Gluon TMDs: from small to large x

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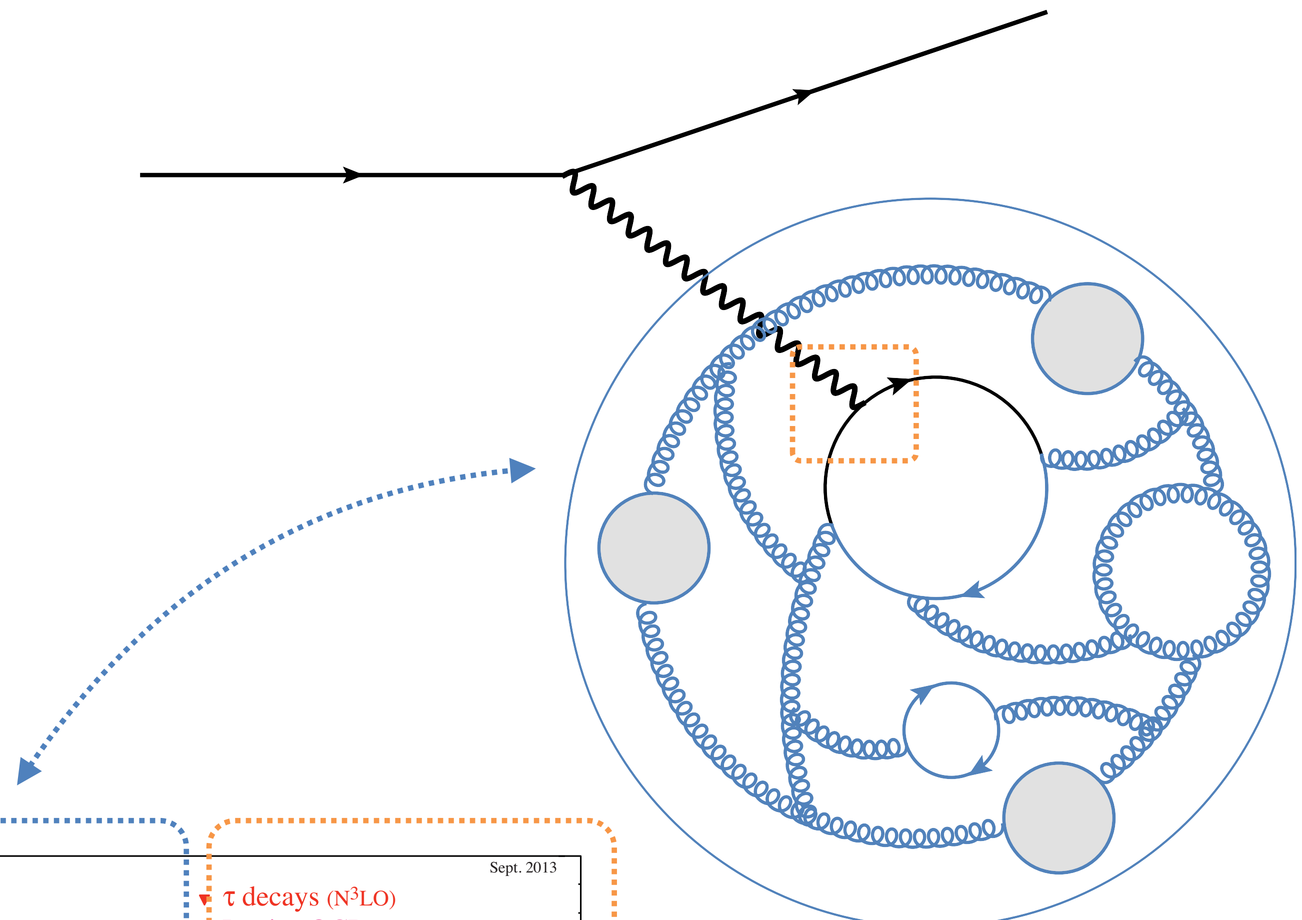
Director's Theory Seminar
Jefferson Lab
December 2, 2015

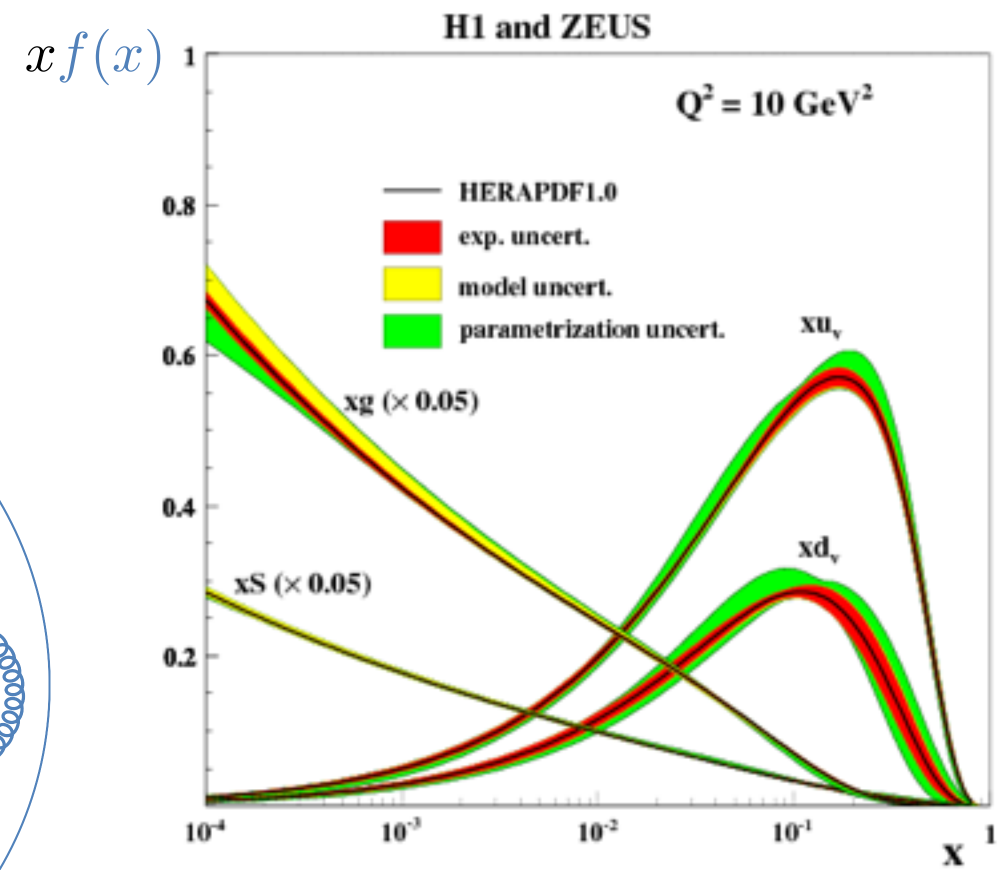
Postdoctoral fellow (2013-Present)
Thomas Jefferson National Accelerator Facility (Jefferson Lab)
Newport News, USA
Evolution of gluon transverse momentum dependent (TMD) distribution functions
Collaborator: Ian Balitsky

