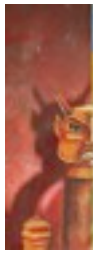
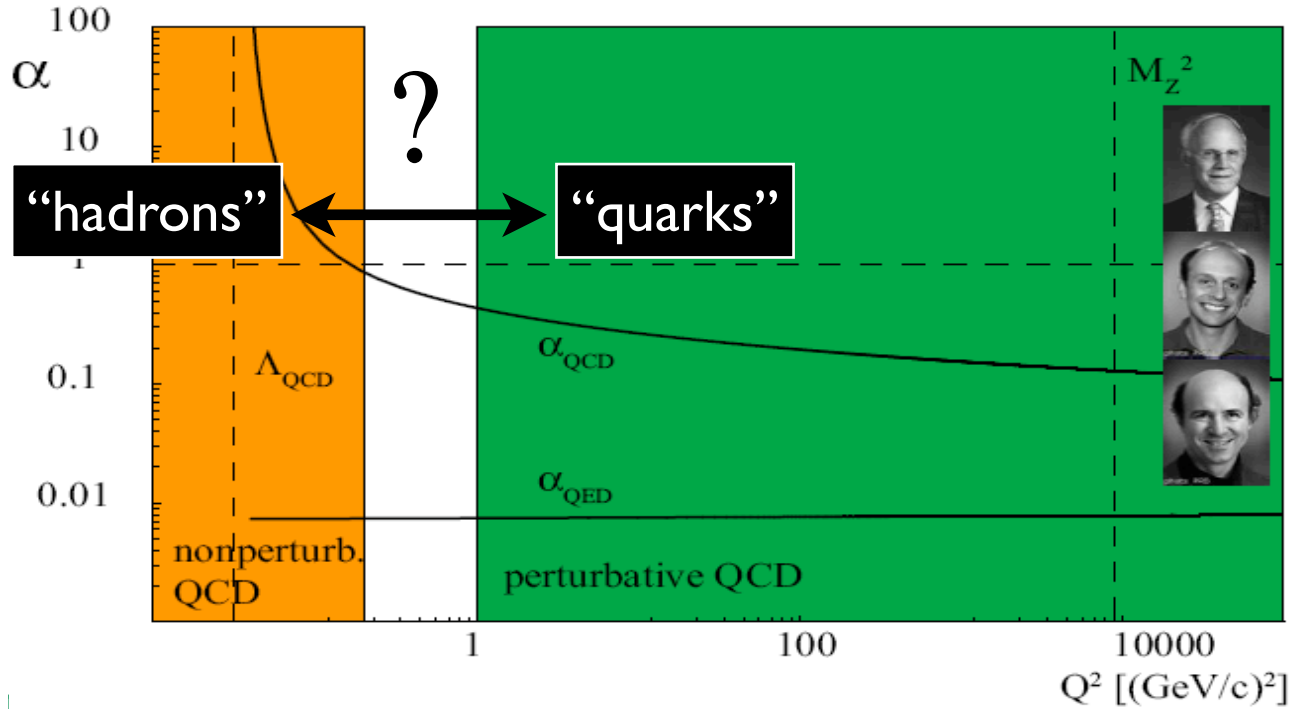
N 2013 Conference*
Peñiscola, Spain, May 27, 2013

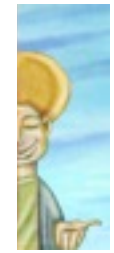
*N** Resonances and Duality in Deep-Inelastic Scattering

Wally Melnitchouk

Jefferson Lab







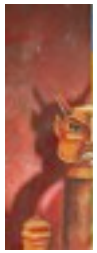
low energy *long distance*

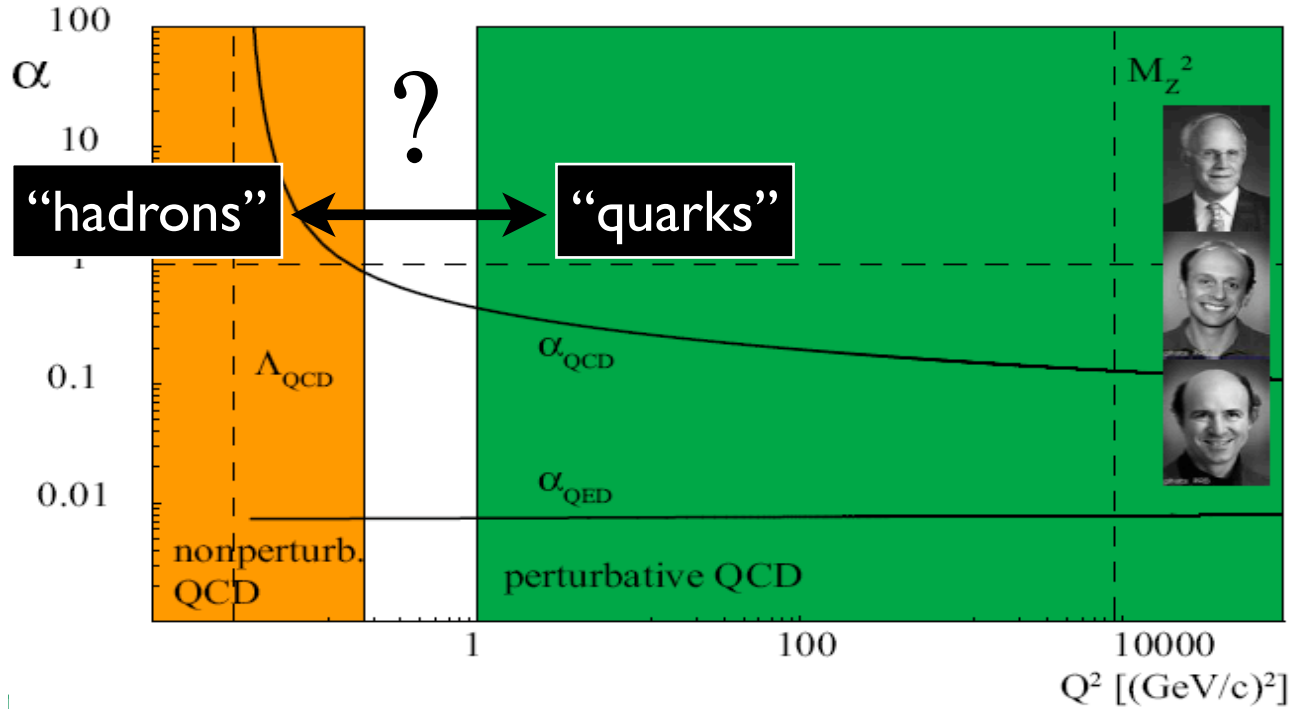
high energy *short distance*

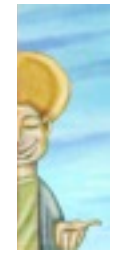
- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

→ can use either set of *complete* basis states to describe physical phenomena







low energy *long distance*

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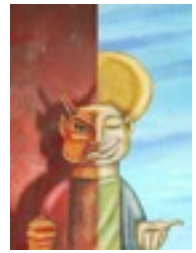
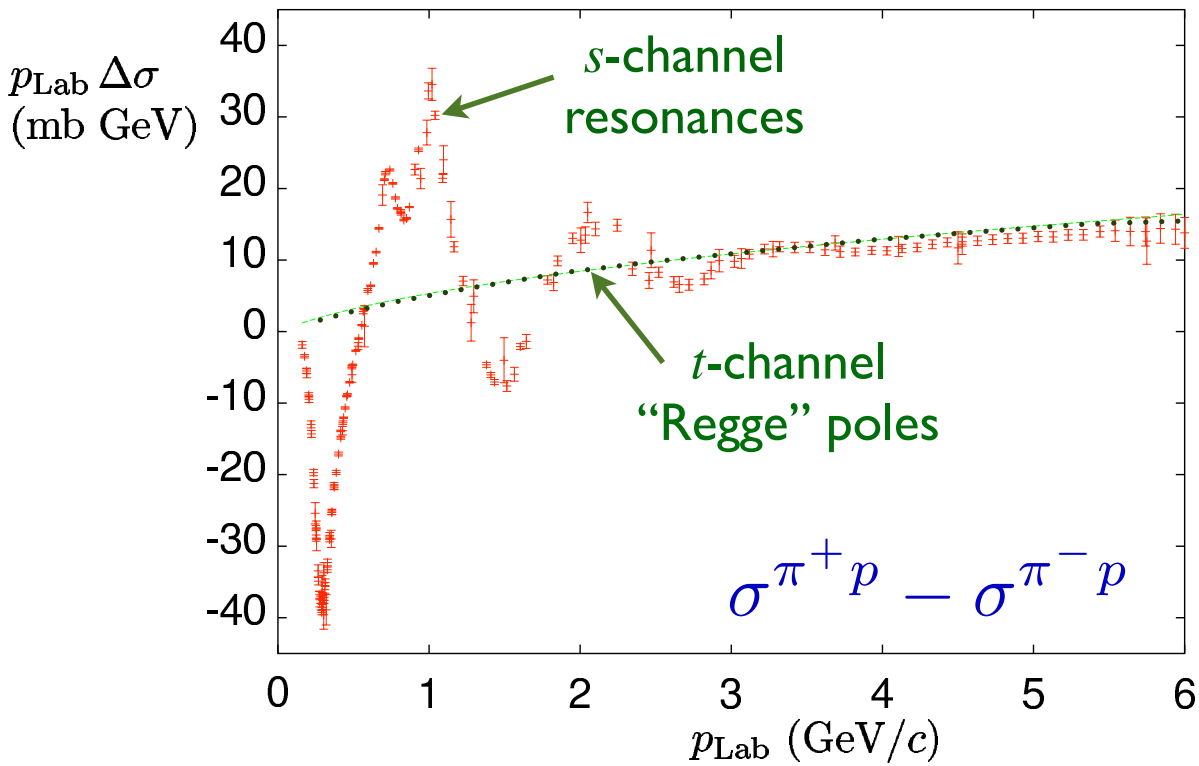
- In practice, at finite energy typically have access only to *limited* set of basis states



