Matching heavy-light currents with NRQCD and HISQ quarks

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Outline

- B physics, CKM unitarity and lattice QCD
- HPQCD's HISQ-NRQCD perturbative matching programme
- Calculating f_B, f_{B_s}: the need for a new matching calculation
- Matching HISQ and NRQCD: automated LPT
- f_B and f_{B_s} results

  [H. Na, Weak Decays & Matrix Elements session, Mon. 14:50]
- Extension to mixed action heavy-heavy currents
- Summary

B physics, CKM unitarity and lattice QCD

$|V_{ub}|$ from

- Semileptonic decays, $B \to \pi \ell \nu$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} \propto |V_{ub}|^2 |f_+(q^2)|^2$$

- Leptonic decays, $B \to \ell \nu$

$$\Gamma \propto |V_{ub}|^2 f_B^2$$

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}| & \begin{matrix} |V_{ub}| \\ B\to\tau\nu \\ B\to\pi\ell\nu \end{matrix} \\ |V_{cd}| & |V_{cs}| & \begin{matrix} |V_{cb}| \\ B\to D^{(*)}\ell\nu \end{matrix} \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

nonperturbative parameters

$$\langle 0|A_\mu|B(p)\rangle = i f_B p^\mu$$

overconstrain parameters $\to$ tensions $\Rightarrow$ new physics?

HPQCD's HISQ-NRQCD matching programme

B physics HISQ-NRQCD perturbative matching programme:

1. massless HISQ-NRQCD
   - $B_{(s)}$ leptonic decays: f_B and f_{B_s}
     [H. Na, Weak Decays & Matrix Elements session, Mon. 14:50]
   - match to $\mathcal{O}(\alpha_s, \alpha_s a\Lambda_{\mathrm{QCD}}, \alpha_s/(aM_b), \alpha_s\Lambda_{\mathrm{QCD}}/M_b)$
   - $B_{(s)}$ semileptonic decays
     [C. Bouchard, Weak Decays & Matrix Elements session, Wed. 08:50]
   - match to $\mathcal{O}(\alpha_s, \alpha_s/(aM_b))$
2. massive HISQ-NRQCD
   - HISQ renormalisation parameters: Z_M, Z_Q, ϵ_1
   - scattering channel for $B_{(s)} \to D_{(s)}$ semileptonic decays
   - annihilation channel for $B_c^{(*)}$ decays
   - match to $\mathcal{O}(\alpha_s, \alpha_s/(aM_b))$
3. HISQ-NRQCD four-fermion operator matching for $B_{(s)}$ mixing

Calculating f_B and f_{B_s}:

the need for a new matching calculation

f_{B_q} parameterises decay $B_q \to \ell\nu$

$$\Gamma \propto |V_{ub}|^2 f_B^2$$

Previous results from HPQCD collaboration:

1. f_{B_d}, f_{B_s}: NRQCD b and improved relativistic d/s (ASQTad)
    - 2 lattice spacings arXiv:0902.1815
    - uncertainties of $6-7\%$ in f_{B_d}, f_{B_s} and $\sim 2\%$ in f_{B_s}/f_{B_d}
2. preliminary studies with HISQ b, s

New results from HPQCD collaboration (since Lattice 2011):

- f_{B_s}: highly-improved relativistic b, s (HISQ) arXiv:1110.4510
  - 5 lattice spacings
  - $1/M_b$ expansion up to physical b quark mass
  - uncertainties of $\sim 2\%$ in f_{B_s}
- f_B: ideally calculate with relativistic b, s arXiv:1202.4914
  - but fine lattices and light masses $\Rightarrow$ expensive
  - (currently) more efficient to update NRQCD calculation
  - ASQTad $\rightarrow$ HISQ valence u/d and s
  - taste-breaking discretisation errors reduced by factor of ~ 3
  - uncertainty in f_B and f_{B_s} largely cancels in f_{B_s}/f_B
  - calculate $f_B/f_{B_s}^{(NRQCD)} \times f_{B_s}^{(HISQ)}$
  - result has errors $\sim 2\%$

require new operator matching calculation

Operator matching

f_{B_q} defined through

$$\langle 0|A_\mu|B(p)\rangle = i f_B p^\mu$$

Simulations carried out with effective lattice operators

$$J_0^{(0)} = \overline{\Psi}_q \Gamma_0 \Psi_Q \qquad J_0^{(1)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \Gamma_0 \gamma \cdot \overrightarrow{\nabla}\Psi_Q$$

$$J_0^{(2)}(x) = -\frac{1}{2M_b}\overline{\Psi}_q \gamma \cdot \overleftarrow{\nabla}\gamma_0 \Gamma_0 \Psi_Q$$

