Nucleon Structure on a Lattice

Sergey Syritsyn (Jefferson Lab)

2006–2010 PhD, advisor: John Negele (MIT Center for Theoretical Physics)
2010–2013 Postdoctoral Fellow (Nuclear Science Division, Berkeley National Lab)
2013–2015 RIKEN Foreign Postdoctoral Researcher (Brookhaven National Lab)
From Sep 2016 Nathan Isgur Fellow at Theory Center, Jefferson Lab

Jefferson Lab Director's Seminar,
December 2, 2015
Outline

- Nucleon Form Factors
  - $G_E, G_M$ on a lattice at the physical point
  - Outlook: High-momentum Form Factors
  - Strange quark contributions to the nucleon charge & magnetization

- Neutron-Antineutron Oscillations & BSM Physics
  - Physics motivation
  - Calculations at the physical point

- Summary
QCD on Lattice

Numerical Path integral over quark & gluon fields

\[ \langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}(\text{Glue}) \int \mathcal{D}(\text{Quarks}) \ e^{-S_{\text{Glue}} - \bar{q}(\mathcal{D} + m)q} \ [q_x \bar{q}_y \ldots] \]

\[ = \int \mathcal{D}(\text{Glue}) \ e^{-S_{\text{Glue}}} \text{Det}(\mathcal{D} + m) \ (\mathcal{D} + m)^{-1}_{x,y} \ldots \]

Markov Chain Monte Carlo sampling

Euclidean lattice

\[ U_\mu \approx e^{ig a A_\mu} \]

\[ U_P \approx e^{ig a^2 F_{\mu\nu}} \quad \text{("curl" } A_\mu) \]

Computation challenges:
- finite volume,
- simulations with light quarks,
- excited states in the correlation functions

quark covariant derivative
Hadron Spectrum & Structure in Lattice QCD

Hadron Spectrum:

\[ C_{2\text{pt}}(T) = \langle N(T)\bar{N}(0) \rangle = \sum Z_n^2 e^{-E_n T} \]

\[ \langle N(T)N(0) \rangle = \sum_n |Z_n|^2 e^{-E_n T} \]

Hadron Matrix Elements:

\[ C_{3\text{pt}}^O(T) = \langle N(T)O(\tau)\bar{N}(0) \rangle = \sum Z_m e^{-E_n (T-\tau)} \langle n|O|m \rangle e^{-E_m \tau} Z_n^* \]

\[ \langle N(T)O(\tau)N(0) \rangle = \sum_{n,m} Z_m e^{-E_n (T-\tau)} \langle n|O|m \rangle e^{-E_m \tau} Z_n^* \]

Dealing with excited states:
- multi-exponential fits
- variational methods
Proton Electromagnetic Structure

Electromagnetic probes $e^\pm, \mu^\pm$

Proton electromagnetic form factors

\[
\langle P + q q^\mu q | P \rangle = \bar{U}_{P+q} \left[ F_1(Q^2) \gamma^\mu + \frac{F_2(Q^2)}{2M_N} i\sigma^{\mu\nu}q_\nu \right] U_P
\]

Related to Fourier transform of charge and magnetization distribution in the nucleon (*)

On-going: continuum limit study

Distribution of charge ...

\[
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)
\]

... and magnetization

\[
G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
\]

Lattice data near the physical point ($m_\pi = 149 \text{ MeV}$):

[J.R.Green, SNS, et al (LHPc) PRD90:074507 (2014)]
Proton Charge Radius Puzzle

Proton radius discrepancy:
7σ difference between e-p scattering and μ-p levels

planned experiments
- JLab pRAD experiment
- MUSE@PSI : e± / μ±-p scattering

Lattice QCD precision for \[ (r_1^2)_p - (r_1^2)_n \]
~10% uncertainty at physical point; agreement with muonium result

ChPT divergence \( \sim \log m_{\pi}^2 \)

Lattice data near the physical point (\( m_{\pi} = 149 \text{ MeV} \)):
[J.R.Green, SNS et al (LHPC), 1209.1687]
Disconnected Light & Strange Quarks

"Hierarchical probing": In sum over $2^{dk+1}$ vectors ($d=3$), nearest terms cancel exactly:

$$\frac{1}{N} \sum_{i}^{N} z_i(x) z_i^\dagger(y) = 0, \quad x \neq y$$

[K. Orginos, A. Stathopoulos]

Disconnected Light & Strange Quarks

"Hierarchical probing": In sum over $2^{dk+1}$ vectors (d=3), nearest terms cancel exactly:

$$\frac{1}{N} \sum_{i}^{N} z_i(x) z_i^\dagger(y) = 0, \quad x \neq y$$

[K. Orginos, A. Stathopoulos]

![Diagram showing disconnected and strange contributions to EM tensor and spin]

- Experiment: Strange part in the elastic $e-p$ asymmetry (forward angles)
- Lattice: $m_\pi = 320$ MeV
- Underway: disconnected & strange contributions to EM tensor and spin