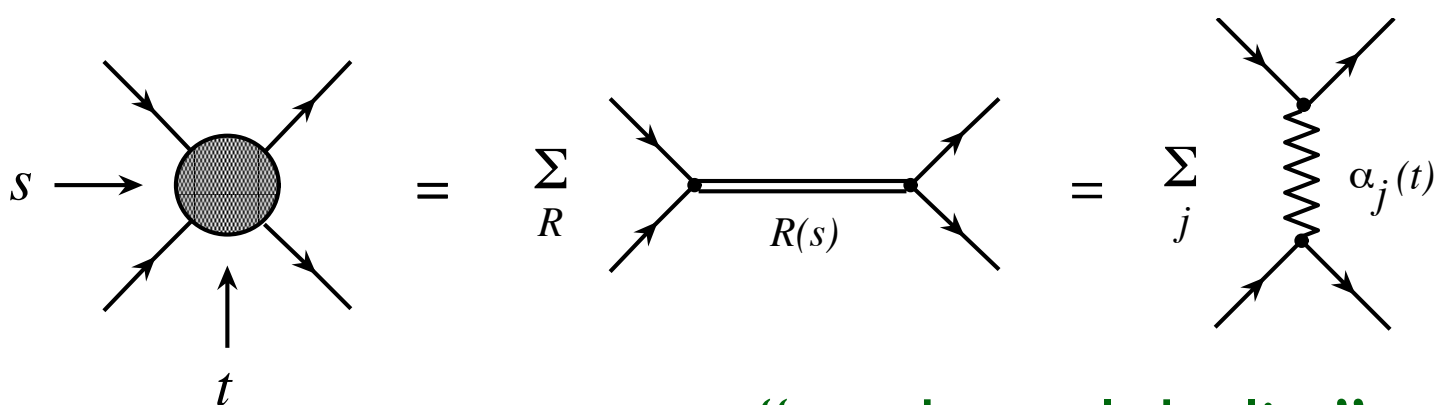
- In practice, at finite energy typically have access only to *limited* set of basis states

- Question is not *why* duality exists, but *how* it arises where it exists, and how we can *make use of it*

Duality in hadron-hadron scattering



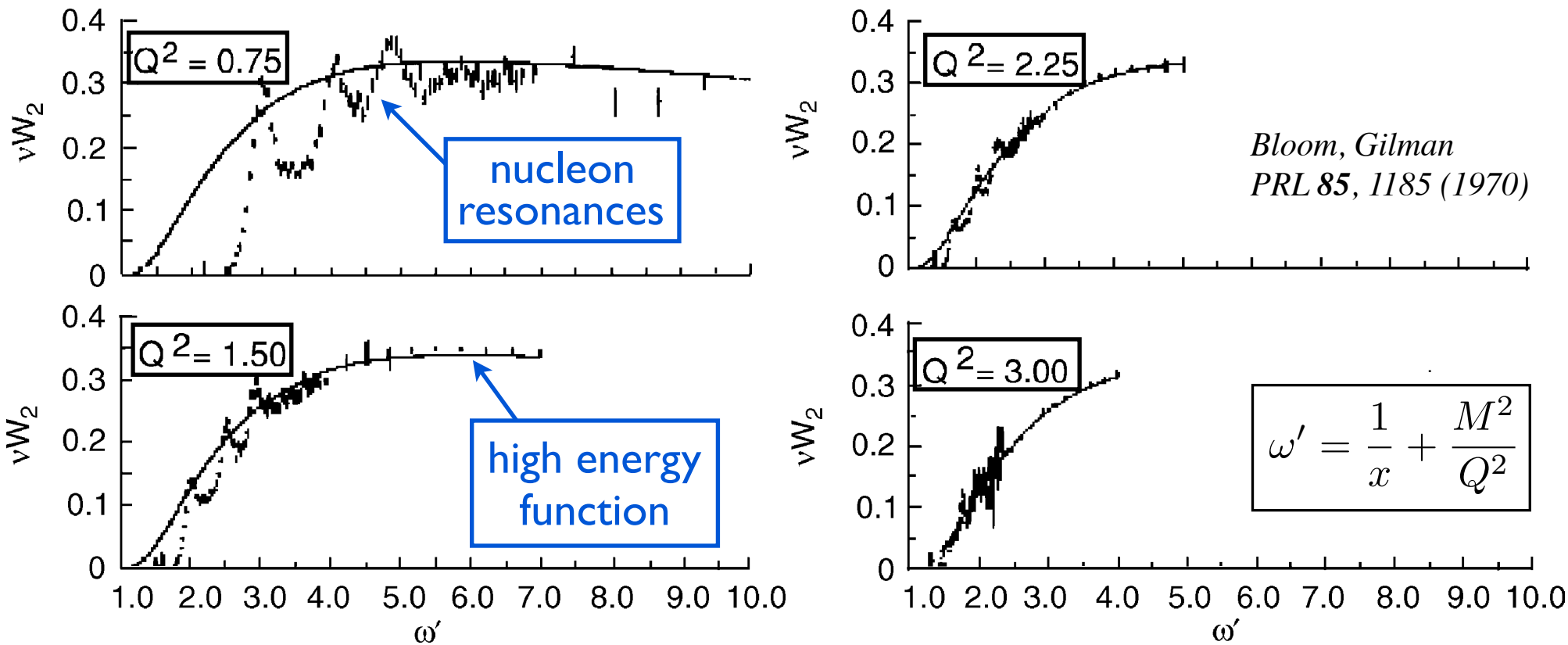
Igi (1962)
Dolen, Horn, Schmidt (1968)



"s-t channel duality"

Duality in electron-nucleon scattering

"Bloom-Gilman duality"

