Nucleon Structure at Large Bjorken x (HiX2014)
Laboratori Nazionali di Frascati
November 18, 2014

Finite Q^2 corrections to structure functions

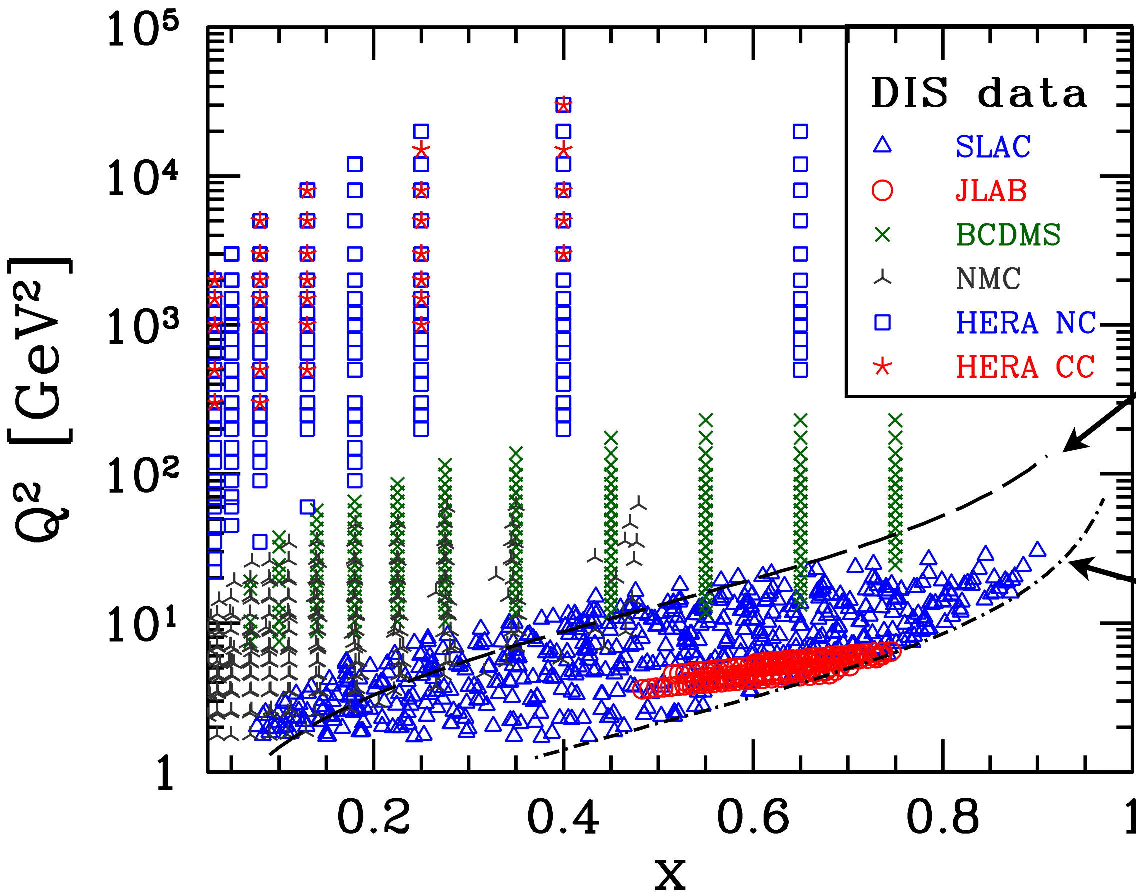
Wally Melnitchouk

Jefferson Lab

Outline

- DIS at finite Q^2
- Target mass corrections (TMC) in DIS
- Hadron mass corrections (HMC) in SIDIS
- Intrinsic charm at low (and high) Q^2
- Outlook

■ To probe high-x region at finite energy requires use of data at lower W & Q^2



$$W^2 = M^2 + Q^2 \frac{(1-x)}{x}$$

strong cut:

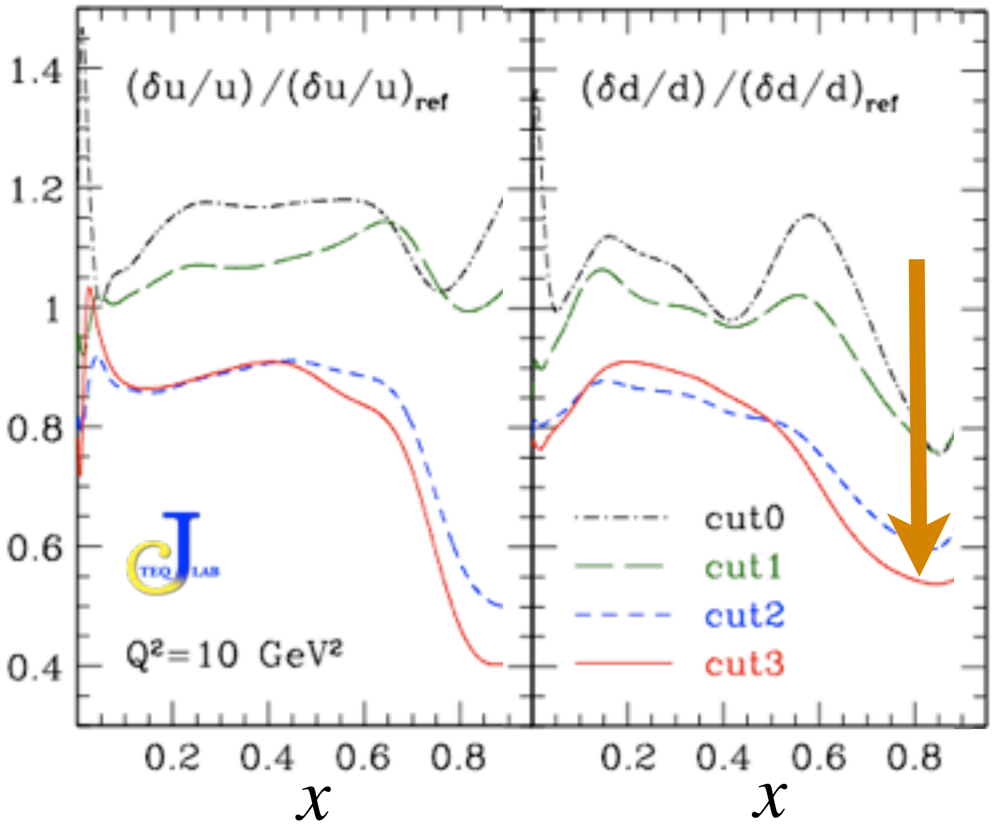
$Q > 2$ GeV, $W > 3.5$ GeV

weak cut:

$Q > 1.3$ GeV , $W > 1.73$ GeV

→ factor 2 increase in # of DIS data points when relax **strong cut** (excludes most SLAC, all JLab data) → **weak cut**

- To probe high-x region at finite energy requires use of data at lower W & Q^2



$$W^2 = M^2 + Q^2 \frac{(1-x)}{x}$$

cut0: strong cut

cut3: weak cut

Accardi et al.
PRD ***81****, 034016 (2010)*

→ significant error reduction when cuts extended to low-W region

Issues at low Q^2

- Need for systematic study of effects of W & Q^2 cuts to ensure stability of PDF analysis
  - → CJ 2010 analysis (also ABM) — *talks by J. Owens & A. Accardi*
  - → JAM 2013 spin PDF analysis — *talk by N. Sato*

- Understand role of subleading $1/Q^2$ corrections, such as
  - → kinematical target mass corrections (TMCs)
  - → dynamical higher twists; multi-parton correlations

- Interplay of nucleon resonances and scaling
  - → quark-hadron duality — *talks by C. Keppel & I. Niculescu*

Target mass corrections in DIS

Operator product expansion

- Compton amplitude

$$T^{\mu\nu} = \sum_{k=1}^{\infty} \Big(- g^{\mu\nu} q_{\mu_1} q_{\mu_2} C_1^{2k} + g^{\mu}_{\mu_1} g^{\nu}_{\mu_2} Q^2 C_2^{2k} - i\epsilon^{\mu\nu\alpha\beta} g_{\alpha\mu_1} q_\beta q_{\mu_2} C_3^{2k} + \frac{q^\mu q^\nu}{Q^2} q_{\mu_1} q_{\mu_2} C_4^{2k}$$

$$+ \ \left(g^{\mu}_{\mu_1} q^\nu q_{\mu_2} + g^{\nu}_{\mu_1} q^\mu q_{\mu_2} \right) C_5^{2k} \Big) \, q_{\mu_3} \cdots q_{\mu_{2k}} \, \frac{2^{2k}}{Q^{4k}} \, A_{2k} \, \Pi^{\mu_1 \cdots \mu_{2k}}$$

where matrix elements of local operators

$$\langle N | \, \mathcal{O}^{\mu_1 \cdots \mu_{2k}} \, | N \rangle = A_{2k} \, p^{\mu_1} \cdots p^{\mu_{2k}} \ - \ \text{traces}$$

related to moments of structure functions, $e.g.$ for F_2

$$M_2^{(n)}(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \mu^j \binom{n+j}{j} \frac{n(n-1)}{(n+2j)(n+2j-1)} \, C_2^{n+2j} \, A_{n+2j}$$

$$\mu = M^2 / Q^2$$

Operator product expansion

- Standard approach (Georgi-Politzer) defines parton distribution f_i

$$C_i^{2k}\, A_{2k} = \int_0^1 dy\, y^{2k-1}\, f_i(y)$$

with massless limit function $F_2^{(0)} \equiv \lim_{\mu\to 0} F_2 = x f_2$

- Inverse Mellin transform (+ generalized binomial theorem) gives

$$F_2^{\rm OPE}(x,Q^2) = x^2 \frac{\partial^2}{\partial x^2}\left(\frac{x g_2(\xi)}{\xi(1+\mu\xi^2)}\right)$$

Nachtmann variable $\xi = \dfrac{2x}{1+\rho}, \quad \rho^2 = 1 + 4\mu x^2$

with $h_i(\xi) = \displaystyle\int_\xi^1 du\, \frac{f_i(u)}{u}, \quad g_i(\xi) = \int_\xi^1 du\, h_i(u)$

