



# New limits on intrinsic charm in the nucleon from global analysis

Wally Melnitchouk



# BHPS model

- Possibility of intrinsic charm (IC) component in nucleon suggested by Brodsky, Hoyer, Peterson, Nakai (BHPS) ~ 35 years ago

## THE INTRINSIC CHARM OF THE PROTON

*Phys. Lett.* **93B**, 451 (1980)

S.J. BRODSKY<sup>1</sup>

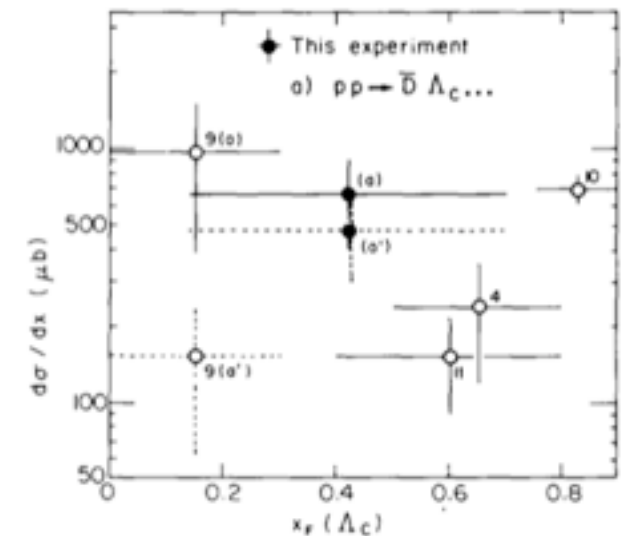
*Stanford Linear Accelerator Center,  
Stanford, California 94305, USA*

and

P. HOYER, C. PETERSON and N. SAKAI<sup>2</sup>

*NORDITA, Copenhagen, Denmark*

→ inspired by larger than expected production cross sections in *e.g.*  $pp \rightarrow \bar{D}\Lambda_c X$  at CERN's ISR (100s  $\mu\text{b}$  *cf.* 10s  $\mu\text{b}$ )



*Phys. Lett.* **99B**, 495 (1981)

# BHPS model

- Significant (nonperturbative) 5-quark component of nucleon wave function, estimated at “O(1%)”, could account for magnitude of new data
- Transition probability (in infinite momentum frame)

$$P(p \rightarrow uudc\bar{c}) \sim \left[ M^2 - \sum_{i=1}^5 \frac{k_{\perp i}^2 + m_i^2}{x_i} \right]^{-2} \quad i = 4, 5 \text{ for } c, \bar{c}$$

- Neglecting transverse momentum and assuming heavy quark limit,  $m_{c,\bar{c}} \gg M, m_{1,2,3}$

→ probability to produce a single charm quark

$$P(x_5) = \frac{N x_5^2}{2} \left[ \frac{(1 - x_5)}{3} (1 + 10x_5 + x_5^2) + 2x_5(1 + x_5) \ln(x_5) \right]$$

# Scalar 5-quark model

## ■ Generalization to include finite size of nucleon

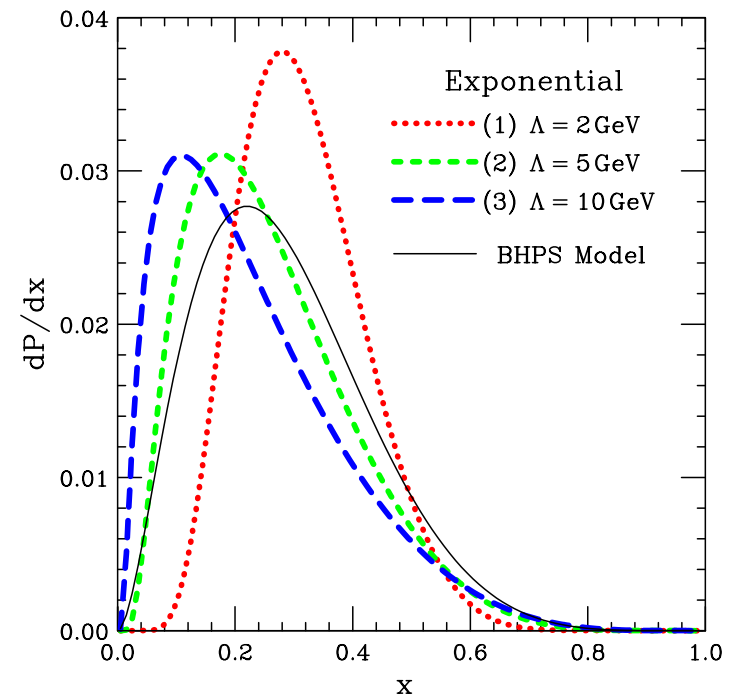
$$dP = \frac{g^2}{(16\pi^2)^{N-1}(N-2)!} \prod_{j=1}^N dx_j \delta \left( 1 - \sum_{j=1}^N x_j \right) \int_{s_0}^{\infty} ds \frac{(s - s_0)^{N-2}}{(s - m_0^2)^2} |F(s)|^2$$

*Pumplin, PRD 73, 114015 (2006)*

invariant mass squared  $s_0 = \sum_{j=1}^N \frac{m_j^2}{x_j}$

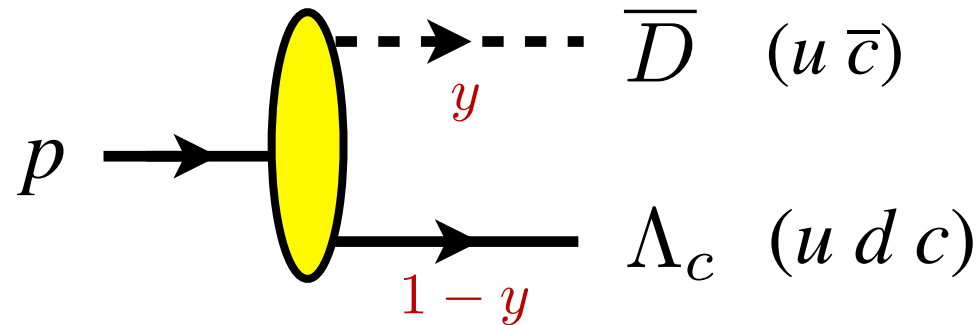
form factor at  $N$ - $qqqc\bar{c}$  vertex

$$|F(s)|^2 = \exp \left[ -(s - m_0^2)/\Lambda^2 \right]$$



# Meson-baryon model

- Fluctuations of nucleon to virtual states with meson & baryon quantum numbers



$$|N\rangle = \sqrt{Z_2} |N\rangle_0 + \sum_{M,B} \int dy d^2 k_{\perp} \phi_{MB}(y, k_{\perp}^2) |M(y, k_{\perp}); B(1-y, -k_{\perp})\rangle$$

wave function renormalization      “bare” 3-quark state       $N \rightarrow M+B$  probability amplitude      longitudinal (or light-cone) momentum fraction

# Meson-baryon model

- Charm distributions in nucleon as convolutions of  $N \rightarrow MB$  splitting functions and distributions inside charmed meson & baryons

$$\bar{c}(x) = \sum_{M,B} \int_x^1 \frac{dy}{y} f_{MB}(y) \bar{c}_M\left(\frac{x}{y}\right)$$

$$c(x) = \sum_{B,M} \int_x^1 \frac{d\bar{y}}{\bar{y}} f_{BM}(\bar{y}) c_B\left(\frac{x}{\bar{y}}\right) \quad \bar{y} \equiv 1 - y$$