Need to match lattice operators to continuum

match perturbatively $\Rightarrow$ Lattice perturbation theory (LPT)

Matching relation:

$$\langle A_0 \rangle_{QCD} = (1 + \alpha_s \rho_0) \langle J_0^{(0)} \rangle + (1 + \alpha_s \rho_1) \langle \widetilde{J}_0^{(1)} \rangle + \alpha_s \rho_2 \langle \widetilde{J}_0^{(2)} \rangle$$

Use improved currents with better power law behaviour.

$$\widetilde{J}_0^{(i)} = J_0^{(i)} - \alpha_s \zeta_{10} J_0^{(0)}$$

Matching coefficients:

$$\rho_0 = B_0 - \frac{1}{2}\underbrace{(Z_H + Z_q)}_{\text{wavefn. renorm.}} - \zeta_{00}$$

$$\rho_1 = B_1 - \frac{1}{2}\overbrace{(Z_H + Z_q)} - Z_M - \overbrace{\zeta_{01} - \zeta_{11}}^{\text{mixing matrix elements}}$$

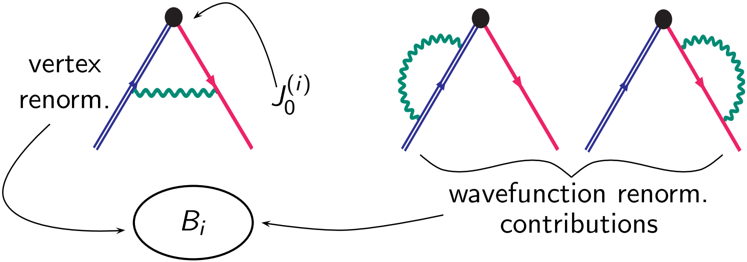
mass. renorm.

$$\rho_2 = B_2 - \underbrace{\zeta_{02} - \zeta_{12}}$$

cont. contributions

Calculating matching coefficients

Diagrams for continuum $\langle A_0 \rangle_{QCD}$:

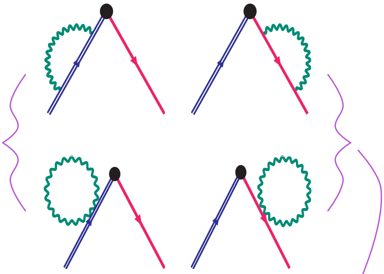


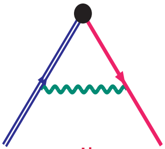
$$\rho_0 = B_0 - \frac{1}{2}(Z_H + Z_q) - \zeta_{00}$$

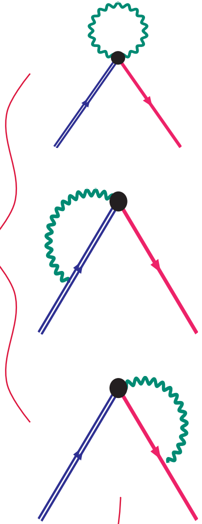
$$\rho_1 = B_1 - \frac{1}{2}(Z_H + Z_q) - Z_M - \zeta_{01} - \zeta_{11}$$

$$\rho_2 = B_2 - \zeta_{02} - \zeta_{12}$$

Diagrams for lattice $\langle J_0^{(i)} \rangle$:







$$\rho_0 = B_0 - \frac{1}{2}\left(Z_H + Z_q\right) - \zeta_{00}$$

$$\rho_1 = B_1 - \frac{1}{2}\left(Z_H + Z_q\right) - Z_M - \zeta_{01} - \zeta_{11}$$

$$\rho_2 = B_2 - \zeta_{02} - \zeta_{12}$$

Lattice perturbation theory

Independent determinations of lattice quantities:

1. automated lattice perturbation theory: `HiPPy` and `HPsrc`
   - `HiPPy` – `python` routines produce Feynman rules encoded as "vertex files" Hart, von Hippel, Horgan arXiv:0904.0375
   - `HPsrc` – `Fortran 90` routines reconstruct diagrams and evaluate integrals with `VEGAS`
2. "by hand" calculation
   - `Mathematica` file handles Dirac algebra
   - `Fortran` suite extracts Feyman rules via iterated convolution
   - integrals evaluated numerically with `VEGAS`

