Space-time picture of chiral dynamics with nucleons

C. Weiss (JLab), Nuclear Dynamics with EFTs, Bochum, 01–Jul–13

- Parton picture of nucleon structure
  Fields vs. particles
  Transverse densities from elastic FFs

- Peripheral transverse densities
  Dispersion representation
  Peripheral densities from chiral EFT
  Chiral vs. non–chiral component
  $\Delta$ isobar and large–$N_c$ QCD

- Time–ordered formulation of chiral EFT
  Chiral $\pi N$ light–cone wave functions
  Particle picture of chiral processes

- Connections and extensions
  GPDs and peripheral quark/glueon structure
  Nuclear structure in high–energy processes

Spatial representation of nucleon:

$\rho(b)$

charge, magnetization

$\sim 1/M_\pi$

b

chiral component

New insights into chiral EFT:
Heavy–baryon expansion,
chiral vs. non–chiral contributions

Connection with QCD structure:
GPDs, peripheral $ep/pp$ processes
Nucleon structure: Parton picture

- QCD vacuum not empty
  
  Strong non-perturbative gluon fields of size $\ll 1 \text{ fm}$ → Lattice QCD, analytic models

  Chiral symmetry breaking: $\bar{q}q$ pair condensate, $\pi$ as collective excitation

- Slow–moving nucleon $P \sim \mu_{\text{vac}}$
  
  $\langle N| J_\mu |N \rangle$ from Euclidean correlation functions

  No concept of particle content!
  Cannot separate "constituents" from vacuum fluctuations

- Fast–moving nucleon $P \gg \mu_{\text{vac}}$
  
  Closed system: Wave function, Gribov, Feynman

  variable particle number, $x_i, k_{T,i}$

  Physical properties:
  Longitudinal momentum densities PDFs
  Transverse distributions → Form factors, GPDs

  QCD operator definitions: Renormalization, scale dependence

  Alt. view: Observer moves with velocity $v \to 1$

  Light–front quantization, time $x^+ = x^0 + x^3$
Nucleon structure: Transverse densities

- Current matrix element parametrized by invariant form factors
  \[ \langle N'|J_\mu|N \rangle \rightarrow F_1(t), F_2(t) \text{ Dirac, Pauli} \]

- Transverse densities \( t = -\Delta_T^2 \) Soper 76, Miller 07
  \[ F_{1,2}(t) = \int d^2b \ e^{i\Delta_T b} \rho_{1,2}(b) \] 2D Fourier
  \[ \rho_1(b) \] charge density
  \[ \tilde{\rho}_2(b) = \frac{d}{db} \left[ \frac{\rho_2(b)}{2M_N} \right] \] spin–dependent current density
  \( b \) displacement from transverse C.M.

- Proper densities for relativistic system
  Overlap of light–front wave functions.
  Breit frame distributions not densities.

- Reduction of quark GPDs
  \[ \rho_1(b) = \int dx f_{q-\bar{q}}(x, b) \]
  Elastic FFs \( \leftrightarrow \) QCD structure, high–energy processes
Nucleon structure: Peripheral densities

- Empirical transverse densities from spacelike form factor data

  Experimental and incompleteness errors estimated  Venkat, Arrington, Miller, Zhan 10

  Recent low– and high–$|t|$ data incorporated
  MAMI: Vanderhaeghen, Walcher 10. JLab Hall A Riordan et al.

  Many interesting questions: Neutron, flavor structure, charge vs. magnetization

- Peripheral densities $b = O(M_{\pi}^{-1})$

  Governed by chiral dynamics: universal, model–independent, calculable using EFT methods

  Theoretical interest:
  Parametric control, space–time picture of EFT dynamics, chiral vs. non-chiral contributions

  Practical interest:
  Low–$|t|$ form factors, connection w. peripheral quark/gluon structure
Peripheral densities: Dispersion representation

- Dispersion representation of form factor

$$ F_{1,2}(t) = \int_{4m^2_\pi}^{\infty} \frac{dt'}{t' - t - i0} \frac{\text{Im} F_{1,2}(t')}{\pi} $$

Spectral function $\text{Im} F_{1,2}(t')$ describes “process” current $\rightarrow$ hadronic states $\rightarrow N\bar{N}$