→ meson-baryon splitting function

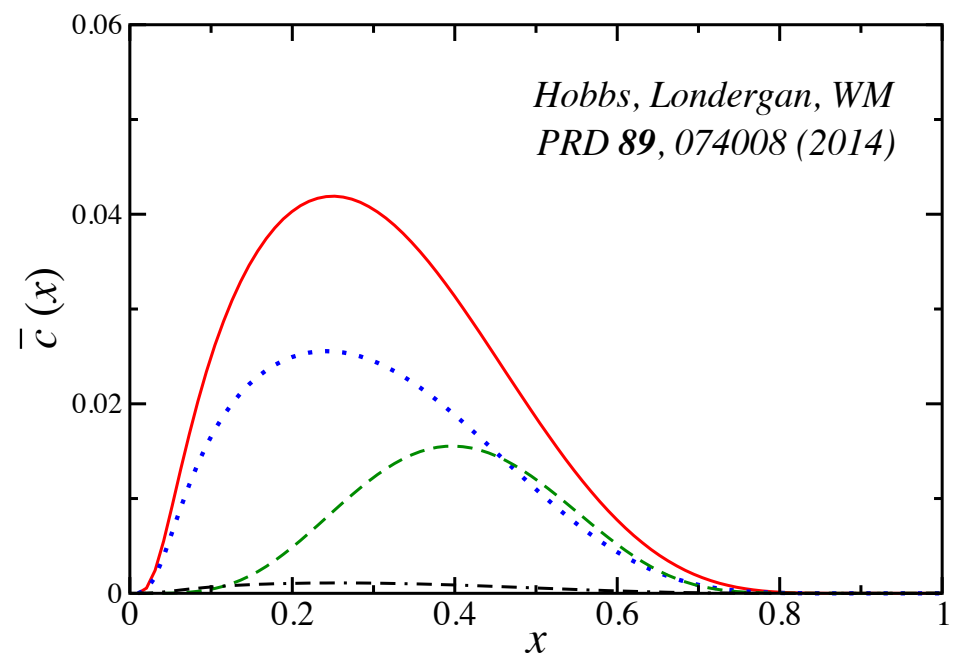
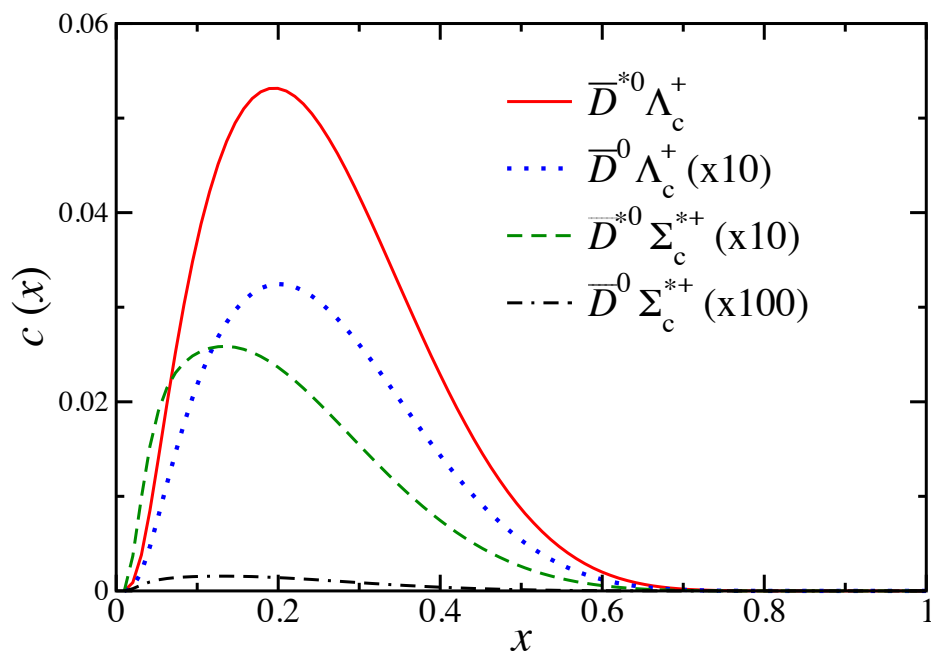
$$f_{MB}(y) = \int_0^\infty d^2 k_\perp |\phi_{MB}(y, k_\perp^2)|^2 = f_{BM}(\bar{y})$$

→ naturally predicts asymmetric charm distributions

$$c(x) \neq \bar{c}(x)$$

# Meson-baryon model

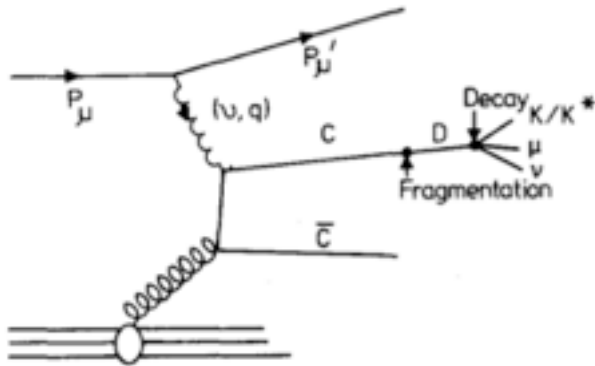
- Charm distributions in nucleon as convolutions of  $N \rightarrow MB$  splitting functions and distributions inside charmed meson & baryons



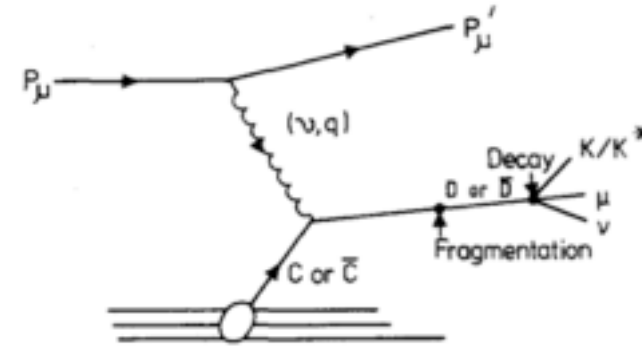
- normalized to  $\langle n \rangle_{MB}^{c+\bar{c}} = 2.4\%$  (for comparison)
- typically  $\bar{c}$  peaks at slightly larger  $x$  than  $c$

# Intrinsic charm in DIS

- European Muon Collaboration (EMC) measured open charm production  $\mu "N" \rightarrow \mu D X$  in early 1980s

