Polarized Electron Beams
MIT, Mar. 15, 2013

Constrained γZ corrections to PVES

Wally Melnitchouk

“AJM” collaboration

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Tony Thomas (Adelaide), Ross Young (Adelaide)

Outline

- γZ corrections in atomic parity violation
  - $\longrightarrow$ dispersive axial-vector hadron correction for p, $^{133}\mathrm{Cs}$

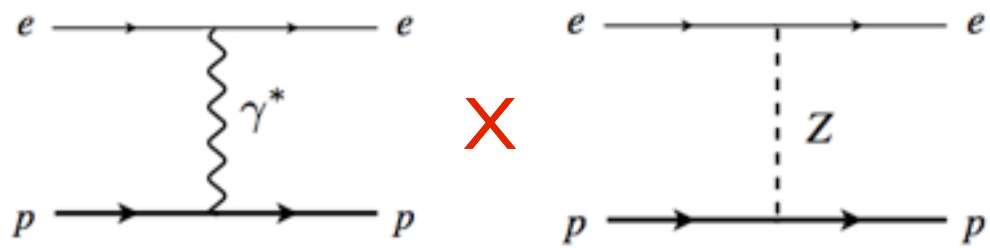
- Constrained vector hadron correction to Qweak
  - $\longrightarrow$ constraints from PDFs, new PVDIS data
  - $\longrightarrow$ significant reduction in uncertainty on γZ correction

Parity-violating e scattering

- Left-right polarization asymmetry in $\vec{e}\, p \to e\, p$ scattering

$$A_{\mathrm{PV}} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \longrightarrow \frac{G_F\, Q_W^p}{4\sqrt{2}\pi\alpha}\, t \qquad t = (k_e - k_e')^2 \to 0$$

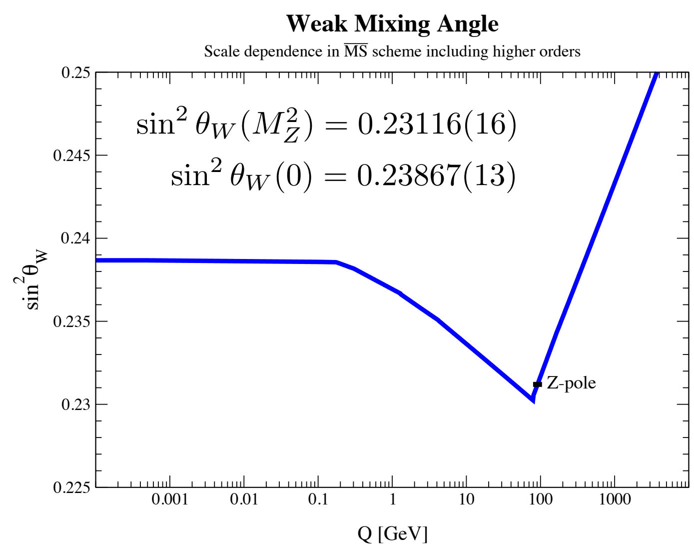
→ measures interference between e.m. and weak currents



→ in forward limit, gives proton weak charge

$$Q_W^p = 1 - 4\sin^2\theta_W$$

(tree level)



Corrections to proton weak charge

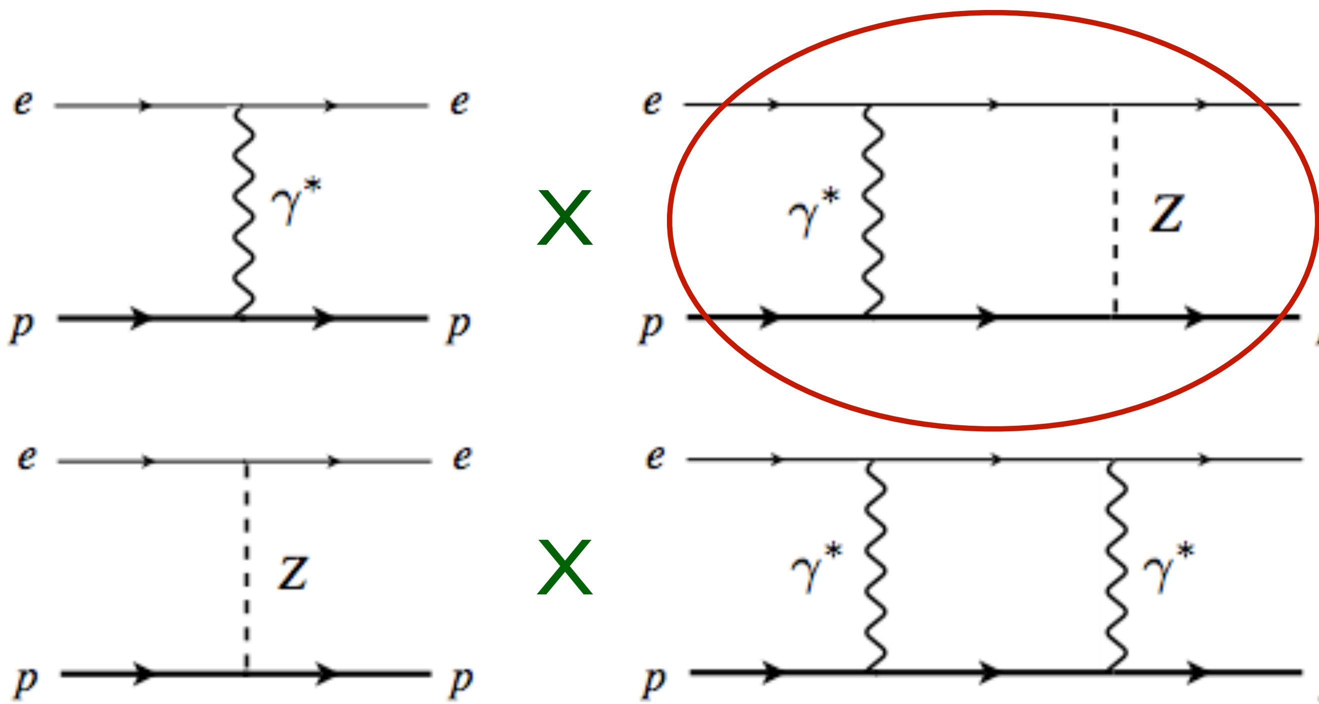
- Including higher order radiative corrections

$$
\begin{aligned}
Q_W^p &= (1+\Delta\rho+\Delta_e)(1-4\sin^2\theta_W(0)+\Delta_e') \\
&\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}
\end{aligned}
$$

box diagrams

$$= 0.0713 \pm 0.0008$$

Erler et al., PRD ***72****, 073003 (2005)*



Corrections to proton weak charge

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&= 0.0713 \pm 0.0008
\end{aligned}
$$

box diagrams

Erler et al., PRD ***72****, 073003 (2005)*

→ WW and ZZ box diagrams dominated by short distances, evaluated perturbatively (*WW* box gives ~ 25% correction!)

→ γZ box diagram sensitive to long distance physics, has two contributions

$$\Box_{\gamma Z} = \Box_{\gamma Z}^{A} + \Box_{\gamma Z}^{V}$$

vector e – axial h (finite at E=0)

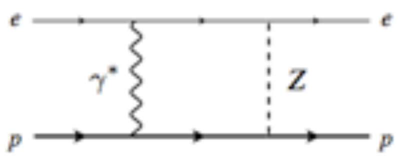
axial e – vector h (vanishes at E=0)

Axial-vector hadron γZ correction

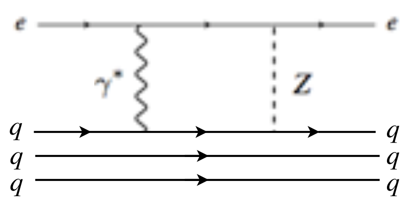
Axial h correction

- Axial h correction $\Box_{\gamma Z}^{A}$ dominant in atomic parity violation at very low (zero) energy

→ seminal work by Marciano & Sirlin (1980s):



★ low-energy part approximated by *Born* contribution (elastic intermediate state)



★ high-energy part (above scale $\Lambda \sim 1$ GeV) computed in terms of scattering from *free quarks*

$$\Box_{\gamma Z}^{A} = \frac{5\alpha}{2\pi}\left(1 - 4\sin^2\theta_W\right)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

$$\approx \; 0.0052(5)$$

short−distance

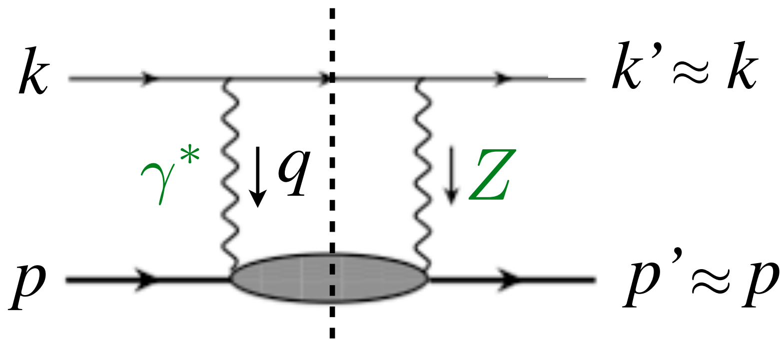
long−distance $\approx 3/2 \pm 1$

Marciano, Sirlin, PRD ***29****, 75 (1984); Erler et al., PRD* ***68****, 016006 (2003)*

Axial h correction

- Axial h correction $\Box^A_{\gamma Z}$ dominant in atomic parity violation at very low (zero) energy

→ evaluate using *forward dispersion relations* with realistic input (inclusive structure function)



forward limit

$t = (k - k')^2 \to 0$

$s = (k+p)^2$
$\quad = M(M + 2E)$

★ axial h contribution *anti*symmetric under $E' \leftrightarrow -E'$:

$$\Re e\, \Box^A_{\gamma Z}(E) = \frac{2}{\pi}\int_0^\infty dE' \frac{E'}{E'^2 - E^2}\, \Im m\, \Box^A_{\gamma Z}(E')$$

★ negative energy part corresponds to crossed box (crossing symmetry $s \to u$)

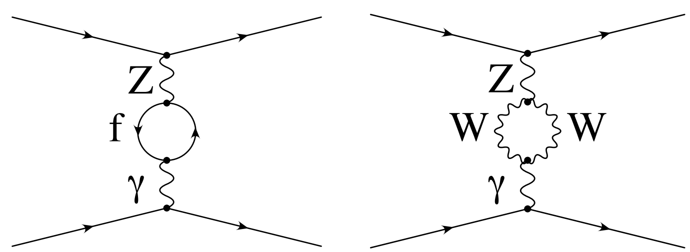
Axial h correction

- Imaginary part given by interference $F_3^{\gamma Z}$ structure function

$$\mathcal{I}m\ \Box^A_{\gamma Z}(E) = \frac{1}{(2ME)^2}\int_{M^2}^{s} dW^2 \int_0^{Q^2_{\rm max}} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2} \times \left(\frac{2ME}{W^2-M^2+Q^2}-\frac{1}{2}\right)\boxed{F_3^{\gamma Z}}$$

with $v_e(Q^2) = 1 - 4\kappa(Q^2)\sin^2\theta_W(Q^2)$

→ scale dependence of v_e, α given by vacuum polarization corrections, *e.g.*



Jegerlehner, arXiv:1107.4683 [hep-ph]

$$\frac{\alpha}{\alpha(Q^2)} = 1 - \Delta\alpha_{\rm lep}(Q^2) - \Delta\alpha^{(5)}_{\rm had}(Q^2)$$