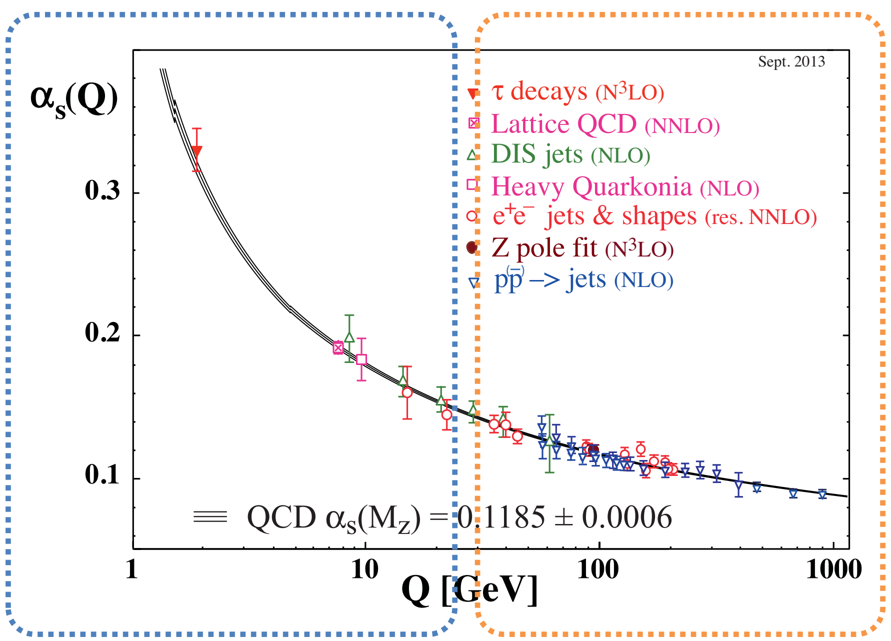
PhD in theoretical high-energy physics (2008-2012)
St. Petersburg State University, Russia
Pomeron loop calculation
Advisor: Mikhail Braun

