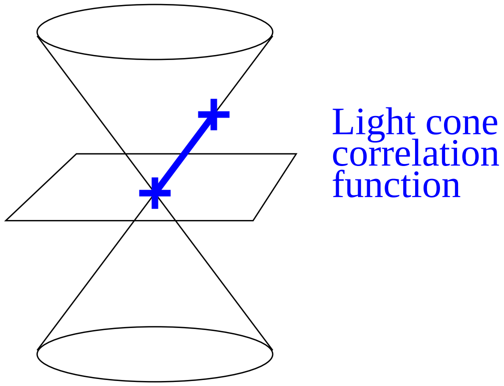
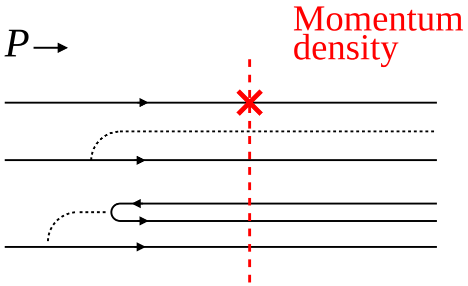
PDFs as momentum densities: Lessons from large–N_c model field theories

C. Weiss (JLab), MCFP Workshop on Lattice Parton Physics, U. Maryland, 31–Mar–14



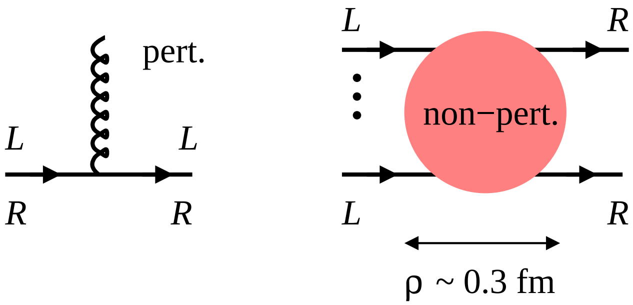


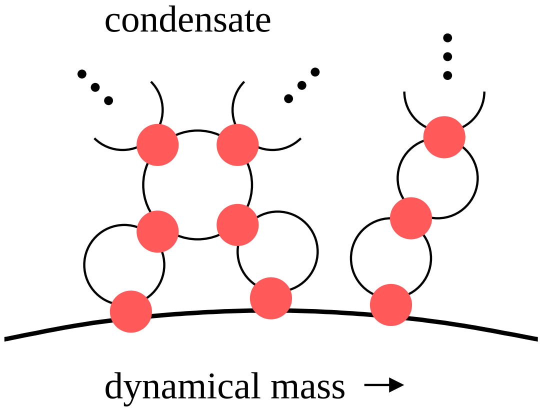
Equivalence, physical requirements
studied in simpler dynamical setting!

Quantum field theory: UV divergences,
particle number fluctuations, ...

- Model field theory and nucleon
  - Dynamics from spontaneous χSB
  - Solution in $1/N_c$ expansion
- Parton distributions
  - LC correlation functions
  - Momentum densities at $P \to \infty$
  - Equivalence
- UV regularization
  - Power divergences of moments
  - Large–x tails and anomalies
  - Completeness of states and causality!
  - Physical regularization by Pauli–Villars cutoff

Dynamics: Chiral symmetry breaking





- Chiral symmetry breaking in QCD

  Non-pert. gluon fields can flip chirality
  Topological gauge fields, instantons

  Condensate of $q\bar{q}$ pairs $\langle\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\rangle$,
  pion as collective excitation
  Order parameter, Goldstone boson

  Dynamical mass generation:
  Constituent quarks, hadron structure
  Euclidean correlation functions → Lattice, analytic methods

  Range of χSB iteractions $\rho \ll 1\,\mathrm{fm}$
  New dynamical scale: Shuryak; Diakonov, Petrov 80's

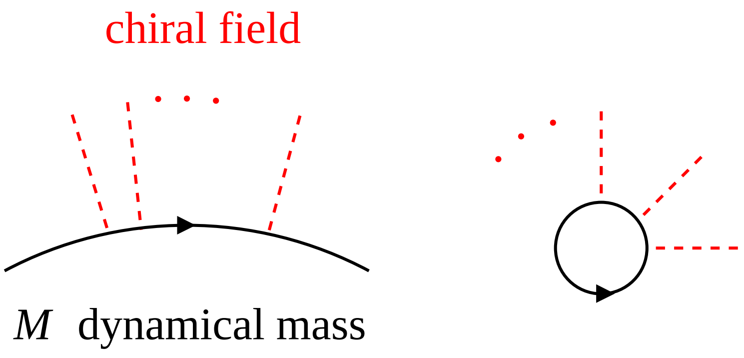
  Gauge–invariant measure of $q\bar{q}$ pair size
  $\langle\bar{\psi}\nabla^2\psi\rangle/\langle\bar{\psi}\psi\rangle \sim 1\,\mathrm{GeV}^2$
  Lattice: Teper 87, Doi 02, Chiu 03. Instantons: Polyakov, CW 96