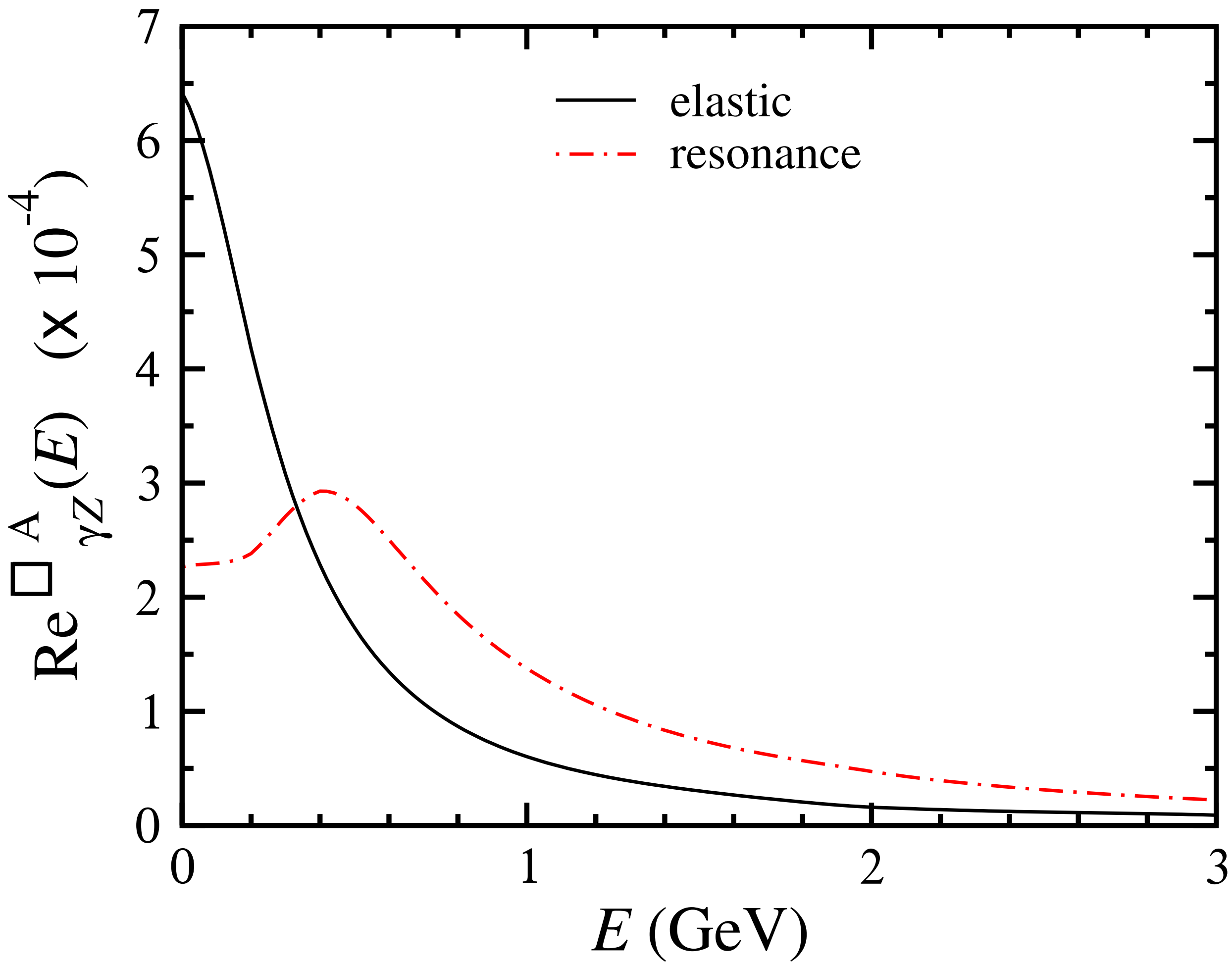
$$\alpha^{-1}(M_Z^2) = 128.94$$

... similarly for weak charges

Axial h correction

- **elastic part** $F_3^{\gamma Z(\mathrm{el})} = -Q^2\, G_M^p(Q^2)\, G_A^Z(Q^2)\, \delta(W^2 - M^2)$
- **resonance part from parametrization of ν scattering data**

Lalakulich, Paschos
*PRD **74**, 014009 (2006)*



Blunden, WM, Thomas
*PRL **107**, 081801 (2011)*

Axial h correction

★ DIS part dominated by leading twist PDFs at high W (small x)

$$e.g.\text{ at LO},\quad F_3^{\gamma Z(\mathrm{DIS})} = \sum_q 2e_q\, g_A^q \left(q(x,Q^2) - \bar{q}(x,Q^2)\right)$$

→ expand integrand in $1/Q^2$ in DIS region $(Q^2 \gtrsim 1\ \mathrm{GeV}^2)$

$$\mathcal{R}e\ \Box_{\gamma Z}^{\mathrm{A(DIS)}}(E) = \frac{3}{2\pi}\int_{Q_0^2}^{\infty} dQ^2\, \frac{v_e(Q^2)\,\alpha(Q^2)}{1+Q^2/M_Z^2}$$

$$\times\left[M_3^{\gamma Z(1)} - \frac{2M^2}{9Q^4}(5E^2-3Q^2)M_3^{\gamma Z(3)}\right]$$

moments $\quad M_3^{\gamma Z(n)}(Q^2) = \int_0^1 dx\, x^{n-1} F_3^{\gamma Z}(x,Q^2)$

Axial h correction

- Structure function moments

$\underline{n = 1}$ $$M_3^{\gamma Z(1)}(Q^2) = \frac{5}{3}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ γZ analog of Gross-Llewellyn Smith sum rule

$$\mathcal{R}e\ \square_{\gamma Z}^{A(\text{DIS})} \approx (1 - 4\hat{s}^2)\frac{5\alpha}{2\pi}\int_{Q_0^2}^{\infty}\frac{dQ^2}{Q^2(1 + Q^2/M_Z^2)}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)$$

$\longrightarrow$ precisely result from Marciano & Sirlin!

(result depends on lowest moment of *valence* PDF,
with model-independent normalization!)

$\underline{n = 3}$ $$M_3^{\gamma Z(3)}(Q^2) = \frac{1}{3}\left(2\langle x^2\rangle_u + \langle x^2\rangle_d\right)\left(1 + \frac{5\alpha_s(Q^2)}{12\pi}\right)$$

$\longrightarrow$ related to x^2-weighted moment of valence PDFs

Axial h correction

- “DIS” region at $Q^2 < 1\ \mathrm{GeV}^2$ does not afford PDF description

  $\longrightarrow$ in absence of data, consider models with general constraints

  - $F_3^{\gamma Z}(x_{\max}, Q^2)$ should not diverge in limit $Q^2 \to 0$
  - $F_3^{\gamma Z}(x, Q^2)$ should match PDF description at $Q^2 = 1\,\mathrm{GeV}^2$

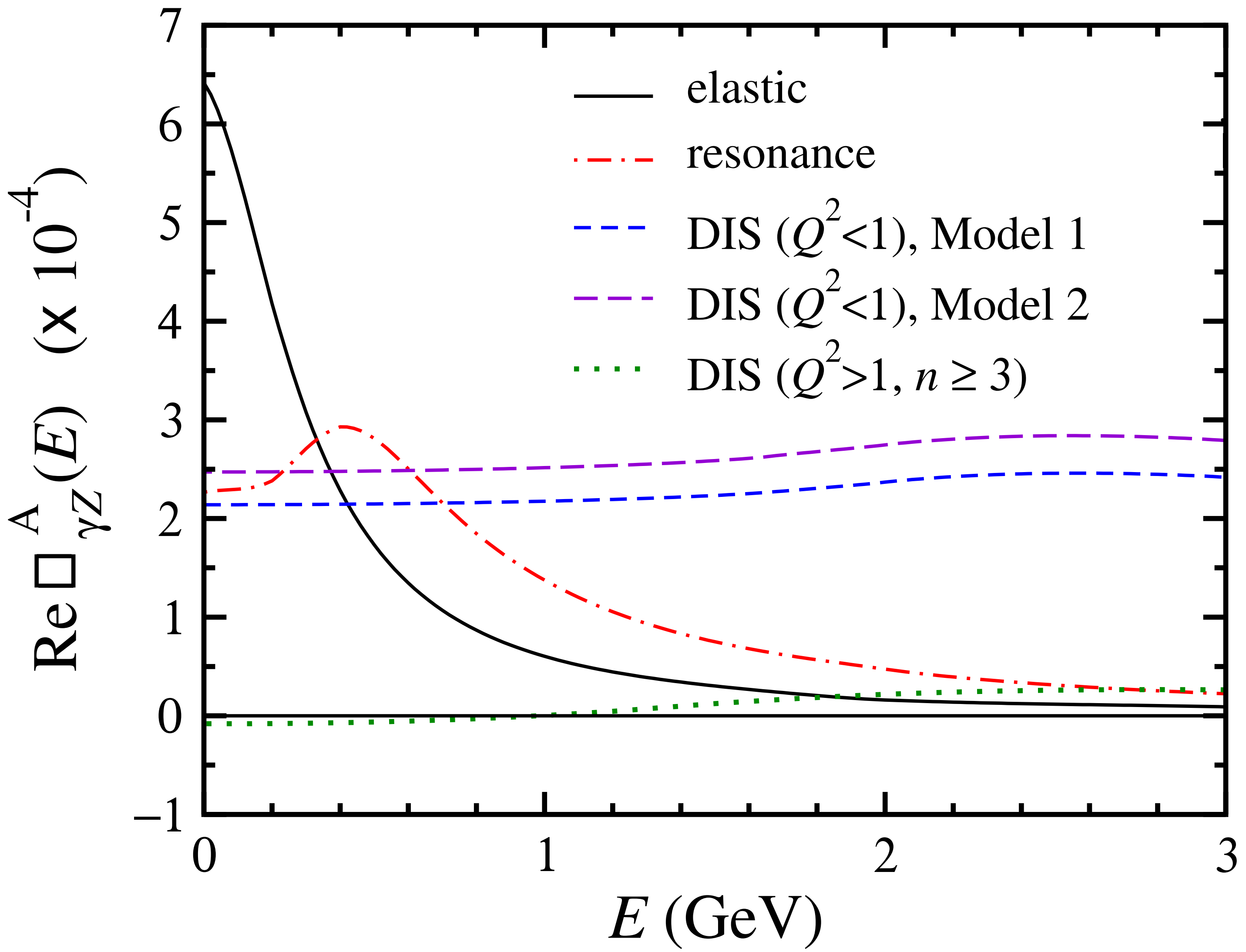
<u>Model 1</u> $\quad F_3^{\gamma Z}(x, Q^2) = \left(\frac{1 + \Lambda^2/Q_0^2}{1 + \Lambda^2/Q^2}\right) F_3^{\gamma Z}(x, Q_0^2)$

$$F_3^{\gamma Z} \sim (Q^2)^{0.3} \text{ as } Q^2 \to 0$$

<u>Model 2</u> $\quad F_3^{\gamma Z}$ frozen at $Q^2 = 1$ value for all W^2

$$F_3^{\gamma Z} \text{ finite as } Q^2 \to 0$$

Axial h correction



Blunden, WM, Thomas
*PRL **107**, 081801 (2011)*

→ dominated by $n = 1$ DIS moment: $\underline{32.8 \times 10^{-4}}$
(weak E dependence)