Cross checks for the matching results

Consistency checks for the matching calculation

1. Both methods agree!
2. Reproduce expected infrared behaviour:
   - Z_q, ζ_{00} and ζ_{11} logarithmically divergent
   - fit to gluon mass regulator
   - control divergence with subtraction function
3. Run in Feynman and Landau gauges to confirm Z_M and e.g. ζ_{10} gauge independent
4. Reproduce previous matching calculation results with massless ASQTad-NRQCD results in PRD **69** (2004) 074501

advantage of automated LPT: simply change input files

f_B and f_{B_s} results

We find arXiv:1202.4914

$$f_B = 0.191(9)\,\mathrm{GeV} \qquad \text{and} \qquad f_{B_s} = 0.228(10)\,\mathrm{GeV}$$

so

$$\frac{f_{B_s}}{f_B} = 1.188(18)$$

Agreement with previous HPQCD HISQ result a non-trivial consistency check: $f_{B_s}^{(HISQ)} = 0.225(4)\,\mathrm{GeV}$

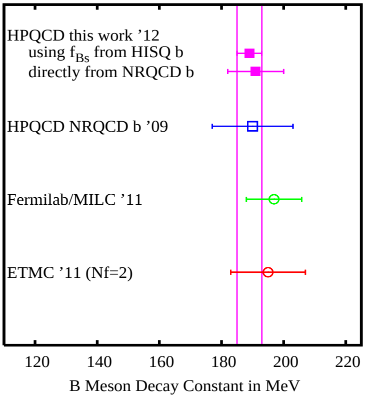
Combining NRQCD-HISQ ratio with HISQ $f_{B_s}^{(HISQ)}$

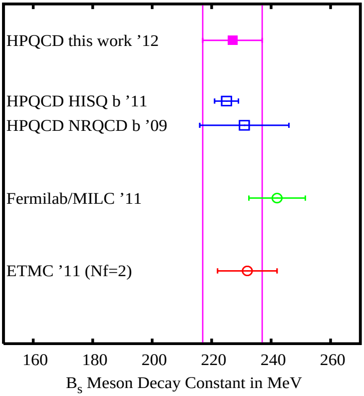
$$f_B = \frac{f_B}{f_{B_s}} \times f_{B_s}^{(HISQ)} = 0.189(4)\,\mathrm{GeV}$$

Error budget

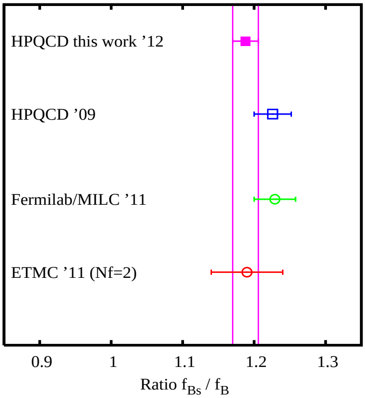
	f_B	f_{B_s}	f_{B_s}/f_B
statistical	1.2	0.6	1.0
$\mathcal{O}(\alpha_s)$ operator matching	4.1		0.1
relativistic	1.0		0.0
r_1 scale	1.1		-
continuum extrapolation	0.9		0.9
chiral extrapolation	0.2	0.5	0.6
mass tuning	0.2	0.1	0.2
finite volume	0.1	0.3	0.36
total	4.7	4.4	1.6

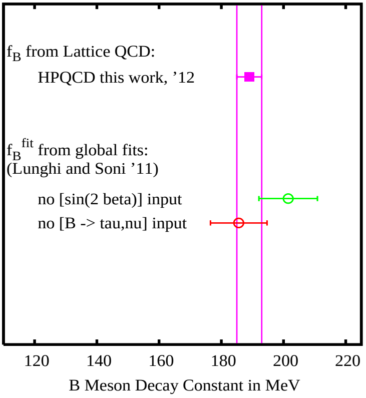
f_B and f_{B_s} results





f_{B_s}/f_B and $f_B^{(\mathrm{fit})}$ results





Mixed action heavy-heavy currents

Massless HISQ action:

crossing symmetry $\Rightarrow$ scattering and annihilation equivalent

Mixed action heavy-heavy currents

Massless HISQ action:

crossing symmetry $\Rightarrow$ scattering and annihilation equivalent

Massive HISQ action:

crossing symmetry $\Rightarrow$ scattering and annihilation equivalent

Mixed action heavy-heavy currents

Massless HISQ action:

crossing symmetry ⇒ scattering and annihilation equivalent

Massive HISQ action:

~~crossing symmetry ⇒ scattering and annihilation equivalent~~

Scattering channel a simple extension of massless case:

1. set quarks onshell: $(G_0^{\text{HISQ}})^{-1} = 0$ and $(i\tilde{\not{p}} + m)u(p) = 0$
2. update vertex files ...
3. ... then just rerun code!