- Dynamical model of chirally broken phase

  Valid at momenta $p_{\mathrm{eucl}} \lesssim \rho^{-1}$

  Based on large–N_c limit of QCD

Dynamics: Field-theoretical model



$$L_{\text{eff}} = \bar{\psi}\left(i\not{\partial} - M e^{i\gamma_5 \boldsymbol{\tau}\boldsymbol{\pi}/f_\pi}\right)\psi$$

$$Z = \int [d\pi] \int [d\bar{\psi}\, d\psi]\; e^{-S_{\text{eff}}}$$

- Effective description of χSB
  Diakonov, Eides 83; Diakonov, Petrov 86

  Quark field acquires dynamical mass $M \sim$ 0.3-0.4 GeV

  Coupled to chiral field (Goldstone boson) with eff. coupling $M/f_\pi = 3$–4 strong!

  Valid up to χSB scale ρ^{-2}, implemented as UV cutoff

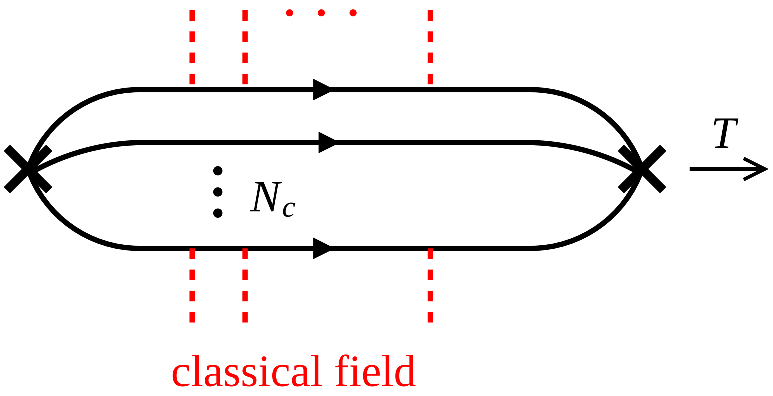
- Solved non–perturbatively using $1/N_c$ expansion

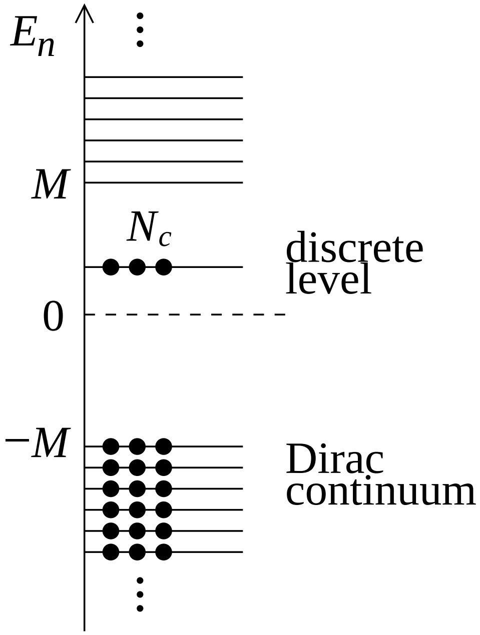
  Correlation functions of composite operators

  Functional integral evaluated in saddle point approximation $S_{\text{eff}} \sim N_c$
  Semiclassical approximation

  Parametric approximation, defined accuracy

Dynamics: Nucleon





- Nucleon correlation function
  Diakonov, Petrov, Pobylitsa 88

  Classical chiral field — soliton
  "Hedgehog" $\boldsymbol{\pi} \parallel \boldsymbol{r}$ in rest frame, cf. skyrmion