Georgi, Politzer
*PRD **14**, 1829 (1976)*

Operator product expansion

- Standard approach (Georgi-Politzer) defines parton distribution f_i

$$C_i^{2k}\, A_{2k} = \int_0^1 dy\, y^{2k-1}\, f_i(y)$$

with massless limit function $F_2^{(0)} \equiv \lim_{\mu \to 0} F_2 = x f_2$

- Inverse Mellin transform (+ generalized binomial theorem) gives

$$F_2^{\mathrm{OPE}}(x,Q^2) = \frac{(1+\rho)^2}{4\rho^3} F_2^{(0)}(\xi,Q^2) + \frac{3x(\rho^2-1)}{2\rho^4}\left[h_2(\xi,Q^2) + \frac{\rho^2-1}{2x\rho} g_2(\xi,Q^2)\right]$$

Operator product expansion

- Problem with standard approach:

→ if $f_2(y) \sim (1-y)^\beta$ at large y

then since $\xi_0 \equiv \xi(x=1) < 1$ ⇒ $F_2^{(0)}(\xi_0) > 0$

→ target mass corrected function nonzero at $x = 1$

$$F_2^{\rm OPE}(x=1, Q^2) > 0$$

"threshold problem"

→ momentum nonconservation symptomatic of problem with matching partonic and hadronic thresholds

→ various remedies proposed
(threshold factors [Tung *et al.* 1979], $1/Q$ expansion [Kulagin, Petti 2006], PDF redefinition [Steffens, WM 2006])
– but all have drawbacks

Other TMC approaches

- **Collinear factorization** (Ellis, Furmanski, Petronzio) *NP **B212**, 29 (1983)*

  → diagrammatic approach, impose $x < 1$ by definition; include parton virtuality & k_T; to $\mathcal{O}(1/Q^2)$ only

$$F_2^{\rm EFP}(x,Q^2) = \frac{1}{\rho^2} F_2^{(0)}(\xi,Q^2) + \frac{3\xi(\rho^2-1)}{\rho^2(1+\rho)} h_2(\xi,Q^2)$$
$$= F_2^{\rm OPE}(x,Q^2) + \mathcal{O}(1/Q^4)$$

- **Collinear factorization** (Accardi, Qiu) *JHEP **07**, 090 (2008)*

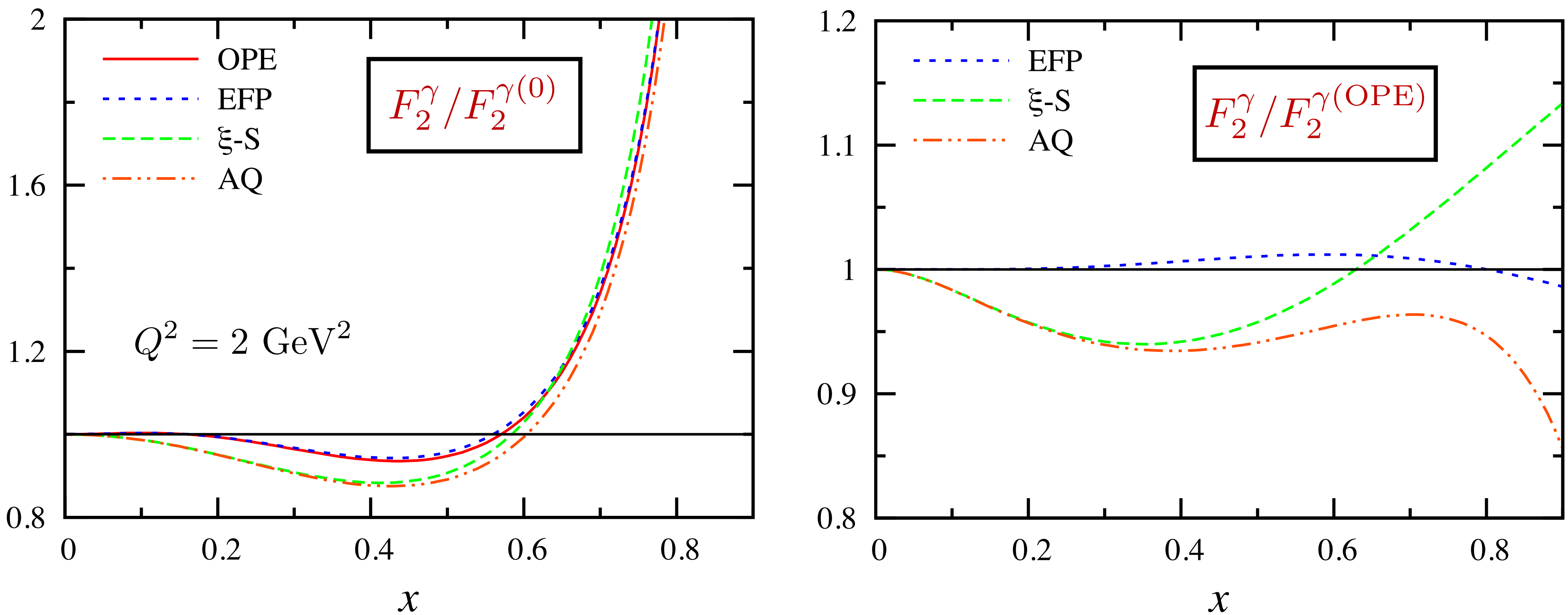
  → similar to EFP, but include only 2-leg (*cf.* 4-leg) diagrams; allow for jet-mass corrections, NLO effects

$$F_2^{\rm AQ}(x,Q^2) = \frac{1+\rho}{2\rho^2} \widetilde{F}_2^{(0)}(\xi,Q^2)$$

- **ξ-scaling** (Aivazis *et al.*, Kretzer-Reno) *PRD **69**, 034002 (2004)*

  → similar to AQ, but at leading order $\widetilde{F}_2^{(0)} \to F_2^{(0)}$

Other TMC approaches



Brady, Accardi, Hobbs, WM
*PRD **84**, 974008 (2011)*

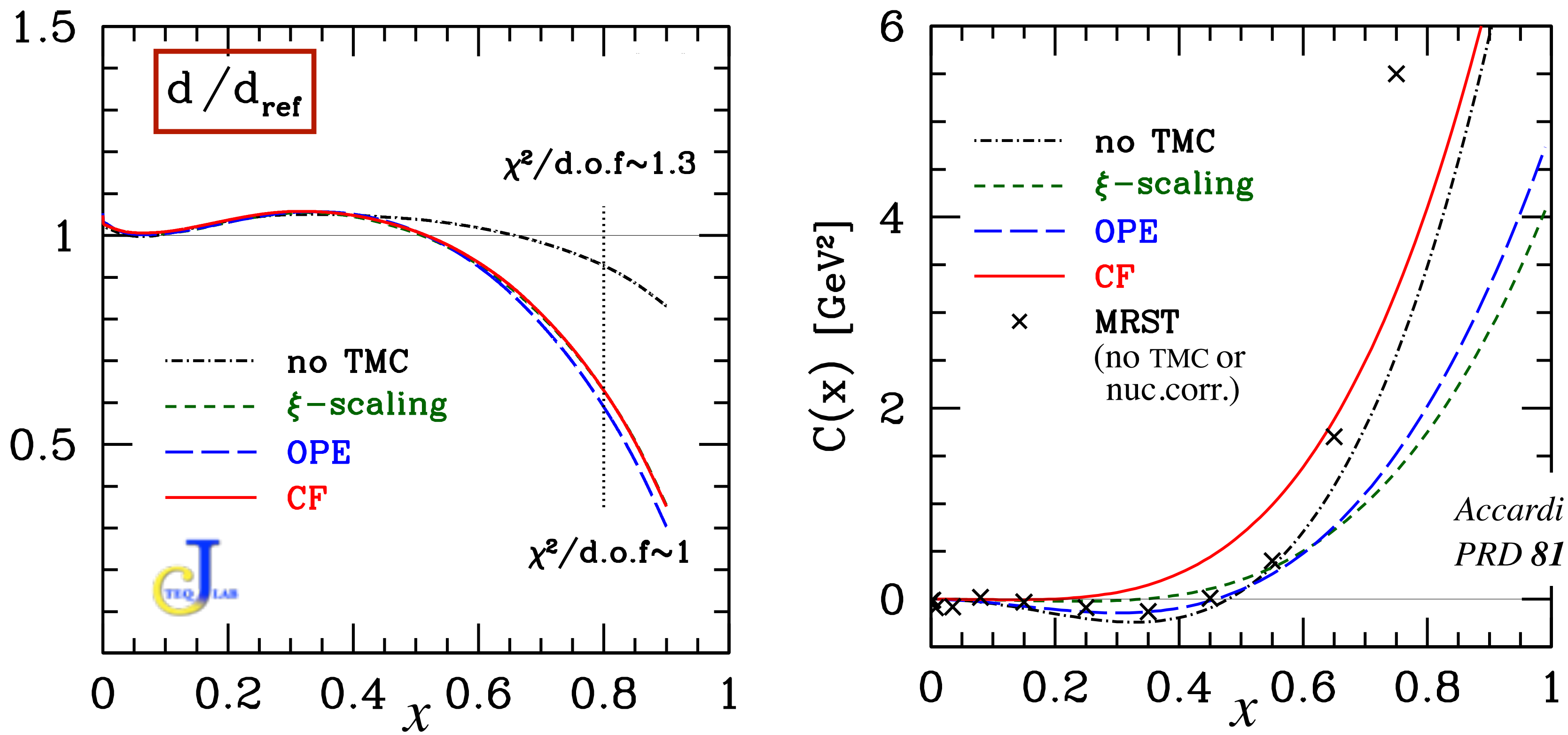
→ EFP generally tracks OPE result

→ AQ and ξ-scaling similar at lower x

Effect of $1/Q^2$ corrections on global fits

- If higher twists parametrized phenomenologically, *e.g.*

$$F_2(x,Q^2) = F_2^{\rm LT}(x,Q^2)\left(1+\frac{C(x)}{Q^2}\right)$$ $C(x)$ polynomial



Accardi et al. PRD ***81****, 034016 (2010)*

→ stable leading twist when *both* TMCs and HTs included

→ extraction of HTs depends on TMC prescription...

