$
Duality and QCD

Operator product expansion

\[ M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) \]

\[ = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

de Rujula, Georgi, Politzer
Ann. Phys. 103, 315 (1975)

If moment \( \approx \) independent of \( Q^2 \)

\[ \text{“higher twist” terms } A_n^{(\tau > 2)} \text{ small} \]

Duality \( \longleftrightarrow \) suppression of higher twists
Truncated moments of $F_2^p$ in resonance region

higher twists < 10–15% for $Q^2 > 1$ GeV$^2$

Malace et al., PRC 80, 035207 (2009)
[JLab Hall C]
On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
→ nontrivial interference between resonances

Total “higher twist” is small at scales $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$

Can we understand this dynamically, at quark level?
→ is duality an accident?

Can we use resonance region data to learn about leading twist structure functions (and vice versa)?
→ expanded data set has potentially significant implications for global quark distribution studies
Consider simple quark model with spin-flavor symmetric wave function

**low energy**

\[ d\sigma \sim \left( \sum_i e_i \right)^2 \]

**high energy**

\[ d\sigma \sim \sum_i e_i^2 \]

For duality to work, these must be equal

\[ \text{how can } \textit{square of a sum} \text{ become } \textit{sum of squares?} \]
Dynamical cancellations

- For toy model of two quarks bound in a harmonic oscillator potential, structure function given by
  \[ F(\nu, q^2) \sim \sum_n |G_{0,n}(q^2)|^2 \delta(E_n - E_0 - \nu) \]

- Charge operator \( \sum_i e_i \exp(iq \cdot r_i) \) excites even partial waves with strength \( \propto (e_1 + e_2)^2 \)
  odd partial waves with strength \( \propto (e_1 - e_2)^2 \)

- Resulting structure function
  \[ F(\nu, q^2) \sim \sum_n \{(e_1 + e_2)^2 G_{0,2n}^2 + (e_1 - e_2)^2 G_{0,2n+1}^2\} \]

- If states degenerate, cross terms \( \sim e_1 e_2 \) cancel when averaged over nearby even and odd parity states

Close, Isgur, PLB 509, 81 (2001)
Dynamical cancellations

- duality is realized by summing over at least one complete set of even and odd parity resonances

Close, Isgur, PLB 509, 81 (2001)

- in NR Quark Model, even & odd parity states generalize to 56 (L=0) and 70 (L=1) multiplets of spin-flavor SU(6)

<table>
<thead>
<tr>
<th>representation</th>
<th>$^28[56^+]$</th>
<th>$^410[56^+]$</th>
<th>$^28[70^-]$</th>
<th>$^48[70^-]$</th>
<th>$^210[70^-]$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9 \rho^2$</td>
<td>$8 \lambda^2$</td>
<td>$9 \rho^2$</td>
<td>0</td>
<td>$\lambda^2$</td>
<td>$18 \rho^2 + 9 \lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3 \rho + \lambda)^2/4$</td>
<td>$8 \lambda^2$</td>
<td>$(3 \rho - \lambda)^2/4$</td>
<td>$4 \lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9 \rho^2 + 27 \lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda \ (\rho) =$ (anti) symmetric component of ground state wave function

PRC 79, 055202 (2009)
Accidental cancellations of charges?

**cat’s ears diagram** \((4\text{-fermion higher twist} \sim 1/Q^2)\)

\[
\propto \sum_{i \neq j} e_i e_j \sim \left( \sum_i e_i \right)^2 - \sum_i e_i^2
\]

\[\text{coherent} \quad \text{incoherent}\]

**proton**

\[\text{HT} \sim 1 - \left( 2 \times \frac{4}{9} + \frac{1}{9} \right) = 0 !\]

**neutron**

\[\text{HT} \sim 0 - \left( \frac{4}{9} + 2 \times \frac{1}{9} \right) \neq 0\]

\(S. \ Brodsky \ (2000)\)

→ **duality in proton a coincidence!**

→ **should not** hold for neutron !!
Neutron: the smoking gun

- Duality in *neutron* more difficult to test because of absence of free neutron targets

- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of $F^{n}_{2}$ in resonance region & test of neutron duality

*Malace, Kahn, WM, Keppel*  
Neutron: the smoking gun

→ “theory”: fit to $W > 2$ GeV data
  
  Alekhin et al., 0908.2762 [hep-ph]

→ locally, violations of duality in resonance regions < 15–20% (largest in $\Delta$ region)

→ globally, violations < 10%

Malace, Kahn, WM, Keppel
PRL 104, 102001 (2010)

duality is **not accidental**, but a general feature of resonance–scaling transition!
Neutron: the smoking gun