Unphysical region: $\text{Im} F_{1,2}(t')$ from theory, FF fits Höhler et al. 76; Belushkin, Hammer, Meissner 06

- Transverse densities

$$ \rho_{1,2}(b) = \int_{4m^2_\pi}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{tb}) \frac{\text{Im} F_{1,2}(t)}{\pi} $$

$$ K_0 \sim e^{-b\sqrt{t}} $$ exponential suppression of large $t$

Distance $b$ selects masses $\sqrt{t} \sim 1/b$: “Filter”
Cf. Borel transformation in QCD sum rules. Strikman, CW 10

Peripheral $\rho(b) \leftrightarrow$ low–mass hadronic states

- Isovector: $\pi, \rho, \rho', \ldots$
- Isoscalar: $\omega, \phi, K\bar{K}, \ldots$
Peripheral densities: Spectral function

- Spectral function near threshold

  \[ \frac{M_\pi^4}{M_N^2} \quad \text{two–pion cut} \]

  Two–pion exchange with \( t - 4M_\pi^2 = O(M_\pi^2) \)

  Subthreshold singularity on unphysical sheet from \( N \) pole in \( \pi N \) scattering amplitude

  Anomalously small scale \( \frac{M_\pi^4}{M_N^2} \)

  Dominates behavior of spectral function near threshold!

- Parametric regions of distances

  \[ b \sim M_\pi^{-1} \quad t - 4M_\pi^2 \sim M_\pi^2 \quad \text{“chiral”} \]

  \[ \sim \frac{M_N^2}{M_\pi^3} \quad \sim \frac{M_\pi^4}{M_N^2} \quad \text{“molecular”} \]
Peripheral densities: Chiral component

- Spectral function from relativistic $\chi$EFT
  
  \begin{align*}
  &\text{Becher, Leutwyler 99; Kubis, Meissner 01; Kaiser 03} \\
  &\text{Efficient calculation: } t\text{–channel cut only,} \\
  &\text{Cutkosky rules, no regularization} \\
  &\text{Compact analytic expressions}
  \end{align*}

- Chiral component of isovector densities

  \begin{align*}
  &\text{Strikman, CW 10; Granados CW 13} \\
  &\rho_{V,1,2}(b) = e^{-2M_\pi b} P_{1,2}(M_N, M_\pi, b) \\
  &\text{Yukawa tail with range } 2M_\pi \\
  &\text{Pre-exponential factor varies strongly,} \\
  &\text{exhibits rich structure} \\
  &\text{Heavy–baryon expansion for } b = O(M_\pi^{-1}): \\
  &\text{Convergence limited by subthreshold singularity,} \\
  &\text{but good numerical accuracy } \sim 10\% \\
  &\text{Granados CW 13. Cf. Becher, Leutwyler 99} \\
  &\text{Molecular region } b = O(M_N^2/M_\pi^3): \\
  &\text{Asymptotic behavior derived explicitly} \\
  &\text{Very large distances } \sim \text{ several 10 fm. Practical applications?}
  \end{align*}
Peripheral densities: Chiral vs. non-chiral

- At what distances does the chiral component of densities become numerically dominant?  
  Strikman, CW 10

  Model higher mass states in spectral function by $\rho$ meson pole  
  Refined estimates w. empirical spectral functions  
  Miller, Strikman, CW 11

  Chiral component dominates only at $b > 2$ fm. Surprisingly large!

  Reasons are strength of $\rho$, suppression of $\pi\pi$ near threshold

- Spatial representation as new way of identifying chiral component

  Model–independent, fully relativistic  
  Impact parameter $b$ objectively defined, observable in exclusive processes  
  $\leftrightarrow$ Breit frame radius
Peripheral densities: $\Delta$ isobar

- Two–pion component with intermediate $\Delta$

  Large coupling due to spin/isospin

  $N$ and $\Delta$ degenerate in large–$N_c$ limit of QCD:
  \[ M_\Delta - M_N = O(N^{-1}_c) \]

  $\Delta$ contribution to spectral functions and densities calculated in relativistic Rarita–Schwinger formalism
  Strikman, CW 10, Granados, CW 13