Term-wise expansion

- Expanded OPE inversion (Steffens, Brown, WM, Sanches) *PRC* **86**, 065208 (2012)

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \frac{(-x)^j}{j!} \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x) \right]$$

→ avoid introducing ξ variable altogether
– no unphysical $(x > 1)$ regions, or "threshold" problem

→ the only approach that explicitly respects

$$M^{\rm CN}(F_i^{\rm LT}) = M^{\rm Nacht}(F_i^{\rm LT+TMC})$$

Cornwall−Norton   Nachtmann

$$M_2^{{\rm Nacht}(n)}(Q^2) = \int_0^1 dx \, \frac{\xi^{n+1}}{x^3} \left\{ \frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right\} \times F_2(x,Q^2)$$

removes higher spin operators from definition of leading twist moments

Term-wise expansion

- Expanded OPE inversion (Steffens, Brown, WM, Sanches) *PRC **86**, 065208 (2012)*

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \, \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x) \right]$$

similarly for other structure functions

$$F_1(x,Q^2) = x \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \, \frac{\partial^j}{\partial x^j} \left[x^{2j-2} \left(\frac{1}{2} x f_1(x) + j g_2(x) \right) \right]$$

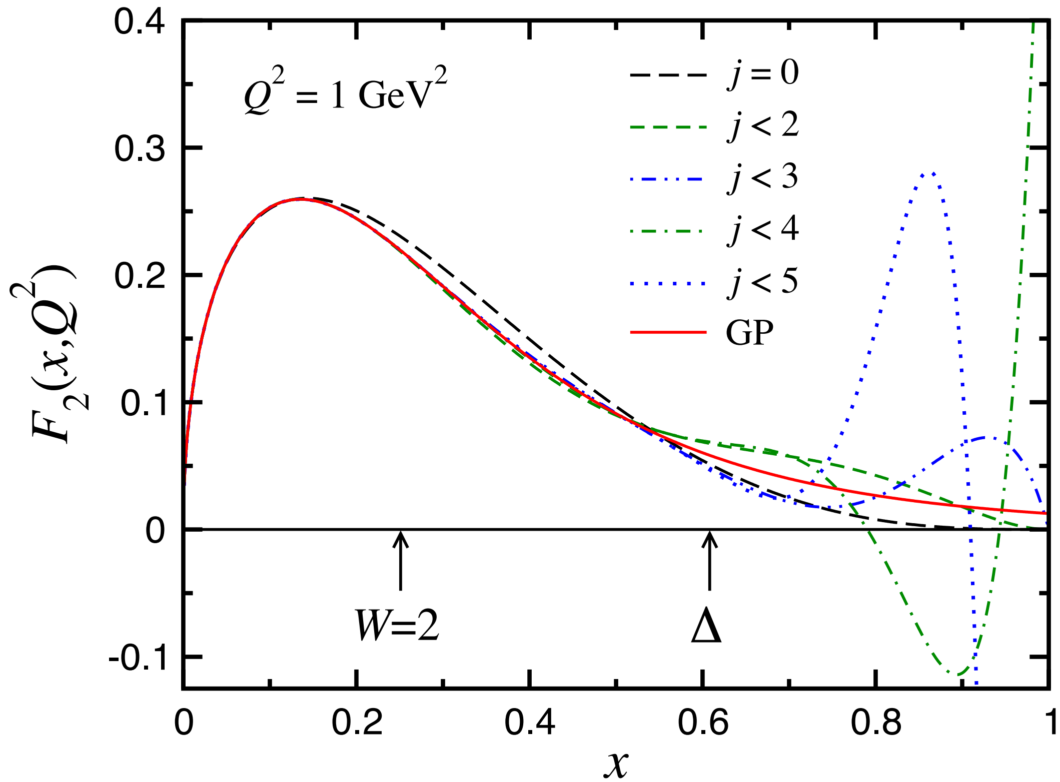
$$F_L(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \, \frac{\partial^j}{\partial x^j} \left[x^{2j-2} \left(x f_2(x) - x f_1(x) \right.\right.$$
$$\left.\left. + 4 j g_2(x) \right) \right]$$

$$F_3(x,Q^2) = \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^{1+j}}{j!} \, \frac{\partial^{1+j}}{\partial x^{1+j}} \left[x^{2j} h_3(x) \right]$$

Term-wise expansion

- Expanded OPE inversion (Steffens, Brown, WM, Sanches) *PRC* **86**, 065208 (2012)

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x) \right]$$

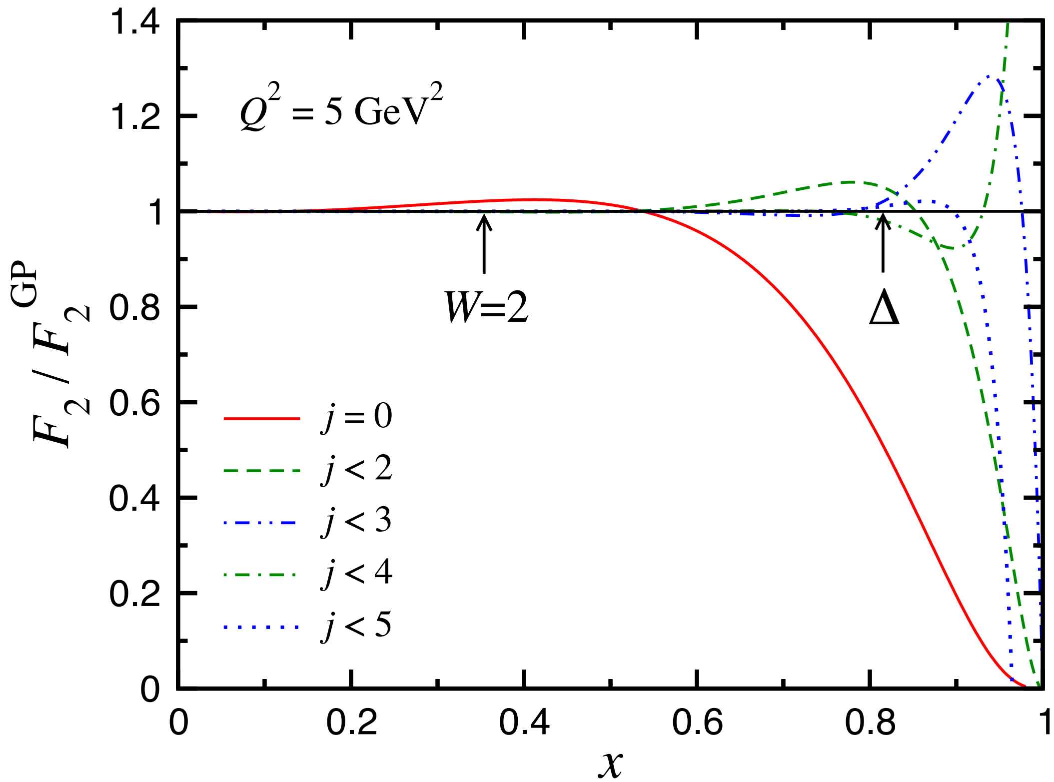


→ fast convergence in DIS region

Term-wise expansion

- **Expanded OPE inversion** (Steffens, Brown, WM, Sanches) *PRC* ***86***, *065208 (2012)*

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \frac{(-x)^j}{j!} \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x)\right]$$

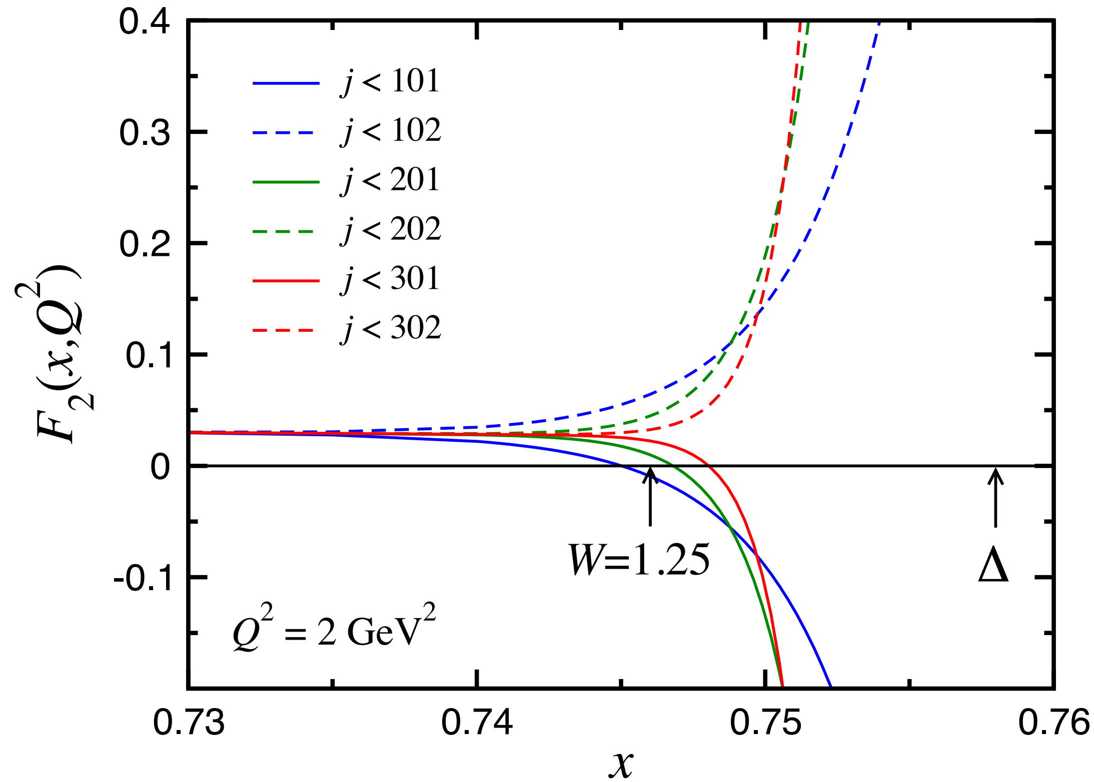


→ differences only in resonance region

Term-wise expansion

- Expanded OPE inversion (Steffens, Brown, WM, Sanches) *PRC **86**, 065208 (2012)*