Annihilation channel (slightly) more involved:

1. replace quarks with antiquarks
2. isolate linear infrared divergence with subtraction function

For both cases recalculate HISQ renormalisation parameters

Massive HISQ renormalisation parameters

Relate m_Q, Z_Q to parameters in action via quark propagator.

Quark two-point function

quark propagator

$$\langle \psi(t,\mathbf{p}')\overline{\psi}(0,\mathbf{p})\rangle = (2\pi)^3\delta(\mathbf{p}-\mathbf{p}')G(t,\mathbf{p}),$$

Starting point: quark field creates single- and multi-particle states

$$G(t,\mathbf{p}) = \mathcal{Z}_2(\mathbf{p})e^{-E(\mathbf{p})t}\Gamma_{\text{proj}} + \cdots .$$

Define:

- mass renormalisation $m_Q \equiv Z_m m_0 \equiv E(\mathbf{p}=\mathbf{0})$
- wavefunction renormalisation $Z_Q \equiv \mathcal{Z}_2(\mathbf{p}=\mathbf{0})$

renormalise at $p = (iE, \mathbf{0})$

$$G(t,\mathbf{0}) = Z_Q e^{-Et}\frac{1+\gamma_0}{2} + \cdots .$$

Introduce self energy via

$$G^{-1}(p) = G_0^{-1}(p) - \Sigma(p),$$

For the HISQ action

HISQ tuning parameter

$$G_0^{-1}(p) = \sum_\mu i\gamma_\mu \sin(p_\mu)\left(1 + \frac{1+\epsilon}{6}(\sin p_\mu)^2\right) + m_0$$

Write one loop self energy as

$$\Sigma^{(1)}(p) = \sum_\mu i\gamma_\mu \sin(p_\mu)\,\Sigma_\mu^{(a)}(p) + \Sigma^{(b)}(p)$$

At $p = (iE, \mathbf{0})$ pole condition is

m_Q

$$\sinh(E)\left(1 - \frac{1+\epsilon}{6}(\sinh(E))^2 - \alpha_s \Sigma_0^{(a)}\right) = m_0 - \alpha_s \Sigma^{(b)},$$

$\epsilon = \epsilon_{\text{tree}} + \alpha_s \epsilon_1$ $\quad$ $E = m_{\text{tree}} + \alpha_s m_1$

- Tree level

$$\sinh(m_{\text{tree}})\left[1 - \frac{1+\epsilon_{\text{tree}}}{6}(\sinh(m_{\text{tree}}))^2\right] = m_0.$$

Fix ϵ_{tree}: set pole mass = kinetic mass

$$\epsilon_{\text{tree}} = -1 + \frac{2}{(\sinh(m_{\text{tree}}))^2}\left[2 - \sqrt{1 + \frac{3m_{\text{tree}}}{\cosh(m_{\text{tree}})\sinh(m_{\text{tree}})}}\right].$$

Solve simultaneously for m_{tree}

- Repeat at one loop ...

Match expressions for propagator

Z_Q

$$\int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}p_0}{2\pi} e^{-ip_0 t} G(p_0, \mathbf{0}) = Z_Q e^{-Et} \frac{1+\gamma_0}{2} + \cdots .$$

Re-express as

$$-i \oint \frac{\mathrm{d}z}{2\pi} z^{t-1} G(z, \mathbf{0}) \equiv -i \oint \frac{\mathrm{d}z}{2\pi} \frac{g_1(z)}{g_2(z)}$$

Z_Q defined as residue at $\mathbf{p} = \mathbf{0}$ or $z = z_1 = e^{-E}$

$$\operatorname*{Res}_{z=z_1} G(z, \mathbf{0}) = z_1^t \frac{g_1(z_1)}{z_1 g_2'(z_1)} = e^{-Et} \frac{1+\gamma_0}{2} f(E)$$