- Peripheral densities in large–$N_c$ limit of QCD

  Transverse distances $b = O(M^{-1}_\pi) = O(N^0_c)$

  $\rho_1(N \text{ alone}) = O(N^2_c)$ too large!
  $\rho_1(N + \Delta) = O(N_c)$ correct

  $\Delta$ restores proper $N_c$–scaling of isovector charge density

  $\rho_2(N + \Delta) = O(N^2_c) = \frac{3}{2}\rho_2(N \text{ alone})$

  $\Delta$ enhances isovector magnetization density by $3/2$

  Agrees with findings for isovector electric/magnetic radii
  Cohen, Broniowski 92; Cohen 96
Time–ordered formulation: Wave functions

\[ \psi_{\pi N}(y, r_T) = \lim_{P \to \infty} \frac{\langle \pi N | \mathcal{L}_\chi | N \rangle}{E_{N_f} + E_{\pi} - E_{N_i}} \]

- **Time–ordered formulation of \( \chi \)EFT**
  Infinite–mom. frame \( P \to \infty \)
  Light–front time \( x^+ = x^0 + x^3 \) equivalent!

- **Wave function of chiral \( \pi N \) system**
  Describe transition \( N \to N\pi \),
  calculable from chiral Lagrangian
  Universal, frame–independent,
  also in high–energy processes, \( \bar{u} - \bar{d} \)
  Pion momentum fraction \( y \sim M_\pi / M_N \),
  parametrically small
  Orbital angular momentum \( L = 0, 1 \)

- **Densities as overlap integrals**
  Contact terms \( \delta(y) \) represent high–mass intermediate states in TOPT.
  Coefficient \( (1 - g_A^2) \) reflects “compositeness” of nucleon
  Equivalent to invariant formulation

Granados, CW 13
Time–ordered formulation: Few–body picture

- Light–front time evolution of $\chi$EFT

  Bare $N$ fluctuates into $\pi N$ system via $\chi$EFT interaction

  Peripheral densities result from charge/current carried by pion at $b = O(M_\pi)$

  Light–front formulation frame–independent: Interpretation in rest frame

  “Few–body picture” of chiral nucleon
  Fully relativistic!

- Explains peripheral densities

  Nucleon polarized in $y$–direction

  $\langle J^+(b) \rangle = \rho_1(b) + (2S_y) \cos \phi \tilde{\rho}_2(b) \geq 0$

  for current carried by quasi–real pion, therefore $|\tilde{\rho}_2| \leq \rho_1$

  $\tilde{\rho}_2/\rho_1 \sim v_\pi$ pion velocity
Outlook: Quark/gluon structure, nuclei

- Peripheral quark/gluon structure of nucleon
  
  Parton densities at $b \sim M_{\pi}^{-1}$ and $x \sim M_{\pi}/M_N$

  Calculable from $\chi$EFT $\pi N$ wave functions and empirical quark/gluon densities in pion
  
  Same $\pi N$ WFs as in transverse charge/current densities!

  Experimental probes: $x$–dependent transverse size, peripheral pion knockout in high–energy $ep/pp$

- Light–front structure of light nuclei in $\chi$EFT

  High–energy $eA/hA$ scattering processes sensitive to low–energy nuclear structure

  Light–front formulation essential: Factorization, momentum conservation, sum rules

  I) Inclusive quark/gluon structure: EMC effect, antishadowing

  II) High–energy processes with detected spectators: Neutron structure, nuclear modifications
  
  Becomes feasible with medium–energy Electron–Ion Collider

  New application of nuclear EFT? Great need for theoretical control
Summary

- Light–front (or partonic) formulation provides concise spatial representation of relativistic system
  
  Elastic FFs reveal transverse densities
  Independent of dynamics; can be applied to QCD, $\chi$EFT

- Peripheral transverse densities from $\chi$EFT
  
  Chiral expansion justified by $b = O(M_\pi^{-1})$, new parameter
  Chiral and non–chiral components identified by spatial size
  Chiral component dominant only at large $b \gtrsim 2 \text{ fm}$
  Inclusion of $\Delta$ ensures proper $N_c$ scaling of densities