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x) \right]$$

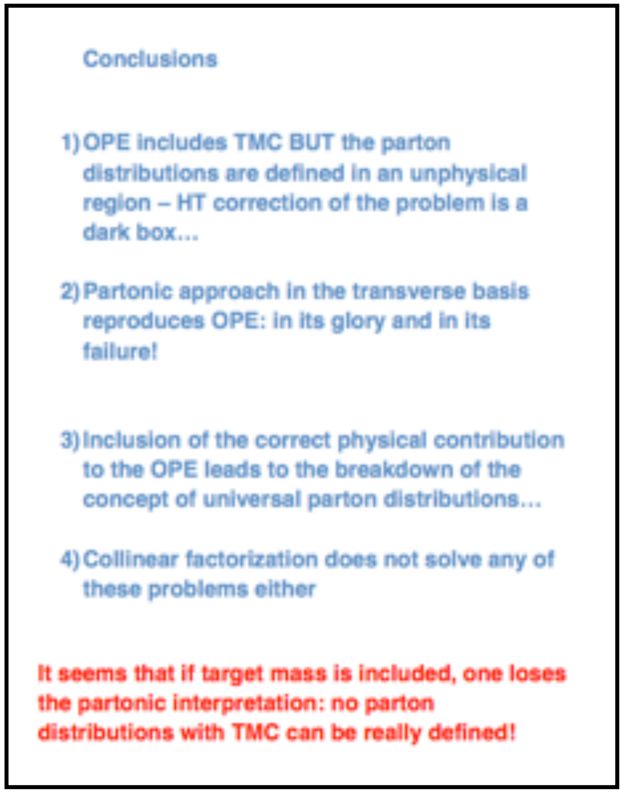


→ differences only in resonance region

Term-wise expansion

- Expanded OPE inversion (Steffens, Brown, WM, Sanches) *PRC* ***86****, 065208 (2012)*

$$F_2(x,Q^2) = x^2 \sum_{j=0}^{\infty} \mu^j \, \frac{(-x)^j}{j!} \, \frac{\partial^{2+j}}{\partial x^{2+j}} \left[x^{2j} g_2(x) \right]$$



Conclusions

1) OPE includes TMC BUT the parton distributions are defined in an unphysical region – HT correction of the problem is a dark box...

2) Partonic approach in the transverse basis reproduces OPE: in its glory and in its failure!

3) Inclusion of the correct physical contribution to the OPE leads to the breakdown of the concept of universal parton distributions...

4) Collinear factorization does not solve any of these problems either

It seems that if target mass is included, one loses the partonic interpretation: no parton distributions with TMC can be really defined!

F. Steffens, HiX 2010

→ in contrast to more pessimistic outlook at HiX2010, now expect minimal uncertainty for global PDF analysis and HT extraction!

Hadron mass corrections in SIDIS

Hadron mass corrections

- Target mass corrections also relevant for semi-inclusive DIS
  - → dependence on both mass of target hadron (nucleon) and produced hadron (pion, kaon, ...)
  - → no OPE – must use $e.g.$ collinear factorization
  - → potentially even more important than for inclusive DIS, since relatively more data at lower Q^2

- Collinear framework ("EFP") – expand parton momenta about on-shell and collinear limits

$$\tilde{k}^\mu = xp^+ \bar{n}^\mu + \frac{\tilde{k}^2}{2xp^+} n^\mu$$

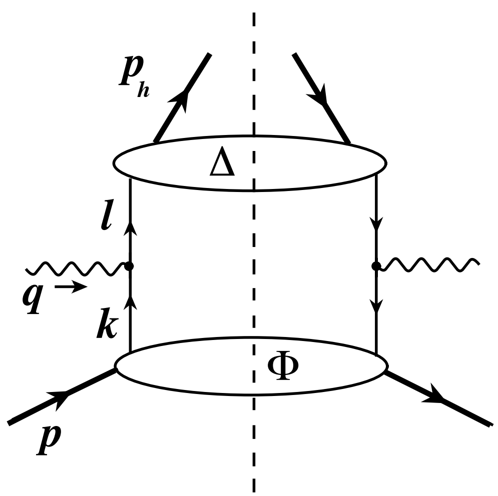
$$\tilde{l}^\mu = \frac{\tilde{l}^2 + p_{h\perp}^2/z^2}{2p_h^-/z} \bar{n}^\mu + \frac{p_h^-}{z} n^\mu + \frac{p_{h\perp}^\mu}{z}$$

$x = k^+/p^+$

$z = p_h^-/l^-$

$n^\mu,\ \bar{n}^\mu$

light-cone vectors



Hadron mass corrections

- Parton distributions and fragmentation functions defined through correlators Φ_q and Δ_q^h

$$\int dk^- d^2k_\perp \, \Phi_q(p,k) = \frac{1}{2} q(x) \, \not{n} \; + \; \cdots$$

$$\int dl^+ d^2l_\perp \, \Delta_q^h(l,p_h) = \frac{1}{2z} D_q^h(z) \, \not{n} \; + \; \cdots$$

→ differential SIDIS cross section (at leading order)

$$\frac{d\sigma^h}{dx\, dQ^2\, dz_h} = \frac{2\pi\alpha^2}{Q^4} \frac{y^2 \mathcal{C}}{1-\varepsilon} \sum_q e_q^2 \, q(\xi_h) \, D_q^h(\zeta_h)$$

inter-dependent, breaks factorization!

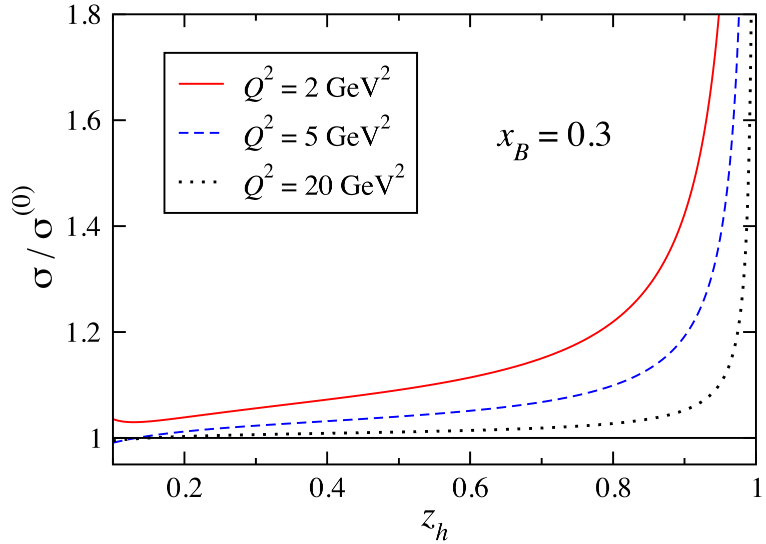
with finite-Q^2 scaling variables

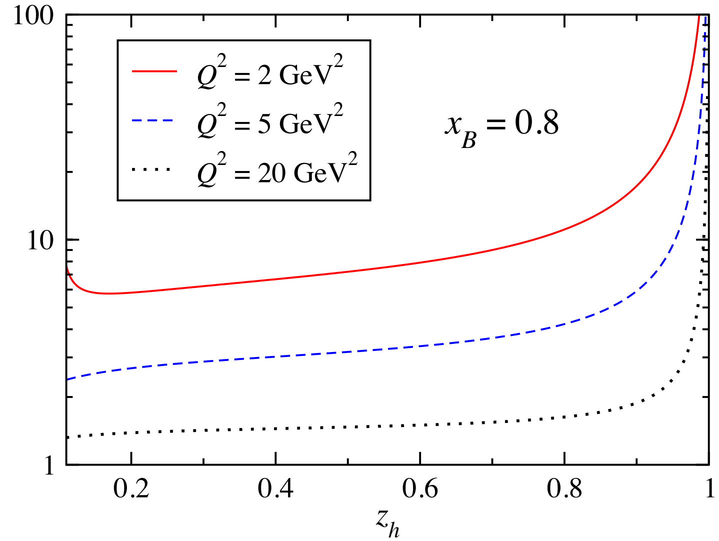
$$\xi_h = \xi \left(1 + \frac{\tilde{l}^2}{Q^2} \right), \qquad \zeta_h = \frac{z_h}{2} \frac{\xi}{x} \left(1 + \sqrt{1 - \frac{(\rho^2-1)m_{h\perp}^2}{z_h^2}} \right)$$

$$z_h = \frac{p_h \cdot p}{q \cdot p}, \qquad \mathcal{C} = \frac{\nu + M}{\nu} \frac{d\zeta_h}{dz_h}$$

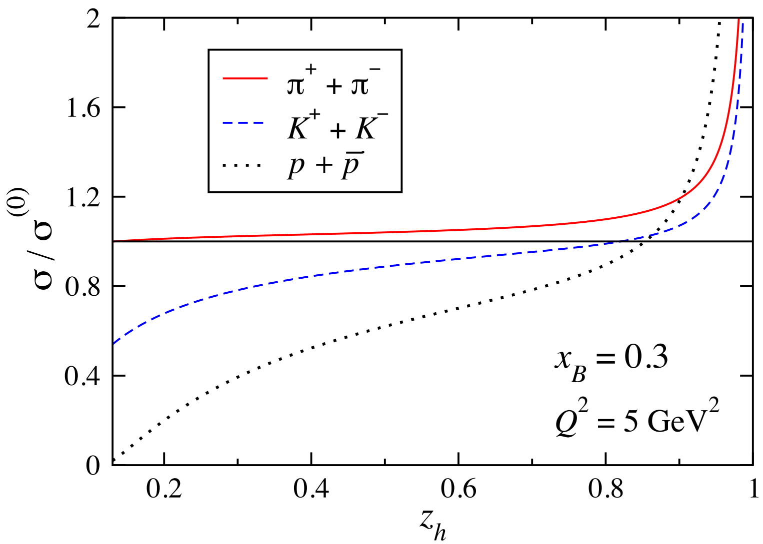
Accardi, Hobbs, WM
JHEP ***0911***, *084 (2009)*