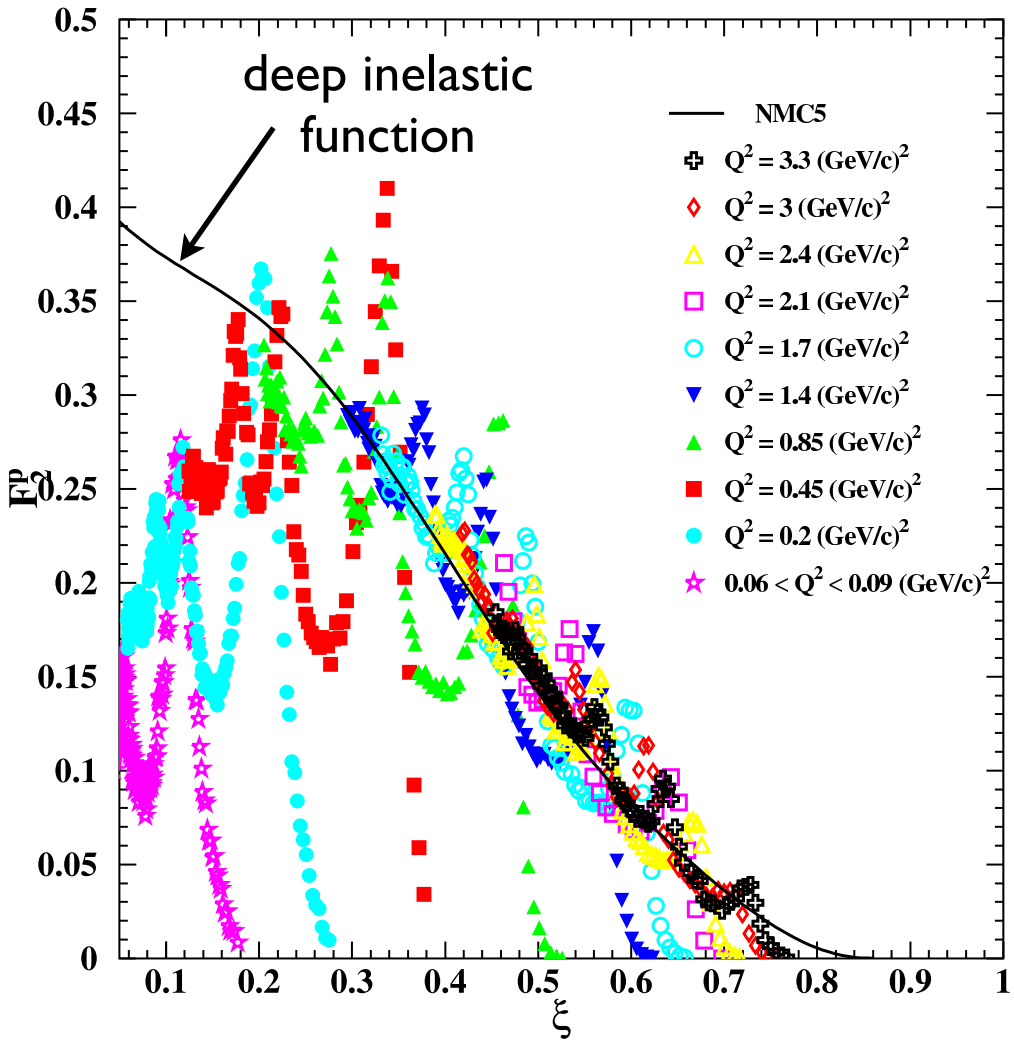


$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

"hadrons" "quarks"

finite-energy sum rules

Duality in electron-nucleon scattering



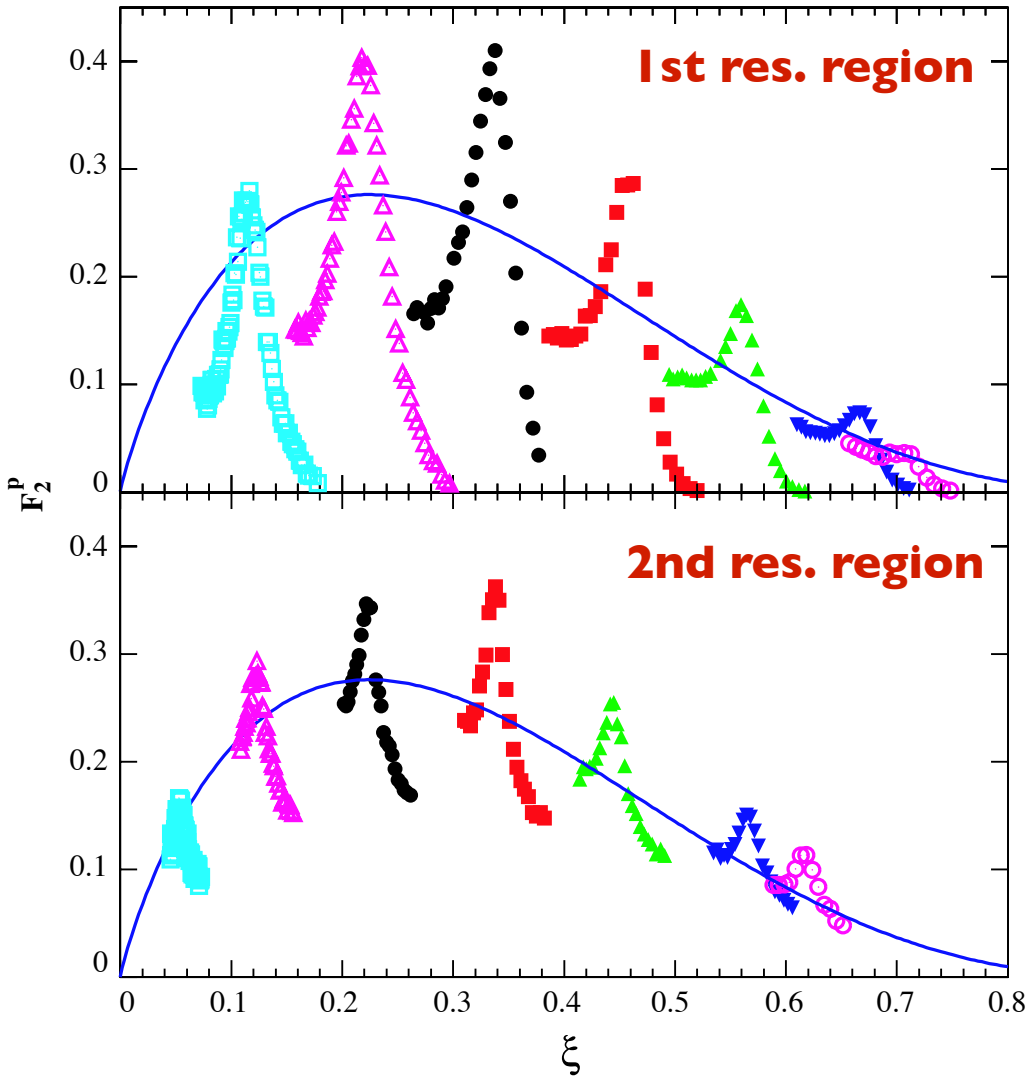
Niculescu et al., PRL ***85****, 1182 (2000)*
WM, Ent, Keppel, PRep. ***406****, 127 (2005)*

average over
(strongly Q^2 dependent)
resonances
$\approx Q^2$ independent
scaling function

"Nachtmann" scaling variable

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

Duality in electron-nucleon scattering

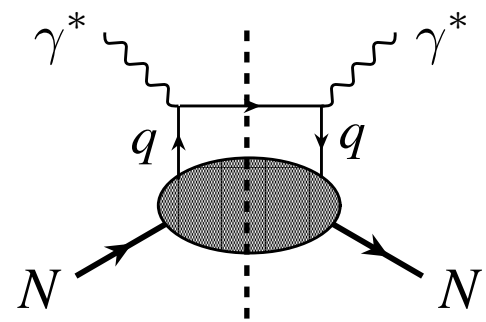


→ also exists *locally* in individual resonance regions

Duality in electron-nucleon scattering

- In *deep-inelastic* region $(W \gtrsim 2\ \mathrm{GeV},\ Q^2 \gtrsim 1\ \mathrm{GeV}^2)$ structure function given by parton distributions

$$F_2(x, Q^2) = x \sum_q e_q^2\ q(x, Q^2)$$



- In *resonance* region $(W \lesssim 2\ \mathrm{GeV})$, or at low Q^2 $(Q^2 \lesssim 1\ \mathrm{GeV}^2)$ can no longer resolve individual quark structure

- Resonance and DIS regions intimately connected

  $\longrightarrow$ resonances an *integral* part of scaling structure function

  e.g. in large-N_c limit, spectrum of zero-width resonances is "maximally dual" to quark-level (smooth) structure function

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- Earliest attempts predate QCD

→ $e.g.$ harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = $1/2$, ..., n+$1/2$
(n even: $I = 1/2$, n odd: $I = 3/2$)

Domokos et al., PRD ***3****, 1184 (1971)*

→ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2/M_n^2)^2} \qquad r^2 \approx 1.41$$

→ in Bjorken limit, $\sum_n \longrightarrow \int dz\,, \quad z \equiv M_n^2/Q^2$

$$F_2 \sim (\omega' - 1)^{1/2}(\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

→ scaling function of $\omega' = \omega + M^2/Q^2 \qquad (\omega = 1/x)$

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Domokos et al., PRD ***3****, 1184 (1971)*

→ in $\Gamma_n \to 0$ limit

$$F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2) \frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4}$$

cf. Drell-Yan-West relation

$$G(Q^2) \sim \left(\frac{1}{Q^2}\right)^m \iff F_2(x) \sim (1-x)^{2m-1}$$

→ similar behavior found in many models

Einhorn, PRD ***14****, 3451 (1976)* ('t Hooft model)
Greenberg, PRD ***47****, 331 (1993)* (NR scalar quarks in HO potential)
Pace, Salme, Lev, PRC ***57****, 2655 (1995)* (relativistic HO with spin)
Isgur et al., PRD ***64****, 054005 (2001)* (transition to scaling)
....