Neutron-Antineutron Oscillations & BSM

Motivation:

- One of Sakharov’s conditions for baryogenesis
- Alternative to proton decay ($\Delta B=1$)
  \[\text{Which one (or both?) realized in nature?}\]
- Probing BSM physics: $\Delta(B-L)=2$
  \[\text{Possible connections to lepton number violation: double beta decay,}\]
  \[\text{and seesaw neutrino mass mechanism [R. Mohapatra, R. Marshak (1980)]}\]

Past experimental searches:

- Quasi-free neutron conversion in flight
  \[P_{n\rightarrow\bar{n}}(t) \approx (\delta m t)^2 = (t/\tau_{n\bar{n}})^2\]
  \[\tau_{n\bar{n}} > 0.86 \cdot 10^8 \text{ sec}\]
  ILL Grenoble reactor, 1990 [M. Baldo-Ceolin et al, 1994]

- Nuclear matter stability
  \[T_d = R\tau_{n\bar{n}}^2\]
  \[\tau_{n\bar{n}} > 1.4 \cdot 10^8 \text{ s [Soudan 2]}\]
  \[\tau_{n\bar{n}} > 3.3 \cdot 10^8 \text{ s [Super-K]}\]
  \[\tau_{n\bar{n}} > 1.96 \cdot 10^8 \text{ s [SNO]}\]
  (limited by irreducible neutrino background)

Experiment at ESS proposed to improve limits on osc.time $x10^2 \ldots 10^3$. 
Neutron ↔ Antineutron Operators

Effective 6-quark operators From Beyond the Standard Model:
interaction with a massive Majorana lepton, unified theories, etc

\[ \mathcal{H}_{n\bar{n}} = \begin{pmatrix} E + V & \delta m \\ \delta m & E - V \end{pmatrix} \quad \text{and} \quad \tau_{n\bar{n}} = (2\delta m)^{-1} \]

\[ \mathcal{L}_{\text{eff}} = \sum_i [c_i \mathcal{O}_i^{6q} + \text{h.c.}] \]

\[ \delta m = -\langle \bar{n} | \int d^4x \mathcal{L}_{\text{eff}} | n \rangle = - \sum_i \frac{c_i}{M_X^5} \langle \bar{n} | \mathcal{O}_i^{6q} | n \rangle \]

BSM scale suppression of 6-quark Dim-9 operators

Sensitivity of matter to BN-violating terms is determined by nuclear scale physics and non-perturbative QCD

What is the scale for physics behind \( n \leftrightarrow \bar{n} \)?

Current lower bound on \( \tau_{n\bar{n}} \) requires\(^(*)\) \( M_X \gtrsim \text{few} \cdot 10^2 \text{ TeV} \)

\(^(*)\) Based on nucleon model-dependent matrix elements calculations
Neutron ↔ Antineutron Matrix Elements

Effective operators: \((R-, L\text{-diquarks})^3\)

singlets in color & electroweak symmetries

\[
O_{1\chi_1\{\chi_2\chi_3\}} = T_{ijklmn}^s [u_{\chi_1}^T C u_{\chi_1}^j] [d_{\chi_2}^k T C d_{\chi_2}^l] [a_{\chi_3}^m T C a_{\chi_3}^n]
\]

\[
O_{2\{\chi_1\chi_2\}\chi_3} = T_{ijklmn}^s [u_{\chi_1}^T C d_{\chi_2}^j] [u_{\chi_2}^T C d_{\chi_2}^l] [d_{\chi_3}^m T C a_{\chi_3}^n]
\]

\[
O_{3\{\chi_1\chi_2\}\chi_3} = T_{ijklmn}^a [u_{\chi_1}^T C d_{\chi_2}^j] [u_{\chi_2}^T C d_{\chi_2}^l] [d_{\chi_3}^m T C d_{\chi_3}^n]
\]

\[\chi_{1,2,3} = R, L\]

[T.Kuo, S.Love, PRL45:93 (1980)]

\[
SU(3)_c \otimes U(1)_{\text{em}} \left[ \otimes SU(2)_L \right]
\]

Chiral \(SU(2)_{L,R}\) multiplet classification:

| \([RRR]_3\) | \(O^1_{R(RR)} + 4O^2_{(RR)R}\) | \(3_R \otimes 0_L\) |
| \([RRR]_1\) | \(O^2_{(RR)R} - O^1_{R(RR)} \equiv 3O^3_{(RR)R}\) | \(1_R \otimes 0_L\) |
| \([RL]_2\) | \(O^2_{(LL)R} - O^1_{L(LR)} \equiv 3O^3_{(LL)R}\) | \(1_R \otimes 0_L\) |
| \([RR]_0\) | \(3O^3_{(LR)R}\) | \(1_R \otimes 0_L\) |

\[\begin{align*}
[(RR)_{1L}]^{(1)} & : O^1_{L(RR)} \\
[(RR)_{1L}]^{(2)} & : O^2_{(LR)R} \\
[(RR)_{1L}]^{(3)} & : O^1_{R(LR)} + 2O^2_{(RR)L}
\end{align*}\]