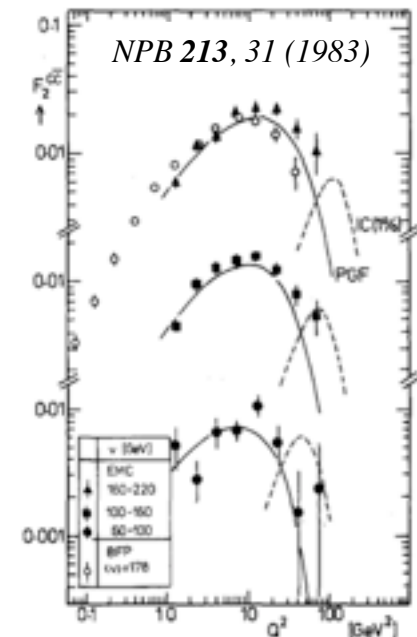
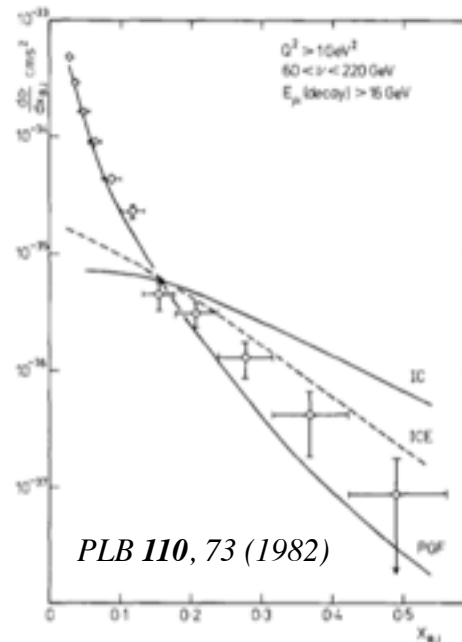


photon-gluon fusion



intrinsic charm

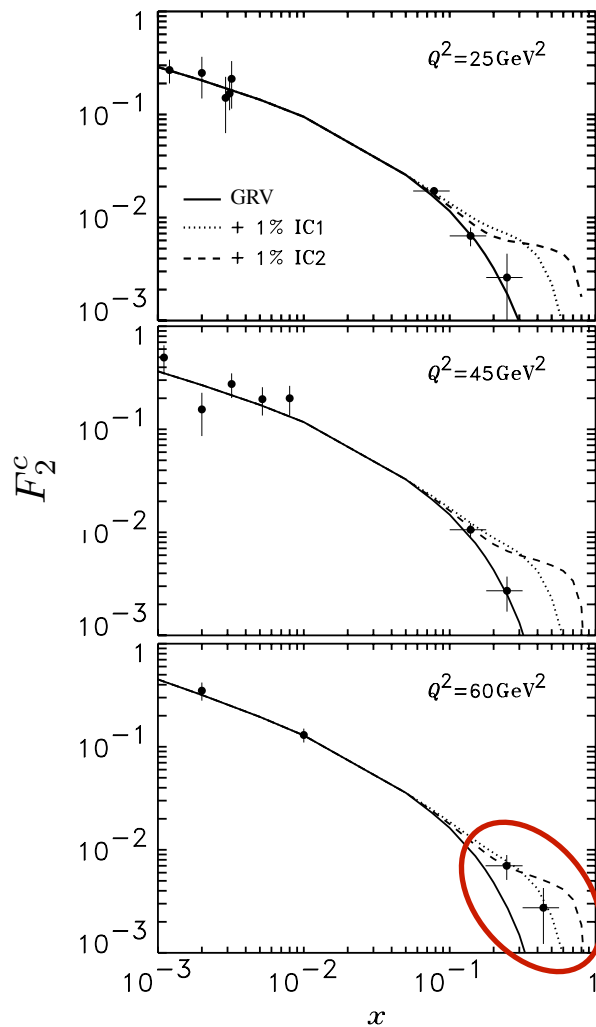
→ EMC data inconclusive!





# Intrinsic charm in DIS

- Some hint of excess charm at highest  $x$  and  $Q^2$   
*cf.* perturbative QCD contribution



Steffens, WM, Thomas  
*EPJC* **11**, 673 (1999)

$$F_2^c(x, Q^2) = \frac{4x}{9} [c(x, Q^2) + \bar{c}(x, Q^2)]$$

at LO in  $\alpha_s$

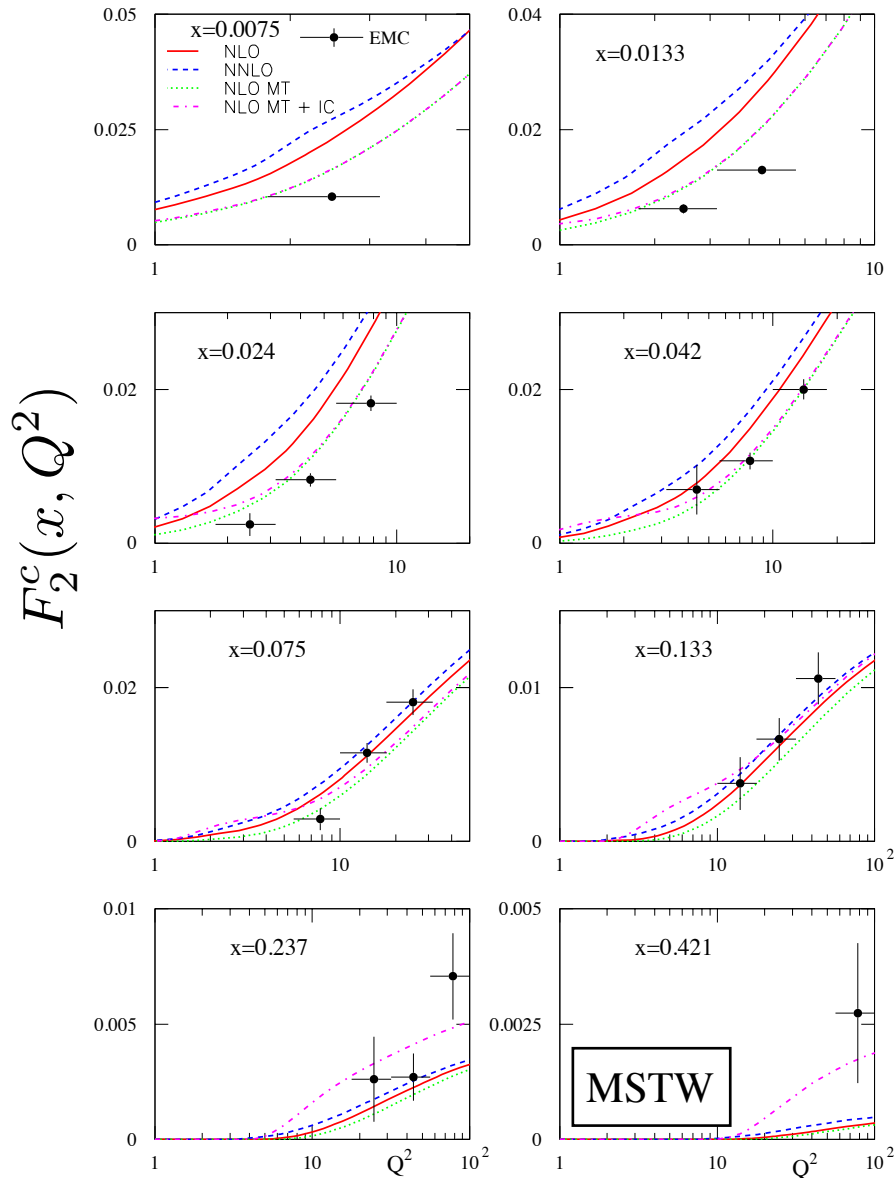
IC1 = BHPS model

IC2 = meson-baryon model

- these data frequently cited as evidence for large IC in nucleon
- definitive study requires systematic global QCD analysis