- Light–front time evolution of $\chi$EFT
  
  “Few–body picture” of low–energy chiral nucleon structure
  Connection with quark/gluon structure, high–energy processes

- Light–front nuclear structure new challenge for nuclear $\chi$EFT
  
  High–energy processes with tagged spectators: Great potential, need theoretical control
Supplementary material
Spectral analysis: Isovector charge density

- Empirical isovector spectral function
  
  Near–threshold $\pi\pi$ from chiral dynamics
  $\rho$ region from $\pi\pi$ phase shifts  
  Höhler 76
  High–mass continuum from form factor fits
  Belushkin, Hammer, Meissner 07

- Spectral analysis of isovector density
  Strikman, CW 10; Miller, Strikman, CW 11
  
  Near–threshold $\pi\pi$ relevant only at $b > 2$ fm
  
  Intermediate $b = 0.5 - 1$ fm dominated by $\rho$, with $\sim 10\%$ correction from higher masses
  "Vector dominance" quantified

- Isoscalar density
  
  $\omega$ dominates at $b > 1.5$ fm.
  
  Large cancellations between $\omega$ and higher–mass states at $b = 0.5 - 1$ fm

Model–independent identification of chiral component, “vector dominance” in QCD
Spectral analysis: Isoscalar charge density

- Isoscalar spectral function
  \[ \omega \text{ exhausts strength below } 1 \text{ GeV}^2 \]
  Non-resonant $3\pi$ negligible

  Large negative strength above $1 \text{ GeV}^2$, dynamical origin unclear
  \[ \phi NN \text{ coupling } \leftrightarrow s\bar{s} \text{ content of nucleon} \]

  High–mass continuum from form factor fits
  Belushkin, Hammer, Meissner 07

- Spectral analysis of isoscalar density
  Miller, Strikman, CW 11

  \[ \omega \text{ dominates at } b > 1.5 \text{ fm} \]
  Fit uncertainty in $\omega NN$ coupling $\pm 15\%$

  Large cancellations between $\omega$ and higher–mass states at $b = 0.5 - 1 \text{ fm}$

- Impact of future form factor data
  Sensitivity to $\omega NN$ coupling broadly distributed at spacelike $|t| \lesssim 1 \text{ GeV}^2$
  Does not require measurements at extremely small $|t|$
Peripheral hard processes

- **Hard exclusive process on peripheral pion**
  
  \[ k_\pi^2 \sim M_\pi^2 \] quasi-real
  
  Requires \( x \ll M_\pi/M_N \sim 0.1 \)

- **Kinematics with**\( p_T(\pi) \gg p_T(N) \)
  
  suppresses production on nucleon

  \[ F_{\pi NN}(t) \text{ softer than } \text{GPD}_{\pi}(t) \]

- **Probe gluon GPD in pion at** \( |t_\pi| \sim 1 \text{ GeV}^2 \)

  Fundamental interest

  Moments calculable in Lattice QCD

- **Requires detection of forward nucleon and moderate–\( p_T \) pion**

  Feasible with Electron–Ion Collider EIC

  Direct probe of chiral component of nucleon’s partonic structure!
Chiral component: Effect on form factors

- Moments of transverse charge density

\[
\langle b^2 \rangle = \int d^2 b \, b^2 \rho(b) = 4 F'_1(0)
\]

\[
\langle b^4 \rangle = 32 F''_1(0)
\]

- Contribution of chiral component \textit{isovector}

\[
\langle b^2 \rangle_{\text{chiral}} \approx 0.2 \times \langle b^2 \rangle_{\text{fit}} \quad \text{small}
\]

\[
\langle b^4 \rangle_{\text{chiral}} \approx 1.5 \times \langle b^2 \rangle^2_{\text{fit}} \quad \text{sizable}
\]

Chiral component should be visible in “unnatural” second and higher derivatives of FF at \( Q^2 = 0 \)
Can we extract it?

- Analyticity of form factor fit is essential

Needs dispersion analysis: Belushkin et al. 07

- Affects extrapolation to \( t \to 0 \)

CLAS/PRIMEX 12 GeV experiment at \( Q^2 = 10^{-4} - 10^{-2} \text{GeV}^2 \)
PR12-11-106 Gasparian et al.