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2

→ additional constraints from threshold behavior at $\mathbf{q} \to 0$ and asymptotic behavior at $Q^2 \to \infty$

Davidovsky, Struminsky, Phys. Atom. Nucl. ***66****, 1328 (2003)*

$$\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M\nu \; \left[|G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2\right] \; \delta(W^2 - M_R^2)$$

→ 21 isospin-$1/2$ & $3/2$ resonances (with mass < 2 GeV)

$$\left|G_\pm^R(Q^2)\right|^2 = \left|G_\pm^R(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_1} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_\pm} \qquad m_{+,0,-} = 3, 4, 5$$

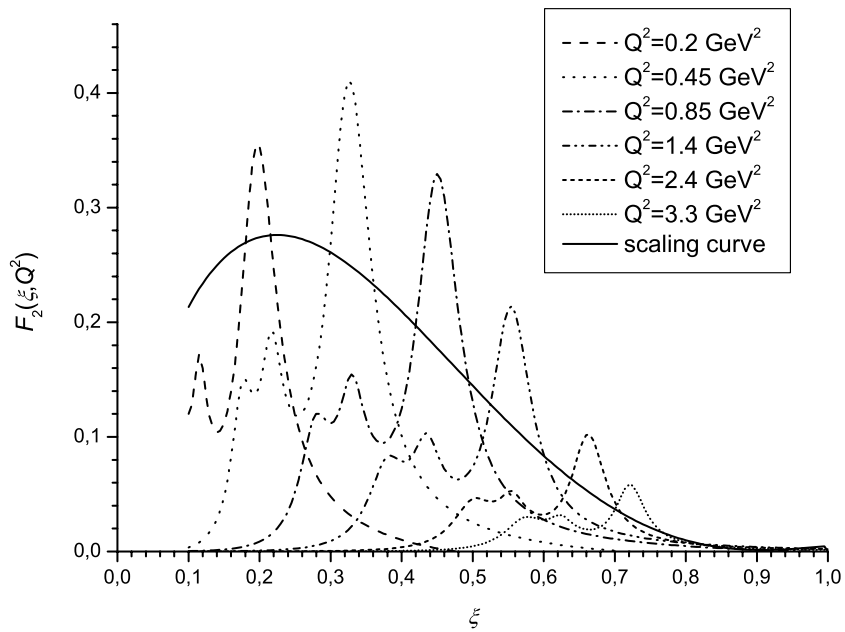
$$\left|G_0^R(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + \Lambda''^2}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_2} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_0}$$

→ in $x \to 1$ limit,

$$F_2(x) \sim (1 - x)^{m_+}$$

How to build up a scaling structure function from $\gamma^* N N^*$ transitions?

- More recent phenomenological analyses at finite Q^2



Davidovsky, Struminsky, Phys. Atom. Nucl. ***66****, 1328 (2003)*

→ valence-like structure of dual function suggests "two-component duality":

- valence (Reggeon exchange) dual to resonances $F_2^{(\mathrm{val})} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background $F_2^{(\mathrm{sea})} \sim x^{-0.08}$

Freund, PRL ***20****, 235 (1968)*
Harari, PRL ***20****, 1395 (1969)*

Duality and QCD

- Operator product expansion

→ expand *moments* of structure functions in powers of $1/Q^2$

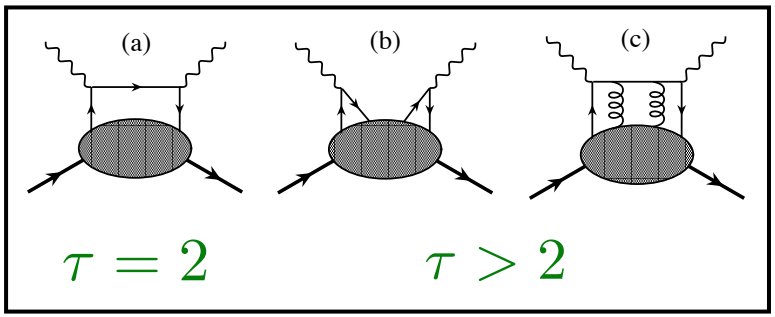
$$x \to 1 \iff W \to M$$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



Duality and QCD

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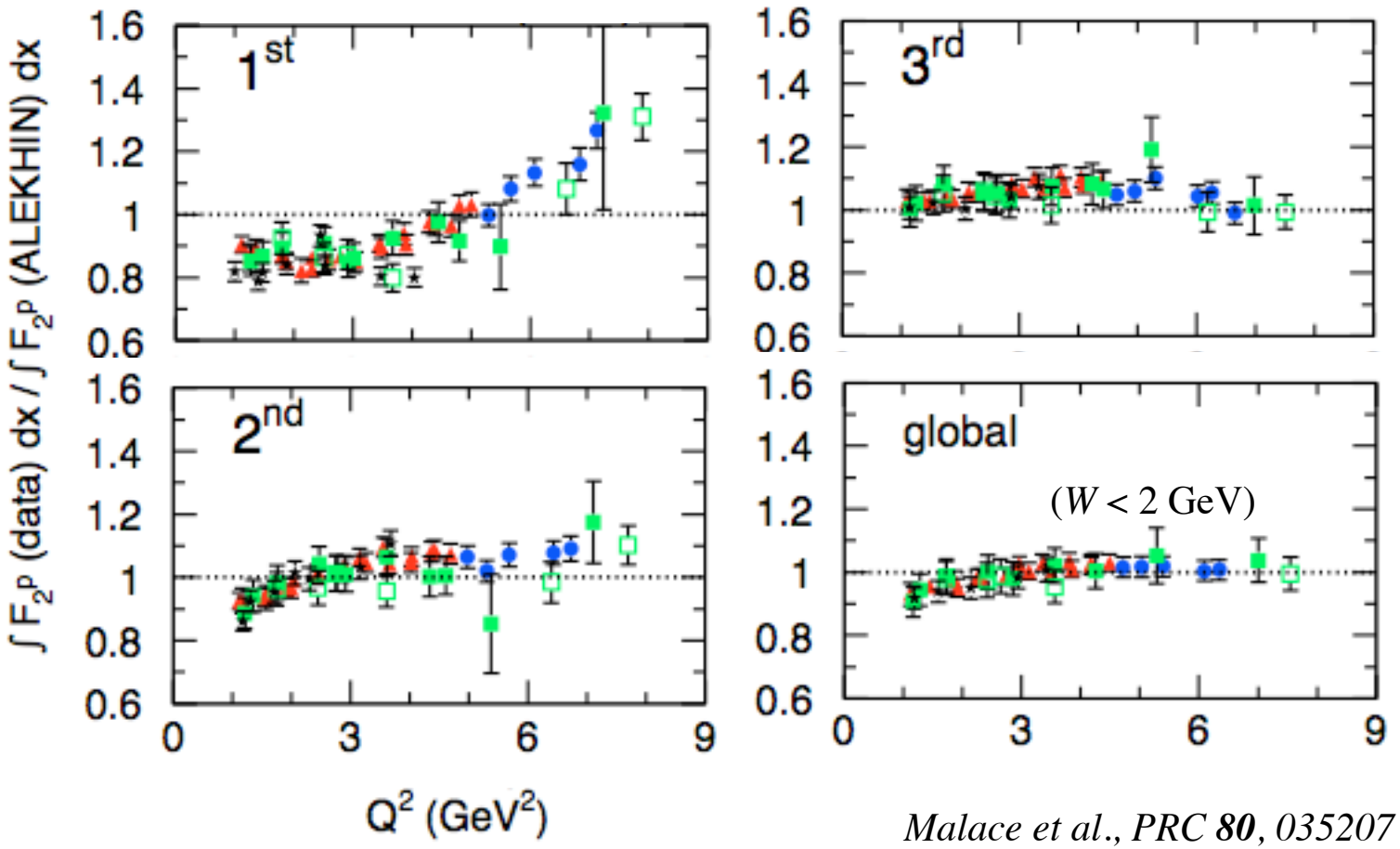
de Rujula, Georgi, Politzer
Ann. Phys. ***103***, *315 (1975)*

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

Truncated moments of F_2^p in resonance region



*Malace et al., PRC **80**, 035207 (2009)*
[JLab Hall C]

→ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total "higher twist" is *small* at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  $\longrightarrow$ nontrivial interference between resonances

---

- Can we understand this dynamically, at quark level?

  $\longrightarrow$ is duality an accident?

- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?