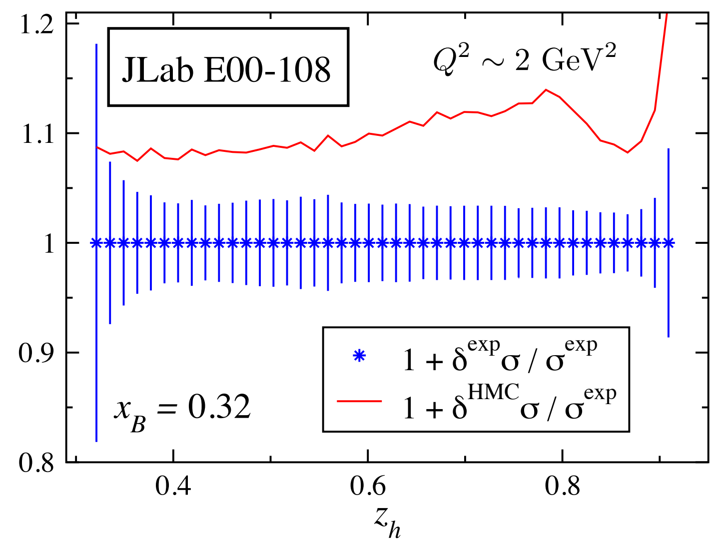
Hadron mass corrections





→ important at large values of z_h & x





→ more important for heavier hadrons, and lower energies

Hadron mass corrections

- Similar analysis for spin-dependent semi-inclusive DIS

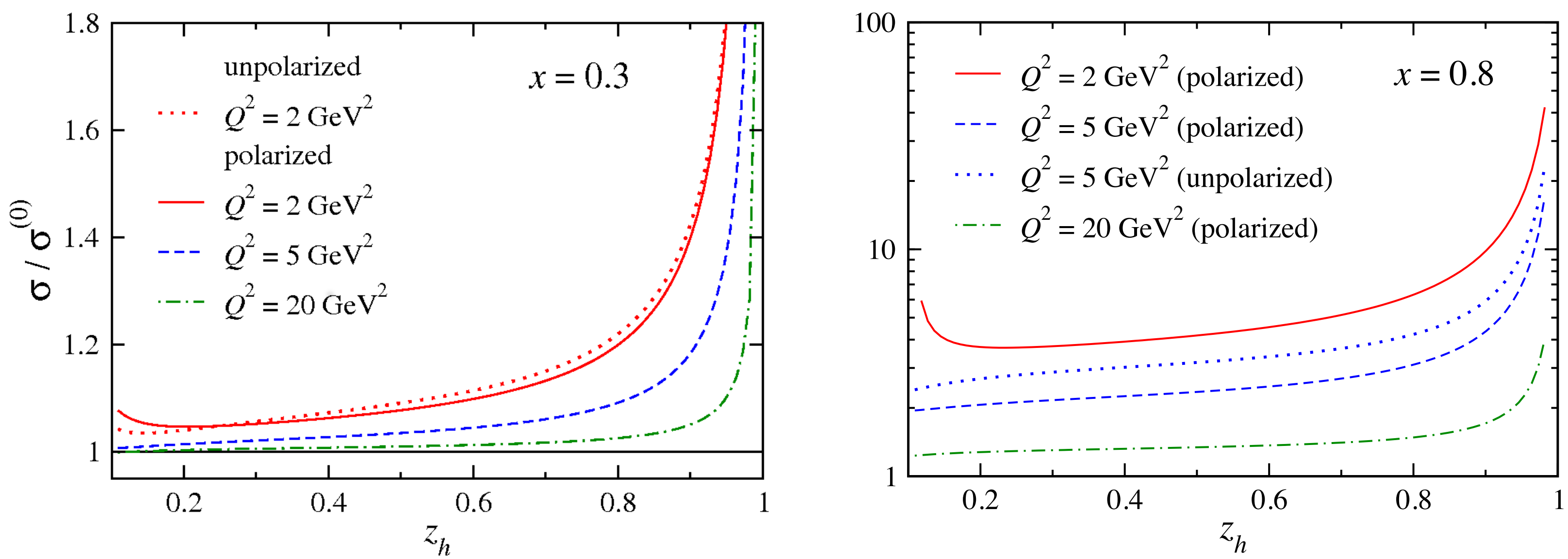
$$\int dk^- d^2k_\perp \, \Phi_q(p,S,k) = \frac{1}{2} q(x) \not{n} \; + \; \frac{1}{2} S_L \Delta q(x) \gamma_5 \not{n} \; + \; \cdots$$

longitudinal N polarization

→ spin-dependent differential SIDIS cross section

$$\frac{d\sigma^{h(\uparrow\uparrow-\uparrow\downarrow)}}{dx\, dQ^2\, dz_h} = \frac{2\pi\alpha^2}{Q^4} \frac{y^2 \mathcal{C}\sqrt{1-\varepsilon^2}}{1-\varepsilon} \lambda S_L \sum_q e_q^2 \, \Delta q(\xi_h)\, D_q^h(\zeta_h)$$

Guerrero, Ethier, Accardi, WM (2014)

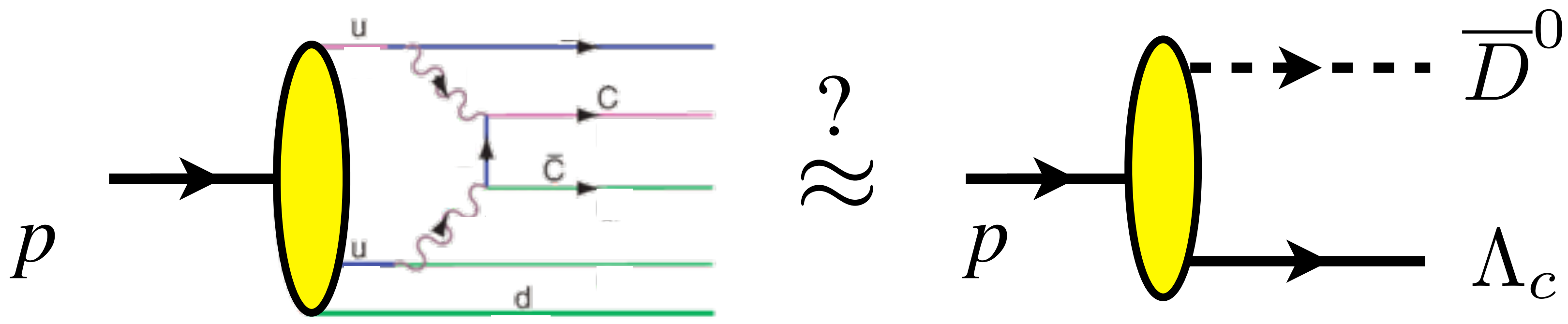


→ important for polarized SIDIS, where more data at lower Q^2

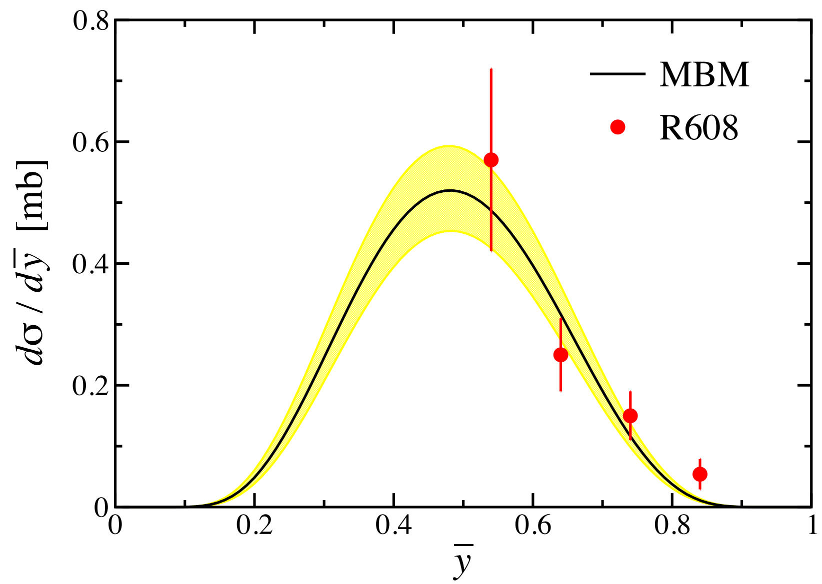
Intrinsic charm in DIS

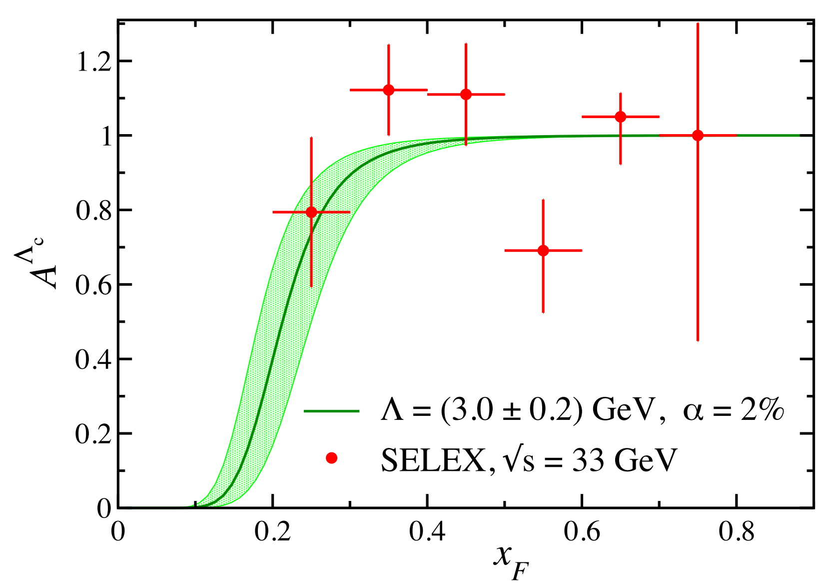
Intrinsic charm

- Possibility of intrinsic (nonperturbative) charm component in nucleon suggested ~30 years ago