→ “theory”: fit to $W > 2$ GeV data
  *Alekhin et al., 0908.2762 [hep-ph]*

→ *locally*, violations of duality in resonance regions $< 15$–$20\%$ (largest in $\Delta$ region)

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*Malace, Kahn, WM, Keppel
PRL 104, 102001 (2010)*

→ analysis using recent (model-independent) BoNuS data in progress
Neutron: the smoking gun

“theory”: fit to $W > 2$ GeV data
Malace et al., 0908.2762 [hep-ph]

locally, violations of duality in resonance regions < 15–20%
(largest in $\Delta$ region)

globally, violations < 10%

use resonance region data to learn about leading twist structure functions?
CTEQ-JLab (CJ) global PDF analysis *

- New global NLO analysis of expanded set of $p$ and $d$ data ($\text{DIS}, pp, pd$) including large-$x$, low-$Q^2$ region

- Systematically study effects of $Q^2$ & $W$ cuts
  - down to $Q \sim m_c$ and $W \sim 1.7$ GeV

| cut0: $Q^2 > 4$ GeV$^2$, $W^2 > 12.25$ GeV$^2$ |
| cut1: $Q^2 > 3$ GeV$^2$, $W^2 > 8$ GeV$^2$ |
| cut2: $Q^2 > 2$ GeV$^2$, $W^2 > 4$ GeV$^2$ |
| cut3: $Q^2 > m_c^2$, $W^2 > 3$ GeV$^2$ |

Owens, Accardi WM
arXiv:1212.1702 (PRD, in print)

*CJ collaboration: http://www.jlab.org/CJ*
Larger database with weaker cuts leads to significantly reduced errors, especially at large $x$

→ up to 40–60% error reduction when cuts extended into resonance region

Accardi et al.  
PRD 81, 034016 (2010)
Vital for large-$x$ analysis, which currently suffers from large uncertainties (mostly due to nuclear corrections)

uncertainty in $d$ feeds into larger uncertainty in $g$ at high $x$ (important for LHC physics!)

Accardi et al., PRD 84, 014008 (2011)
Large Hadron Collider (CERN)

$pp$ collisions at $\sqrt{s} = 7$ TeV
Observation of new physics signals requires accurate
determination of QCD backgrounds — depend on PDFs!
(since $x_{1,2} \sim M_{Z',W'}$, large-$x$ uncertainties scale with mass!)

- for $W'$ production

-dominated by $d \ast \bar{u}$

-dominated by $d \ast u + u \ast d$

> 100% uncertainties at large $y$!

Brady et al., JHEP 1206, 019 (2012)
Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction

\[ \sum_{N_1^*, N_2^*} \gamma^* N \rightarrow N_1^* \rightarrow N_1^* N_2^* \rightarrow N_2^* M = \sum_{q, X} q N \rightarrow q X \rightarrow X M \]

\( s \)-channel resonance excitation and decay

parton level scattering and fragmentation

\[
\sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \rightarrow N_1^*} (Q^2, M_1^*) \mathcal{D}_{N_1^* \rightarrow N_2^* M} (M_1^*, M_2^*) \right|^2 = \sum_q e_q^2 q(x, Q^2) D_q^M (z, Q^2)
\]
Duality in exclusive reactions

**Exclusive–inclusive correspondence principle:**

- continuity of dynamics from one (known) region to another (poorly known)

\[
\int_{p_{\text{max}} - M_X^2/4p_{\text{max}}}^{p_{\text{max}}} dp \left. E \frac{d^3 \sigma}{dp^3} \right|_{\text{incl}} \sim \sum_{\text{res}} \left. E \frac{d\sigma}{dp_T^2} \right|_{\text{excl}}
\]

\[
\gamma^* N \rightarrow M \ X \quad \gamma^* N \rightarrow M \ N^*
\]

- resonance contribution to \( d\sigma \) should be comparable to the continuum contribution extrapolated from high energy

\[
\frac{E \ d^3 \sigma}{\sigma \ d p^3} \equiv f(x, p_T^2, sQ^2) \quad \rightarrow \quad f(x, p_T^2, sQ^2) \xrightarrow{s \rightarrow \infty} f(x, p_T^2)
\]

*Bjorken, Kogut, PRD 8, 1341 (1973)*
Conclusion

- **Confirmation of duality** (experimentally & theoretically) suggests an origin in dynamical cancelations between resonances
  - explore more realistic descriptions based on phenomenological $\gamma^* NN^*$ form factors
  - incorporate *nonresonant* background in same framework

- **Practical application of duality**
  - use resonance region data to constrain PDFs at high $x$
    (better knowledge of resonances could be relevant for LHC !)

- **Extend quark-hadron duality concept to electroproduction**
  - application to semi-inclusive DIS, DVCS / GPDs, ...
Gracias!