  $\longrightarrow$ expanded data set has potentially significant implications for global quark distribution studies

- Consider simple quark model with spin-flavor symmetric wave function

<u>low energy</u>

→ *coherent* scattering from quarks $d\sigma \sim \left(\sum_i e_i\right)^2$

<u>high energy</u>

→ *incoherent* scattering from quarks $d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

→ how can <u>*square of a sum*</u> become <u>*sum of squares*</u>?

Dynamical cancellations

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites
even partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

→ resulting structure function

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left\{(e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1}\right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

Close, Isgur, PLB ***509****, 81 (2001)*

Dynamical cancellations

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

Close, Isgur, PLB ***509****, 81 (2001)*

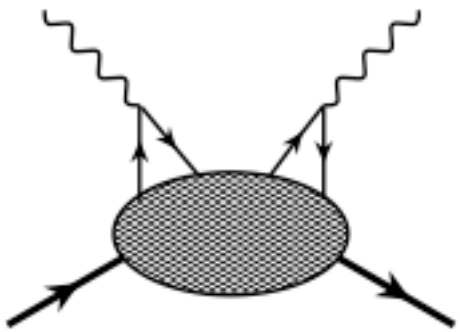
→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wave function

Close, WM, PRC ***68****, 035210 (2003)*
PRC ***79****, 055202 (2009)*

Accidental cancellations of charges?



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent *incoherent*

proton $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0\,!$

neutron $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

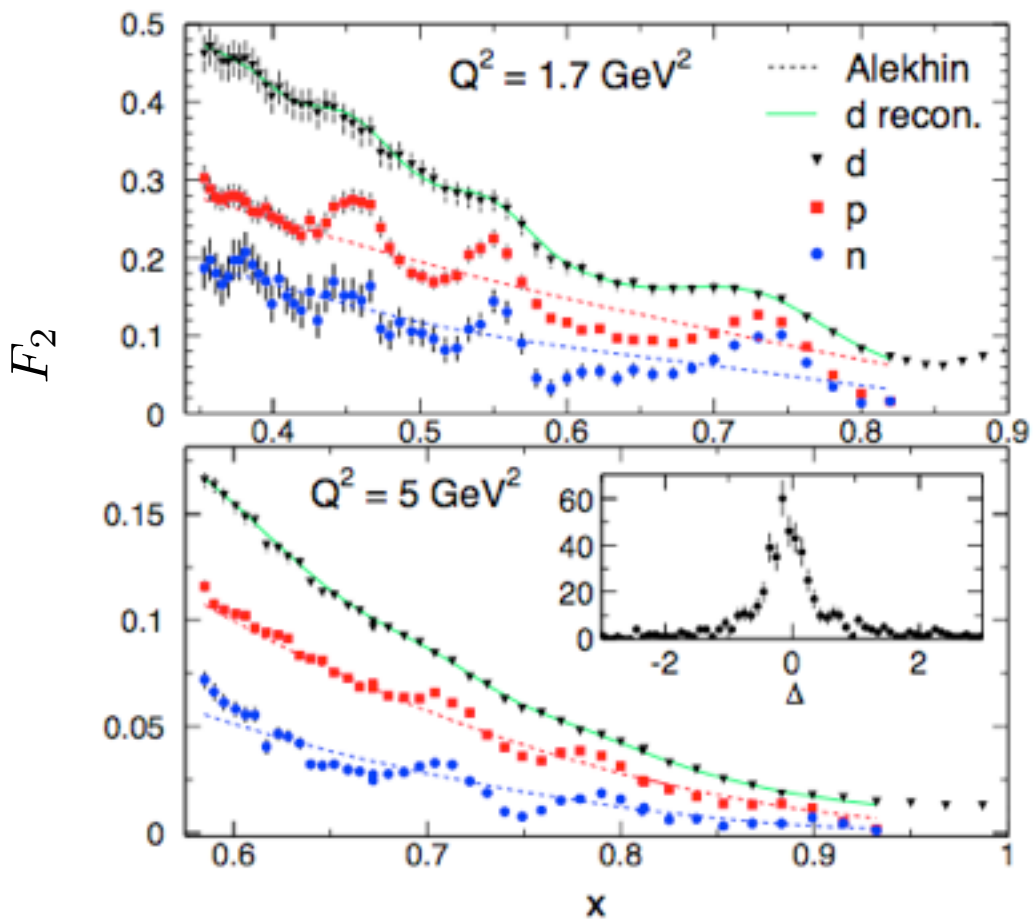
S. Brodsky (2000)

→ duality in proton a *coincidence!*

→ should *not* hold for neutron !!

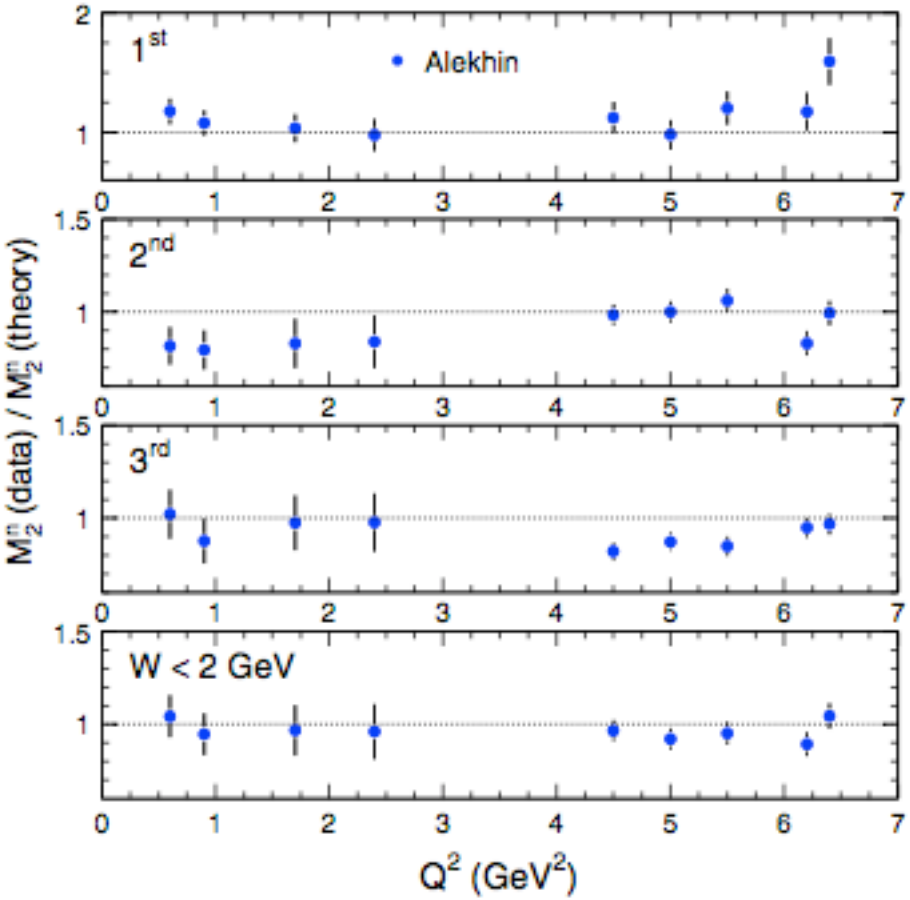
Neutron: the smoking gun

- Duality in *neutron* more difficult to test because of absence of free neutron targets
- New extraction method (using iterative procedure for solving integral convolution equations) has allowed first determination of F_2^n in resonance region & test of neutron duality



Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

Neutron: the smoking gun



→ "theory": fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

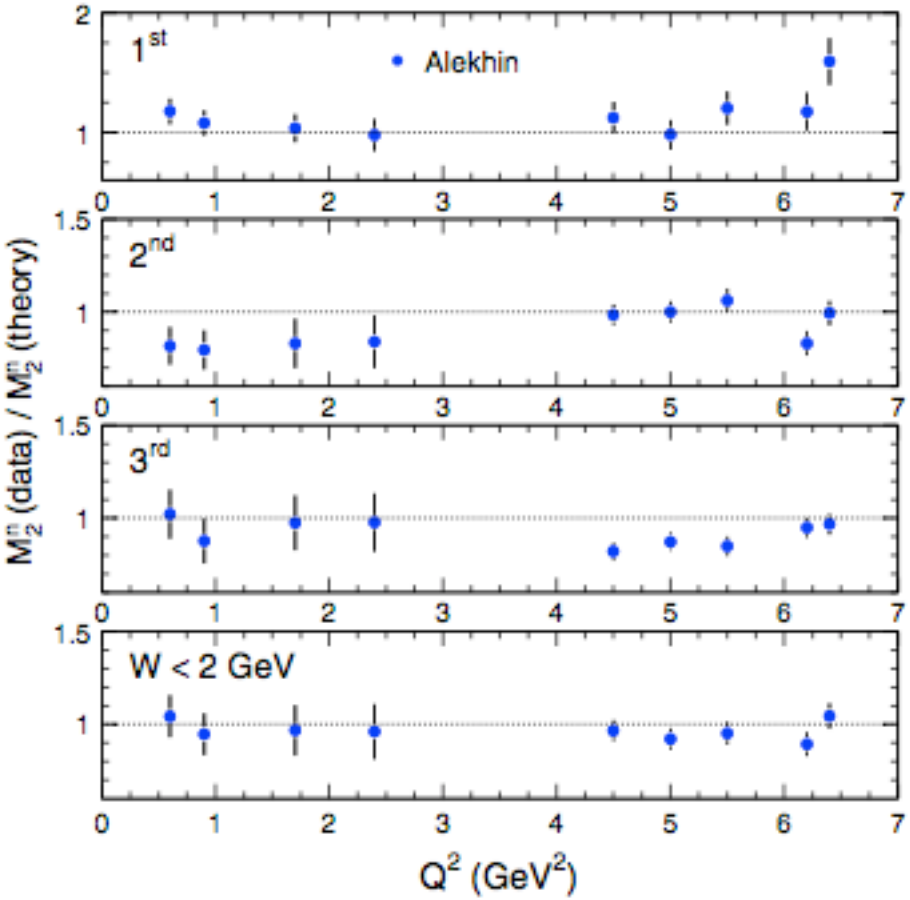
→ *locally*, violations of duality in resonance regions < 15–20% (largest in Δ region)

→ *globally*, violations < 10%

Malace, Kahn, WM, Keppel
PRL ***104****, 102001 (2010)*

➡ duality is *not accidental*, but a general feature of resonance–scaling transition!

Neutron: the smoking gun



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➡ analysis using recent (model-independent) BoNuS data in progress

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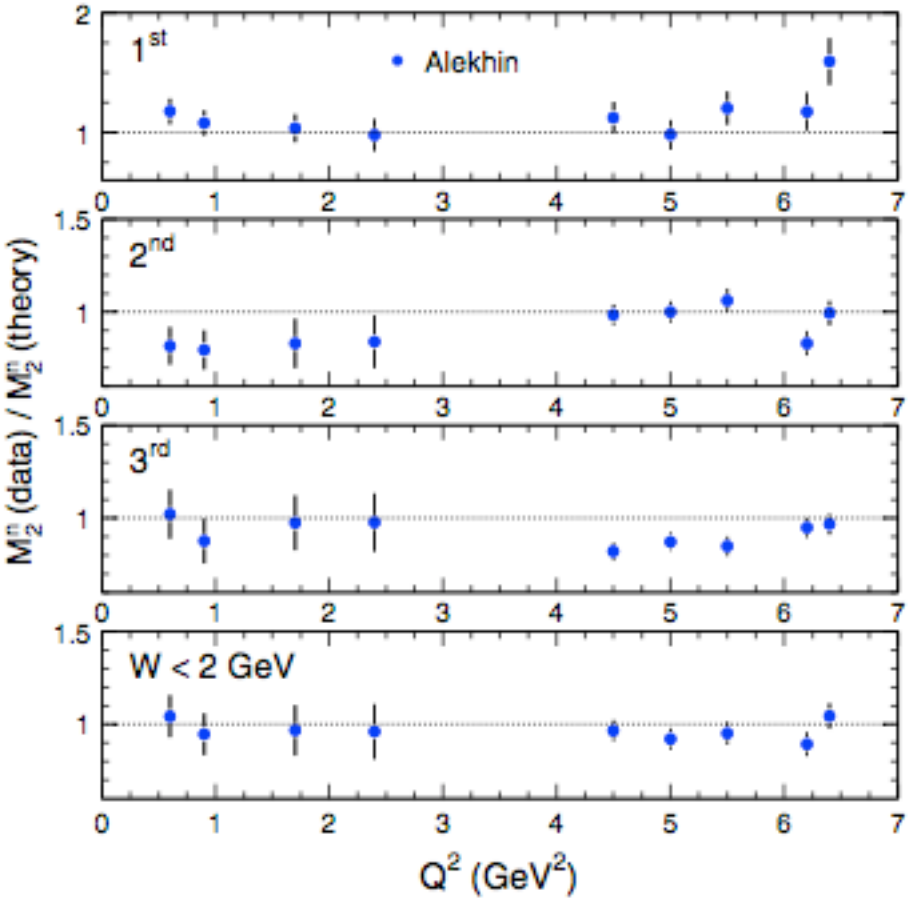

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PRL ***104****, 102001 (2010)*

➡ use resonance region data to learn about *leading twist* structure functions?

CTEQ-JLab (CJ) global PDF analysis *

- New global NLO analysis of expanded set of p and d data (DIS, pp, pd) including large-x, low-Q^2 region

Owens, Accardi WM
arXiv:1212.1702 (PRD, in print)

- Systematically study effects of Q^2 & W cuts

→ down to $Q \sim m_c$ and $W \sim 1.7$ GeV

cut0: $Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2, \quad W^2 > 8\ \mathrm{GeV}^2$

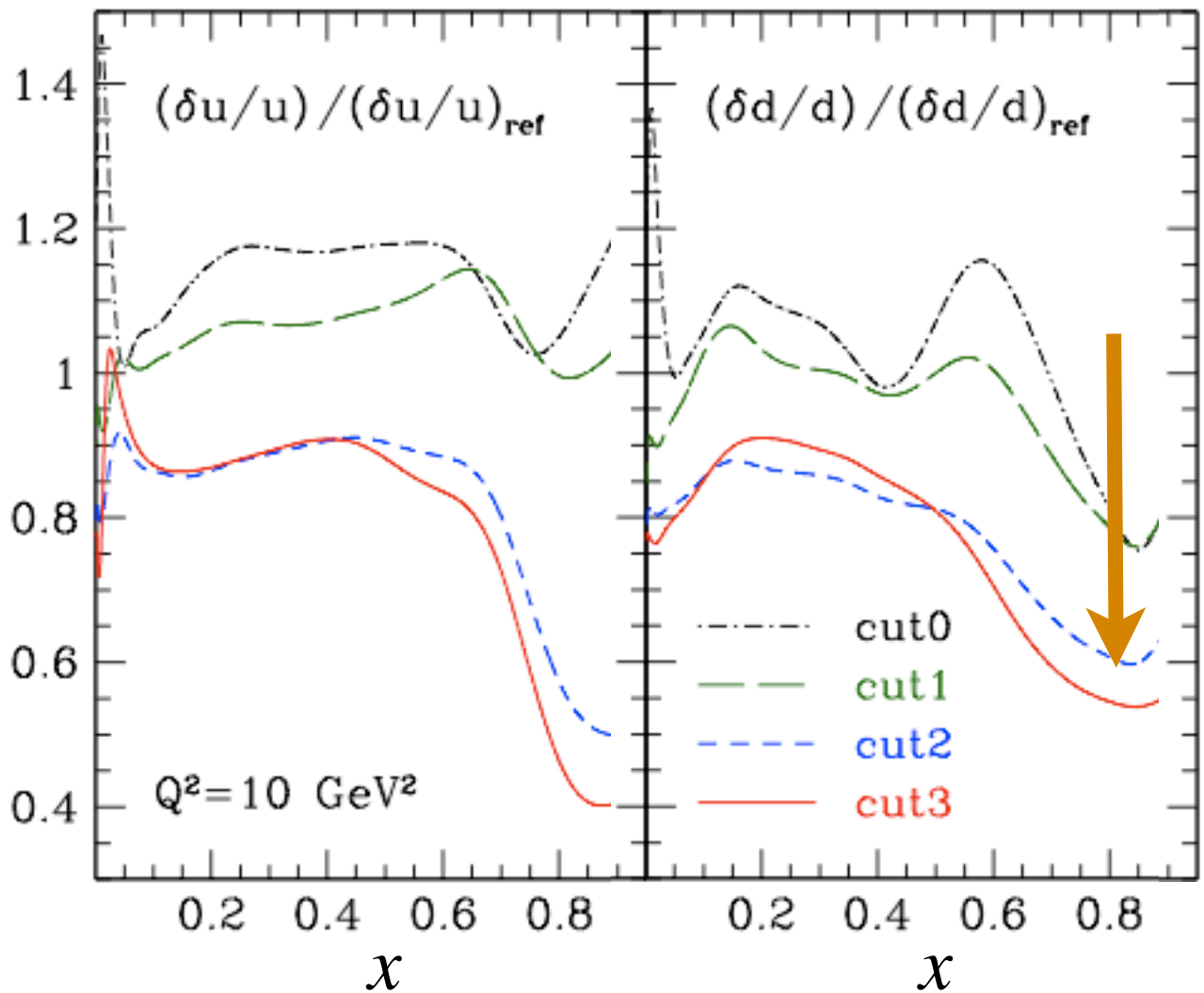
cut2: $Q^2 > 2\ \mathrm{GeV}^2, \quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase in DIS data from cut0 → cut3