Compare equations $\Rightarrow$ $\quad z_Q = f(E)$

$$Z_Q^{-1} = \cosh(E) \left[1 - \frac{1+\epsilon}{2} (\sinh(E))^2 \right] + i\alpha_s \frac{\mathrm{d}}{\mathrm{d}p_0} \left[i \sin(p_0) \Sigma_0^{(a)} + \Sigma^{(b)} \right]$$

$\epsilon = \epsilon_{\text{tree}} + \alpha_s \epsilon_1$ $\qquad$ $E = m_{\text{tree}} + \alpha_s m_1$

Summary

- HISQ-NRQCD operator matching for f_B and f_{B_s}
  $\Rightarrow$ combined results give uncertainties of $\sim 2\%$ for f_B
- B physics HISQ-NRQCD perturbative matching programme:
  1. massless HISQ-NRQCD for $B_{(s)}$ decays ✔
  2. massive HISQ-NRQCD
     - new determination of HISQ renormalisation parameters ✔
     - scattering channel for $B_{(s)} \longrightarrow D_{(s)}$ semileptonic decays ✔
     - annihilation channel for $B_c^{(*)}$ decays
  3. HISQ-NRQCD four-fermion operator matching for $B_{(s)}$ mixing

Thank you!

Fits and correlators

- Delta function and Gaussian smearing used at both source and sink for meson correlators
- Random wall sources in operator-meson correlators
- Correlators fitted between
  - $t_{\rm min} = 2 \sim 4$ and $t_{\rm max} = 16$ on coarse ensembles
  - $t_{\rm min} = 4 \sim 8$ and $t_{\rm max} = 24$ on fine ensembles
- Bayesian multiexponential fits with $t_{\rm min}$, $t_{\rm max}$ fixed and no. exponentials increased until saturation in results

Chiral and lattice spacing fits

Fit to lattice spacing dependence as described by Rachel Dowdall, but include chiral fit.

- Fit to

$$\Phi_q = f_{B_q}\sqrt{M_{B_q}} = \Phi_0\left(1 + \delta f_q + [\text{analytic}]\right)\left(1 + [\text{disc.}]\right)$$

- δf_q includes chiral logs using one-loop χPT and lowest order in $1/M$
- [analytic] - powers of m_{val}/m_c and m_{sea}/m_c, with m_c scale chosen for convenience
- [disc.] - powers of $(a/r_1)^2$ with expansion coefficient functions of aM_b or am_q

ASQTad action correct to $\mathcal{O}(a^2)$, strongly reduced $\mathcal{O}(\alpha_S a^2)$ errors:

$$S_{\mathrm{ASQTad}} = \sum_x \overline{\psi}(x) \left(\gamma^\mu \Delta_\mu^{\mathrm{ASQTad}} + m \right) \psi(x)$$

where

$$\Delta_\mu^{\mathrm{ASQTad}} = \Delta_\mu^F - \frac{1}{6} (\Delta_\mu)^3.$$

F indicates

$$U_\mu \to \mathcal{F}_\mu \widetilde{U}_\mu = u_0^{-1} \left[\prod_{\nu \neq \mu} \left(1 + \frac{\Delta_\nu^{(2)}}{4} \right)_{\mathrm{symm}} - \sum_{\nu \neq \mu} \frac{(\Delta_\nu)^2}{4} \right] U_\mu$$

HISQ action correct to $\mathcal{O}(a^4)$, $\mathcal{O}(\alpha_S a^2)$ with reduced taste=changing:

$$S_{\mathrm{HISQ}} = \sum_x \overline{\psi}(x) \left(\gamma^\mu \Delta_\mu^{\mathrm{HISQ}} + m\right) \psi(x)$$

where

$$\Delta_\mu^{\mathrm{HISQ}} = \Delta_\mu \left[\mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right] - \frac{1+\epsilon}{6}(\Delta_\mu)^3 \left[U \mathcal{F}_\mu^{\mathrm{HISQ}} U_\mu(x)\right].$$

and

$$\mathcal{F}_\mu^{\mathrm{HISQ}} = \mathcal{F}_\mu^{\mathrm{ASQTad}} U_\mu \mathcal{F}_\mu^{\mathrm{ASQTad}}$$