# Global QCD analysis

- Several previous global analyses have considered possibility of intrinsic charm component



$$m_c^2 \rightarrow m_c^2 (1 + \Lambda^2 / m_c^2)$$

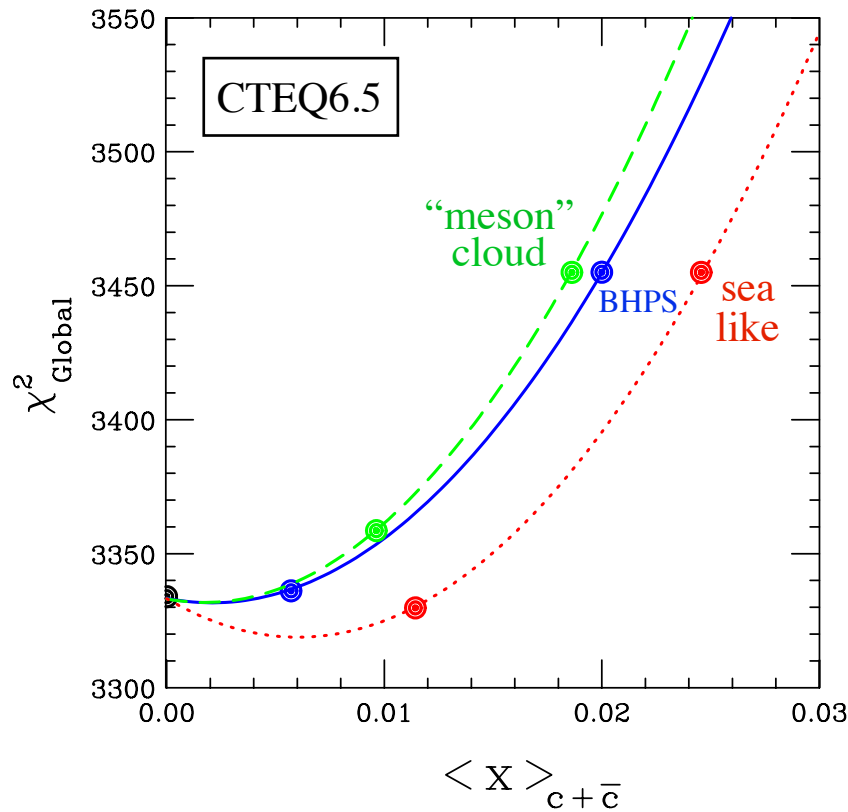
“hadronic threshold” modification

“if the EMC data are to be believed,  
there is no room for a very sizeable  
intrinsic charm contribution”

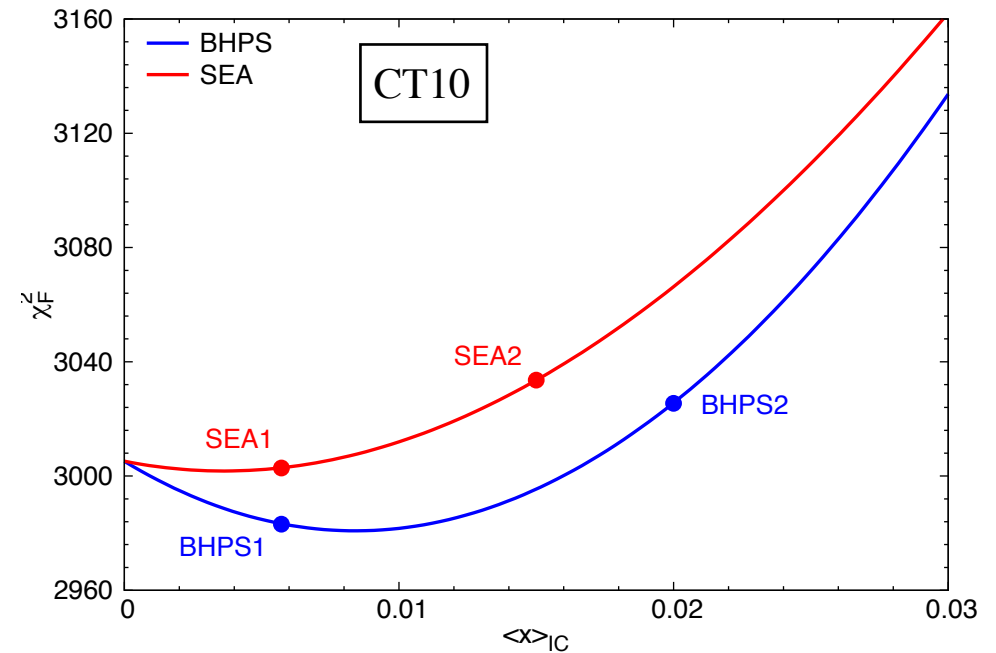
*MSTW, EPJC 63, 189 (2009)*

# Global QCD analysis

- CTEQ/CT *do* find room for  $\sim$  few % IC in their analysis



*Pumplin et al., PRD 75, 054029 (2007)*



*Dulat et al., PRD 89, 073004 (2014)*

$\langle x \rangle_{\text{IC}} \lesssim 0.025$  at 90% CL

→ however, CTEQ/CT use rather strong kinematic cuts, excluding much high- $x$  / low- $W$  data

# Global QCD analysis

- Excluding high- $x$  data (to avoid subleading  $1/Q^2$  effects),  
exclude region where IC expected to be important!
  - several recent analyses (CJ, ABM, JR) have sought better constraints on large- $x$  PDFs by expanding kinematic coverage down to  $Q^2 \sim 1 \text{ GeV}^2$  &  $W^2 \sim 3.5 \text{ GeV}^2$
  - requires careful treatment of higher twist, target mass, nuclear corrections
  - better constraints on light-quark ( $u, d$ ) PDFs at large  $x$ , which are background on which possible IC sits

recall 
$$F_2^p \sim \frac{4x}{9}(u + \bar{u} + c + \bar{c}) + \frac{x}{9}(d + \bar{d} + s + \bar{s}) + \dots$$

# New global QCD analysis


- Using framework of JR14 (NLO) global analysis, most recent analysis has fit all available data for  $Q^2 \geq 1 \text{ GeV}^2$ ,  $W^2 \geq 3.5 \text{ GeV}^2$  allowing for the possibility of IC

*Jimenez-Delgado, Reya  
PRD 89, 074049 (2014)*


$$F_2 = F_2^{u,d,s} + F_2^{c,b}$$


$$F_2^c = F_2^{\text{PGF}} + F_2^{\text{IC}}$$

- $F_2^{\text{PGF}}(x, Q^2, m_c^2) = \frac{Q^2 \alpha_s}{4\pi^2 m_c^2} \sum_i \int \frac{dz}{z} \hat{\sigma}_i(\eta, \xi) f_i\left(\frac{x}{z}, \mu\right)$   
computed in “fixed-flavor number scheme”