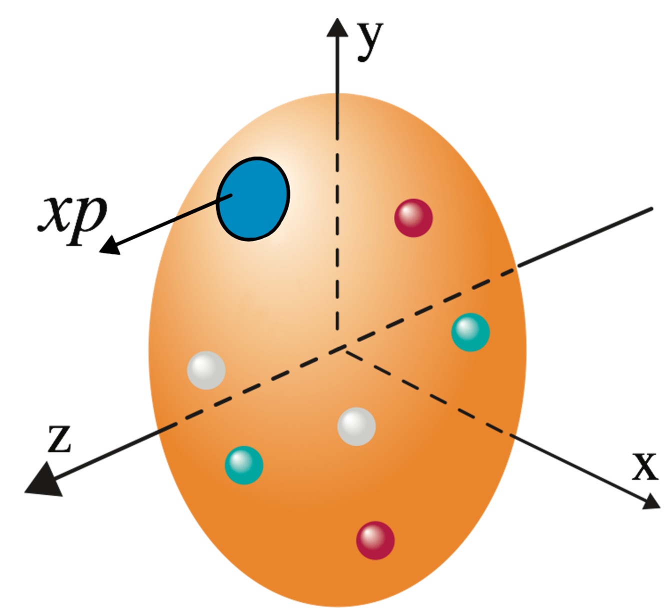
Factorization

Confined system of strongly interacting quarks and gluons



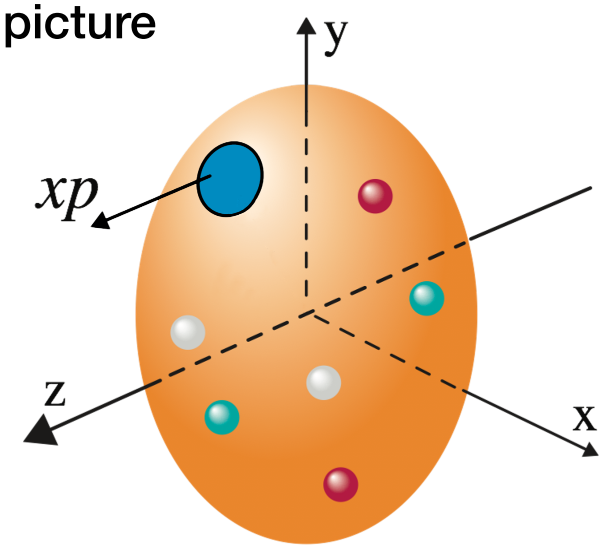


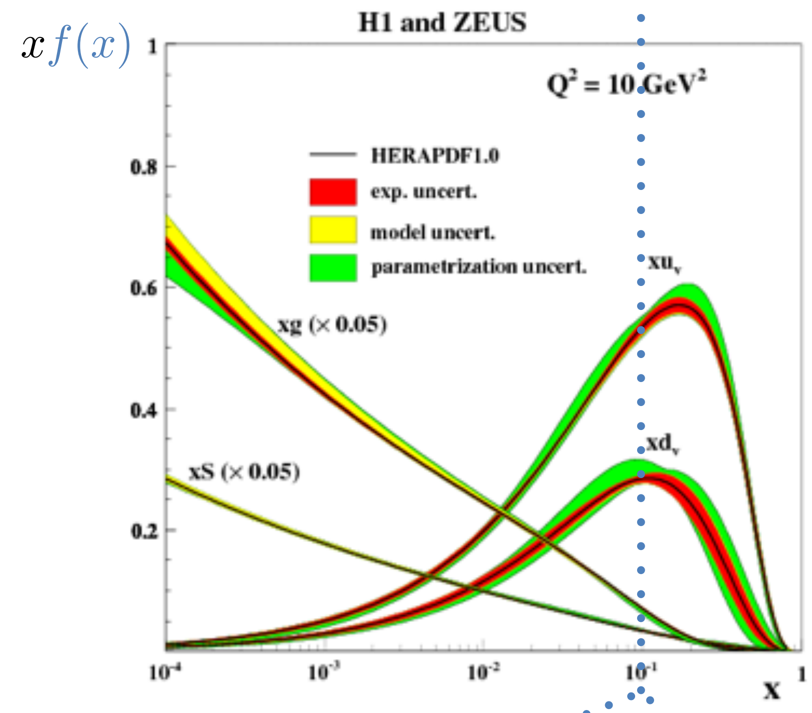




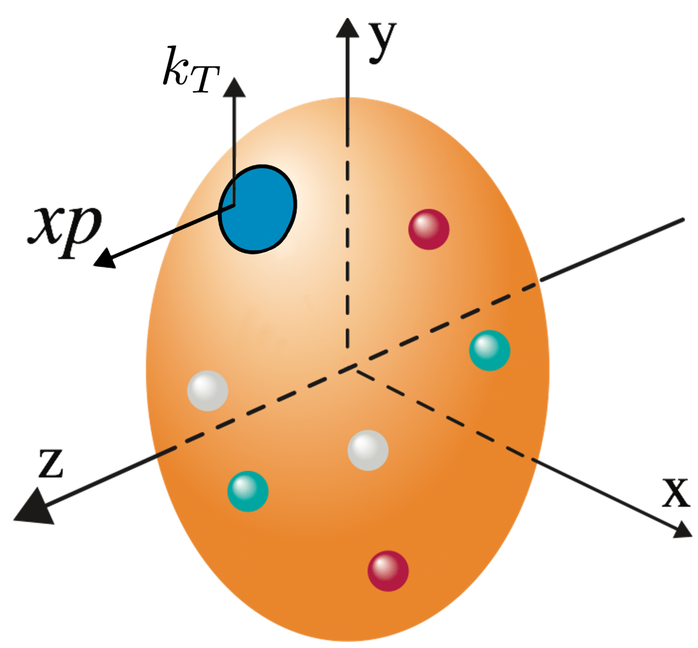
Hadron as a three-dimensional object

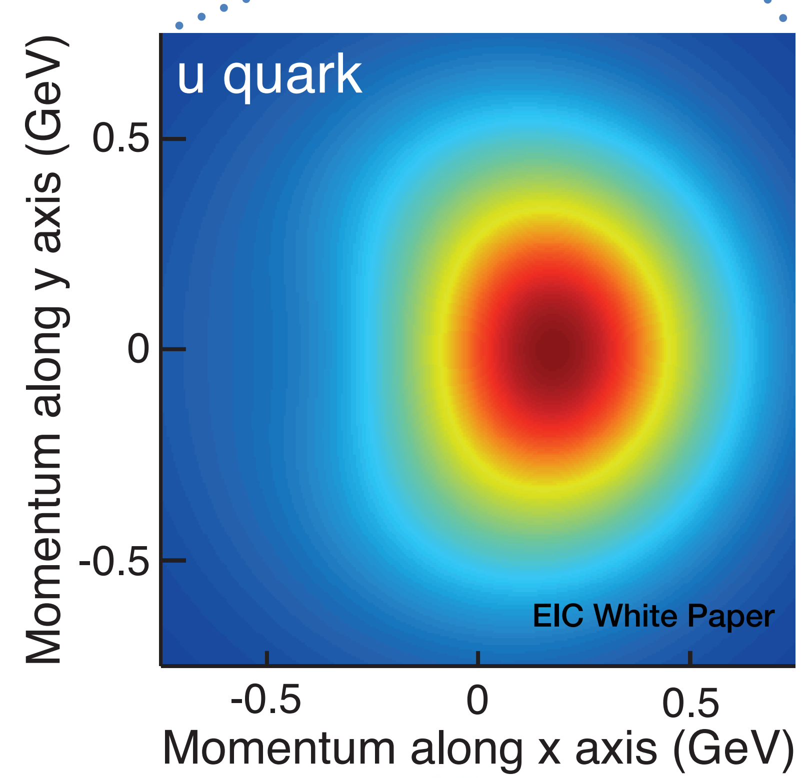
One-dimensional picture



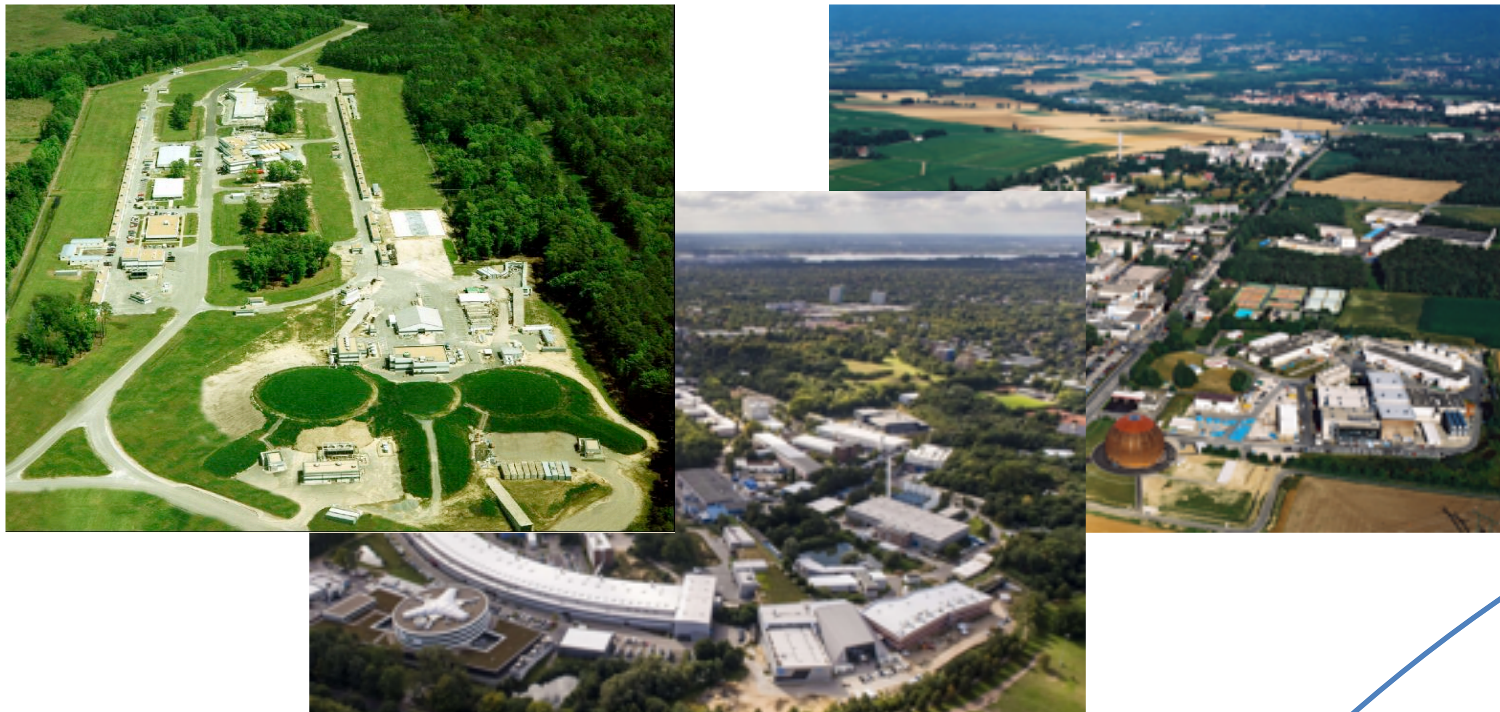


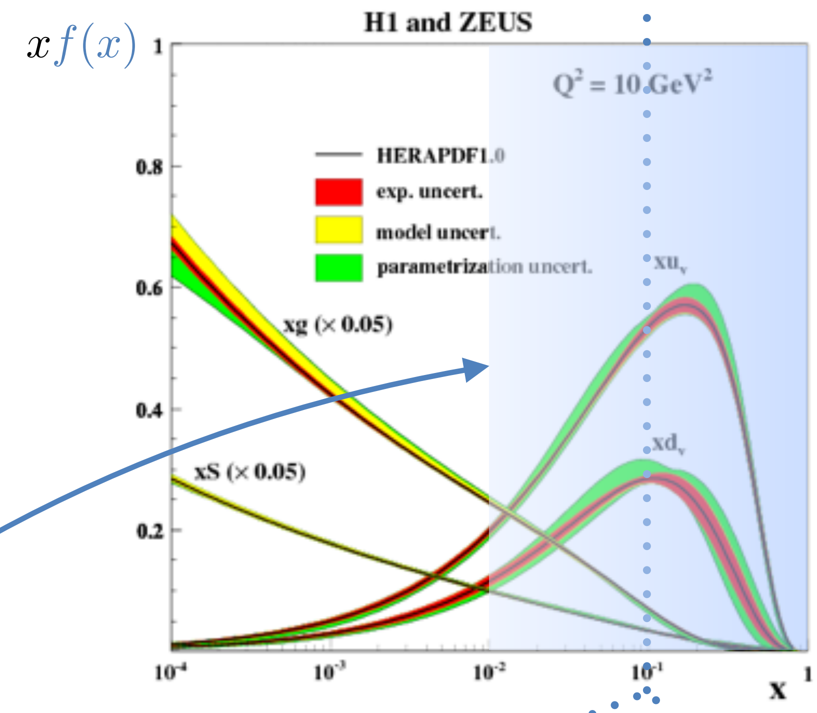
Three-dimensional picture
(Transverse momentum dependent distribution functions)