  Binds N_c quarks, distorts chiral vacuum
  Relativistic mean–field approximation

  Rest frame: Quark Hamiltonian,
  single–particle levels, discrete + continuum

  Quantization of (iso)rotational
  zero modes: N, Δ quantum numbers

  $\langle N | \hat{O} | N \rangle$ from 3–point functions

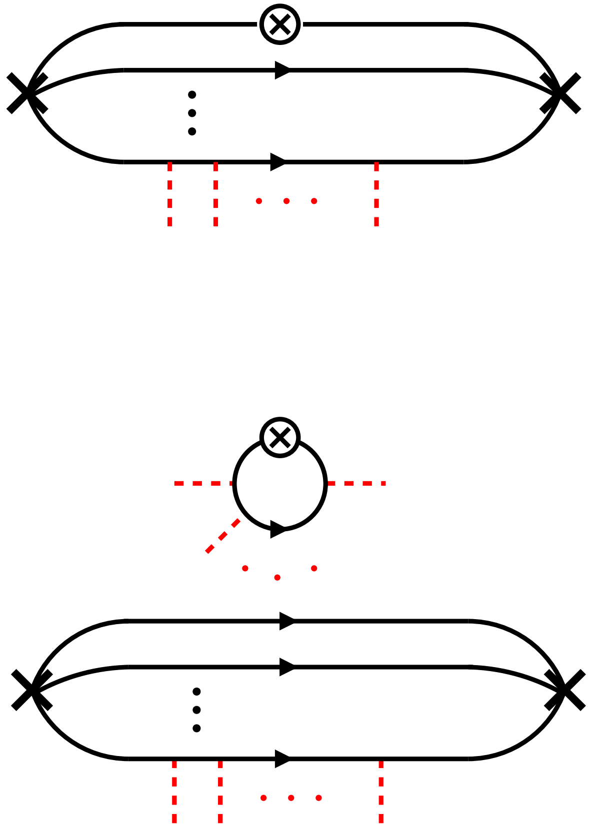
- Field–theoretical description!

  Completeness of single–particle levels
  $\sum_n \Phi_n^\dagger(\boldsymbol{x}) \Phi_n(\boldsymbol{y}) = \delta^{(3)}(\boldsymbol{x} - \boldsymbol{y})$

  No Fock space truncation
  → PDFs, sea quarks

  Relativistically covariant

Dynamics: Matrix elements



- $\langle N|\hat{O}|N\rangle$ from 3–point functions

  Calculated in $1/N_c$ expansion

  Connected and disconnected quark diagrams in classical chiral field

  "Valence" and "sea quark" contribution

- Sums over single–particle levels

  $$\sum_n \langle n|\hat{O}|n\rangle$$

  $n =$ discrete $+$ continuum

- Rich phenomenology: Charges, form factors, $SU(3)$ extension...
  Review Christov et al., Prog. Part. Nucl. Phys. 37, 91 (1996)

PDFs: Light-cone correlation function

- PDF in model as LC correlation function
  Diakonov, Petrov, Pobylitsa, Polyakov, Weiss NPB 480 (1996) 341;
  PRD 56 (1997) 4069; see also Wakamatsu et al. 97+

$$f(x) \;=\; \int \frac{d\xi^-}{8\pi}\, e^{iP^+z^-/2} \langle N|\bar{\psi}(0)\gamma^+\psi(\xi)|N\rangle_{\xi^+=0} \;=\; \left\{ \begin{array}{rr} q(x) & x > 0 \\ -\bar{q}(-x) & < 0 \end{array} \right.$$

Calculate moments or x–dependence; model respects analyticity preserved by UV cutoff → later

Parametrically $x = O(1/N_c)$

- Evaluate in rest frame

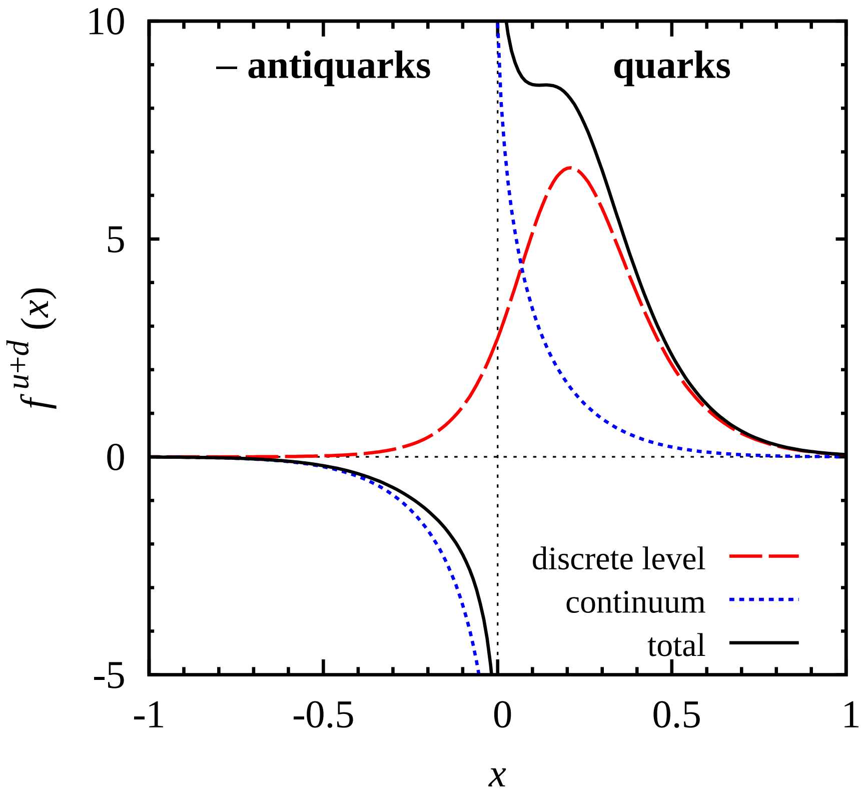
$$f^{u+d}(x) = N_c M_N \sum_n \int \frac{d^3p}{(2\pi)^3}\, \delta(p^3 + E_n - xM_N)\; \Phi_n(\boldsymbol{p})^\dagger\, \gamma^0\gamma^+\Phi_n(\boldsymbol{p})$$