* CJ collaboration: http://www.jlab.org/CJ

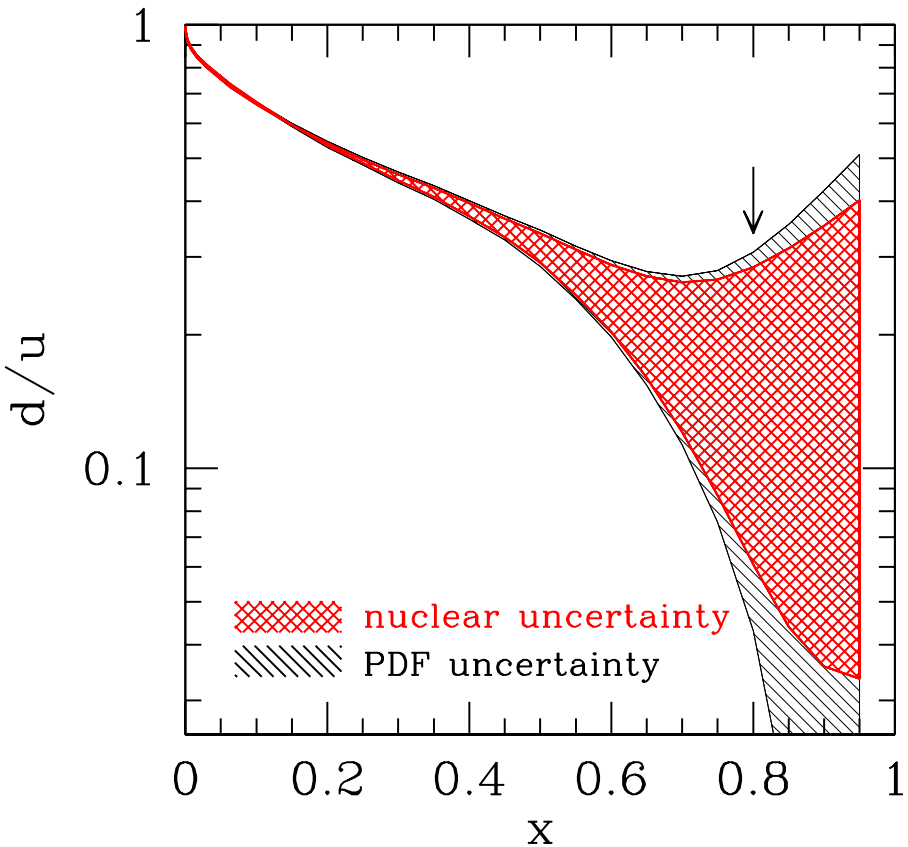
- Larger database with weaker cuts leads to significantly reduced errors, especially at large x

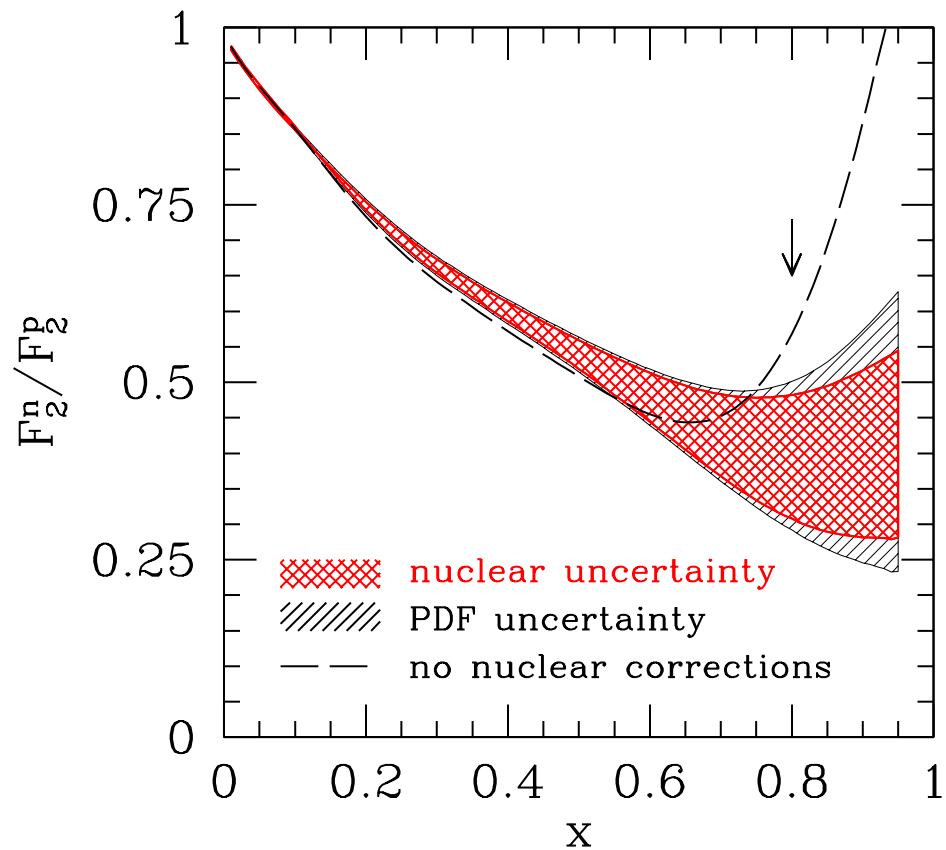


Accardi et al.
PRD ***81****, 034016 (2010)*

→ up to 40–60% error reduction when cuts extended into resonance region

- Vital for large-x analysis, which currently suffers from large uncertainties (mostly due to nuclear corrections)



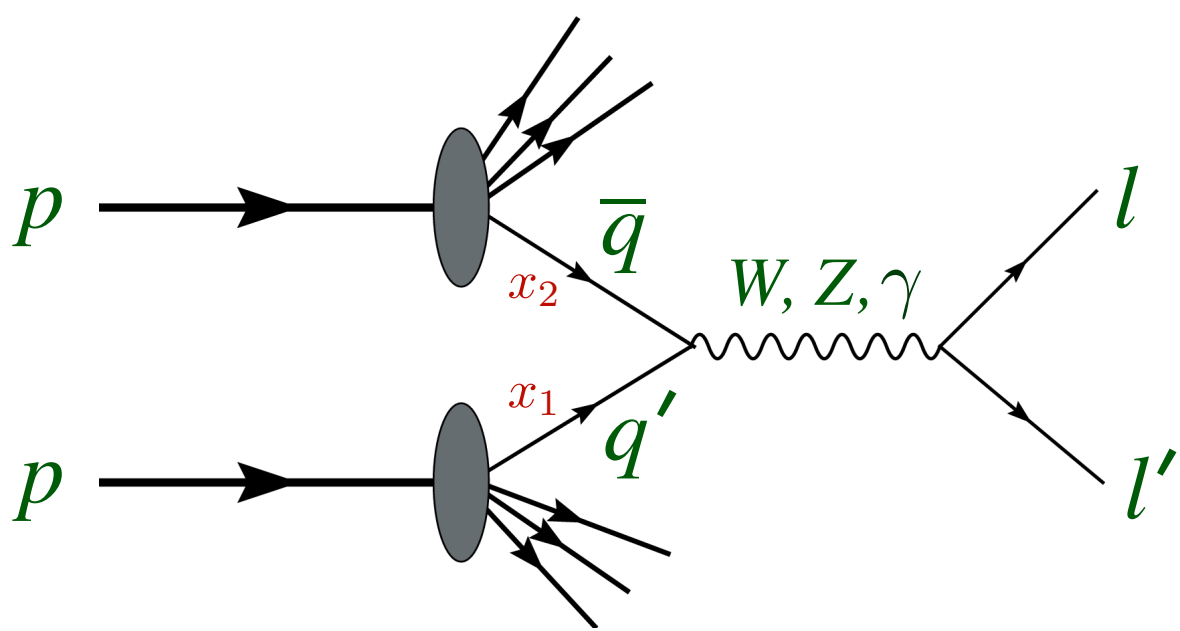


Accardi et al., PRD ***84****, 014008 (2011)*

→ uncertainty in d feeds into larger uncertainty in g at high x (important for LHC physics!)