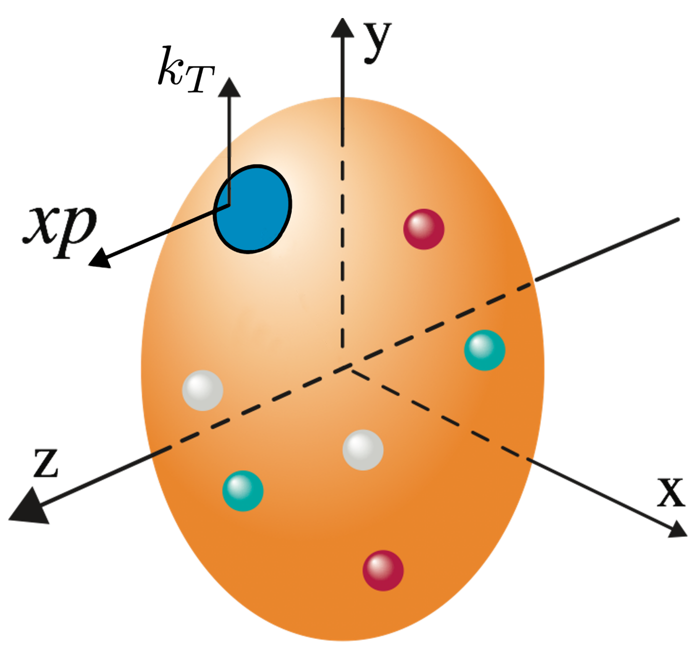


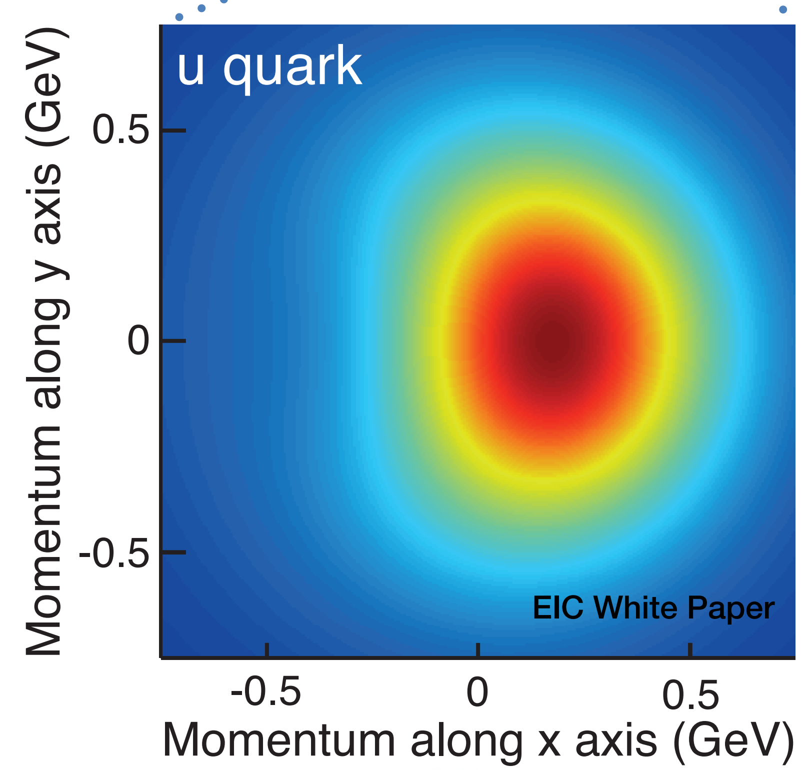
Quark TMDs



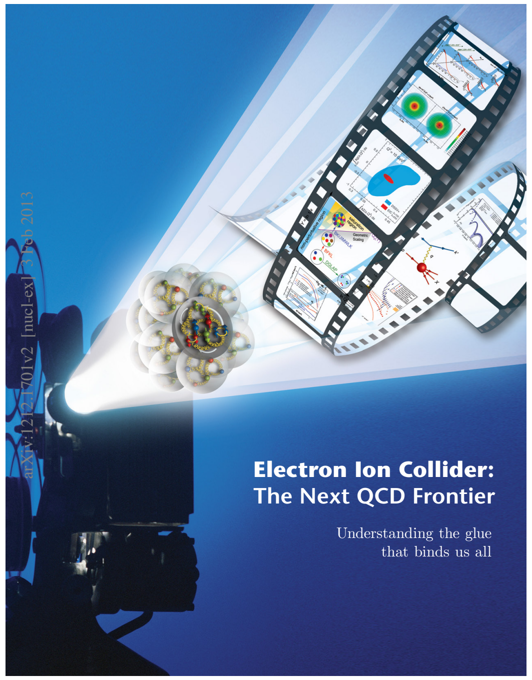


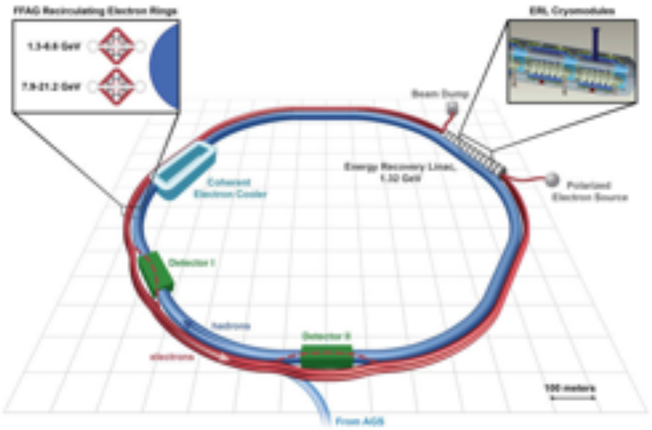
Fixed target experiments

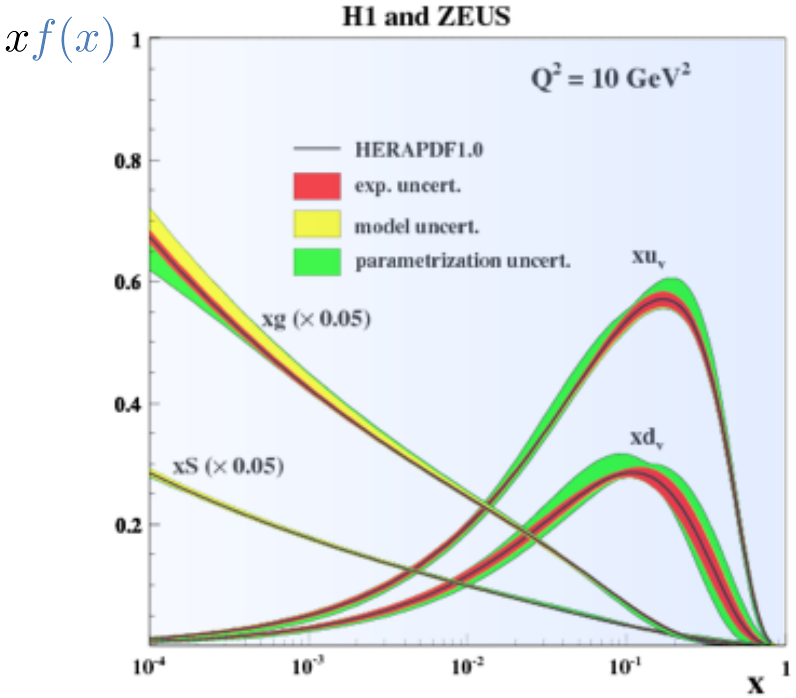