Sum over quark single–particle states, n = discrete level + continuum

Evaluated numercially

Alt. method: Green function, analytic approximations

PDFs: Positivity and sum rules



Completeness of quark single–particle states!

- Positivity
  Diakonov et al. NPB 480 (1996) 341

  Discrete level give negative antiquarks

  Continuum gives positive antiquarks

  Total quark/antiquark PDFs positive!

- Partonic sum rules

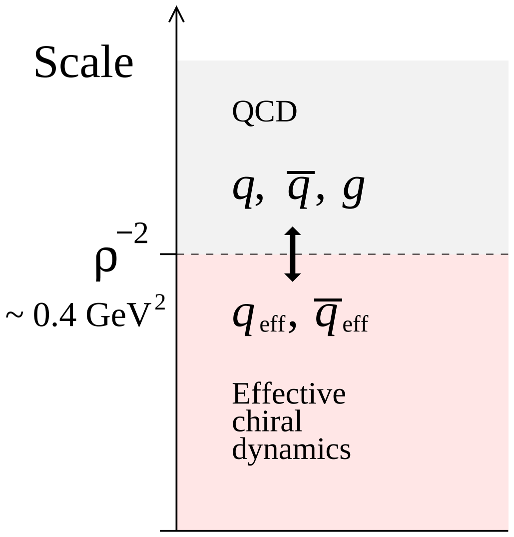
$$\int dx\ f^{u-\bar{u}+d-\bar{d}} = N_c \qquad \text{baryon nr}$$

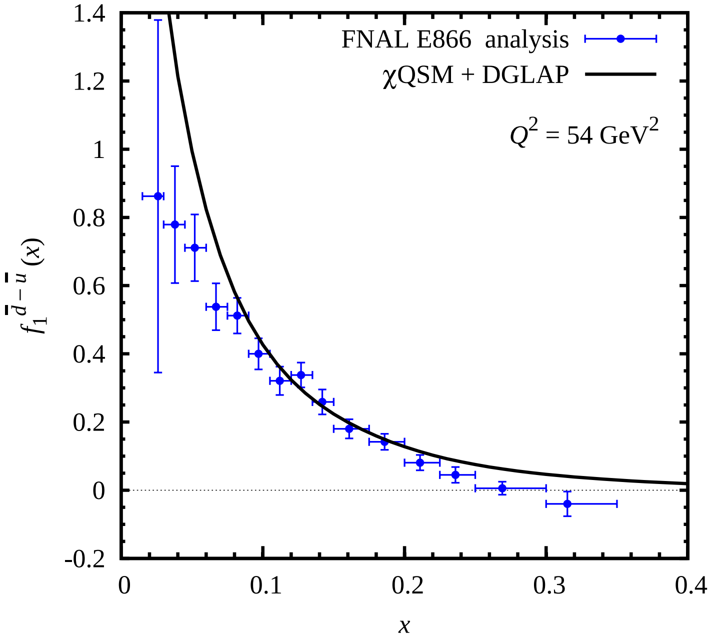
$$\int dx\ x\ f^{u+\bar{u}+d+\bar{d}} = 1 \qquad \text{momentum}$$

$$\int dx\ g^{u+\bar{u}-d-\bar{d}} = g_A \qquad \text{Bjorken}$$

satisfied within model!

