Global Analysis of Polarized PDFs

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Part I: Large $x$ behavior of nucleon spin structure.

Part II: Constraints from direct photon data at large $x$ on PDFs.
Part I: Large $x$ behavior of nucleon spin structure.
Motivations: “Large $x$ behavior of nucleon spin structure”

- Valence PDFs at large $x \rightarrow$ direct connection with models of nucleon structure.

- Opportunity to study the behavior of $\Delta q/q$ ratios at $x \rightarrow 1$.

- Impact of finite $Q^2$ & nuclear corrections to parton distributions at high $x$ (following CJ).

- Non trivial interplay between pQCD and the nuclear models at high $x$. 
Motivations: “Large $x$ behavior of nucleon spin structure”

- pQCD:
  $$q^\downarrow \sim (1 - x)^2 q^\uparrow \sim (1 - x)^5$$

- Some of the consequences:
  $$\Delta q/q \rightarrow 1 \quad \text{(for all flavors)}$$
  $$A_1 \rightarrow 1$$

- To test the above statements, a global analysis of spin PDFs (SPDFs) at large $x$ region is needed.
The bulk of the available data are the polarized inclusive DIS cross section asymmetries:

\[
A_{\parallel} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D(A_1 + \eta A_2)
\]

\[
A_{\perp} = \frac{\sigma^{\uparrow\leftrightarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\uparrow\leftrightarrow} + \sigma^{\uparrow\leftarrow}} = d(A_2 - \xi A_1)
\]

\[
A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \quad A_2 = \gamma \frac{(g_1 + g_2)}{F_1}
\]

Theory predictions for polarized observables depends on unpolarized PDFs.
To ensure that leading twist contribution dominates the scattering process, cuts on $Q^2$ and $W^2$ are imposed.

Typically for unpolarized PDFs $Q^2 \geq 4\text{GeV}^2$ and $W^2 \geq 14\text{GeV}^2$.

These cuts are severe for global SPDF analysis.

$$W^2 = M_N^2 + Q^2\left(\frac{1}{x} - 1\right)$$
polarized DIS data WITHOUT CUTS

COMPASS (npts = 33)

EMC (npts = 10)

HERMES (npts = 122)

JLabHallA (npts = 6)

JLabHallB (npts = 0)

SLAC (npts = 1060)

SMC (npts = 60)

All (npts = 1291)

All (npts = 1291)
polarized DIS data WITH “TYPICAL” CUTS
polarized DIS data WITH JAM13 CUTS

COMPASS (npts = 31)

EMC (npts = 10)

HERMES (npts = 103)

JLabHallA (npts = 6)

JLabHallB (npts = 0)

SLAC (npts = 853)

SMC (npts = 40)

All (npts = 1043)

All (npts = 1043)
In order to maximally utilize the data, the JAM13 analysis sets the cuts to $Q^2 \geq 1\text{GeV}^2$ and $W^2 \geq 3.5\text{GeV}^2$.

The price:

- TM and HT corrections must be included.
- Asymmetries depend on unpolarized PDFs.
- A consistent analysis requires to fit unpolarized PDFs under the same conditions.
The leading twist $\tau$ (NLO) in Mellin-space

$$g_{1,\tau=2}^{(n)} = \frac{1}{2} \sum_{q} e_{q}^{2} (\Delta C_{qq}^{(n)} \Delta q^{(n)} + \Delta C_{g}^{(n)} \Delta g^{(n)})$$

$$g_{2,\tau=2}^{(n)} = -\frac{n-1}{n} g_{1,\tau=2}^{(n)}$$
Higher twist corrections

- Twist-3 part of $g_2$ (PRD 83,094023(2011))

$$g_2^{(\tau=3)} = t_0 \left[ \log x + (1 - x) + \frac{1}{2} (1 - x)^2 \right] + \sum_{i=1}^{4} t_i (1 - x)^{i+2}$$

- Twist-3 part of $g_1$ (NPB,553(1999))

$$g_1^{(\tau=3)} = \gamma^2 \left[ g_2^{(\tau=3)} - 2 \int_x^1 \frac{dy}{y} g_2^{(\tau=3)} \right]$$

- Twist-4 part of $g_1$ (spline parametrization)

$$g_1^{(\tau=4)} = \frac{h(x)}{Q^2}$$
Higher twist corrections

\[ \Delta q^+(LT+HT) / \Delta q^+(LT) \]

\[ x \]

(b)

\[ \Delta q^+(LT+HT) / \Delta q^+(LT) \]
Target Mass corrections (twist-2)

- **TM to** $g_1$

\[
g_1^{(n)} = n \sum_{j=0}^{\infty} \left( \frac{M^2}{Q^2} \right)^j \frac{(n+j)!}{j!(n-1)!(n+2j)^2} g_1^{(n+2j)}
\]

\[
\approx g_1^{(n)} + \frac{M^2}{Q^2} \frac{n^2(n+1)}{(n+2)^2} g_1^{(n+2)} + O \left( \frac{M^4}{Q^4} \right)
\]
Target Mass corrections (twist-2)

\[ \frac{\Delta q^+(\text{TMC})}{\Delta q^+(\text{no TMC})} \]

\( u \)

\( d \)

(a)
Nuclear corrections

- To achieve flavour separation (at least between $\Delta u$ and $\Delta d$) polarized data with deuterium and $^3$He are used.