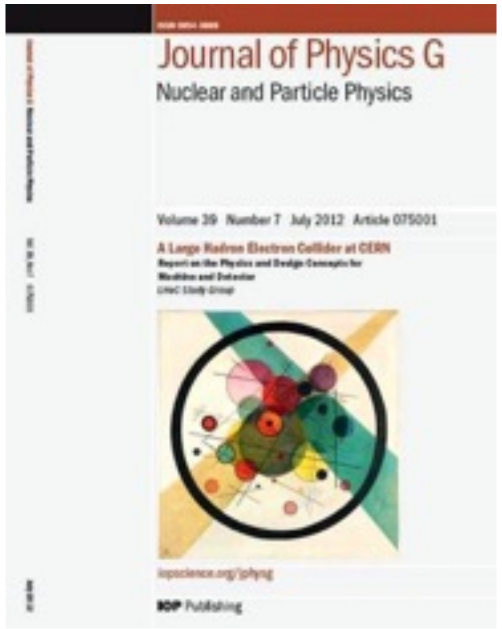


GLUON TMDS: EIC AND LHEC

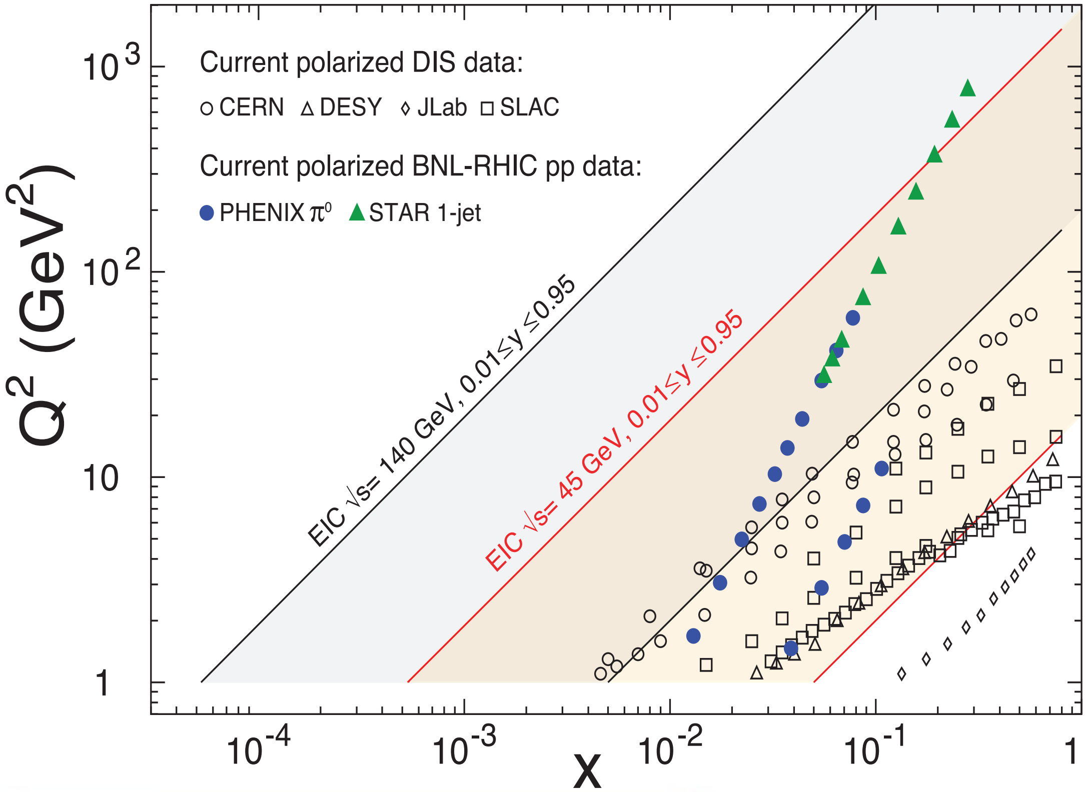




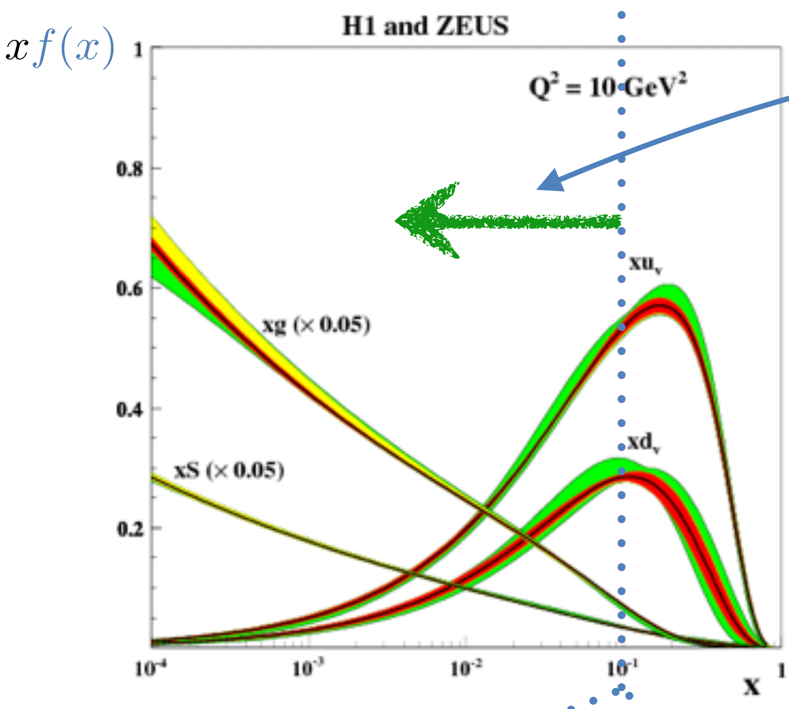




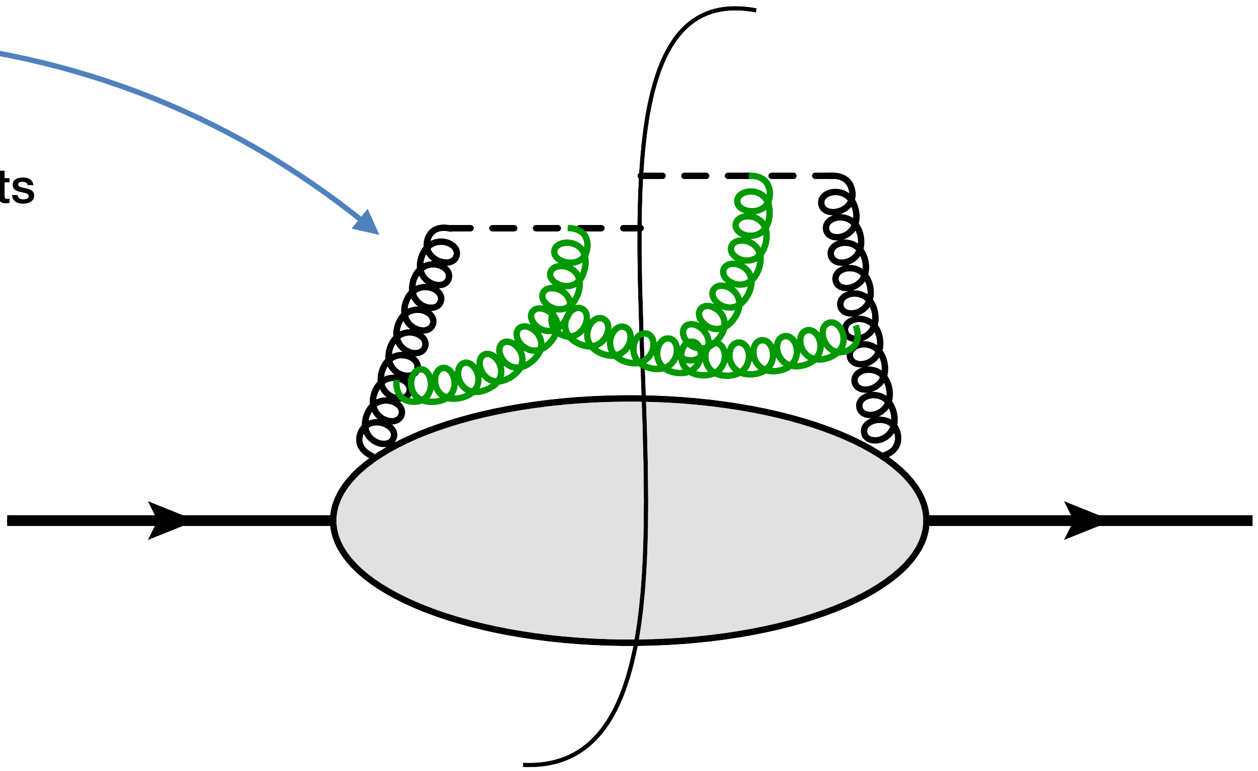
- Region of much smaller x
- We will be able to study gluon-matter distributions