→ constraints from hadronic reactions, $p\,p \to \Lambda_c\, X$

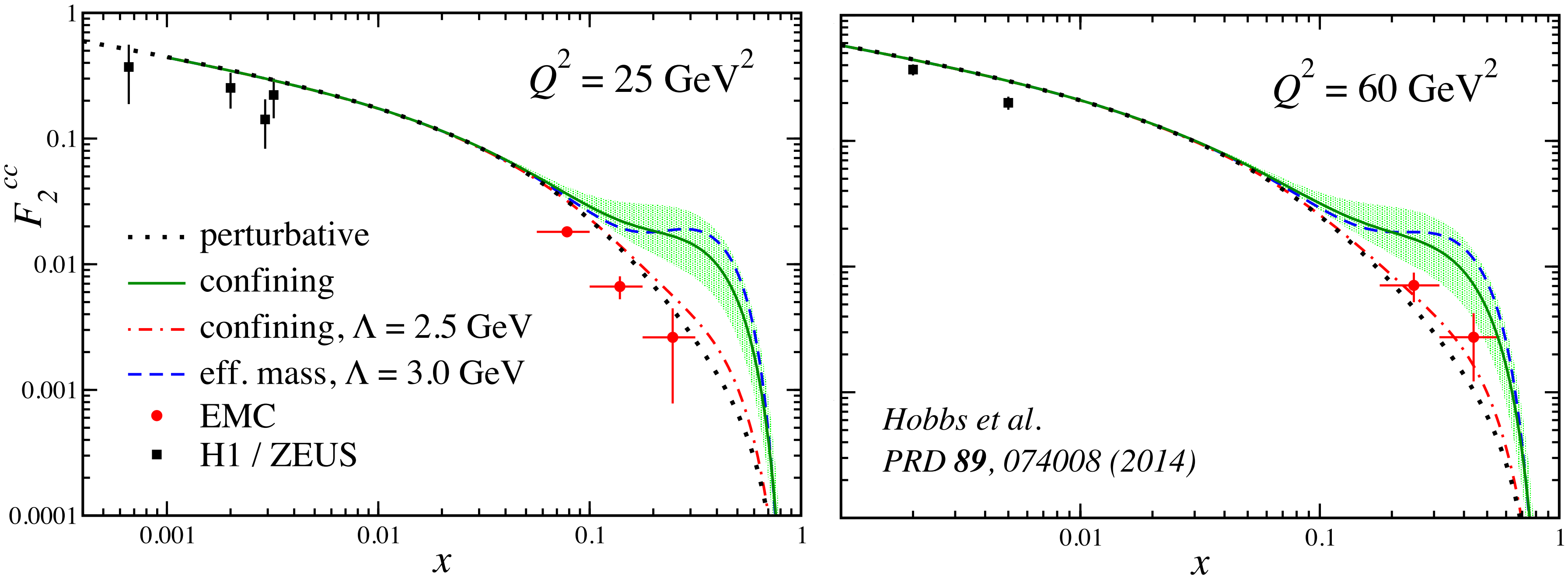




*Hobbs et al., PRD **89**, 074008 (2014)*

Intrinsic charm

- Possibility of intrinsic (nonperturbative) charm component in nucleon suggested ~30 years ago

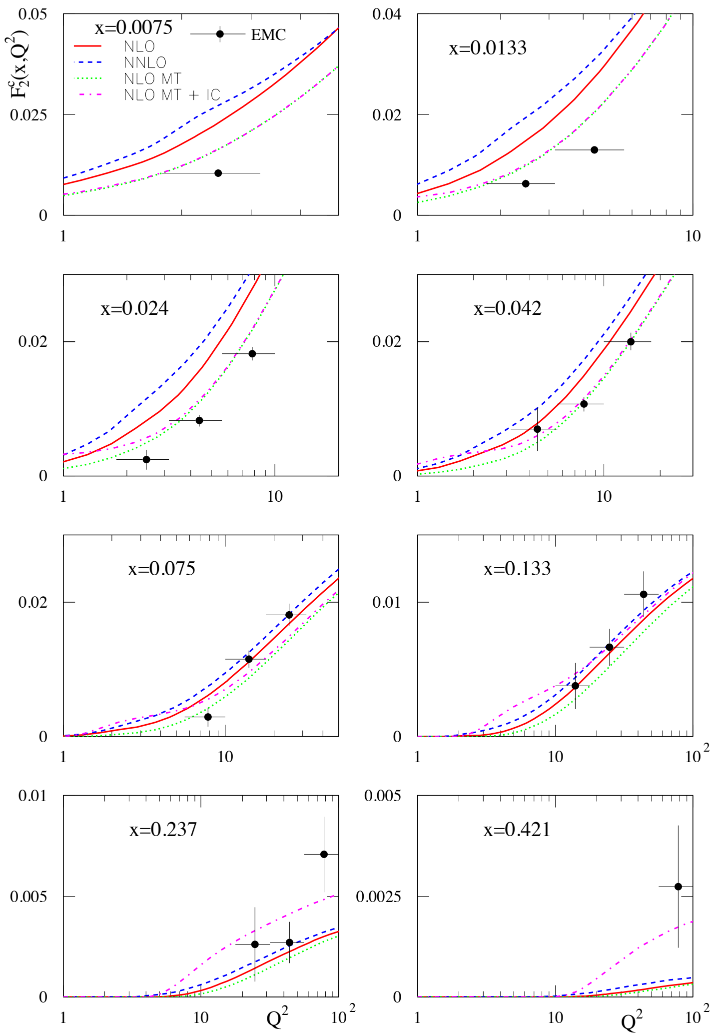


→ hint of excess charm *cf.* pQCD contribution

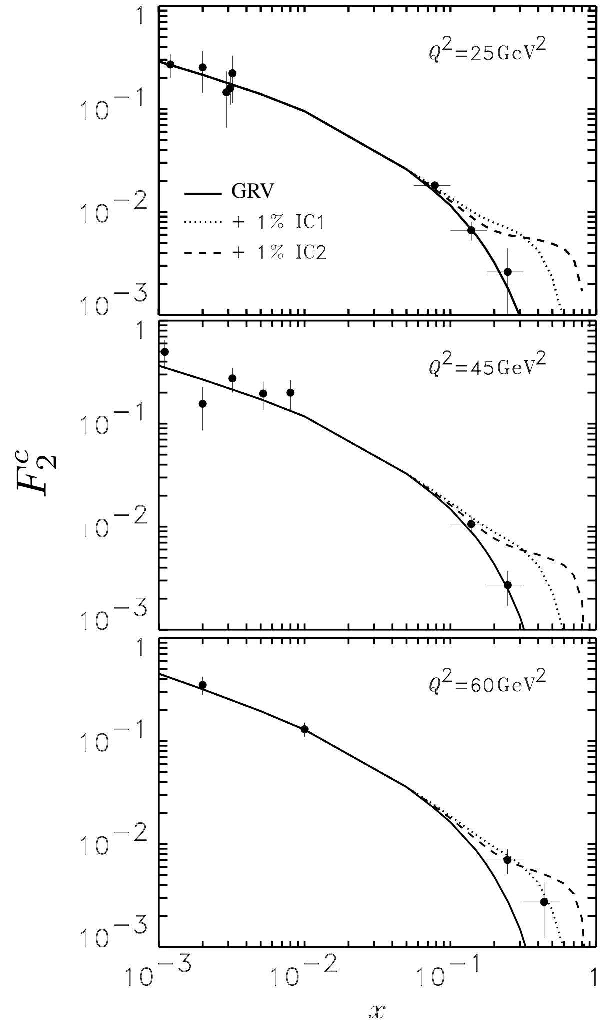
→ requires systematic global QCD analysis…