Axial h correction

- correction at $\underline{E = 0}$

$$\Re e\,\Box^{A}_{\gamma Z} \;=\; \underbrace{0.00064}_{\text{elastic}} \;+\; \underbrace{0.00023}_{\text{resonance}} \;+\; \underbrace{0.00350}_{\text{DIS}} \;\to\; \underline{0.0044(4)}$$

- correction at $\underline{E = 1.165\ \mathrm{GeV}}$ (Qweak)

$$\Re e\,\Box^{A}_{\gamma Z} \;=\; 0.00005 \;+\; 0.00011 \;+\; 0.00352 \;=\; \underline{0.0037(4)}$$

cf. MS* value: $\underline{0.0052(5)}$ (~1% shift in Q_W^p)

* *Marciano, Sirlin, PRD* ***29****, 75 (1984)*

- shifts Q_W^p from $\underline{0.0713(8)} \longrightarrow \underline{0.0705(8)}$

APV in ^{133}Cs

- Parity violating dipole transition $6\,\mathrm{S}_{1/2} - 7\,\mathrm{S}_{1/2}$ sensitive to weak mixing angle $(E \sim 0)$

  → weak charge of Cs

$$Q_W(\mathrm{Cs}) \;=\; 55\,\widetilde{Q}_W^p \;+\; 78\,\widetilde{Q}_W^n$$

weak charge of *bound* p in Cs nucleus

- Nuclear effect on elastic N contribution – Pauli blocking

  → intermediate state N (in target rest frame) must have momentum above Fermi level

$$|\mathbf{q}| > p_F \approx 260\,\mathrm{MeV}$$

$$\Rightarrow \quad Q^2 > Q^2_{\min} = 2M^2\left(\sqrt{1 + p_F^2/M^2} - 1\right) \approx p_F^2$$

APV in ^{133}Cs

- Significantly reduced elastic contribution

$$\Box_{\gamma Z}^{p\,(\mathrm{el})}\ :\ 0.00064\ \to\ 0.00029, \qquad \Box_{\gamma Z}^{n\,(\mathrm{el})}\ :\ 0.00044\ \to\ 0.00020$$

- Total γZ corrections dominated by DIS contributions

	p	n
total	0.0040(4)	0.0032(4)
MS	0.0052(5)	0.0040(4)
$\Delta \widetilde{Q}_W^N$	−0.0012	−0.0008
$\Delta Q_W(\mathrm{Cs})$	−0.065	−0.060

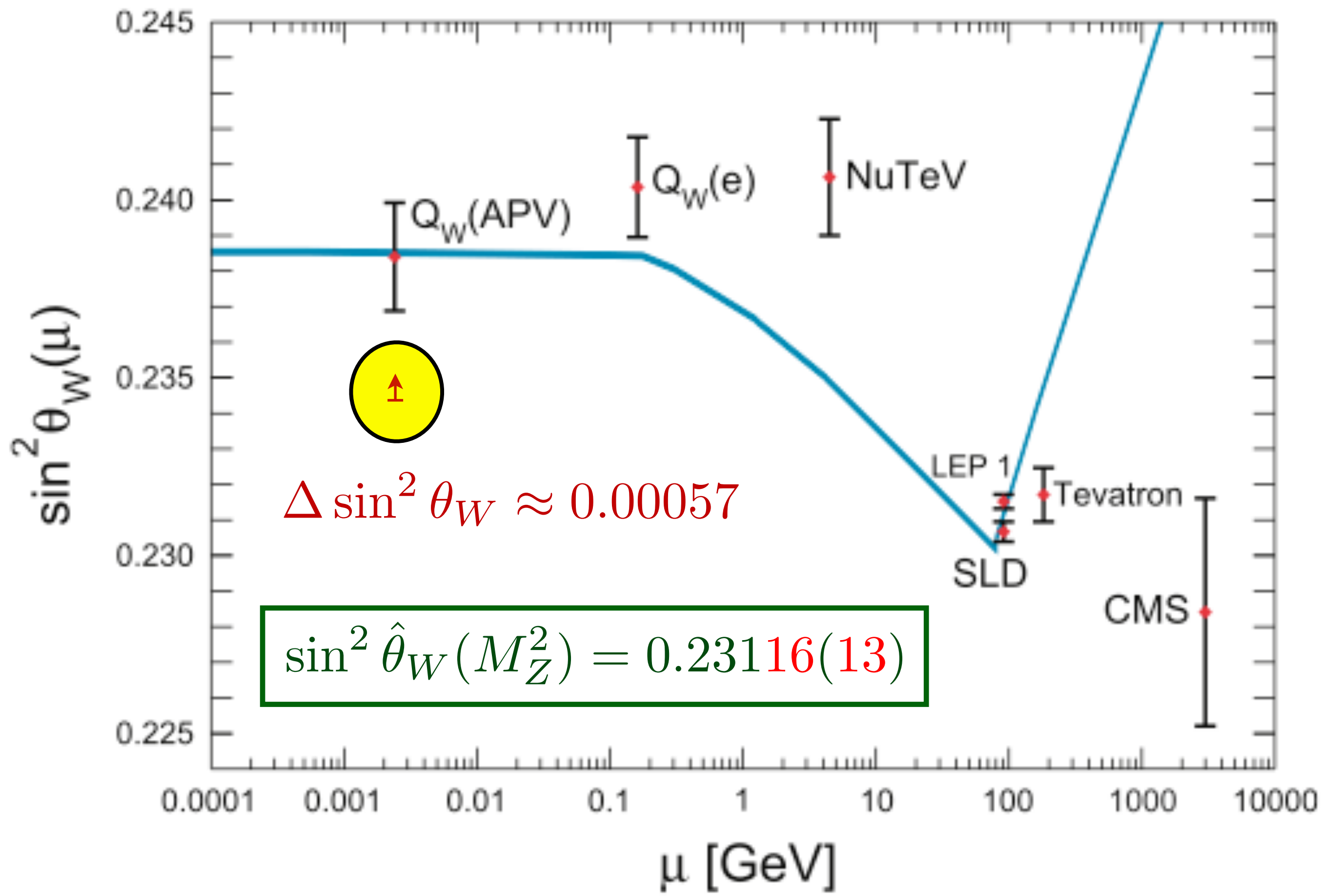
Blunden, WM, Thomas
PRL ***109****, 262301 (2012)*

→ overall shift (relative to MS): $\Delta Q_W(\mathrm{Cs}) = -0.126$

or $\boxed{-\,0.16\%\ \text{ of }\ \mathrm{Q_W^{exp}}(\mathrm{Cs}) = -73.20(35)}$

→ 4 times larger than current SM uncertainty on $\sin^2\theta_W$

APV in ^{133}Cs



Particle Data Group
PRD ***86****, 010001 (2012)*
[*June 18, 2012*]

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Vector hadron γZ correction

Vector h correction

- Vector h correction $\Box^V_{\gamma Z}$ vanishes at $E = 0$, but has sizable energy dependence

→ forward dispersion relation

★ $$\Re e\, \Box^V_{\gamma Z}(E) = \frac{2E}{\pi} \int_0^\infty dE' \frac{1}{E'^2 - E^2}\, \Im m\, \Box^V_{\gamma Z}(E')$$

★ integration over $E' < 0$ corresponds to crossed-box, vector h contribution symmetric under $E' \leftrightarrow -E'$

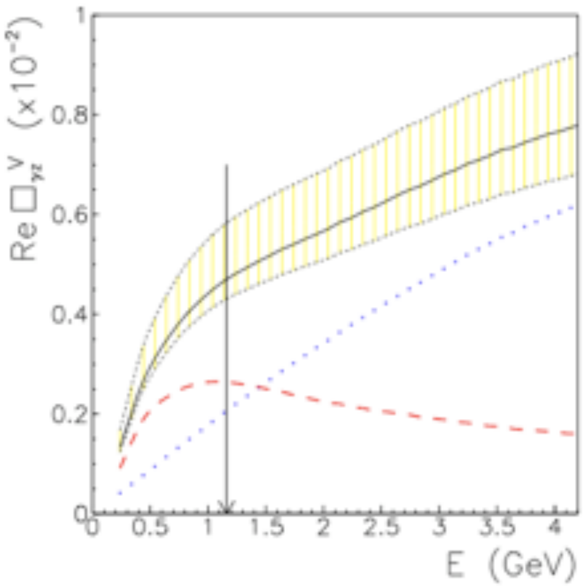
→ imaginary part given by

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\rm max}} \frac{dQ^2}{1 + Q^2/M_Z^2} \times \left(F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\,(Q^2_{\rm max} - Q^2)}{Q^2(W^2 - M^2 + Q^2)} \right)$$

*Gorchtein, Horowitz, PRL **102**, 091806 (2009)*

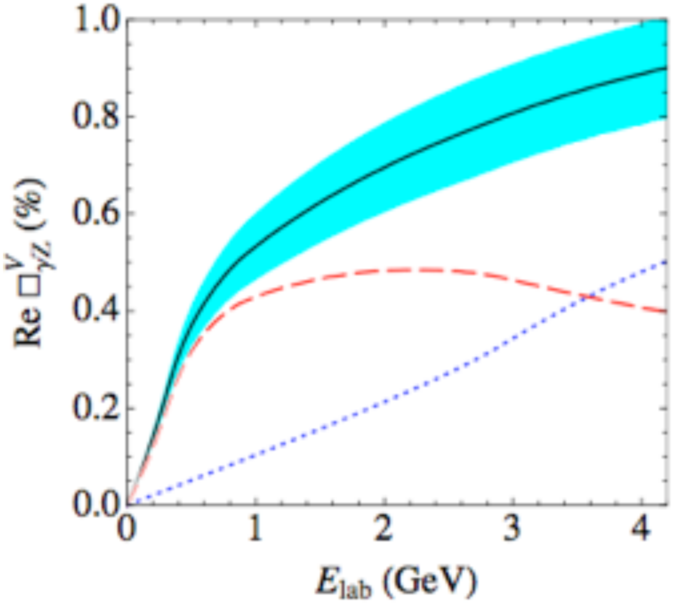
Vector h correction

Sibirtsev *et al.*



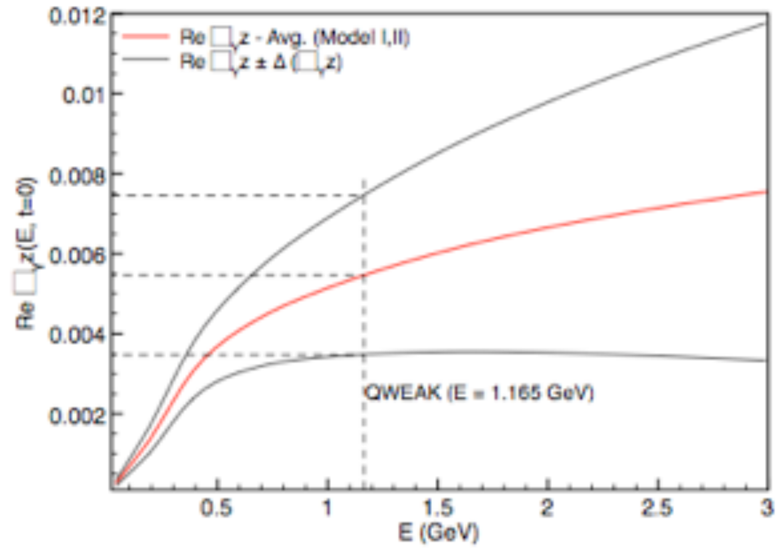
Sibirtsev, Blunden, WM, Thomas
PRD **82**, *013011 (2010)*

Rislow & Carlson



Rislow, Carlson
PRD **83**, *113007 (2011)*

GHRM



Gorchtein, Horowitz, Ramsey-Musolf
PRC **84**, *015502 (2011)*

$$\boxed{\Re e\, \square_{\gamma Z}^{V} = 0.0047^{+0.0011}_{-0.0004}}$$

$$\boxed{\Re e\, \square_{\gamma Z}^{V} = 0.0057 \pm 0.0009}$$

$$\boxed{\Re e\, \square_{\gamma Z}^{V} = 0.0054 \pm 0.002}$$

2 x larger uncertainty

$$\Re e\, \square_{\gamma Z}^{V} = (5.39 \pm 0.27 \pm 1.88^{+0.58}_{-0.49} \pm 0.07) \times 10^{-3}$$

model bckgnd res. t dep.