$\gamma^* g \rightarrow c\bar{c}$

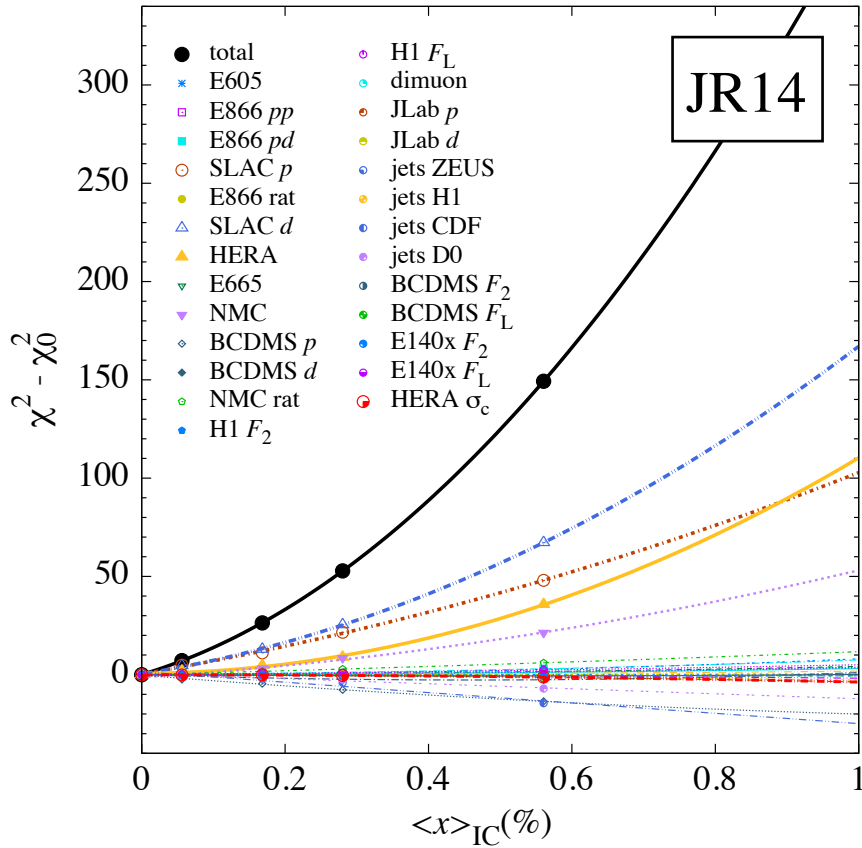


$\mu^2 = 4m_c^2 + Q^2$

- $F_2^{\text{IC}}$  computed from various models (BHPS, MBM)

# New global QCD analysis

Jimenez-Delgado et al., PRL 114, 082002 (2015)



$\chi_0^2 = \chi^2$  value for no IC

→ total  $\chi^2$  has minimum at zero IC and rises rapidly with  $\langle x \rangle_{IC}$

→ strongest constraints from SLAC, HERA, NMC data; others have very little sensitivity

→ full data set gives  $\langle x \rangle_{IC} < 0.1\%$  at  $5\sigma$  CL for  $\Delta\chi^2 = 1$

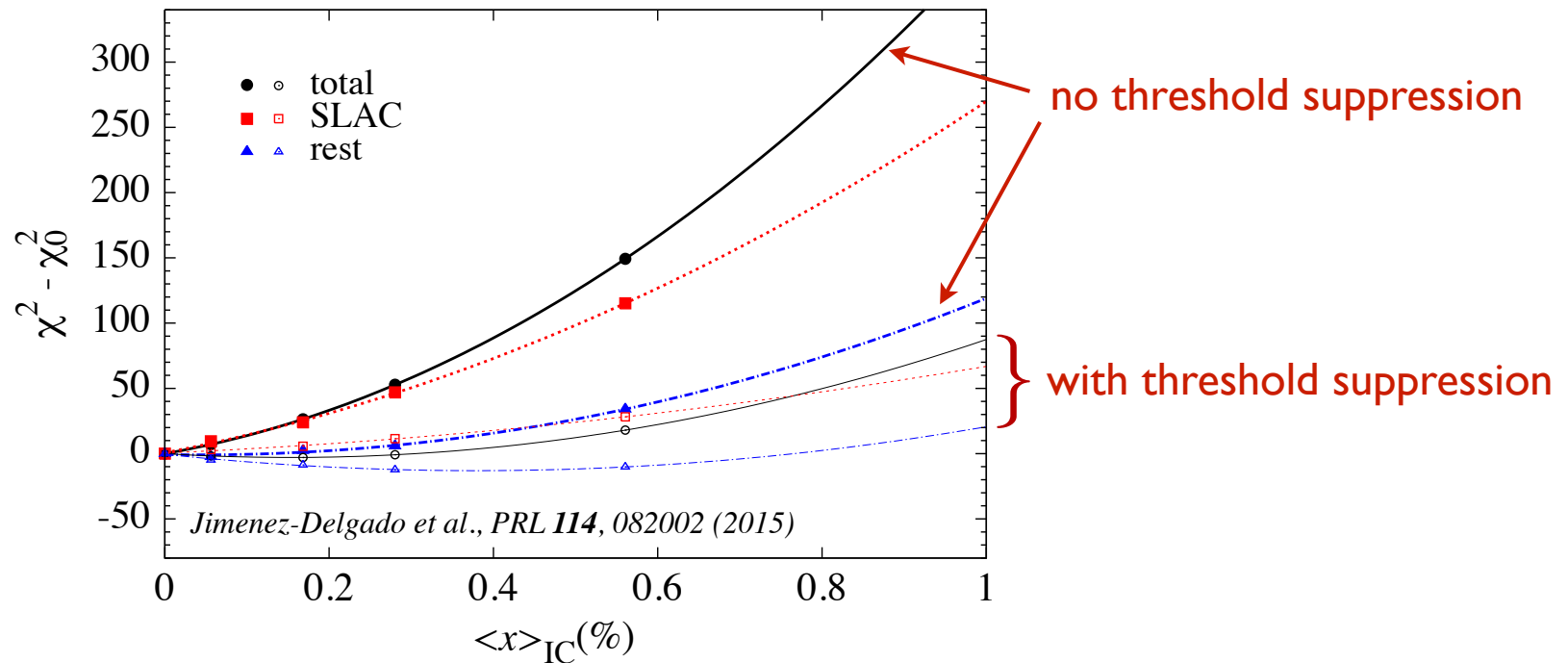
→ for  $\Delta\chi^2 = 100$  (“tolerance”) would have  $\langle x \rangle_{IC} \lesssim 0.4\%$

# Threshold suppression?

- Significant portion of SLAC data lie *below* partonic charm threshold,  $W^2 = 4m_c^2$ , so cannot directly constrain IC
  - through  $Q^2$  evolution, stronger constraints on light-quark PDFs at high  $x$  influence determination of IC in global fit
  - in fact, partonic threshold is lower than physical charm production threshold,  $W^2 \geq (M_N + m_{J/\psi})^2 \approx 16 \text{ GeV}^2$
  - various prescriptions to account for mismatch between partonic & hadronic thresholds
    - MSTW modified threshold with effective charm mass
$$m_c^2 \rightarrow m_c^2(1 + \Lambda^2/m_c^2)$$
    - threshold suppression factor
$$\theta(W^2 - W_{\text{thr}}^2)(1 - W_{\text{thr}}^2/W^2)$$

# Threshold suppression?

- Including hadronic suppression factor generally gives shallower  $\chi^2$  profile