- Ignoring nuclear binding and Fermi motion the “nuclear” structure functions are given by

$$g_i^A(x, Q^2) = P_{p/A} g_i^p(x, Q^2) + P_{n/A} g_i^n(x, Q^2)$$

- This is called *Effective Polarization Approximation* (EPA). (no dependence on $x$)
At large $x$ the nuclear smearing plays an important role (PRC, 88, 5).

Within the weak binding approximation the structure functions are written as

$$g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y, \gamma) g_j^N(x/y, Q^2) \quad i, j = 1, 2$$
Nuclear corrections

\[
\frac{\Delta q^+(\text{smear})}{\Delta q^+(\text{ref})}
\]

\(x\)

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Combining all the effects...
The JAM13 distributions

\[ x\Delta u^+ \]

- ref.
- + smear
- + TMC
- + HT

\[ x\Delta d^+ \]
The $\Delta q/q$ ratio?
The JAM13 distributions

\[ \Delta u^+/u^+ \]

\[ Q^2 = 1 \text{ GeV}^2 \]

\[ \Delta d^+/d^+ \]
### Other groups

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<th>nuclear smearing</th>
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Inclusion of new data sets from

HallA: d2n (DIS from $^3$He)

HallB: EG1-dvcs (DIS from $p,d$)

HallB: EG1b (DIS from $p,d$)
JAM 13 data

COMPASS (npts = 31)

EMC (npts = 10)

HERMES (npts = 103)

JLabHallA (npts = 6)

JLabHallB (npts = 0)

SLAC (npts = 853)

SMC (npts = 40)

All (npts = 1043)
JAM 13 + new data sets

COMPASS (npts = 31)

EMC (npts = 10)

HERMES (npts = 103)

JLabHallA (npts = 38)

JLabHallB (npts = 1901)

SLAC (npts = 853)

SMC (npts = 40)

All (npts = 2976)
Preliminary results

\[ \frac{\Delta u^+}{\Delta u^+_{ref}} \]

\[ \frac{\Delta d^+}{\Delta d^+_{ref}} \]

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Preliminary results: eg1b A1 d

\[ Q_2 = (1.0 - 1.38) \]

\[ Q_2 = (1.38 - 1.76) \]

\[ Q_2 = (1.76 - 2.13) \]

\[ Q_2 = (2.13 - 2.51) \]

\[ Q_2 = (2.51 - 2.89) \]

\[ Q_2 = (2.89 - 3.27) \]

\[ Q_2 = (3.27 - 3.65) \]

\[ Q_2 = (3.65 - 4.02) \]

\[ Q_2 = (4.4 - 4.78) \]
Preliminary results: eg1b A1 p
HTs are very important if we use data down to $Q^2 = 1 \ \text{GeV}^2$.

There is no indication of an upturn in $\Delta d/d$ as indicated by pQCD.
Inclusion of SIDIS data

Inclusion polarized $pp$ data: $\pi^0$, Jets

Study the error correlation between PDFs and SPDFs (Impact of large $x$ PDFs to SPDFs)
Part II: Constraints from direct photon data at large $x$ on PDFs.
Motivations:

- PDFs at collider energies receives contributions from PDFs at large $x$ and low $Q^2$ via DGLAP evolution equations.

- Production of a massive state in $pp$ collisions at forward rapidities are sensitive to PDFs at small and large $x$ ($x \simeq (m/\sqrt{s}) \exp(\pm y)$).

- In particular, gluon distribution is largely unconstrained for $x > 0.6$. Current constraint for the gluon comes from inclusive Jet data.
Motivations:

- The data of direct photon production has the potential to constrain large $x$ gluons.

- Currently, direct photon data are largely excluded from global fits due to inconsistencies among the data and the theory at NLO in pQCD.

- Threshold resummation at NLO+NLL seems to alleviate the disagreement (Catani et al, de Florian, Vogelsang).
Direct photon production.

The \textit{leading-order} (LO)

- In proton-proton collisions direct photon productions is dominated by $qg$ scattering.
Direct photon production.

Do we understand direct photons?
Direct photon production.

- WA70 $\sqrt{s} = 23.0\text{GeV}$ pp
- CDF $\sqrt{s} = 1800.0\text{GeV}$ $p\bar{p}$
- D0 $\sqrt{s} = 1960.0\text{GeV}$ $p\bar{p}$
- E706 $\sqrt{s} = 31.5\text{GeV}$ pp
- E706 $\sqrt{s} = 38.7\text{GeV}$ pp
- PHENIX $\sqrt{s} = 200.0\text{GeV}$ pp
- R110 $\sqrt{s} = 63.0\text{GeV}$ pp
- R806 $\sqrt{s} = 63.0\text{GeV}$ pp
- R807 $\sqrt{s} = 63.0\text{GeV}$ pp
- UA6 $\sqrt{s} = 24.3\text{GeV}$ pp
- UA6 $\sqrt{s} = 24.3\text{GeV}$ $p\bar{p}$

$\text{data/theory (NLO)}$ vs. $x_T$

$\mu_{R, IF, FF} = p_T$

FFs = BFG II
Can we improve the theory predictions?
Direct photon production.