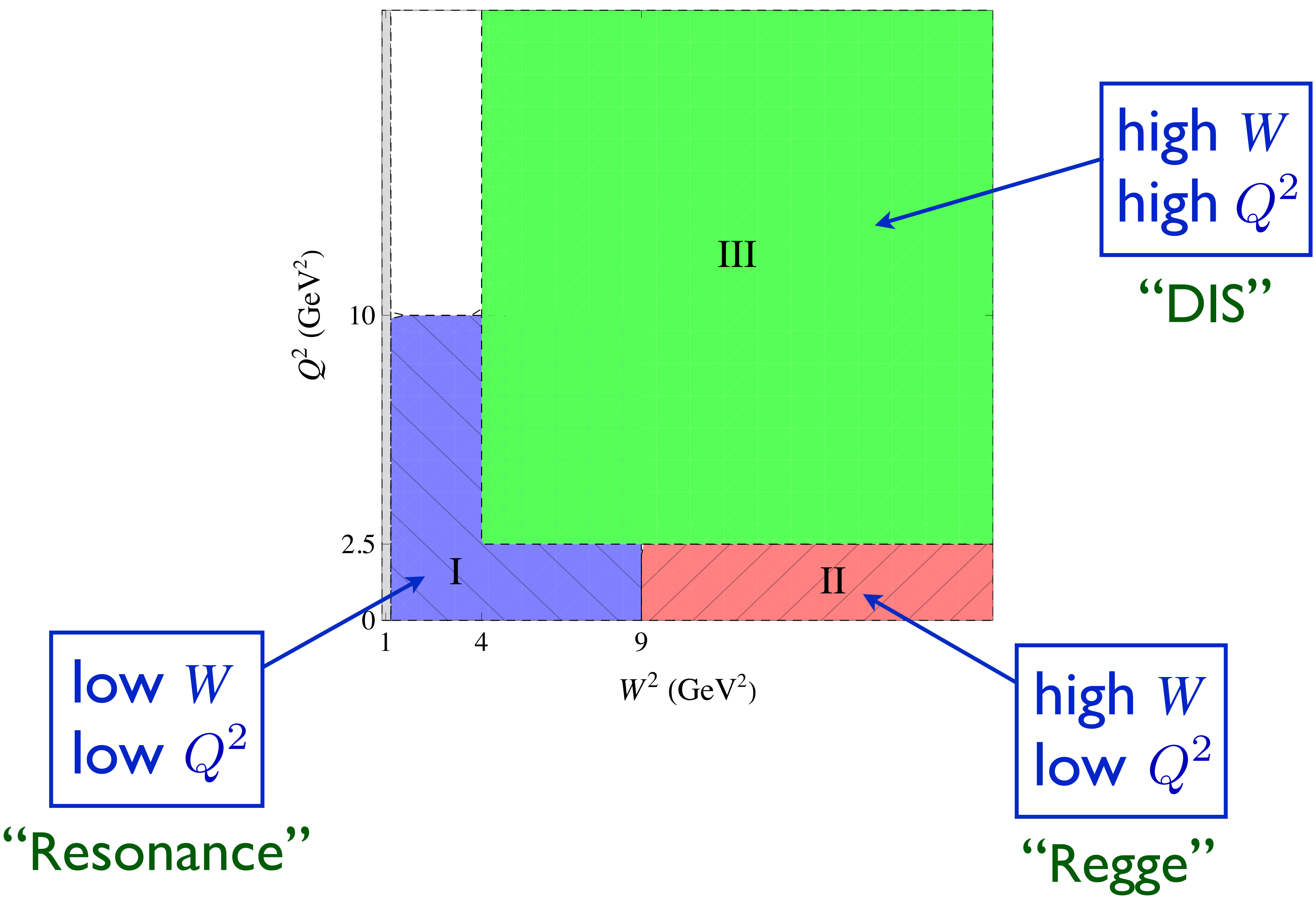
Can the γZ interference be constrained by other observables?

→ Parity-violating inclusive DIS asymmetries

→ Parton distribution functions from global QCD fits

→ New AJM analysis of *constrained* γZ structure functions

AJM γZ model



AJM γZ model

- $F_{1,2}^{\gamma Z}$ structure functions
  - parton model for DIS region

$$F_2^{\gamma Z} = 2x \sum_q e_q \, g_V^q \, (q + \bar{q}) = 2x F_1^{\gamma Z}$$

  - in resonance region use phenomenological input for F_2 ($e.g.$ Christy-Bosted), empirical (SLAC) fit for R

→ for transitions to $I = 3/2$ states ($e.g.$ Δ), CVC and isospin symmetry give $F_i^{\gamma Z} = (1 + Q_W^p) F_i^{\gamma}$

→ for transitions to $I = 1/2$ states, $\gamma\gamma \to \gamma Z$ rotations fixed by CVC and p, n helicity amplitudes

$$\frac{\sigma_p^{\gamma Z}}{\sigma_p^{\gamma\gamma}} = (1 - 4\sin^2\theta_W) - y_R \,, \qquad y_R = \frac{A^p_{R,\frac{1}{2}} A^{n*}_{R,\frac{1}{2}} + A^p_{R,\frac{3}{2}} A^{n*}_{R,\frac{3}{2}}}{|A^p_{R,\frac{1}{2}}|^2 + |A^p_{R,\frac{3}{2}}|^2}$$

*Gorchtein, Horowitz, Ramsey-Musolf, PRC **84**, 015502 (2011)*

AJM γZ model

- $F_{1,2}^{\gamma Z}$ structure functions

  - for background at low Q^2, weak isospin rotation uses VMD

$$\sigma_V^{\gamma Z} = \kappa_V \, \sigma_V^{\gamma\gamma}$$

$$\kappa_\rho = 2 - 4\sin^2\theta_W\,, \quad \kappa_\omega = -4\sin^2\theta_W\,, \quad \kappa_\phi = 3 - 4\sin^2\theta_W$$

$$\frac{\sigma^{\gamma Z}}{\sigma^{\gamma\gamma}} = \frac{\kappa_\rho + \kappa_\omega R_\omega + \kappa_\phi R_\phi + \kappa_C R_C}{1 + R_\omega + R_\phi + R_C}$$

$$R_V = \frac{\sigma^{\gamma^* p \to V p}}{\sigma^{\gamma^* p \to \rho p}}$$

production cross section ratio for vector meson V to ρ meson

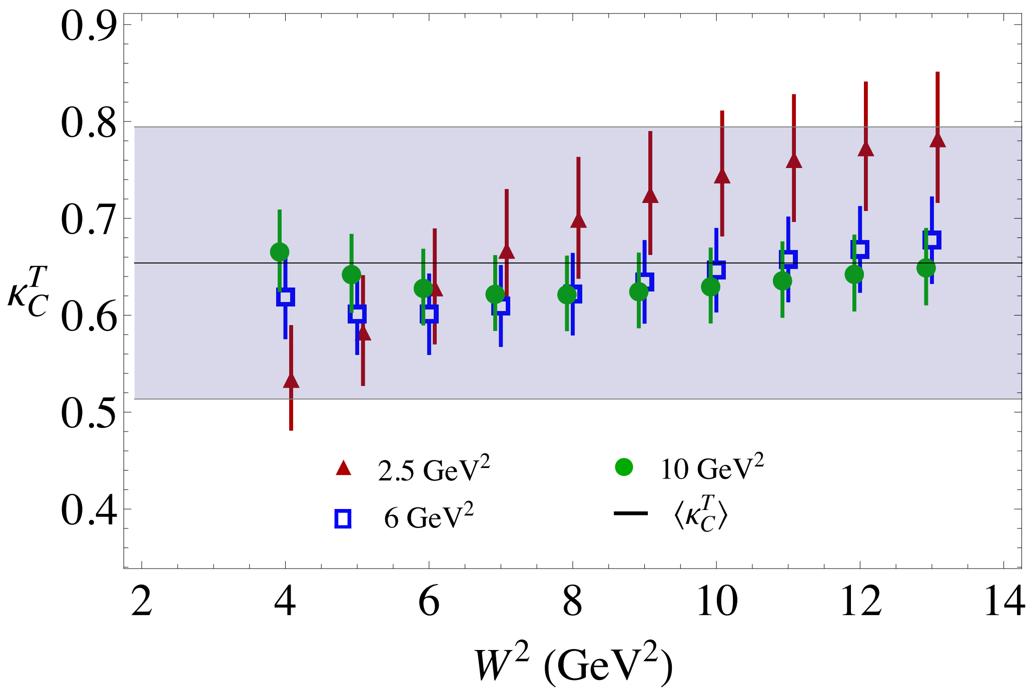
$\longrightarrow$ continuum parameter κ_C not constrained in VMD

$\longrightarrow$ GHRM assume $\kappa_C = 1 \pm 1$ $\longleftarrow$ largest source of error!