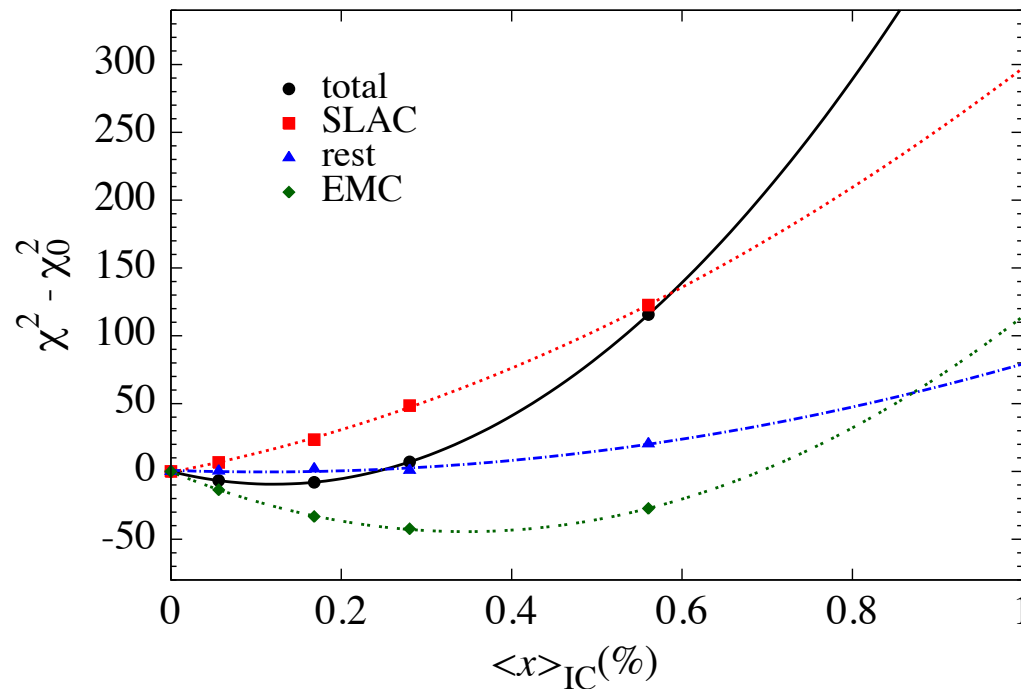
→ minimum  $\chi^2$  at  $\langle x \rangle_{IC} = (0.15 \pm 0.09)\%$

→ exclusion limit  $\langle x \rangle_{IC} \lesssim 0.5\%$  at  $4\sigma$  CL



# Analysis of EMC data

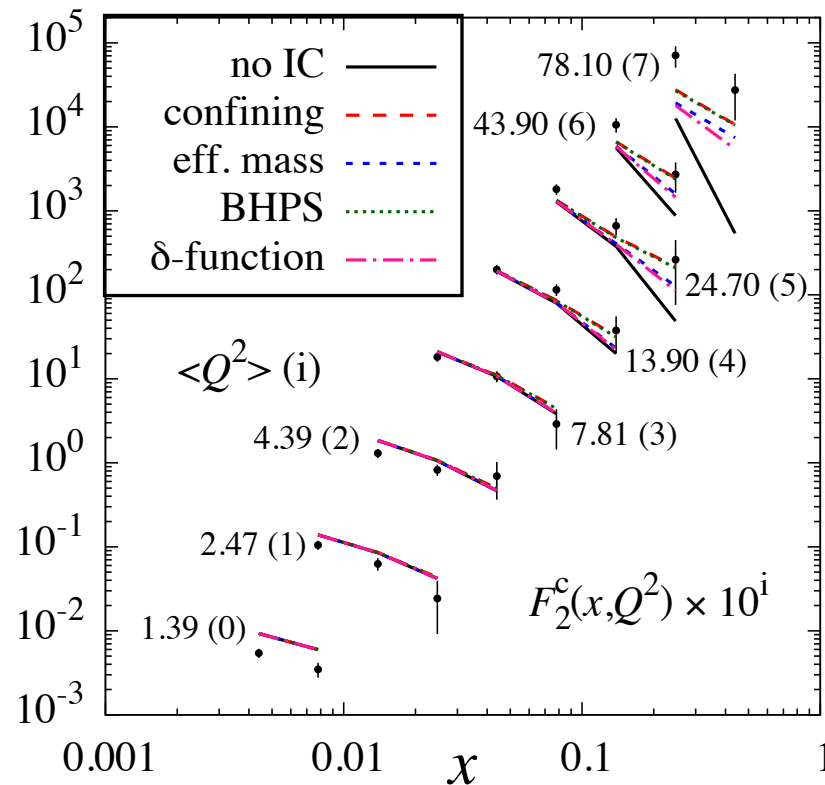
- Including old EMC data on charm structure function favors slightly larger IC



→ EMC alone favors  $\langle x \rangle_{IC} \approx (0.3 - 0.4)\%$   
... but poor description of data,  
with  $\chi^2/N_{\text{EMC}} = 4.3$  for  $N_{\text{EMC}} = 19$

# Analysis of EMC data

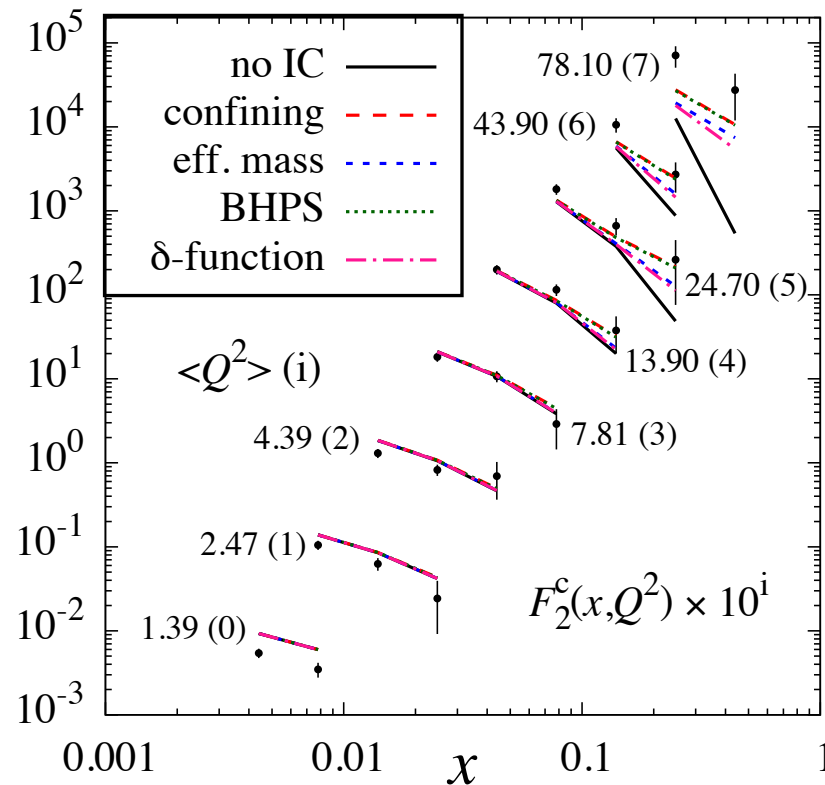
## ■ Closer look at $x$ dependence of EMC $F_2^c$ data



- at small  $x$  ( $x \lesssim 0.02$ ) global fits (constrained by HERA data) overestimate EMC data
- at largest  $x$  ( $x \gtrsim 0.2$ ) fits underestimate EMC data, with or without IC, for all IC models considered

# Analysis of EMC data

## ■ Closer look at $x$ dependence of EMC $F_2^c$ data



- better agreement would require much larger IC at high  $x$  and suppression mechanism (negative IC?) at small  $x$
- because of significant tension with other data sets, IC data usually not included in global PDF analyses

# Outlook

- *No evidence for large intrinsic charm* from global QCD analysis of high-energy data, for range of IC models
- Small amount of IC not excluded, but any more definitive determination requires new data (perhaps from future Electron-Ion Collider, AFTER@LHC?)
  - “smoking gun” would be observation of asymmetric distributions  $c(x) \neq \bar{c}(x)$
- Study of nonperturbatively generated sea quarks remains exciting subject in QCD!
  - novel nonperturbative effects reflected in various asymmetries, *e.g.*  $\bar{d} \neq \bar{u}$ ,  $s \neq \bar{s}$ ,  $\Delta s \neq \Delta \bar{s}$ , ...