Intrinsic charm

- Global QCD analysis with intrinsic charm component



*MSTW, EPJC **63**, 189 (2009)*

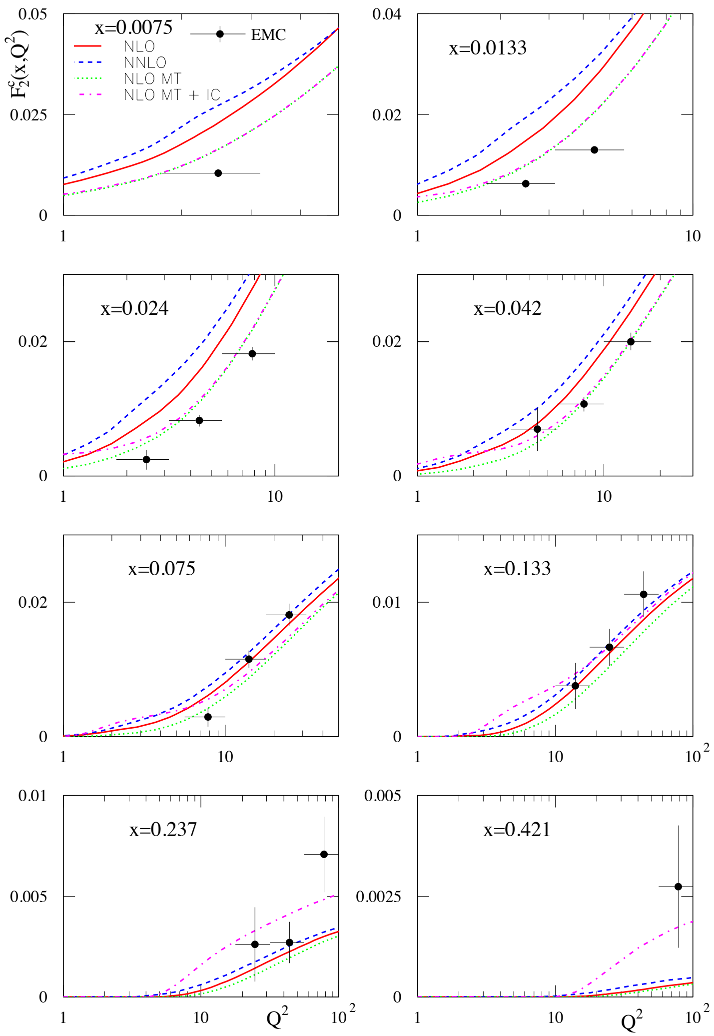


Steffens, WM, Thomas
*EPJC **11**, 673 (1999)*

→ very weak (inconsistent?) evidence for IC from EMC data

Intrinsic charm

Global QCD analysis with intrinsic charm component



*MSTW, EPJC **63**, 189 (2009)*

$$m_c^2 \;\to\; m_c^2(1+\Lambda^2/m_c^2)$$

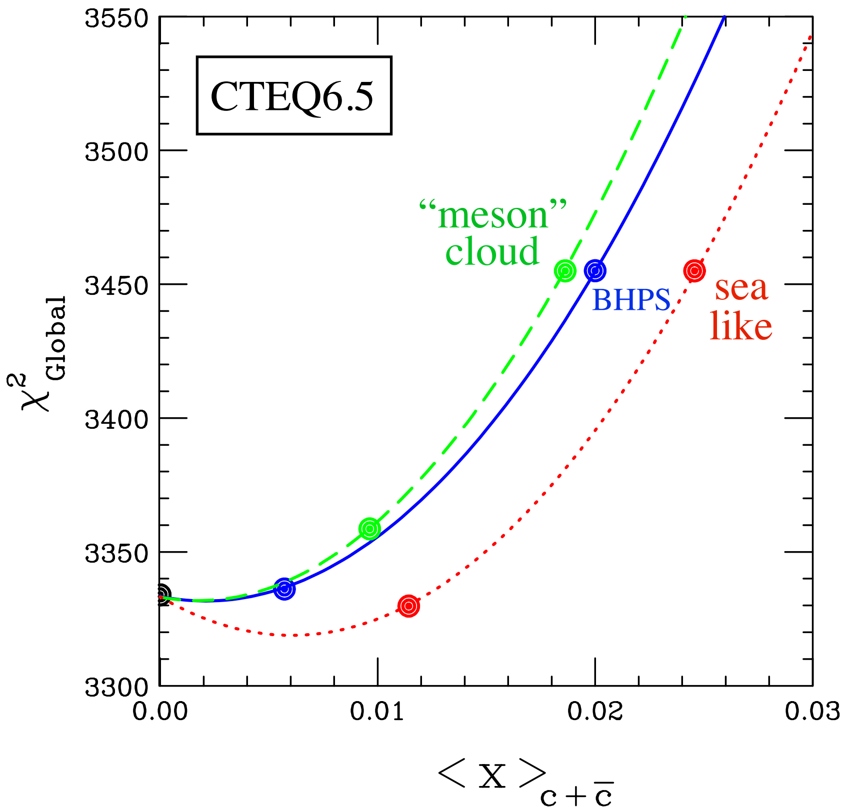
"hadronic threshold" modification

- "if the EMC data are to be believed, there is no room for a very sizeable intrinsic charm contribution"

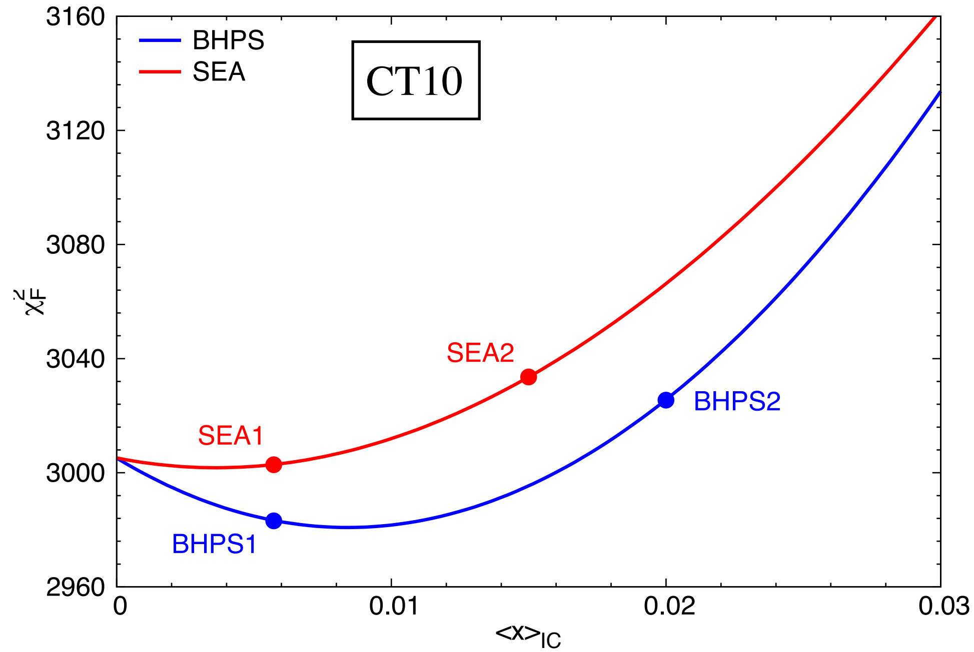
→ very weak (inconsistent?) evidence for IC from EMC data

Intrinsic charm

- Global QCD analysis with intrinsic charm component



*Pumplin et al., PRD **75**, 054029 (2007)*



*Dulat et al., PRD **89**, 073004 (2014)*

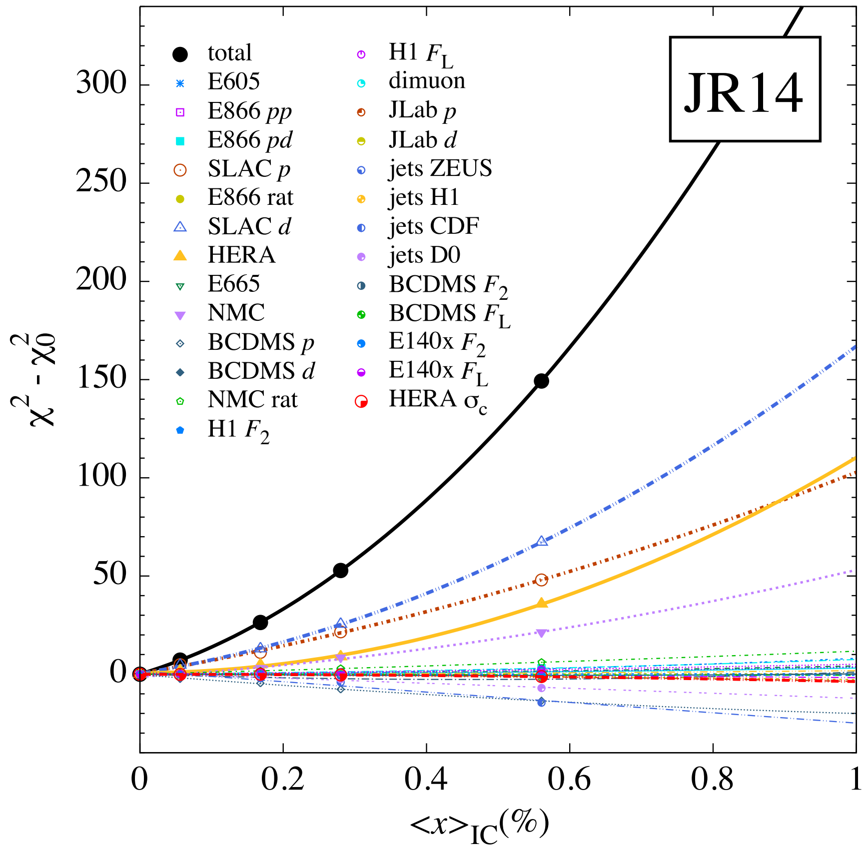
$\langle x\rangle_{\rm IC} \lesssim 0.025$ at 90% CL

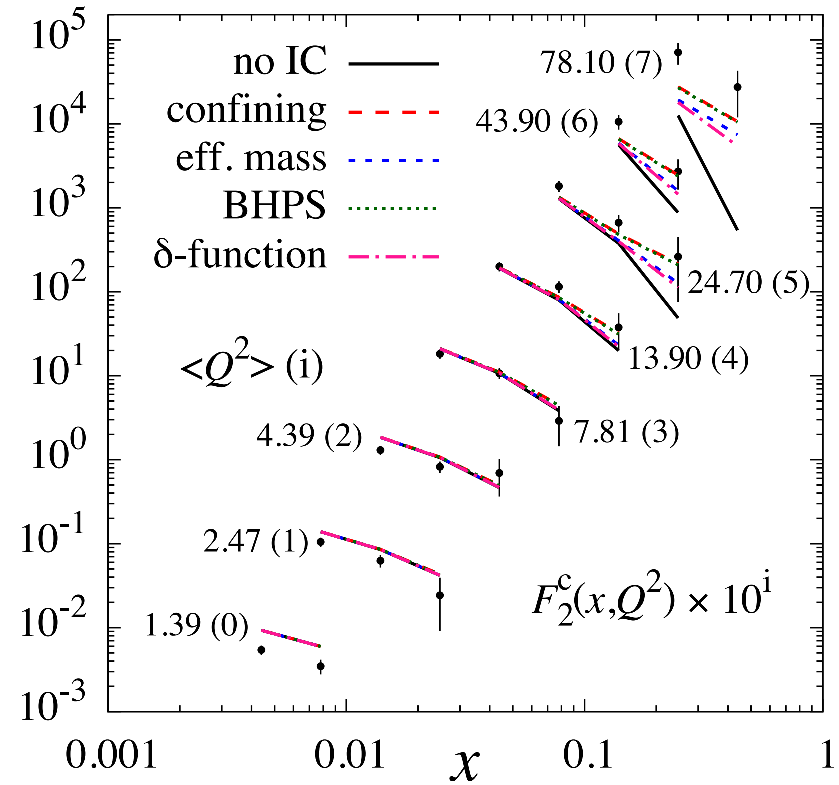
- "global analysis of hard-scattering data provides no evidence either for or against IC up to 0.01"