AJM γZ model

- Region where continuum contributions are relevant overlaps with typical reach of global PDF fits

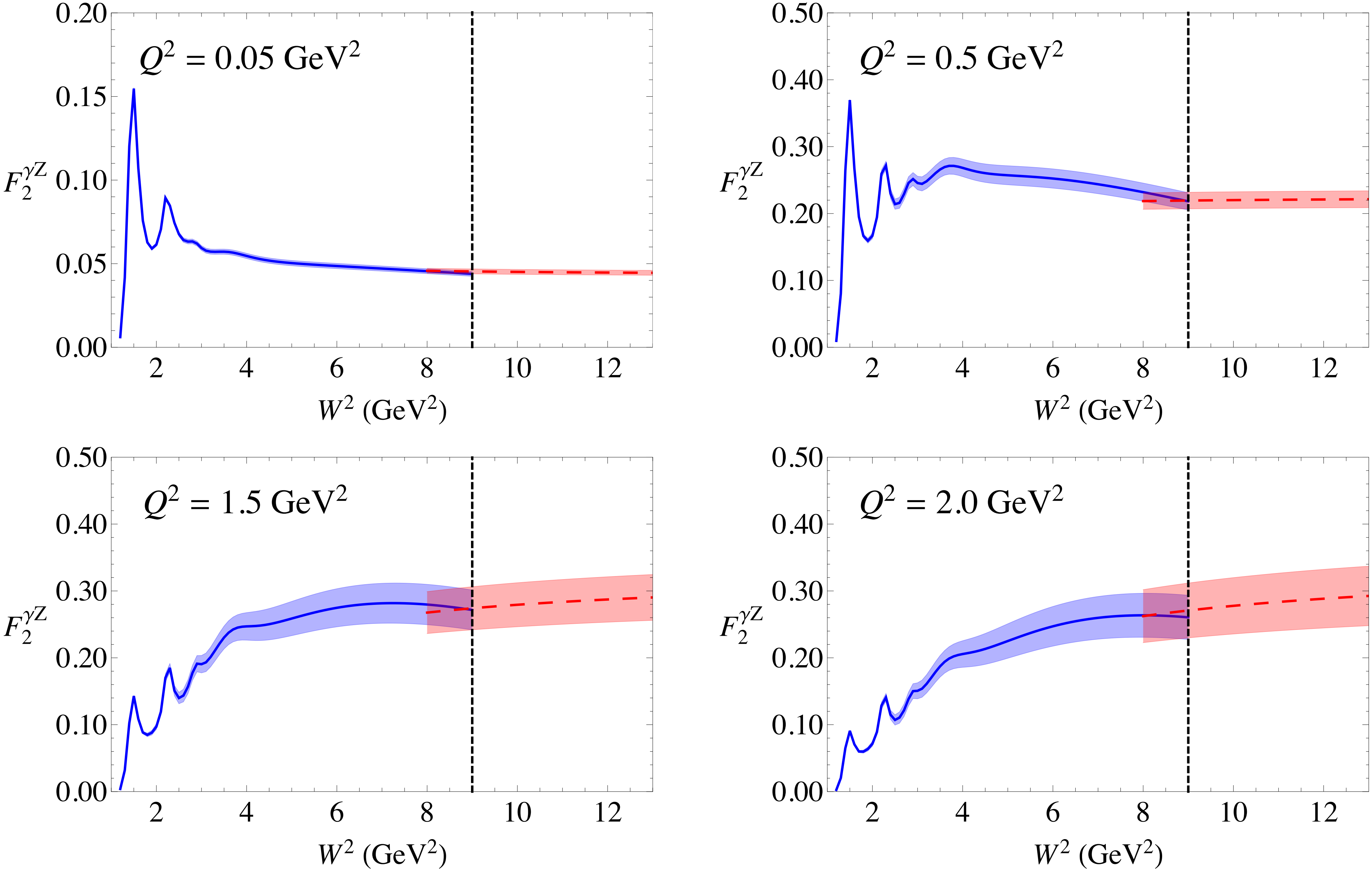
→ constrain κ_C using PDF parametrizations by requiring matching of $F_{1,2}^{\gamma Z}$ to DIS structure functions



→ $\boxed{\kappa_C^T = 0.65 \pm 0.14, \quad \kappa_C^L = -1.3 \pm 1.7}$

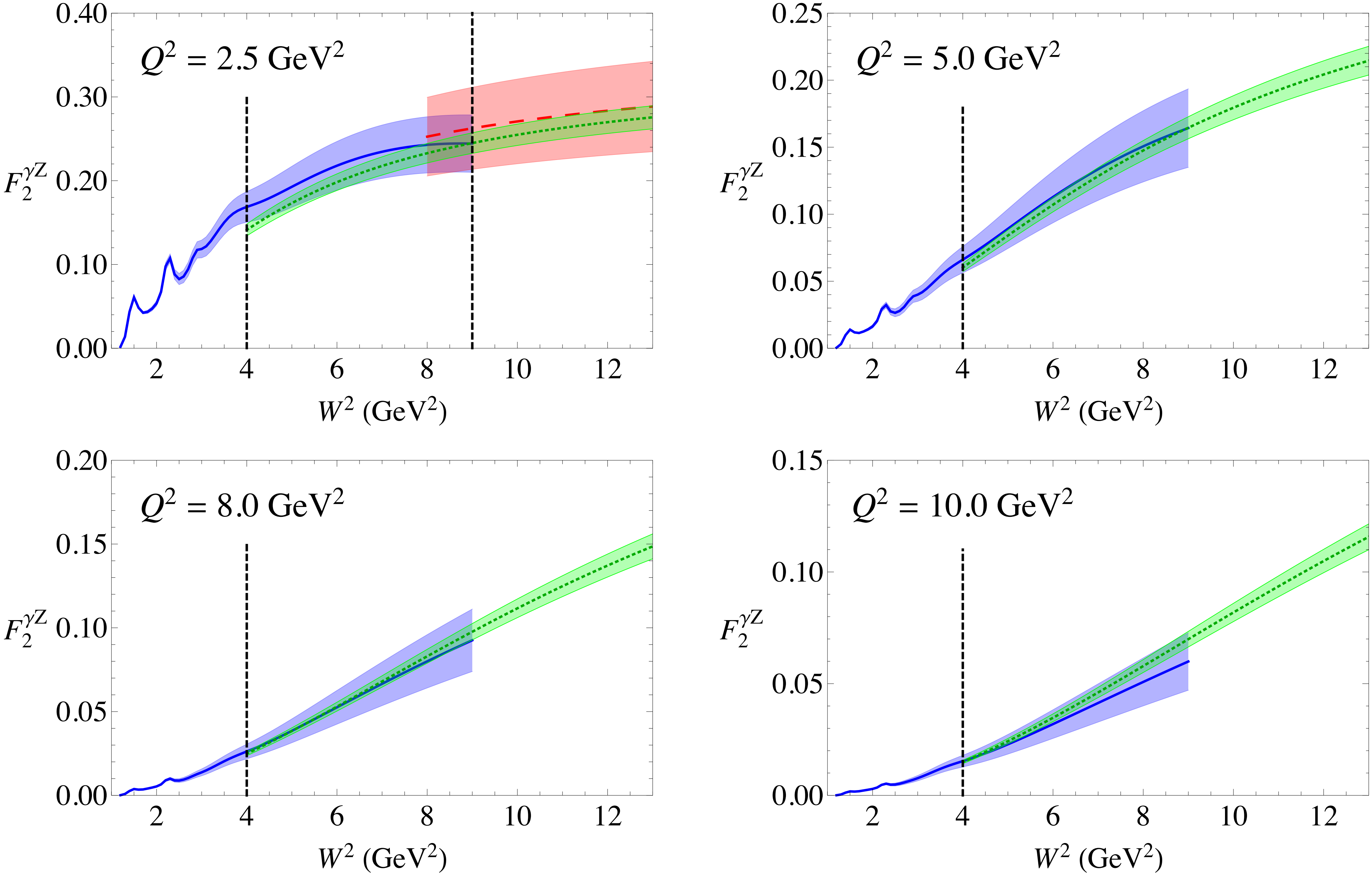
(small contribution to asymmetry)

AJM γZ model



* continuum uncertainty only

AJM γZ model

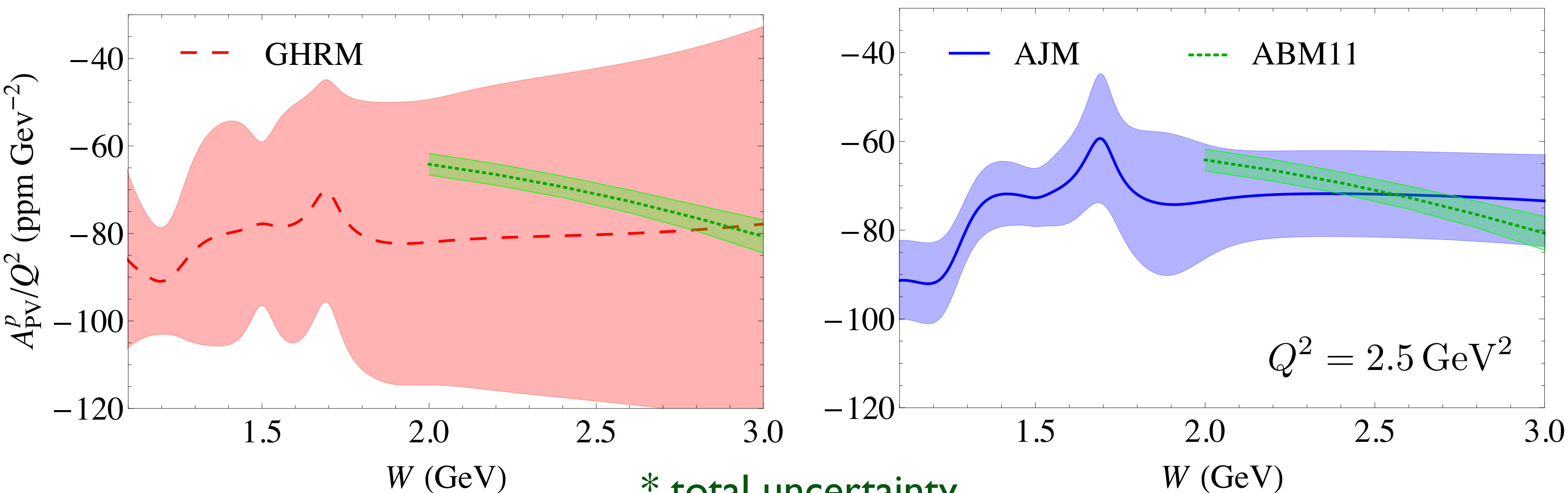


* continuum uncertainty only

AJM γZ model

- PVDIS asymmetry

$$A_{\rm PV} = g_A^e \left(\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\right) \frac{xy^2 F_1^{\gamma Z} + (1-y)F_2^{\gamma Z} + \frac{g_V^e}{g_A^e}(y - y^2/2)xF_3^{\gamma Z}}{xy^2 F_1^{\gamma\gamma} + (1-y)F_2^{\gamma\gamma}}$$