Rapidity integrated cross section

\[ p_T^3 \frac{d\sigma}{dp_T}(x_T^2) = \]
\[ \sum_{a,b,c} \int_{x_T^2}^{1} dx_a f_{a/A}(x_a, \mu_{IF}) \int_{x_T^2/x_a}^{1} dx_b f_{b/B}(x_b, \mu_{IF}) \int_{x_T/\sqrt{x_a x_b}}^{1} dzz^2 D_{\gamma/c}(z, \mu_F) \]
\[ \times \frac{\hat{x}_T^4 \hat{S}}{2} \frac{d\sigma^{(0)}_{a,b}}{d\hat{x}_T^2}(\hat{x}_T^2, \mu_R) \left[ 1 + \alpha_s(\mu_R) \{ A' \ln^2(1 - \hat{x}_T^2) + B' \ln(1 - \hat{x}_T^2) + C' + \ldots \} \right] \]

- \[ x_T = 2p_T / \sqrt{S} \]
- \[ \hat{x}_T = 2p_T / z \sqrt{x_a x_b S} \]
- At large \( x_T \) the role of logs are more relevant.
Direct photon production.

\[ p_T^3 \frac{d\sigma(x_T)}{dp_T} = \sum_{a,b,c} f_{a/A}(x_a, \mu_{IF}) \ast f_{b/B}(x_b, \mu_{IF}) \ast D_{\gamma/c}(z, \mu_{FF}) \ast \hat{\Sigma}(\hat{x}_T, \ldots) \]

\[ \hat{\Sigma}(\hat{x}_T, \ldots) \supset \]

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<td>( \alpha_s L^{2n} )</td>
<td>( \alpha_s L^{2n-1} )</td>
<td>( \alpha_s^2 L^{2n-2} )</td>
<td>\ldots</td>
<td>NNLO</td>
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| LL | NLL | NNLL | \ldots |

- It is possible to include “towers” of logs into the cross section → Threshold Resummation
Global analysis

Example: UA6 experiment \((\sqrt{s} = 24.3 \text{ GeV})\)

\[
\begin{align*}
\text{UA6 experiment} & \quad pp \rightarrow \gamma + X \\
\sqrt{s} & = 24.3 \text{ GeV} \\
PDFs & = \text{Cteq6, FFs = BFGII} \\
NLO & \quad ζ = 0.5 \\
NLO + NLL & \quad ζ = 1.0 \\
UA6 & \quad ζ = 2.0
\end{align*}
\]
Global analysis

All the data sets

![Graph showing data and theory comparisons for NLO and NLL + NLO models. The x-axis represents \( x_T \), and the y-axis represents the ratio of data to theory.]
Global analysis

All the data sets without E706

![Graph showing data and theory comparison for NLO and NLL + NLO with a logarithmic scale on the x-axis and a linear scale on the y-axis. The graph includes error bars and two distinct lines for NLO and NLL + NLO.]
Impact on PDFs: Bayesian Reweighting

The method:

- Construct replica PDFs (Monte Carlo sampling) from the uncertainty band of a given PDF set

- Compute replica cross sections.

- Assign a weight to each replica PDF:

\[
w_k \propto \exp \left( -\frac{1}{2} \sum_e \chi_e^2(k) \right)
\]
Reweighting PDFs

The method:

- Get reweighted theoretical cross sections

\[ E[\sigma] = \sum_k w_k \sigma_k \]
\[ \text{VAR}[\sigma] = \sum_k w_k (\sigma_k - E[\sigma])^2 \]

- Get reweighted PDFs

\[ E[f] = \sum_k w_k f_k \]
\[ \text{VAR}[f] = \sum_k w_k (f_k - E[f])^2 \]
Reweighting PDFs

All the data sets

![Graph showing reweighted and prior distributions](image)
Reweighting PDFs

All the data sets excluding E706

The graph shows the distribution of normalized yield against $\chi^2_{\text{DOF}}$. The categories are:
- NLO prior
- NLO reweighted
- NLL + NLO prior
- NLL + NLO reweighted
Reweighting PDFs

CT10nlo Glue @ Q = 10 GeV

\[
\sigma_{\text{RW}} / \sigma_{\text{prior}}
\]

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Reweighting PDFs

CT10nlo Up @ Q=10GeV

Prior
Reweighted
Reweighting PDFs

CT10nlo Down @ Q = 10 GeV

Radio to central prior reweighted
Summary

- Direct photon data contraints gluon PDFs at large $x$.

- Uncertainties on PDFs (gluon) at large $x$ might have an impact on SPDF at large $x$.

- Threshold correction might have an impact on polarized $pp$ collision. (To be implement in future JAM analysis)
END