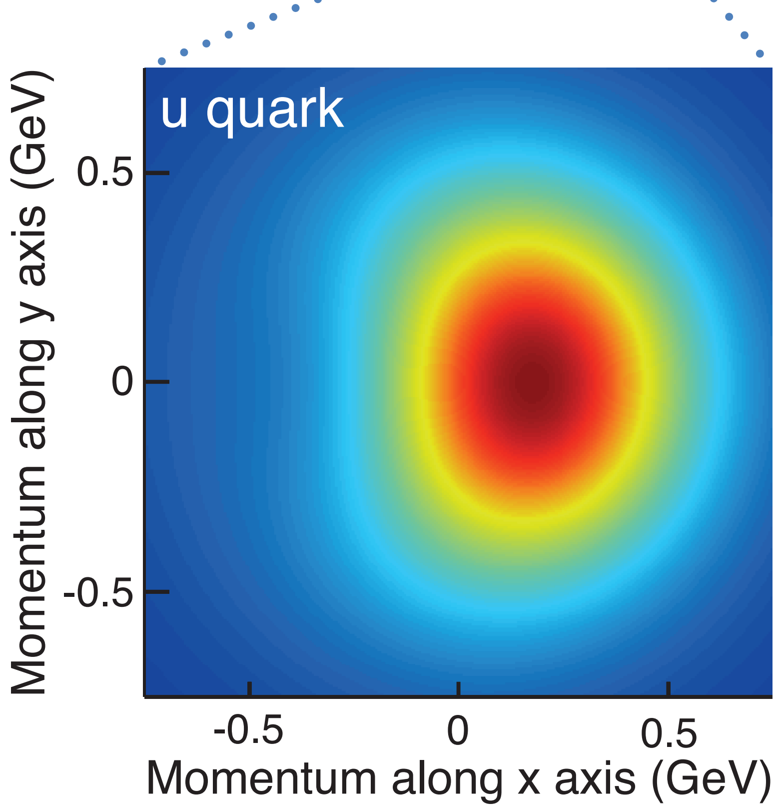


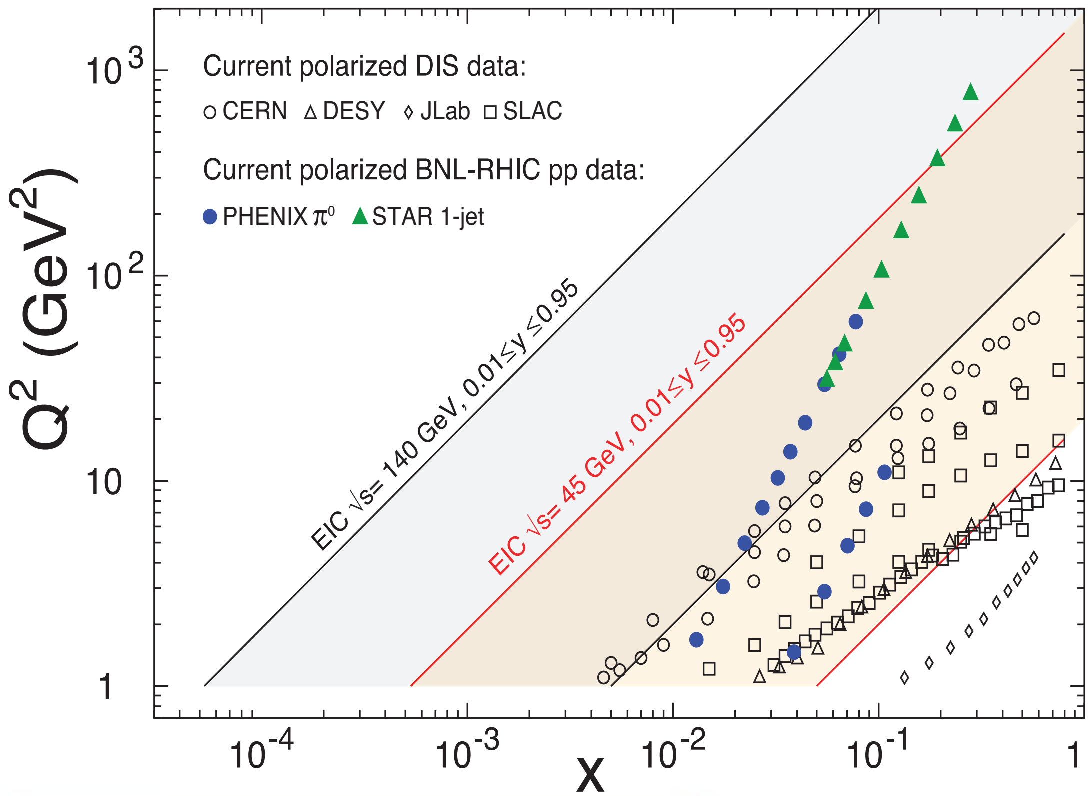
DEFINITION OF GLUON TMDS



Emission connects data together

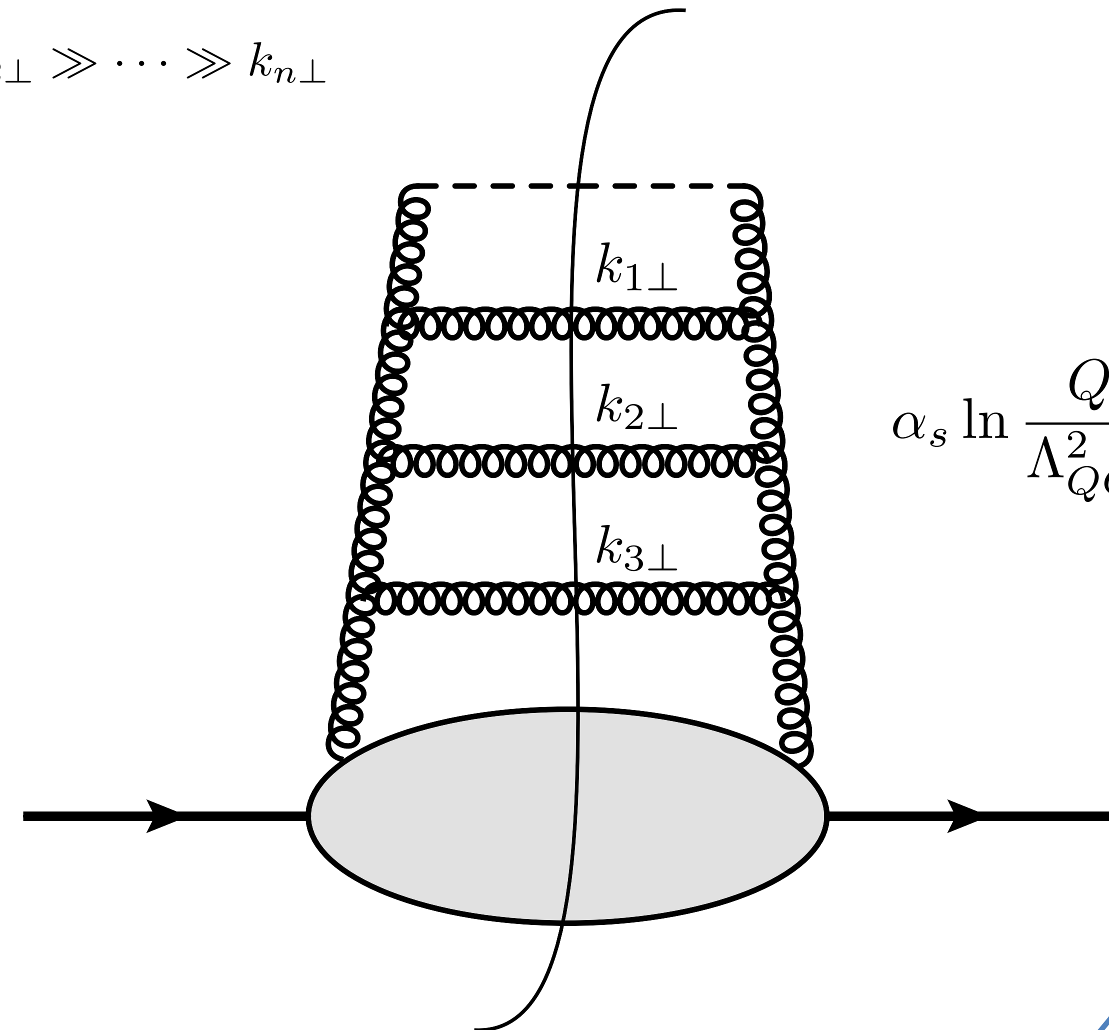






DGLAP EVOLUTION

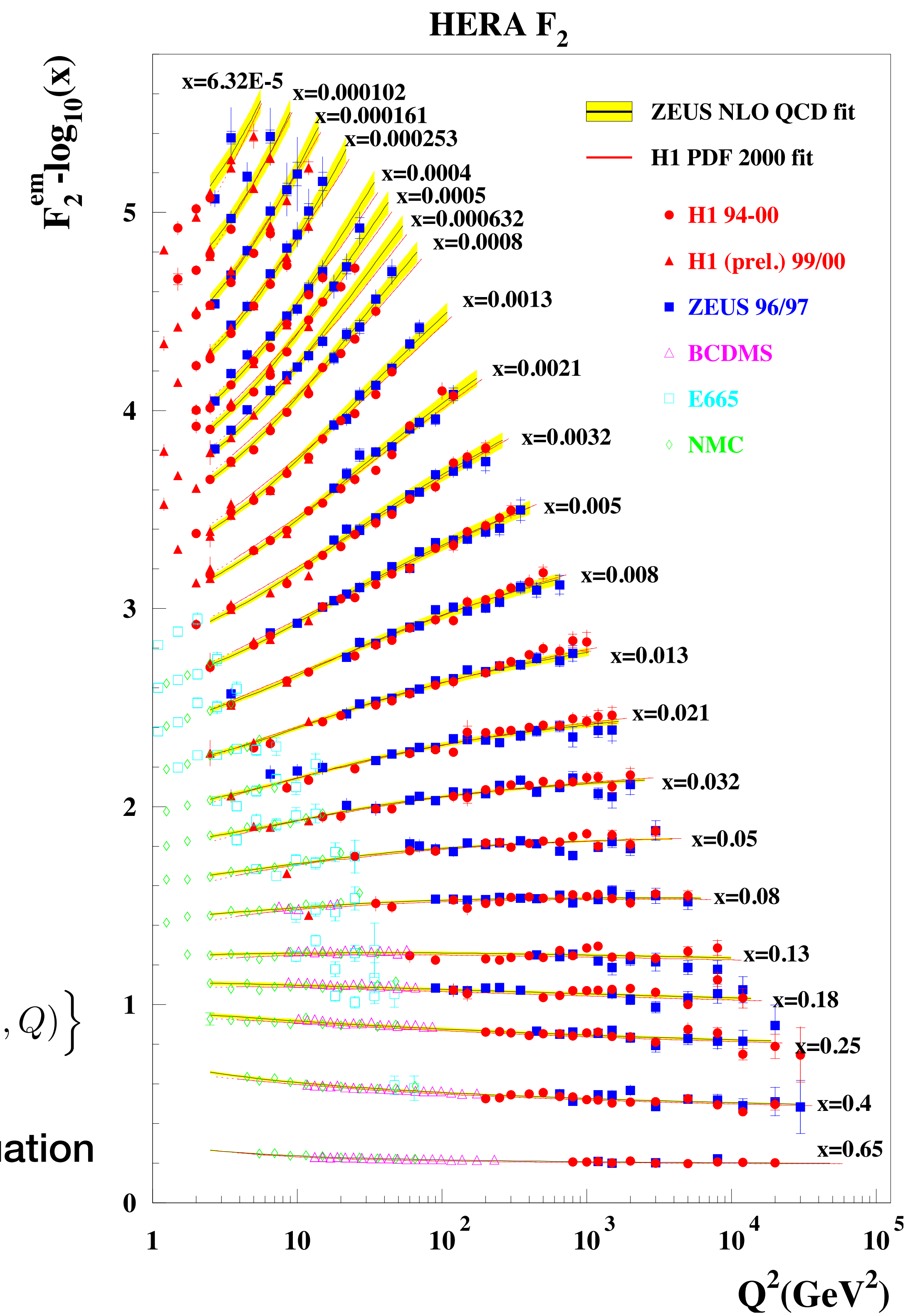
$k_{1\perp} \gg k_{2\perp} \gg \cdots \gg k_{n\perp}$