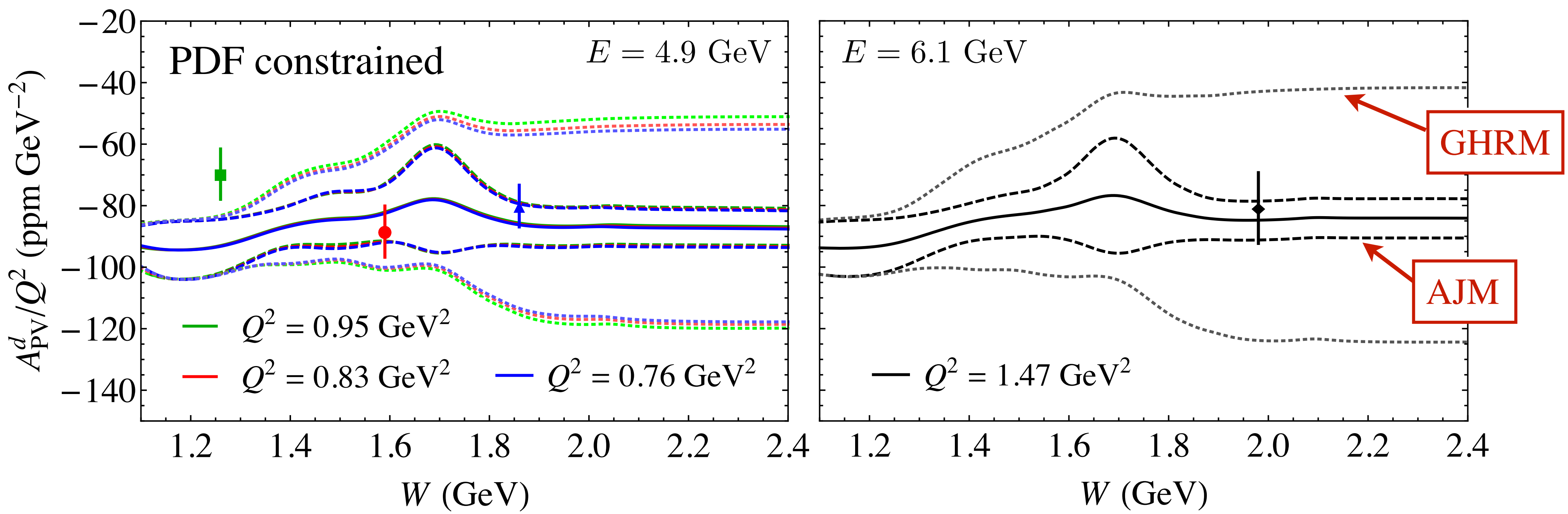


* total uncertainty

→ significantly smaller uncertainties (at typical JLab kinematics) for constrained model

Inclusive PV asymmetries

- Procedure can be tested by comparing with new JLab data on PV asymmetries on deuteron (E08-011*)

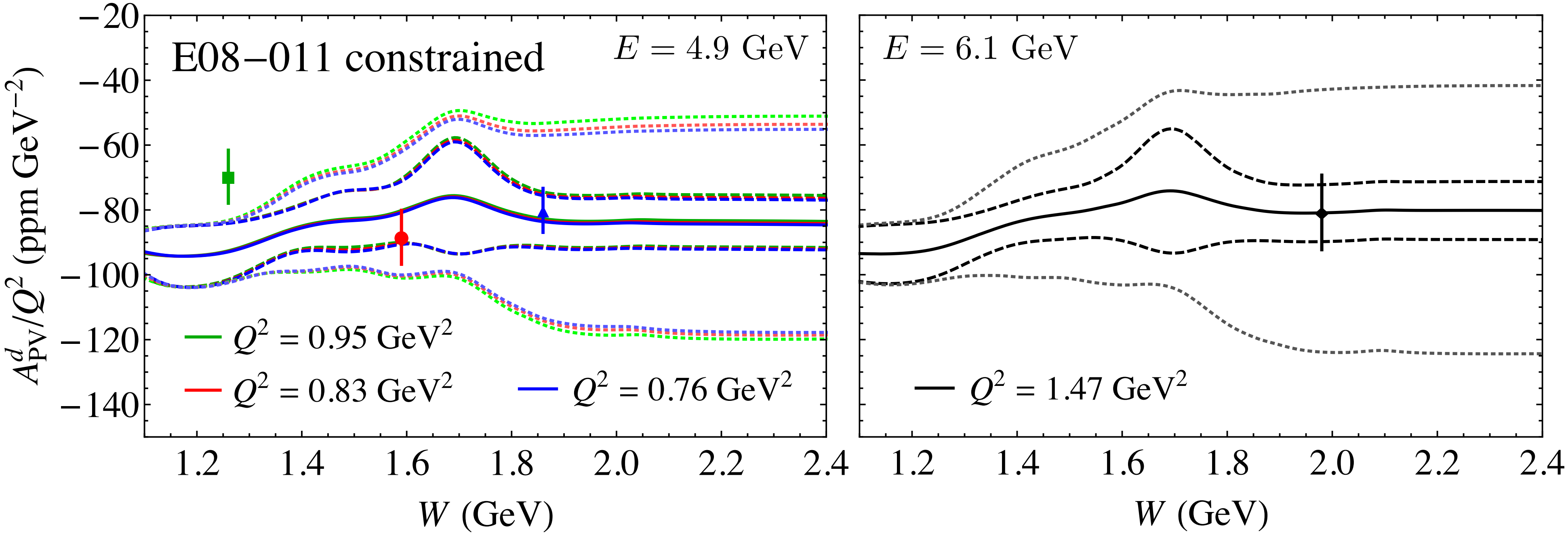


→ agrees well with resonance region PVDIS data
(question about Δ region datum)

* *X. Zheng, P. Reimer, R. Michaels et al.*

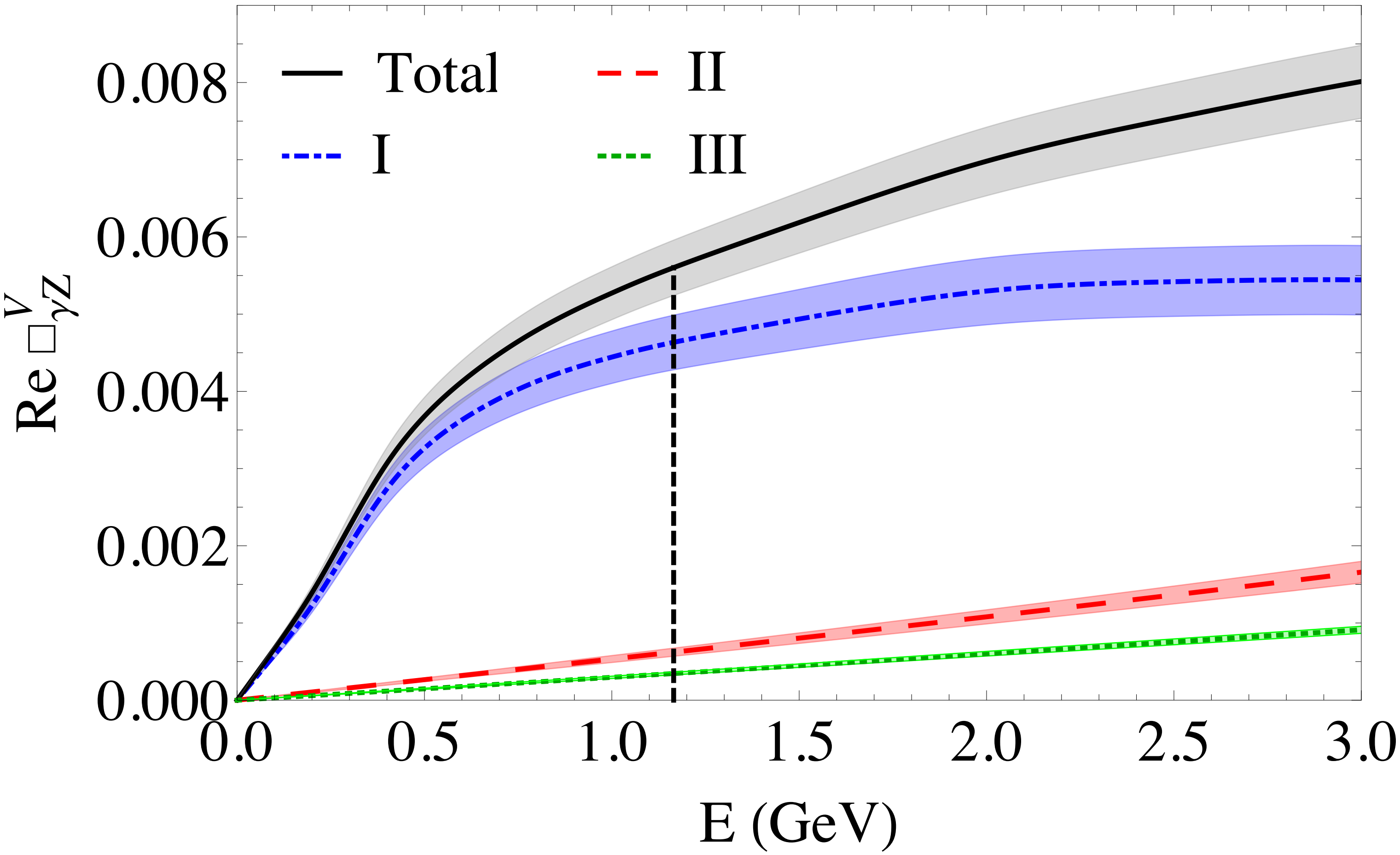
Inclusive PV asymmetries

- Can also use PVDIS-resonance data themselves as constraint, to test consistency of model



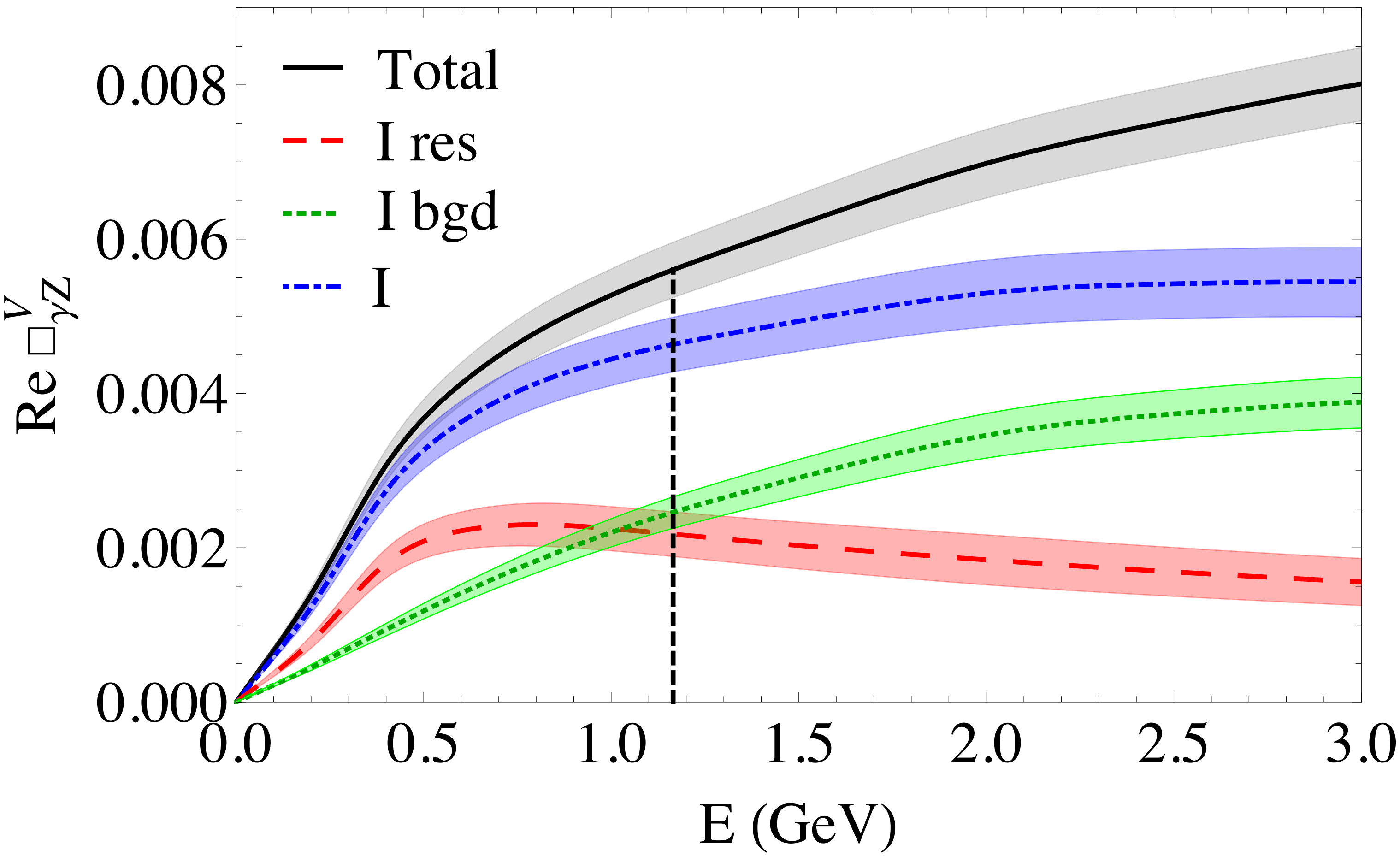
→ slightly larger uncertainties than with PDF constraint, but still ~ 3–4 times smaller (at $W \gtrsim 1.8$ GeV) than GHRM