# ■ Brodsky not happy!



## ■ 3 main criticisms

- $\Delta\chi^2 = 1$  wrong if have  $\sim 30$  params.
- cannot use SLAC data to learn about charm
- should include EMC  $F_2^c$  data in fits

arXiv:1504.00969v1 [hep-ph] 4 Apr 2015

## Comment on “New Limits on Intrinsic Charm in the Nucleon from Global Analysis of Parton Distributions”

 Stanley J. Brodsky<sup>1</sup> and Susan Gardner<sup>2</sup>
<sup>1</sup>SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309

<sup>2</sup>Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055

 A Comment on the Letter by P. Jimenez-Delgado, T. J. Hobbs, J. T. Londergan, and W. Melnitchouk, Phys. Rev. Lett. **114**, 082002 (2015).

Intrinsic heavy quarks in hadrons emerge from the non-perturbative structure of a hadron bound state [1] and are a rigorous prediction of QCD [2, 3]. Lattice QCD calculations also indicate a significant intrinsic charm probability [4, 5]. Since the light-front momentum distribution of the Fock states is maximal at equal rapidity, intrinsic heavy quarks carry significant fractions of the momentum. The presence of Fock states with intrinsic strange, charm, or bottom quarks in hadrons lead to an array of novel physics phenomena [6]. Accurate determinations of the heavy-quark distribution functions in the proton are needed to interpret LHC measurements as probes of physics beyond the Standard Model [7, 8]. Determinations [7, 9, 10] of the momentum fraction carried by intrinsic charm quarks in the proton typically limit  $\langle x \rangle_{IC} \sim \mathcal{O}(1\%)$  at 90% CL, consistent with the analysis of the EMC measurements of the charm structure function [11] and the large rate for high- $p_T$   $\bar{p}p \rightarrow c\gamma X$  reactions at the Tevatron [12]; however, a precise determination of  $\langle x \rangle_{IC}$  has proved elusive. The letter by P. Jimenez-Delgado, T. J. Hobbs, J. T. Londergan, and W. Melnitchouk (JDHLM) [13] is the most recent of such analyses, and it finds a much more severe limit on intrinsic charm  $\langle x \rangle_{IC} \sim \mathcal{O}(0.1\%)$  than the previous such study [7]. JDHLM input different shapes for the intrinsic charm contributions but allow the overall normalization to vary. They include low-energy data from the 1991 single-arm  $ed(p) \rightarrow e'X$  SLAC experiment [14] in their global fit. Ref. [7] did not use the SLAC data and came to much weaker conclusions. Nevertheless, we believe the very stringent conclusions of JDHLM are in error.

JDHLM assess their PDF errors using a tolerance criteria of  $\Delta\chi^2 = 1$  at  $1\sigma$ ; however, the actual value of  $\Delta\chi^2$  to be employed depends on the number of parameters to be simultaneously determined in the fit. This is illustrated in Table 38.2 of Ref. [15] and is used broadly, noting, e.g., Refs. [16–19]. Ref. [7] employs the CT10 PDF analysis [20], so that it contains 25 parameters, plus one for intrinsic charm. Figure 38.2 of Ref. [15] then shows that  $\Delta\chi^2 \approx 29$  at  $1\sigma$  (68% CL), whereas  $\Delta\chi^2 \approx 36$  at 90% CL. Ref. [7] uses the criterion  $\Delta\chi^2 > 100$ , determined on empirical grounds, to indicate a poor fit. JDHLM employs the framework of Ref. [21] which contains 25 parameters for the PDFs and 12 for the higher-twist contributions, so that a much larger tolerance than  $\Delta\chi^2 = 1$  is warranted.

JDHLM find that the SLAC data (on  $d$  and  $p$  targets) give the strongest constraints on intrinsic charm, although, by their count, only 157 of 1021 data points have  $W^2$  in excess of the charm hadronic threshold:  $W_{th}^2 \approx 16 \text{ GeV}^2$ . [JDHLM mention the partonic threshold constraint  $W^2 > 4m_c^2$ , but this is not relevant for the detection of intrinsic charm — if  $x < 1$ , leptons can only scatter off charm quarks when the kinematics permit the formation of charmed hadrons in the final state.] It is possible that JDHLM’s strong rejection of the intrinsic charm hypothesis is driven by sharpened constraints on the non-charm PDFs. However, for the SLAC data set, the theoretical model which is constrained is that of the intrinsic charm PDF combined with the treatment of uncertain higher-twist and threshold corrections. Thus a global analysis cannot reject intrinsic charm *per se*, but rather only the particular model in which it is embedded.

We also note that JDHLM exclude the EMC data — which indicate significant intrinsic charm — citing a “goodness of fit” criterion. Statistical criteria alone cannot allow the exclusion of data sets, as here with the EMC data; additional corrections, however, may exist through their use of an iron target [22, 23].

Finally, we note that the SLAC measurements of  $ed(p) \rightarrow e'X$ , which only detects the scattered electron, has an overall normalization (systematic) error of  $\pm 1.7$  (2.1)%, and a relative normalization error of typically  $\pm 1.1\%$  [14]. The SLAC data points in the  $W^2 > 16 \text{ GeV}^2$  and  $x > 0.1$  regime where intrinsic charm could be directly relevant have even larger statistical uncertainties. Thus it seems implausible that the SLAC data can yield the severe constraint claimed.

JDHLM claim that the momentum fraction carried by intrinsic charm is  $\langle x \rangle_{IC} < 0.1\%$  at the  $5\sigma$  level, and they note in their final summary that  $\langle x \rangle_{IC} \leq 0.5\%$  at  $4\sigma$ . We find neither conclusion is warranted.

We thank B. Plaster for a cross-check of Fig. 38.2 in Ref. [15] and B. Plaster, A. Deur, P. Hoyer, C. Lorcé, J. Pumplin, and R. Vogt for helpful remarks. We acknowledge support from the U.S. Department of Energy under contracts DE-AC02-76SF00515 and DE-FG02-96ER40989.

## ■ Brodsky not happy!



## ■ 3 main criticisms

- $\Delta\chi^2 = 1$  wrong if have  $\sim 30$  params.
- cannot use SLAC data to learn about charm
- should include EMC  $F_2^c$  data in fits

## ■ addressed in arXiv:1504.06304

arXiv:1504.06304v1 [hep-ph] 23 Apr 2015

### Reply to Comment on “New limits on intrinsic charm in the nucleon from global analysis of parton distributions”

P. Jimenez-Delgado<sup>1</sup>, T. J. Hobbs<sup>2</sup>, J. T. Londergan<sup>3</sup>, W. Melnitchouk<sup>1</sup>

<sup>1</sup>Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

<sup>2</sup>Department of Physics, University of Washington, Seattle, Washington 98195, USA

<sup>3</sup>Department of Physics and Center for Exploration of Energy and Matter, Indiana University, Bloomington, Indiana 47405, USA

(Dated: April 24, 2015)

We reply to the Comment of Brodsky and Gardner on our paper “New limits on intrinsic charm in the nucleon from global analysis of parton distributions” [Phys. Rev. Lett. **114**, 082002 (2015)]. We elaborate how global QCD analysis of all available high-energy data provides no evidence for a large intrinsic charm component of the nucleon.