$$\alpha_s \ln \frac{Q^2}{\Lambda^2_{QCD}} \sim 1$$

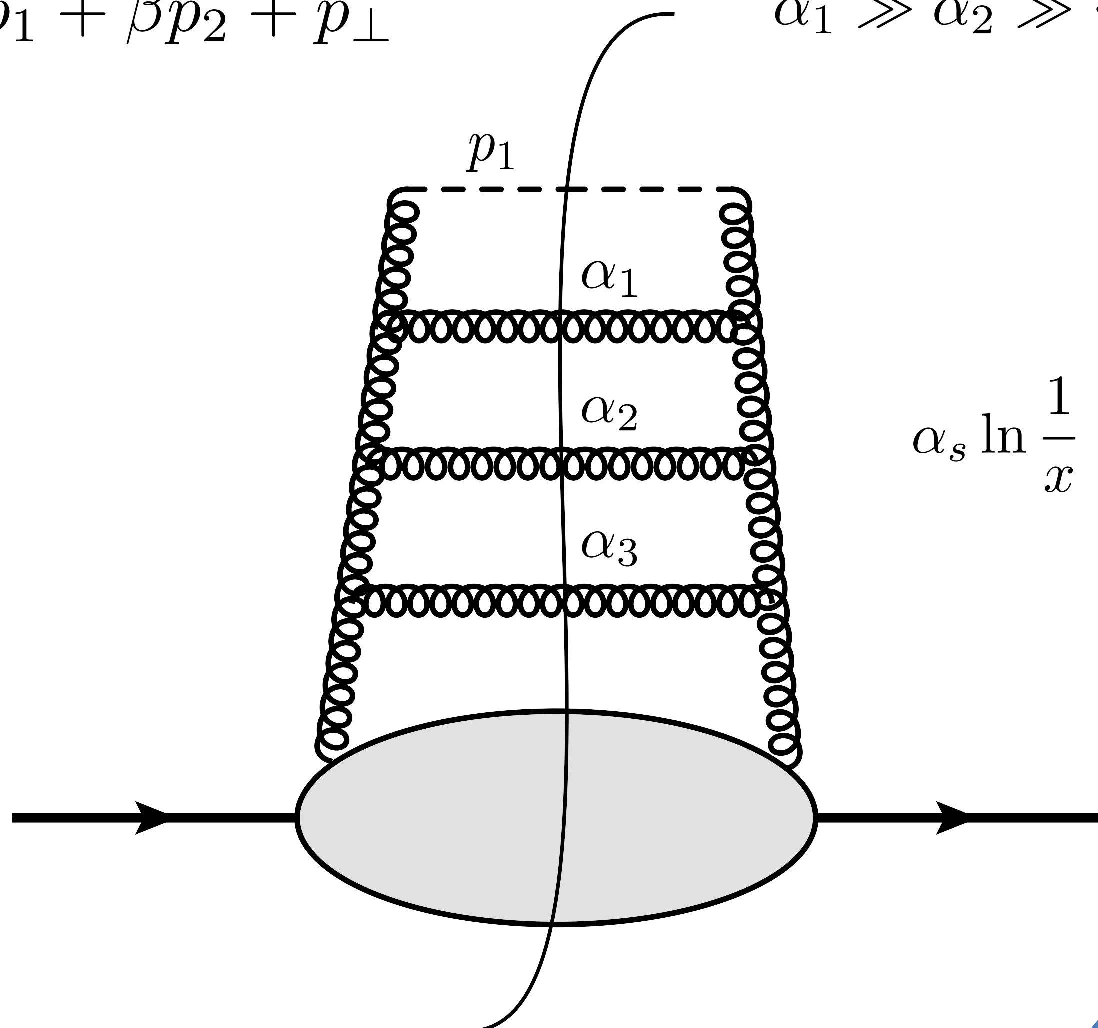
$$\frac{d}{d\log Q} f_g(x,Q) = \frac{\alpha_s(Q^2)}{\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q\to q} \sum_f [f_f(\frac{x}{z},Q) + f_{\bar{f}}(\frac{x}{z},Q)] + P_{g\to g}(z) f_g(\frac{x}{z},Q) \right\}$$