Correction to Qweak



→ Region I dominates correction & its uncertainty

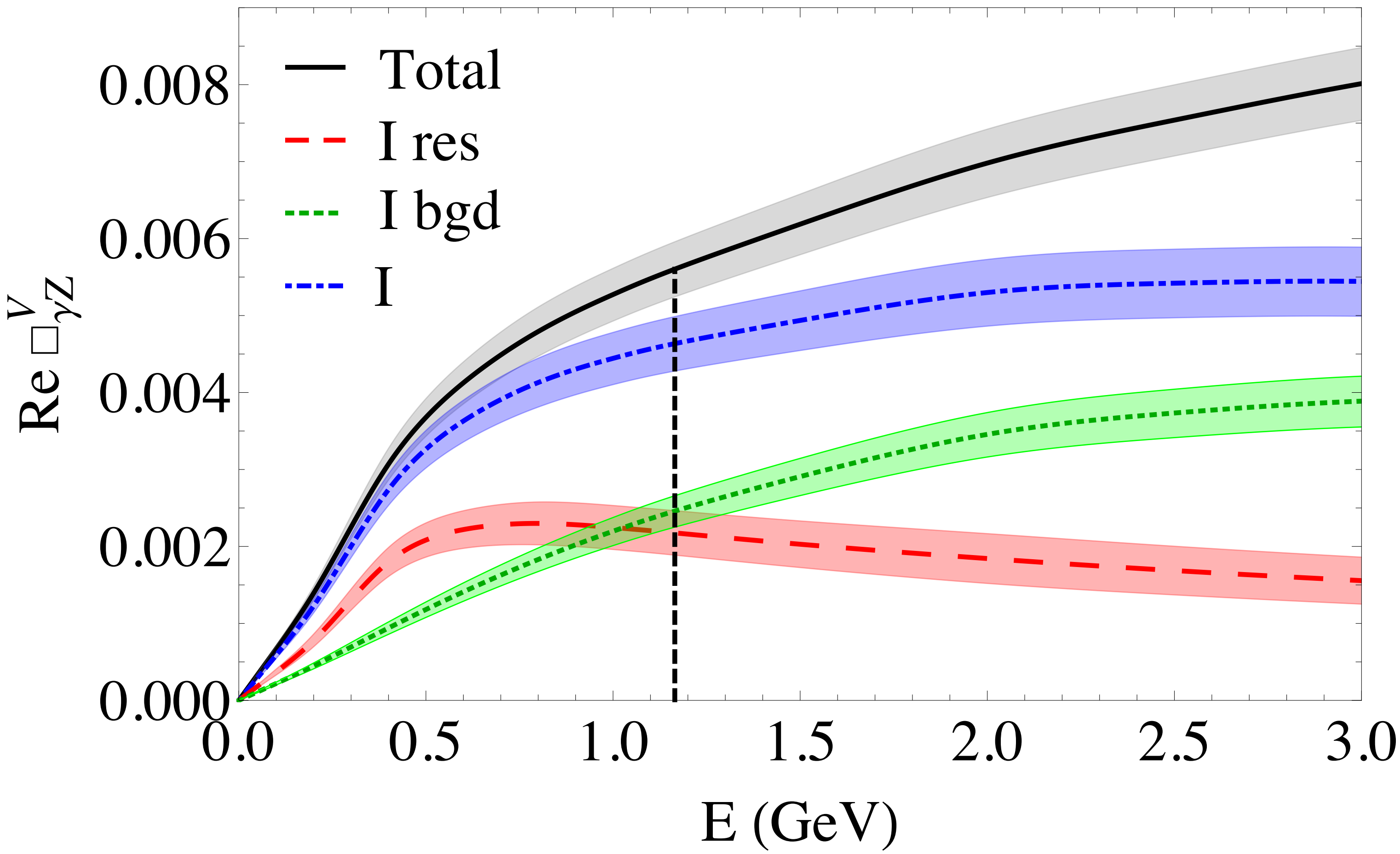
Correction to Qweak



→ Region I dominates correction & its uncertainty

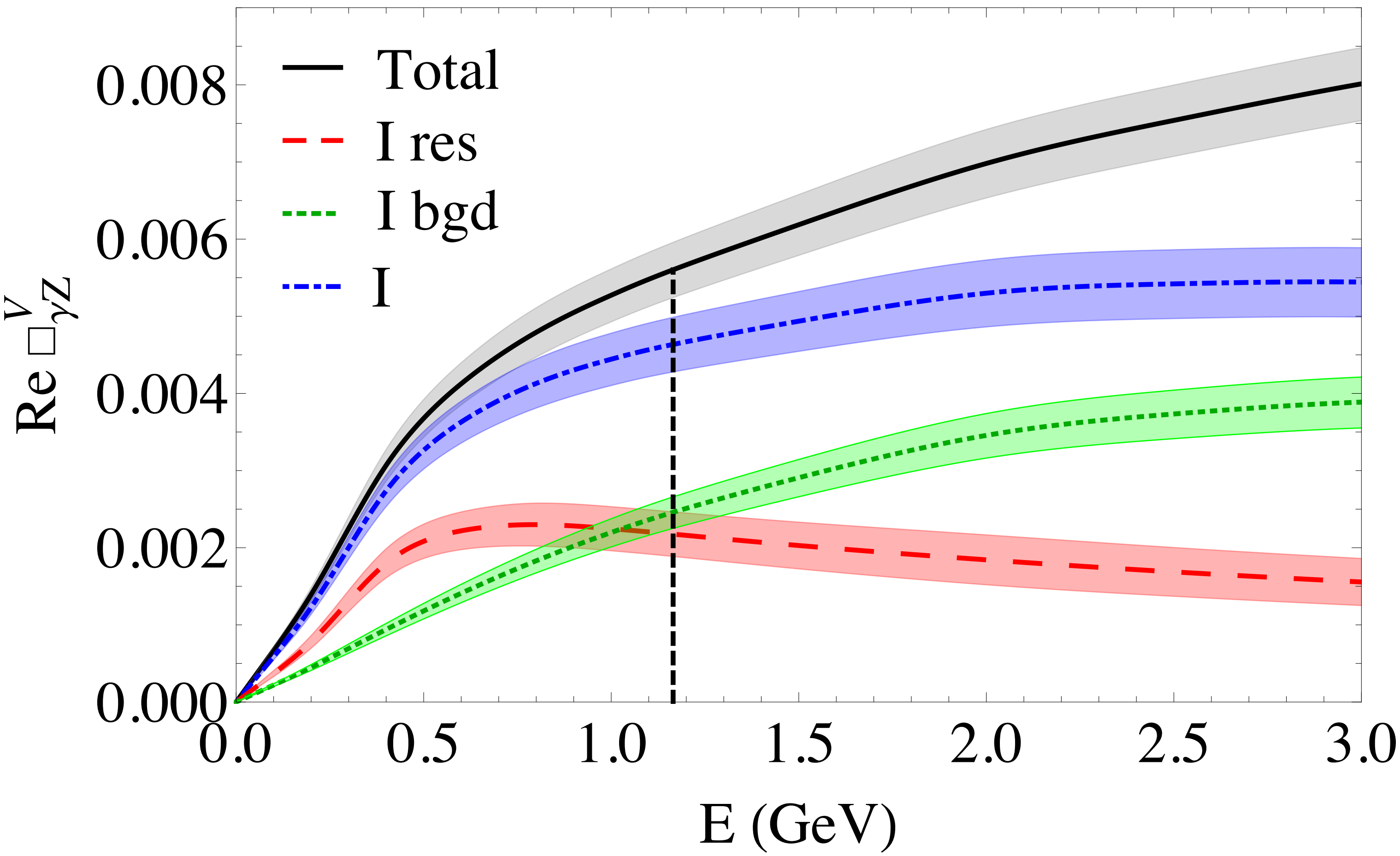
→ resonance & background similar at $E \sim 1$ GeV

Correction to Qweak



$$\rightarrow \quad \Re e\,\Box_{\gamma Z}^{V} = (5.60 \pm \underbrace{0.22}_{\text{background}} \pm \underbrace{0.29}_{\text{resonance}} \pm \underbrace{0.02}_{\text{DIS}}) \times 10^{-3}$$