Lattice NRQCD action

$$S_{\mathrm{NRQCD}} = \sum_{\mathbf{x},\tau} \psi^{+}(\mathbf{x},\tau)\left[\psi(\mathbf{x},\tau) - \kappa(\tau)\psi(\mathbf{x},\tau-1)\right]$$

with

$$\kappa(\tau) = \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n U_4^\dagger \left(1 - \frac{H_0}{2n}\right)^n \left(1 - \frac{\delta H}{2}\right)$$

- Link variable in temporal direction: $U_4^\dagger$
- Leading nonrelativistic kinetic energy: $H_0 = -\Delta^{(2)}/2M$
- Higher order terms in δH:
  - Chromoelectric and chromomagnetic interactions
  - Leading relativistic kinetic energy correction
  - Discretisation error corrections

Automated LPT: HiPPy

`HiPPy` generates Feynman rules, encoded as "vertex files"
To generate vertex files:

- Expand link variables

Lüscher and Weisz, NPB 266 (1986) 309

$$U_{\mu>0}(x) = \exp\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right) = \sum_{r=0}^{\infty}\frac{1}{r!}\left(gA_\mu\left(x+\frac{\hat{\mu}}{2}\right)\right)^r$$

with $U_{-\mu} \equiv U_\mu^\dagger(x-\hat{\mu})$

- Actions built from products of link variables - Wilson lines

$$L(x,y;U) = \sum_r\left(\frac{g^r}{r!}\right)\sum_{k_1,\mu_1,a_1}\cdots\sum_{k_r,\mu_r,a_r}\widetilde{A}_{\mu_1}^{a_1}(k_1)\cdots\widetilde{A}_{\mu_r}^{a_r}(k_r) \times V_r(k_1,\mu_1,a_1;\ldots;k_r,\mu_r,a_r)$$

where the V_r are "vertex functions"

- Vertex functions decomposed into colour structure matrix, C_r and "reduced vertex", Y_r

$$V_r(k_1,\mu_1,a_1;\ldots;k_r,\mu_r,a_r) = C_r(a_1;\ldots;a_r)Y_r(k_1,\mu_1;\ldots;k_r,\mu_r)$$

- Reduced vertices are products of exponentials

$$Y_r(k_1,\mu_1;\ldots;k_r,\mu_r) = \sum_{n=1}^{n_r} f_n \exp\left(\frac{i}{2}\left(k_1\cdot v_1^{(n)} + \cdots + k_r\cdot v_r^{(n)}\right)\right)$$

where the f_n are amplitudes and the $v^{(n)}$ the locations of each of the r factors of the gauge potential

- Feynman rules encoded as ordered lists

$$E = (\mu_1,\cdots,\mu_r; x, y; v_1,\cdots,v_r; f)$$

For example, the product of two links, $L(0, 2x, U) = U_x(0)U_x(x)$, is

$$U_x(0)U_x(x) = \left[\sum_{r_1=0}^{\infty} \frac{1}{r_1!}\left(gA_x\left(\frac{x}{2}\right)\right)^{r_1}\right]\left[\sum_{r_2=0}^{\infty} \frac{1}{r_2!}\left(gA_x\left(\frac{3x}{2}\right)\right)^{r_2}\right]$$

$$= 1 + g\sum_{k_1} \widetilde{A}_x(k_1)e^{ik_1\cdot x/2} + g\sum_{k_2} \widetilde{A}_x(k_2)e^{i2_1\cdot 3x/2} + \dots$$

$$= 1 + g\sum_{k_1}\sum_{a_1} \widetilde{A}_x^{a_1}(k_1)T^{a_1}\left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Vertex function

$$V_1(k_1, x, a_1) \equiv C_1(a_1)Y_1(k_1, x) = T^{a_1}\left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \left(e^{ik_1\cdot x/2} + e^{ik_1\cdot 3x/2}\right)$$

Reduced vertex

$$Y_1(k_1, x) = \sum_{n=1}^{n_1=2} f_n \exp\left(\frac{i}{2}\left(k_1 \cdot v_1^{(n)}\right)\right)$$

So in this case

$$f_1 = f_2 = 1\ ;\ \ v_1^{(1)} = (1,0,0,0)\ ,\ \ v_1^{(2)} = (3,0,0,0)$$

We store this information as the list

$$\begin{aligned} E &= (\mu_1; x, y; v_1^{(1)}, v_1^{(2)}; f) \\ &= (x; (0,0,0,0), (2,0,0,0); (1,0,0,0), (3,0,0,0); (1,1)) \end{aligned}$$