Large Hadron Collider (CERN)

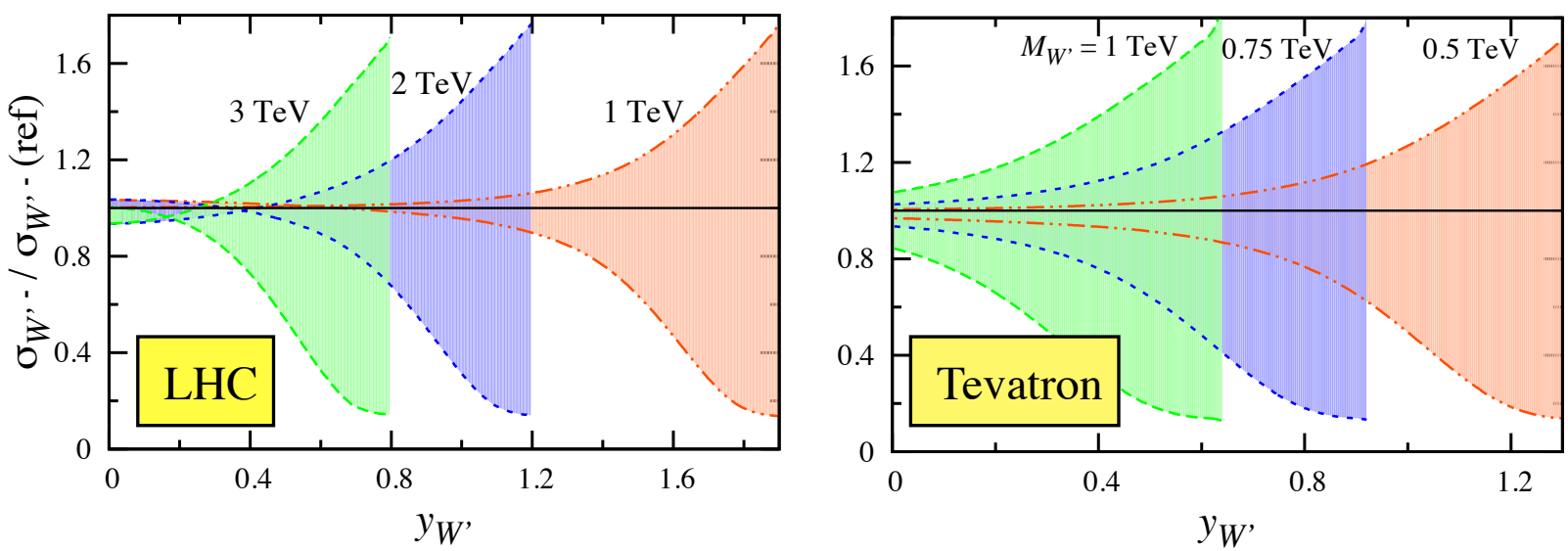
pp collisions
at $\sqrt{s} = 7$ TeV



Heavy Z', W' boson production

- Observation of new physics signals requires accurate determination of QCD backgrounds — depend on PDFs!
  (since $x_{1,2} \sim M_{Z',W'}$, large-x uncertainties scale with mass!)

  - for W'^- production



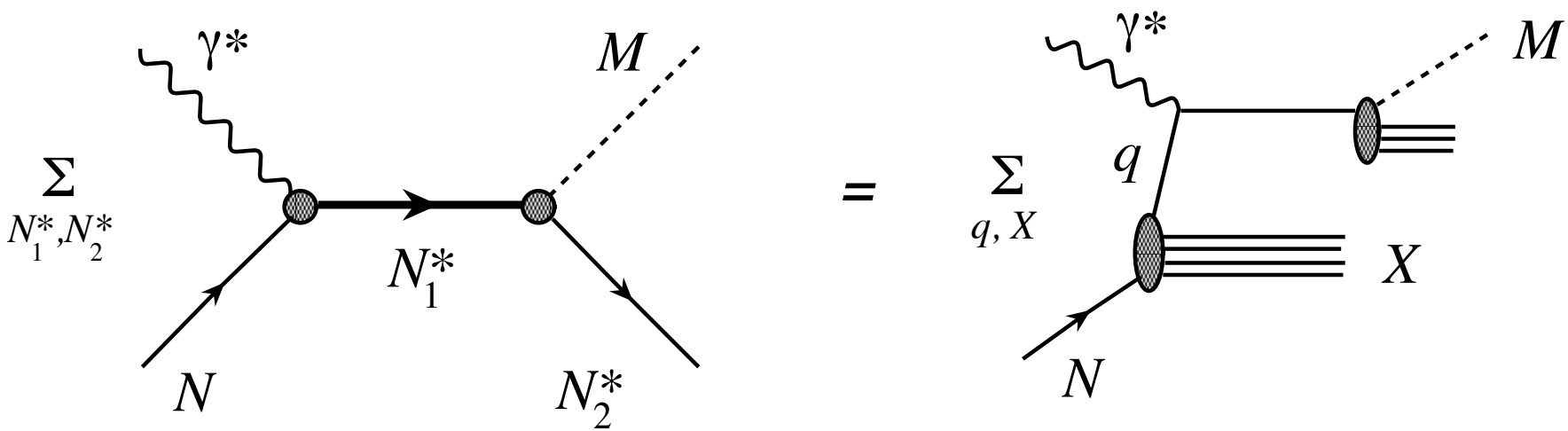
→ dominated by $d * \bar{u}$ → dominated by $d * u + u * d$

> 100% uncertainties at large y !

Brady et al., JHEP ***1206****, 019 (2012)*

Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction



s-channel resonance excitation and decay

parton level scattering and fragmentation

$$\sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \to N_1^*}(Q^2, M_1^*)\, \mathcal{D}_{N_1^* \to N_2^* M}(M_1^*, M_2^*) \right|^2 = \sum_q e_q^2\, q(x, Q^2)\, D_q^M(z, Q^2)$$

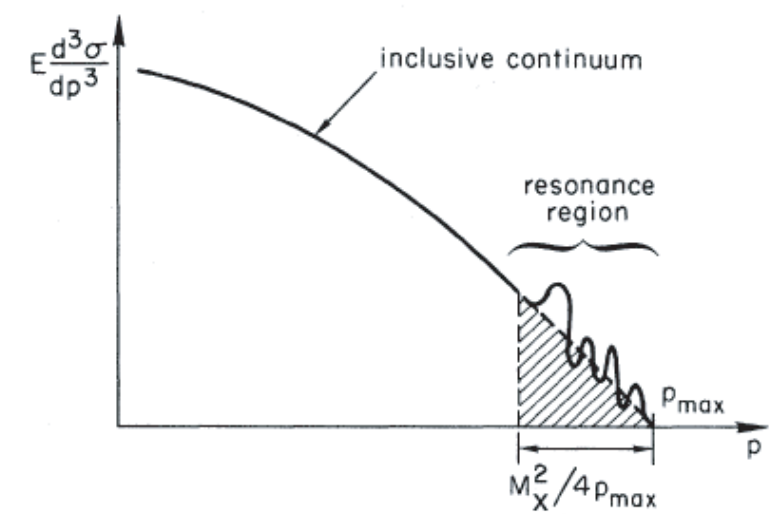
Duality in exclusive reactions

- Exclusive–inclusive correspondence principle:

  → continuity of dynamics from one (known) region to another (poorly known)

$$\int_{p_{\max}-M_X^2/4p_{\max}}^{p_{\max}} dp \;\; E\frac{d^3\sigma}{dp^3}\bigg|_{\text{incl}} \sim \sum_{\text{res}} E\frac{d\sigma}{dp_T^2}\bigg|_{\text{excl}}$$

$$\gamma^* N \to M\ X \qquad\qquad \gamma^* N \to M\ N^*$$



→ resonance contribution to $d\sigma$ should be comparable to the continuum contribution extrapolated from high energy

$$\frac{E}{\sigma}\frac{d^3\sigma}{dp^3} \equiv f(x, p_T^2, sQ^2) \longrightarrow f(x, p_T^2, sQ^2) \overset{s\to\infty}{\longrightarrow} f(x, p_T^2)$$

Bjorken, Kogut, PRD ***8****, 1341 (1973)*

Conclusion

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances

  → explore more realistic descriptions based on phenomenological $\gamma^* N N^*$ form factors

  → incorporate *nonresonant* background in same framework

- Practical application of duality

  → use resonance region data to constrain PDFs at high x
  (better knowledge of resonances could be relevant for LHC !)

- Extend quark-hadron duality concept to electroproduction

  → application to semi-inclusive DIS, DVCS / GPDs, ...

Gracias!