PDFs: Interpretation





- Interpretation of model PDFs

  Model uses effective degrees of freedom:
  Massive quarks and antiquarks, **no gluons**

  Momentum sum rule saturated by effective degrees of freedom

  Matching with QCD quarks, antiquarks, and gluons at χSB scale ρ^{-2}

  PDF fits at $\mu^2 \sim 0.5\,\mathrm{GeV}^2$ show 30% of nucleon momentum carried by gluons
  "Accuracy" of matching

- Flavor asymmetries

  Model describes well measured $\bar{d} - \bar{u}$
  Pobylitsa, Polyakov, Goeke, Watabe, Weiss, PRD59 (1999) 034024

  Predicts sizable asymmetry $\Delta\bar{u} - \Delta\bar{d}$
  Diakonov et al. NPB 480 (1996) 341; PRD 56 (1997) 4069.
  Hints seen in DSSV global fits, RHIC W data

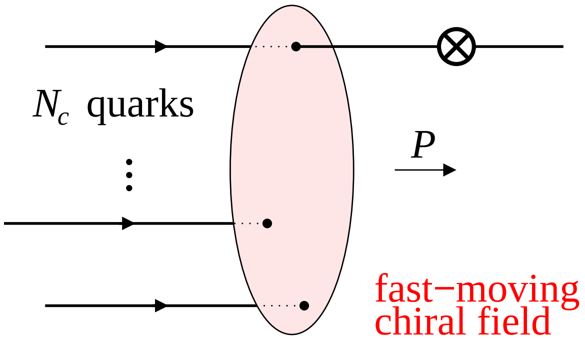
PDFs: Momentum density at $P \to \infty$

$$f(x) = \int \frac{d^3p}{(2\pi)^3}\, \delta\left(x - \frac{p^3}{P}\right) \langle P|a^\dagger a(\boldsymbol{p})|P\rangle$$

$$\bar{f}(x) = \quad \ldots \qquad\qquad b^\dagger b(\boldsymbol{p})$$

- PDF in model as momentum density of quarks/antiquarks
  Diakonov et al. PRD 56 (1997) 4069

  "Parton model" definition



- Nucleon state at $P \to \infty$

  Model dynamics is relativistically invariant

  Boosted chiral field and quark single–particle wave funtions

- Equivalence of PDF formulations within model

  Uses completeness of quark single–particle states $\sum_n \Phi^\dagger_n(\boldsymbol{x})\Phi_n(\boldsymbol{y}) = \delta^{(3)}(\boldsymbol{x} - \boldsymbol{y})$

  Ensures causality $\{\bar{\psi}(x), \psi(y)\}_+ = 0$ if $(x-y)^2 < 0$ spacelike separation

  Preserved by UV cutoff

UV regularization: Physical requirement

- UV cutoff physical ingredient of model

  Represents χSB scale

  Form determined from physical considerations: symmetries, conservation laws,

- PDFs UV divergent, require cutoff

  Logarithmic divergence: dominant contribution from momenta $p \ll$ cutoff

- Physical requirements
  Diakonov et al. NPB 480 (1996) 341; PRD 56 (1997) 4069

  Causality and spectral properties of LC correlation function

  $\rightarrow$ Preserve completeness of quark single–particle states!

  $\rightarrow$ Preserve analyticity properties

- Pauli–Villars subtraction

$$f(x)_{\text{reg}} \quad = \quad f(x|M) - \frac{M^2}{M^2_{\text{reg}}}\, f(x|M_{\text{reg}})$$

UV regularization: Artifacts of unphysical scheme

- Energy cutoff $|E_n| < \Omega$

  Used in numerical calculations in finite model space

  Violates completeness of single–particle states!

- Power–like divergence of moments

$$\mathcal{M}_n \quad \equiv \quad \int dx\, x^{n-1}\, f(x) \quad \sim \quad \Omega^{n-2} \quad (n > 2)$$

- Large–x tail of distribution, extends over range $-\dfrac{2\Omega}{M_N} < x < 0$

- Anomaly phenomenon

$$\left(\sum_{\mathrm{occ}} + \sum_{\mathrm{non-occ}} \right) f(x)_{\mathrm{tail}} \quad \neq \quad 0 \quad \text{even in limit } \Omega \to \infty$$

- All artifacts disappear with Pauli–Villars subtraction – physical regularization!
  Diakonov et al. PRD 56 (1997) 4069

Summary

- Equivalence of “light–cone correlation function” and “momentum density” demonstrated in field–theoretical model

- Causality/completeness as essential prerequisites

Lessons for QCD calculations

- Causality/completeness–violating regularization can produce unphysical tails in x–distribution

- Regularization artifacts may persist even in limit cutoff $\rightarrow \infty$ (“anomalies”)

- Pauli–Villars subtraction as physical regularization

Extensions and applications

- Quasi–PDFs and $1/P$ corrections in large–N_c model