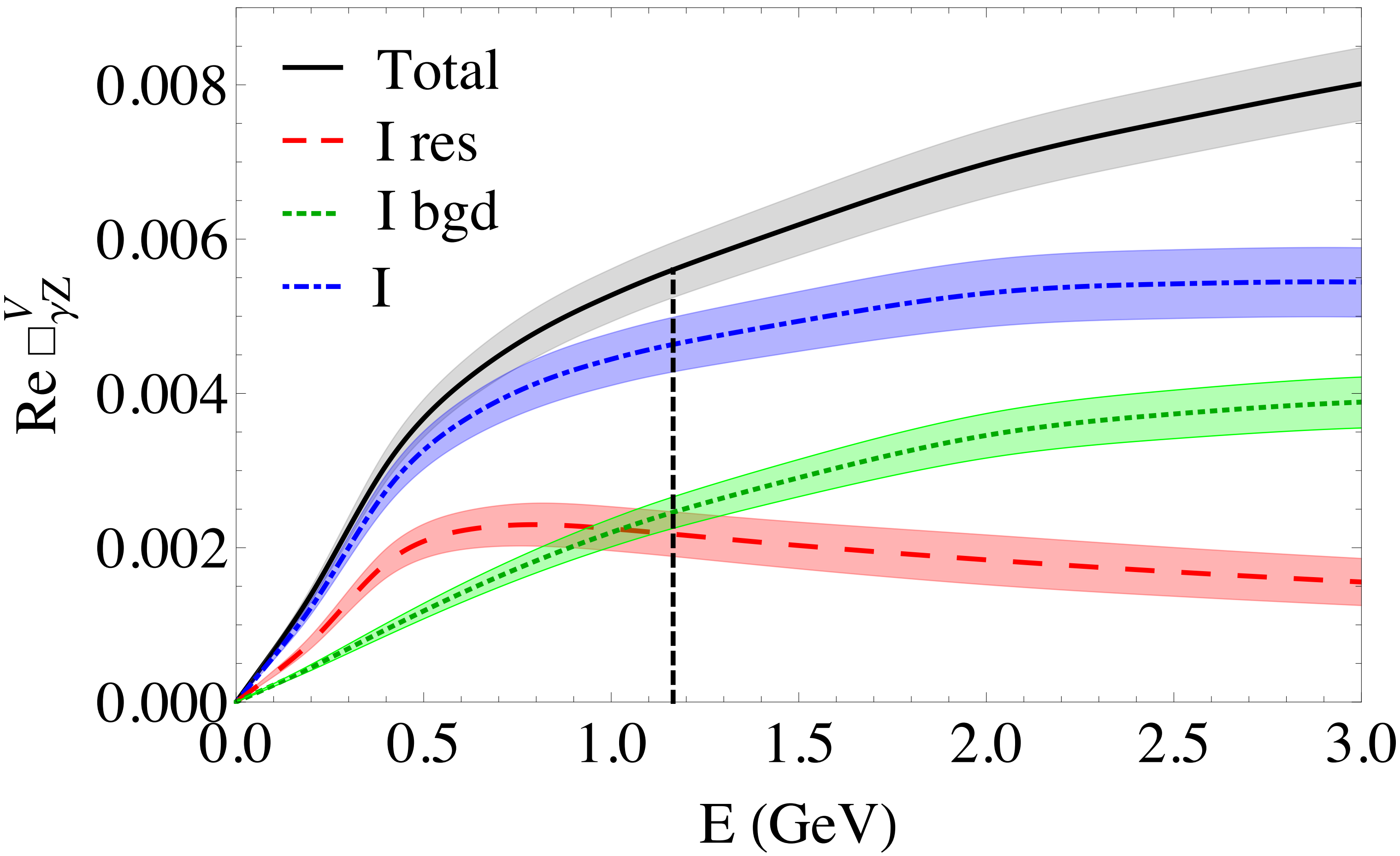
Correction to Qweak



→ $$\Re e\,\Box_{\gamma Z}^{V} = (5.60 \pm 0.36) \times 10^{-3}$$

→ ~ 5 times smaller uncertainty *cf*. GHRM

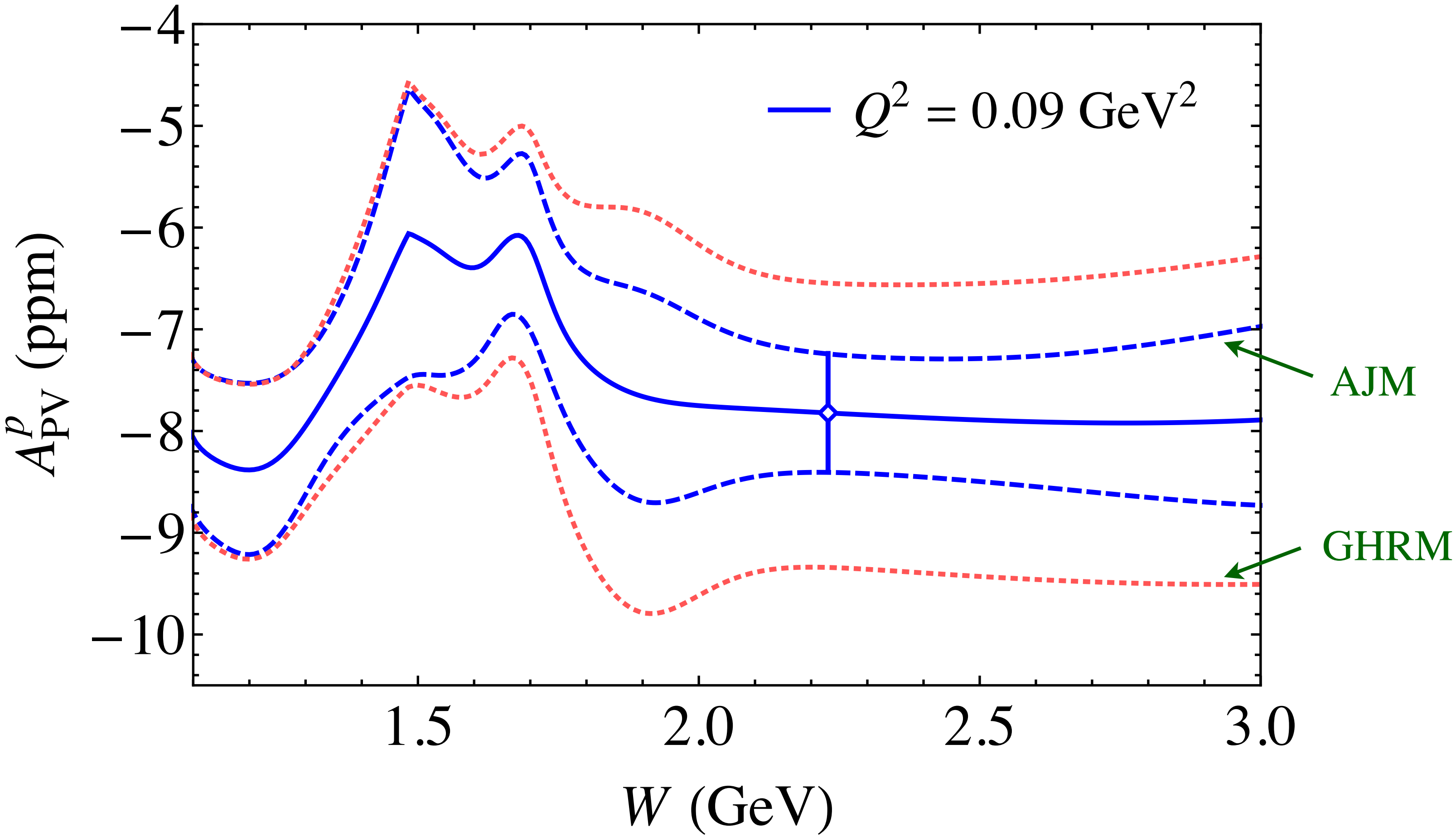
Correction to Qweak



→ $\boxed{\Re e\,\Box_{\gamma Z}^{V} = (5.60 \pm 0.36) \times 10^{-3}}$

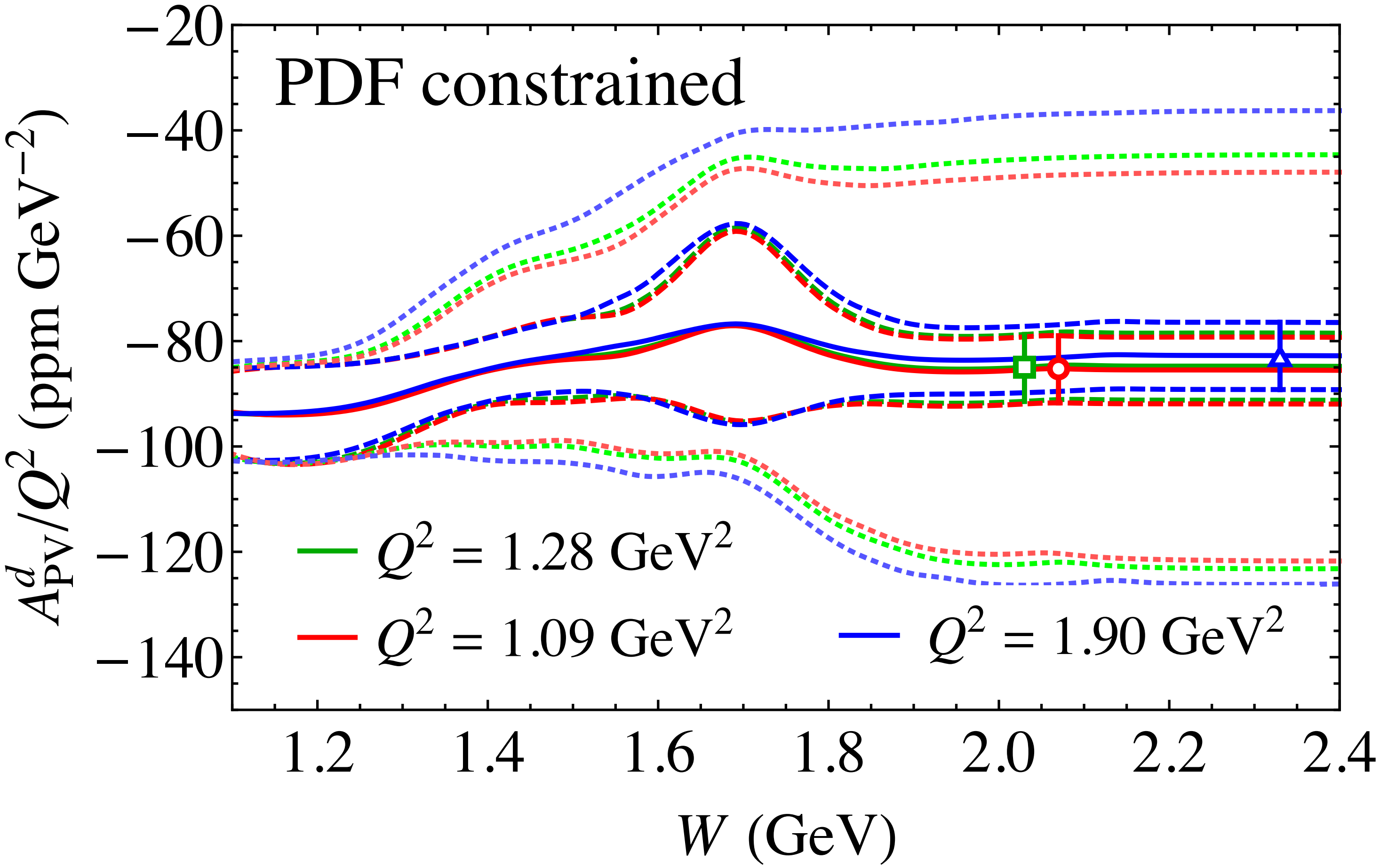
→ ~ 2.5 times smaller uncertainty *cf.* Rislow–Carlson

Expected inelastic asymmetry from Qweak *



* *R. Carlini, M. Dalton et al.*

Expected inelastic asymmetry from E08-011 *



* *X. Zheng, P. Reimer, R. Michaels*

Summary

- γZ box corrections computed *via* dispersion relations from inclusive γZ interference structure functions

  $\longrightarrow$ new formulation in terms of moments puts on firmer footing earlier estimates within free−quark model

- Axial-vector hadron γZ corrections to APV in $^{133}\mathrm{Cs}$

  $\longrightarrow$ shift relative to MS value for $Q_W(\mathrm{Cs})$ of -0.16%
  ($\Delta \sin^2\theta_W \;\approx\; 4$ x SM uncertainty)

- Significant constraints on vector hadron correction from new "PVDIS" asymmetry data & global PDF fits

  $\longrightarrow$ reduces uncertainty on $\Re e\,\Box^V_{\gamma Z}$ by factor ~ $2.5 - 5$

  $\longrightarrow$ additional "PVDIS" data (E08-011, Qweak, SOLID) will further constrain $\Re e\,\Box^V_{\gamma Z}$