- new NNLO analysis, including new HERA data, disfavors "sea-like" IC model

Intrinsic charm

Global QCD analysis with intrinsic charm component





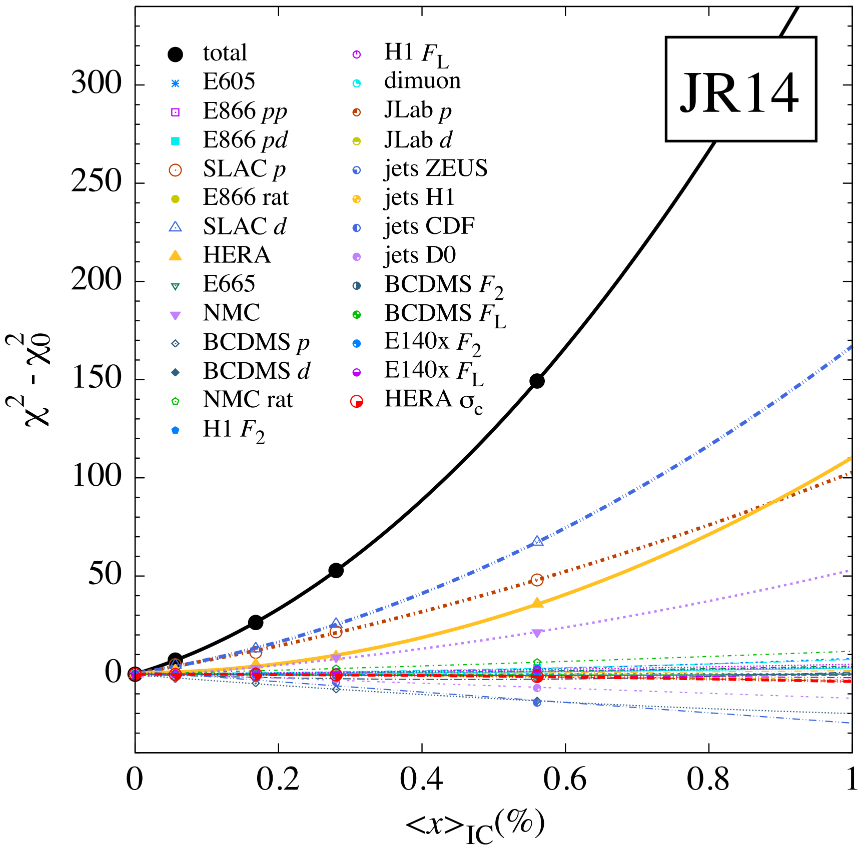
Jimenez-Delgado et al., arXiv:1408.1708

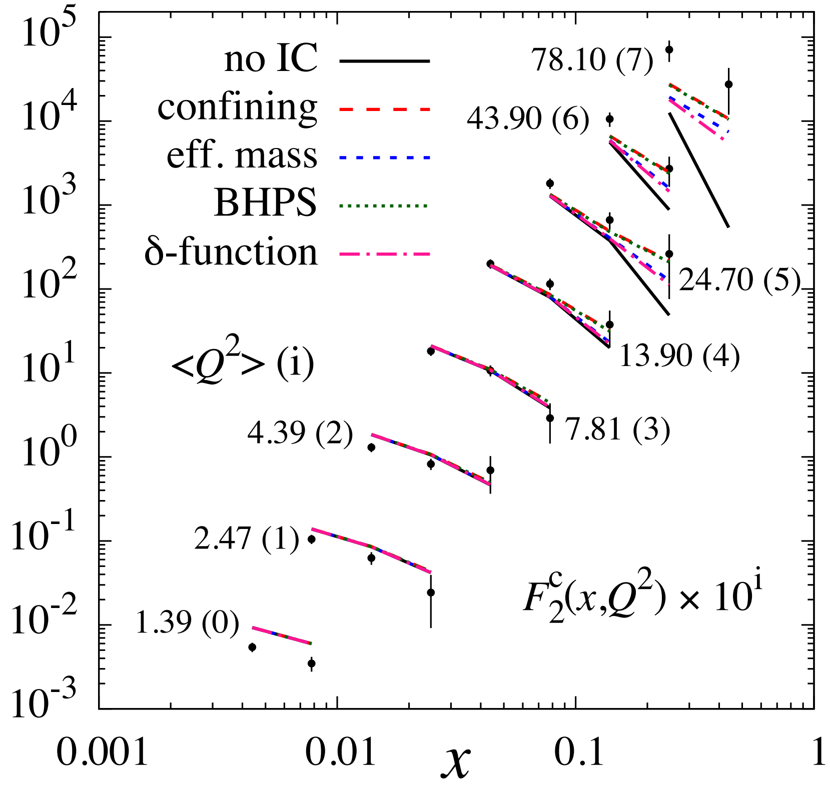
→ $\langle x \rangle_{\rm IC} < 0.1\%$ $\Delta\chi^2 = 1$ at 5σ CL

[$\langle x \rangle_{\rm IC} \sim 0.13(4)\%$ with EMC data, but $\chi^2/\rm{dat} = 4.3$]

Intrinsic charm

■ Global QCD analysis with intrinsic charm component





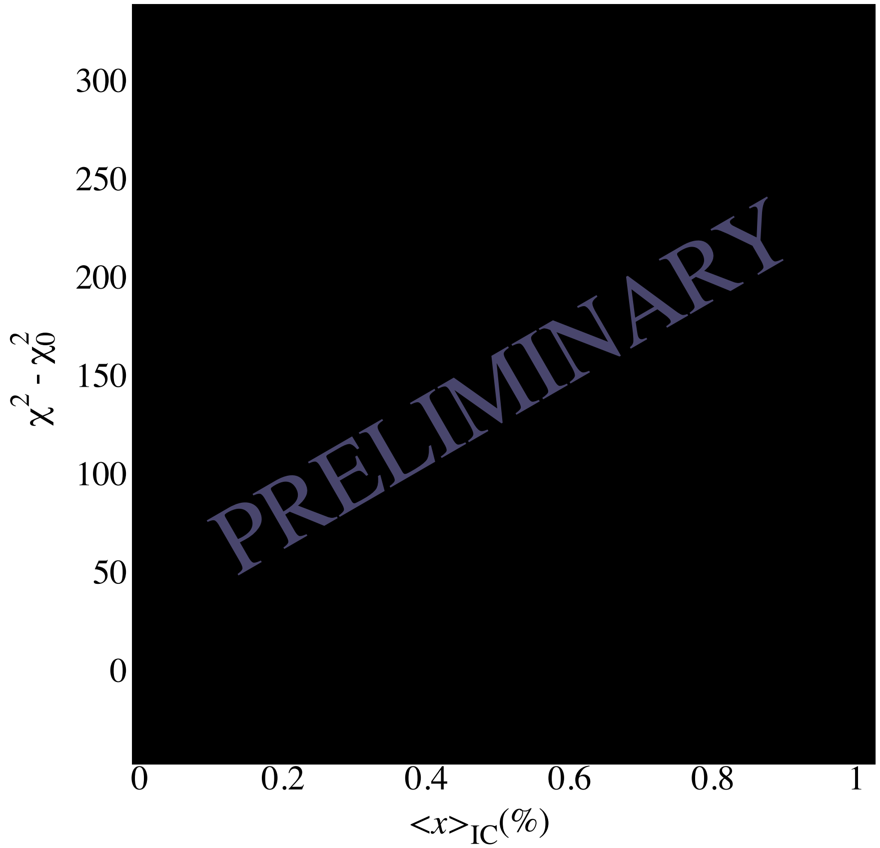
Jimenez-Delgado et al., arXiv:1408.1708

→ $\langle x \rangle_{\rm IC} \lesssim 0.4\%$ $\quad \Delta\chi^2 = 100$

→ some lower energy data (*e.g.* SLAC) near charm threshold – additional suppression?

Intrinsic charm

- Global QCD analysis with intrinsic charm component



- partonic charm threshold

  $W^2 \geq 4m_c^2$

- hadronic charm threshold

  $W^2 \geq (M_N + m_{J/\psi})^2$

- additional suppression factor

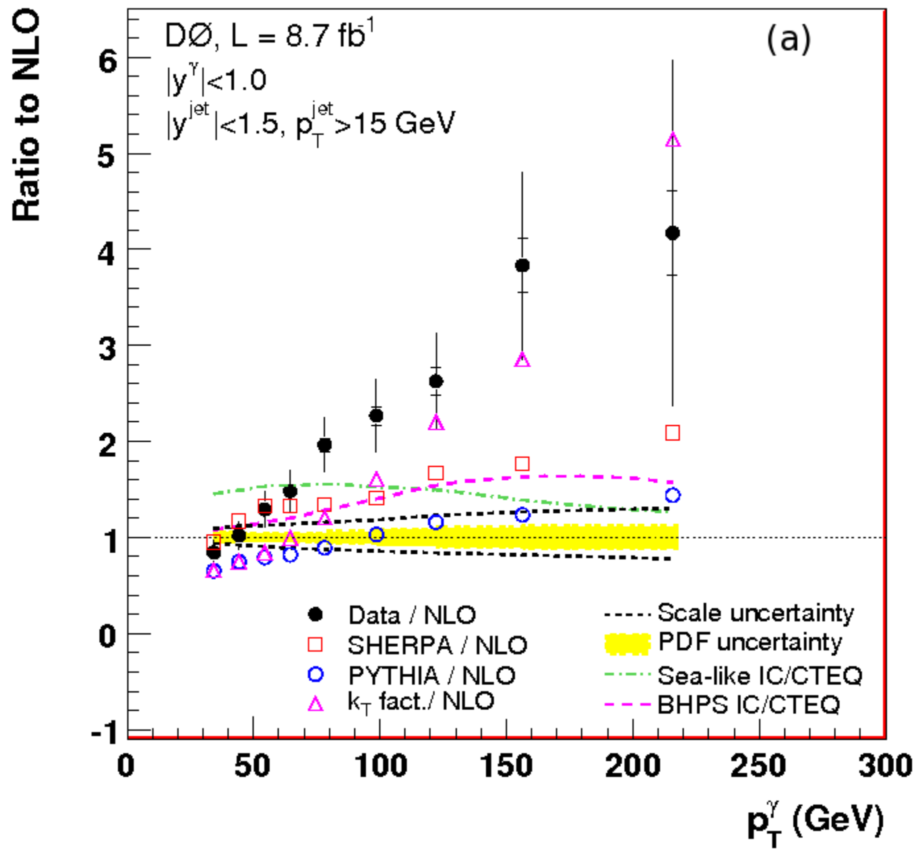
  $(W^2 - W_{\rm th}^2)/W^2$ *Brodsky*

→ $\langle x\rangle_{\rm IC} \lesssim$ [redacted] $\Delta\chi^2 = 100$

→ with threshold factor (no refitting), constraints from lower energy data are weaker, but still *no evidence for large IC*

Intrinsic charm?

$$p\bar{p} \to \gamma + c\text{-jet} + X$$



Mesropian, Bandurin
arXiv:1409.5639

"None of the theoretical predictions considered (QCD NLO, kT factorization, SHERPA and PYTHIA) give good description of the data in all p_T^γ bins. Such a description might be achieved by including higher-order corrections into the QCD predictions. At $p_T^\gamma \gtrsim 80$ GeV, the observed difference from data may also be caused by an underestimated contribution from gluon splitting $g \to c\bar{c}$ in the annihilation process or by contribution from intrinsic charm."

Outlook

- Progress in understanding theoretical foundations (prescription dependence) of TMCs

  → towards (practical) solution of threshold problem through term-wise expansion

- First calculations of HMCs in semi-inclusive DIS

  → important at high x and z_h (and low z_h for heavier h)

  → more important for polarized (more data at lower Q^2)

- Global PDF analysis of all existing high-energy data shows no evidence of large IC in DIS

  → new F_2^c data needed to settle question of IC in DIS