In a recent Comment [1], Brodsky and Gardner (BG) make several criticisms of our global PDF analysis [2] of all available high-energy scattering data, including those from fixed-target experiments at high  $x$  and low  $Q^2$ , which placed strong constraints on the magnitude of intrinsic charm (IC) in the nucleon. For a range of models of IC, the analysis [2] strongly disfavored large magnitudes of IC, with the momentum fraction carried by charm quarks  $\langle x \rangle_{\text{IC}}$  at most 0.5% at the  $4\sigma$  CL confidence level (CL).

BG claim that because our global analysis [2] uses  $\mathcal{O}(30)$  parameters, as is typical in all such fits, one must adopt a much larger tolerance criterion than  $\Delta\chi^2 = 1$ , suggesting that the appropriate  $\Delta\chi^2$  should be  $\sim 30$  for  $1\sigma$ . In fact, it is well known that parameter errors in  $\chi^2$  fits are determined by  $\Delta\chi^2 = 1$ , irrespective of the number of parameters in the fit [3, 4]. The parameter  $m$  in Table 38.2 of Ref. [3] cited by BG is the dimensionality of the error regions for joint distributions ( $m = 1$  for linear errors,  $m = 2$  for error ellipses, etc.), and has nothing to do with the total number of parameters in the fit. For the determination of individual parameter errors, the correct dimension is  $m = 1$ , which gives  $\Delta\chi^2 = 1$  at the 68.3% CL. (For examples of error ellipses with  $m = 2$ , see Fig. 12 of Ref. [5].)

Furthermore, Fig. 38.2 of Ref. [3] referred to by BG involves the number of degrees of freedom of a fit (number of points – number of parameters) and not the number of parameters in the fit. The discussion there deals with criteria to judge the goodness of a fit at a particular CL, rather than for the determination of standard parameter errors.

The parameter errors and  $\chi^2$  profiles related to one-dimensional probability distributions are correctly evaluated using  $\Delta\chi^2 = 1$ . Errors on other quantities are then computed using standard error propagation techniques, such as the Hessian method; they can also be used to produce error regions of different dimensionalities with the appropriate  $\Delta\chi^2$  criteria [3, 4]. Apparently, BG have confused the dimensionality of error regions with the number of independent parameters in a fit. Their claims about  $\Delta\chi^2$  are simply wrong.

Tolerance criteria  $\Delta\chi^2 > 1$  are used by some PDF groups [6–8] on purely phenomenological grounds, to account for tensions among different data sets. Other groups [5, 9, 10] use the standard  $\Delta\chi^2 = 1$ . The  $\chi^2$  profiles in [2] were presented as a function of  $\langle x \rangle_{\text{IC}}$ , so that  $\langle x \rangle_{\text{IC}}$  values for different tolerance choices can be easily compared.

Inclusive DIS cross sections, such as those measured at SLAC, receive contributions from all quark flavors, so they cannot by themselves provide significant constraints on charm. The power of a global fit, however, lies in the correlation between different observables, with different weightings of quark flavors, within the framework of perturbative QCD. While the bulk of the data from SLAC [11] at large  $x$  lie below the charm threshold, cross sections below threshold do provide better constraints on light quark distributions, which indirectly impact the determination of IC at the same kinematics. Our analysis also takes into account the suppression of charm production below and near the hadronic charm threshold [1, 2]. Implementing the suppression involves some model dependence in relating the partonic and hadronic charm thresholds [2, 6], and while this affects the quantitative limits (with partonic threshold factors alone  $\langle x \rangle_{\text{IC}}$  would be  $< 0.1\%$  at the  $5\sigma$  CL), the effects do not alter the overall conclusions about the magnitude of IC supported by the data.

To avoid dealing with complications from thresholds and other hadronic effects at low  $W^2$  and  $Q^2$ , many global PDF analyses impose more severe cuts on  $W^2$  and  $Q^2$  than those in Ref. [2]. While this simplifies the theoretical treatment, it also removes a significant amount of data at large  $x$  that could potentially impact on the question of IC. More recently, some PDF analyses [5, 7, 10] have relaxed the  $W^2$  and  $Q^2$  cuts in order to better constrain large- $x$  PDFs. Such analyses benefit from increased statistics at large  $x$ , but require careful treatment of subleading  $1/Q^2$  and nuclear corrections. Our analysis [2] employs the standard treatment of target mass corrections, phenomenological higher twists determined consistently within the same fit, and the latest technology in nuclear corrections [5, 7]. The global fit

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$\chi^2$  fits are determined by  $\Delta\chi^2 = 1$ , irrespective of the number of parameters in the fit [3, 4]. The parameter  $m$  in Table 38.2 of Ref. [3] cited by BG is the dimensionality of the error regions for joint distributions ( $m = 1$  for linear errors,  $m = 2$  for error ellipses, etc.), and has nothing to do with the total number of parameters in the fit. For the determination of individual parameter errors, the correct dimension is  $m = 1$ , which gives  $\Delta\chi^2 = 1$  at the 68.3% CL. (For examples of error ellipses with  $m = 2$ , see Fig. 12 of Ref. [5].)

Furthermore, Fig. 38.2 of Ref. [3] referred to by BG involves the number of degrees of freedom of a fit (number of points – number of parameters) and not the number of parameters in the fit. The discussion there deals with criteria to judge the goodness of a fit at a particular CL, rather than for the determination of standard parameter errors.

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## ■ 3 main criticisms

→  $\Delta\chi^2 = 1$  wrong  
if have  $\sim 30$  params.

→ cannot use SLAC data  
to learn about charm

→ should include EMC  
 $F_2^c$  data in fits

## ■ addressed in arXiv:1504.06304

arXiv:1504.06304v1 [hep-ph] 23 Apr 2015

### Reply to Comment on “New limits on intrinsic charm in the nucleon from global analysis of parton distributions”

P. Jimenez-Delgado<sup>1</sup>, T. J. Hobbs<sup>2</sup>, J. T. Londergan<sup>3</sup>, W. Melnitchouk<sup>1</sup>

<sup>1</sup>Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA

<sup>2</sup>Department of Physics, University of Washington, Seattle, Washington 98195, USA

<sup>3</sup>Department of Physics and Center for Exploration of Energy and Matter,  
Indiana University, Bloomington, Indiana 47405, USA

(Dated: April 24, 2015)

We reply to the Comment of Brodsky and Gardner on our paper “New limits on intrinsic charm in the nucleon from global analysis of parton distributions” [Phys. Rev. Lett. **114**, 082002 (2015)]. We elaborate how global QCD analysis of all available high-energy data provides no evidence for a large intrinsic charm component of the nucleon.

In a recent Comment [1], Brodsky and Gardner (BG) make several criticisms of our global PDF analysis [2] of all available high-energy scattering data, including those from fixed-target experiments at high  $x$  and low  $Q^2$ , which placed strong constraints on the magnitude of intrinsic charm (IC) in the nucleon. For a range of models of IC, the analysis [2] strongly disfavored large magnitudes of IC, with the momentum fraction carried by charm quarks  $\langle x \rangle_{\text{IC}}$  at most 0.5% at the  $4\sigma$  CL confi-

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BG claim that because our global analysis [2] uses  $\mathcal{O}(30)$  parameters, as is typical in all such fits, one must adopt a much larger tolerance criterion than  $\Delta\chi^2 = 1$ , suggesting that the appropriate  $\Delta\chi^2$  should be  $\sim 30$  for  $1\sigma$ . In fact, it is well known that parameter errors in

**EMC data in conflict  
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