\(+ L\leftrightarrow R\) counterparts

**Chiral symmetry is essential for simple renormalization properties**
\[ \langle N_{\uparrow}^{(+)}(t_2) \mathcal{O}^{6q}(0) N_{\downarrow}^{(-)}(-t_1) \rangle \sim e^{-M_n(t_2 + t_1)} \langle n_\uparrow | \mathcal{O}^{6q} | \bar{n}_\uparrow \rangle \]

Initial calculation with anisotropic Wilson

CURRENT: Calculations with physical quarks masses & chiral symmetry
[SNS et al (LATTICE2015)]
Renormalization: RI-(S)MOM on a lattice

\[
(G_I)^{ijklmn}_{\alpha\beta\gamma\delta \varepsilon \eta}(x, p_1 \ldots p_6) = \langle \mathcal{O}_I \bar{d}^{m}_\eta(p_6) d^{m}_\varepsilon(p_5) \bar{d}^{l}_\delta(p_4) d^{k}_\gamma(p_3) \bar{u}^{j}_\beta(p_2) \bar{u}^{i}_\alpha(p_1) \rangle
\]

\[p_1 = p_3 = p_5 = p\]
\[p_2 = p_4 = p_6 = -p\]

\[
\begin{pmatrix}
    u(p) \\
    d(-p)
\end{pmatrix}
\] is not an SU(2) doublet: mix \(3_R \leftrightarrow 1_R\), etc

Fix: symmetrize external momenta

\[
\frac{1}{5} \langle \mathcal{O}^{6g} \bar{u}(p) \bar{u}(+p) \bar{d}(+p) \bar{d}(-p) \bar{d}(-p) \rangle
\]
\[
\frac{3}{5} \langle \mathcal{O}^{6g} \bar{u}(p) \bar{u}(+p) \bar{d}(+p) \bar{d}(-p) \bar{d}(-p) \rangle
\]
\[
\frac{1}{5} \langle \mathcal{O}^{6g} \bar{u}(+p) \bar{u}(-p) \bar{d}(+p) \bar{d}(+p) \bar{d}(-p) \rangle
\]

Input to pert. QCD matching [M.Buchoff, M.Wagman, arXiv:1506.00647]
Lattice-\(\rightarrow\)MSbar Conversion

\[ Z^{\text{phys}}(\mu) = Z^{\text{lat}}(\mu) / \left( Z(2\text{GeV})^{\text{MS}} \right) \]

Fit \( \sim Z + \alpha \, a^2 \mu^2 \)

Perturbative 1-loop running from

Syst. error: difference between 2–4 GeV and 4–6 GeV fits
Preliminary Results & Comparison to Bag Model

Nucleon Structure on a Lattice

Preliminary Results & Comparison to Bag Model

Lattice Results, PRELIMINARY

<table>
<thead>
<tr>
<th>Bag “A”</th>
<th>LQCD Bag “A”</th>
<th>Bag “B”</th>
<th>LQCD Bag “B”</th>
</tr>
</thead>
<tbody>
<tr>
<td>((RRR)_3)</td>
<td>0</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
<td>((RRR)_{1})</td>
<td>45.4(5.6)</td>
<td>8.190</td>
<td>5.5</td>
</tr>
<tr>
<td>([R_1(LL)]_0)</td>
<td>44.0(4.1)</td>
<td>7.230</td>
<td>6.1</td>
</tr>
<tr>
<td>((RR)_{1}L_0)</td>
<td>-66.6(7.7)</td>
<td>-9.540</td>
<td>7.0</td>
</tr>
<tr>
<td>((RR)_{2}L_1) ((1))</td>
<td>-2.12(26)</td>
<td>1.260</td>
<td>-1.7</td>
</tr>
<tr>
<td>((RR)_{2}L_1) ((2))</td>
<td>0.531(64)</td>
<td>-0.314</td>
<td>-1.7</td>
</tr>
<tr>
<td>((RR)_{2}L_1) ((3))</td>
<td>-1.06(13)</td>
<td>0.630</td>
<td>-1.7</td>
</tr>
</tbody>
</table>

MIT Bag model results from [S.Rao, R.Shrock, PLB116:238 (1982)]

**N-Nbar oscillation is x(5-10) more sensitive to BSM physics than previously thought!**

Outlook:

- Finalize the current results (physical quarks with chiral symmetry)
- Use finer discretization to eliminate potential cutoff effects
Summary

Lattice QCD methods are mature enough for reliable nucleon structure calculations, and

... offer multiple avenues to explore nucleon structure to complement experiments, and

... are crucial for experiments that use nucleons as a lab for new physics searches.