DGLAP evolution equation

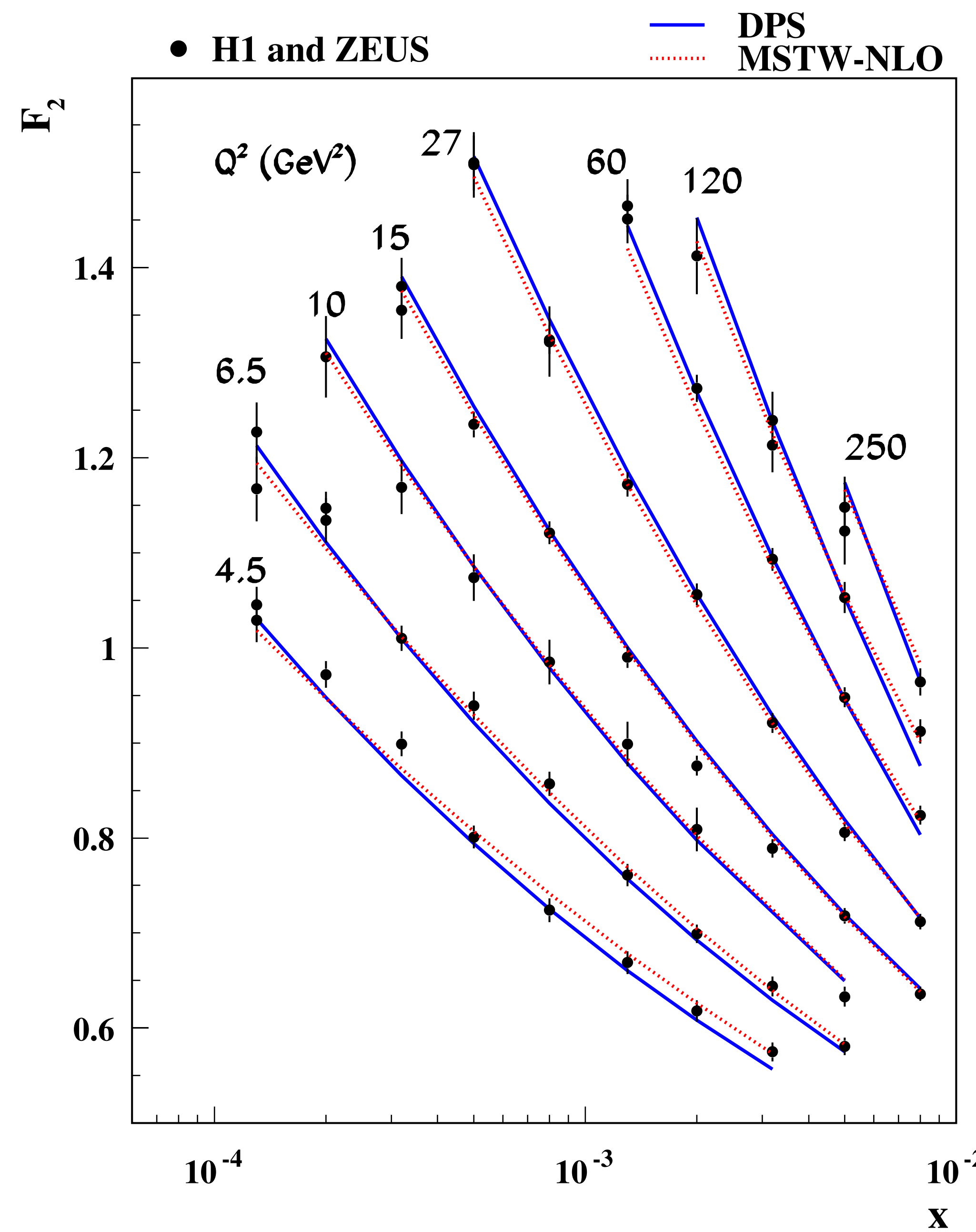


BFKL/BK EVOLUTION

$$p = \alpha p_1 + \beta p_2 + p_\perp \qquad \alpha_1 \gg \alpha_2 \gg \cdots \gg \alpha_n$$



$$\alpha_s \ln \frac{1}{x} \sim 1$$



$$\frac{\partial \phi(x,k_\perp^2)}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int \frac{d^2 q_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2}\left[\phi(x,q_\perp^2) - \frac{k_\perp^2}{2q_\perp^2}\phi(x,k_\perp^2)\right]$$

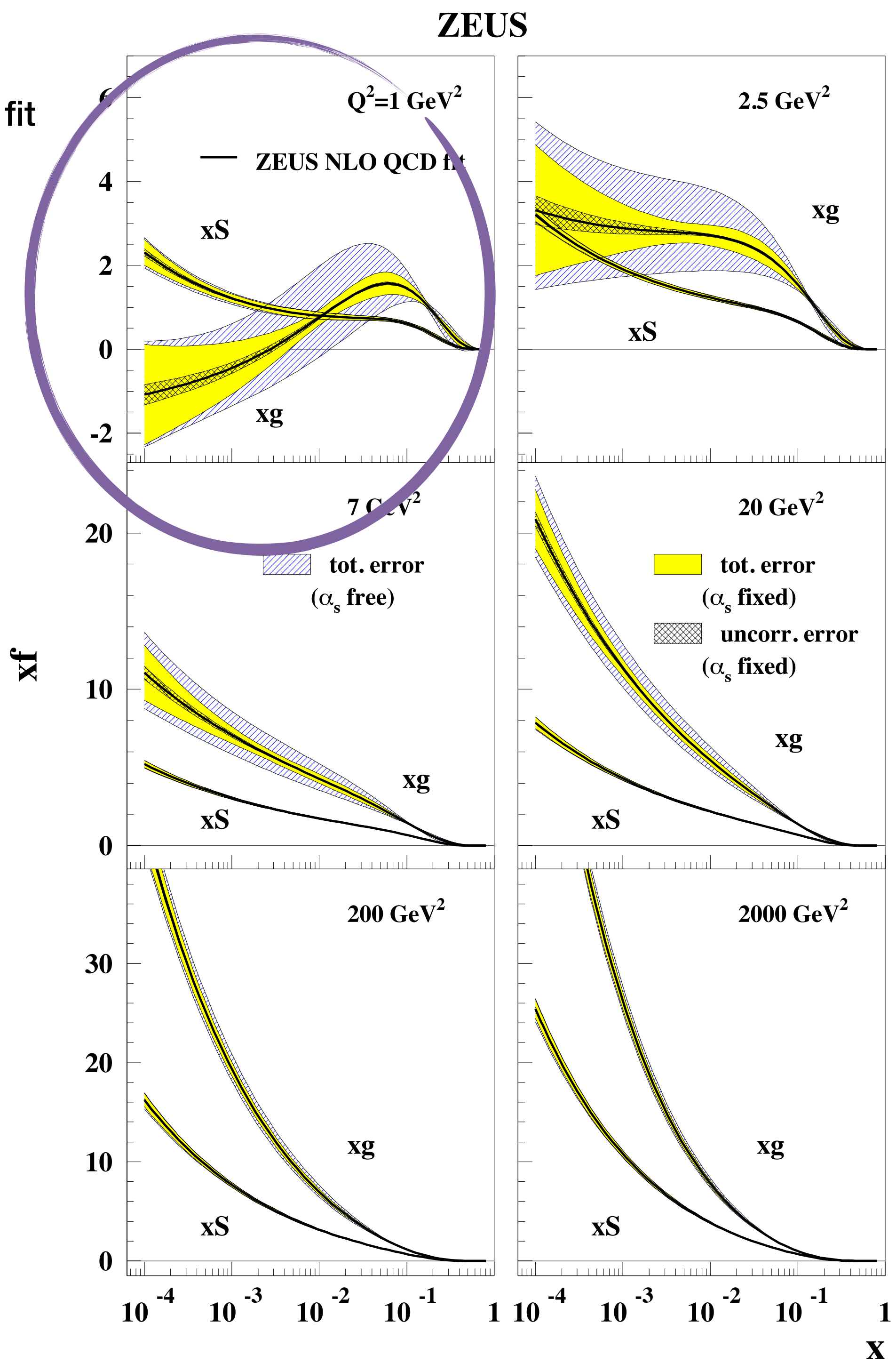
$$\phi(x,Q^2) = \frac{\partial x f_g(x,q^2)}{\partial Q^2}$$

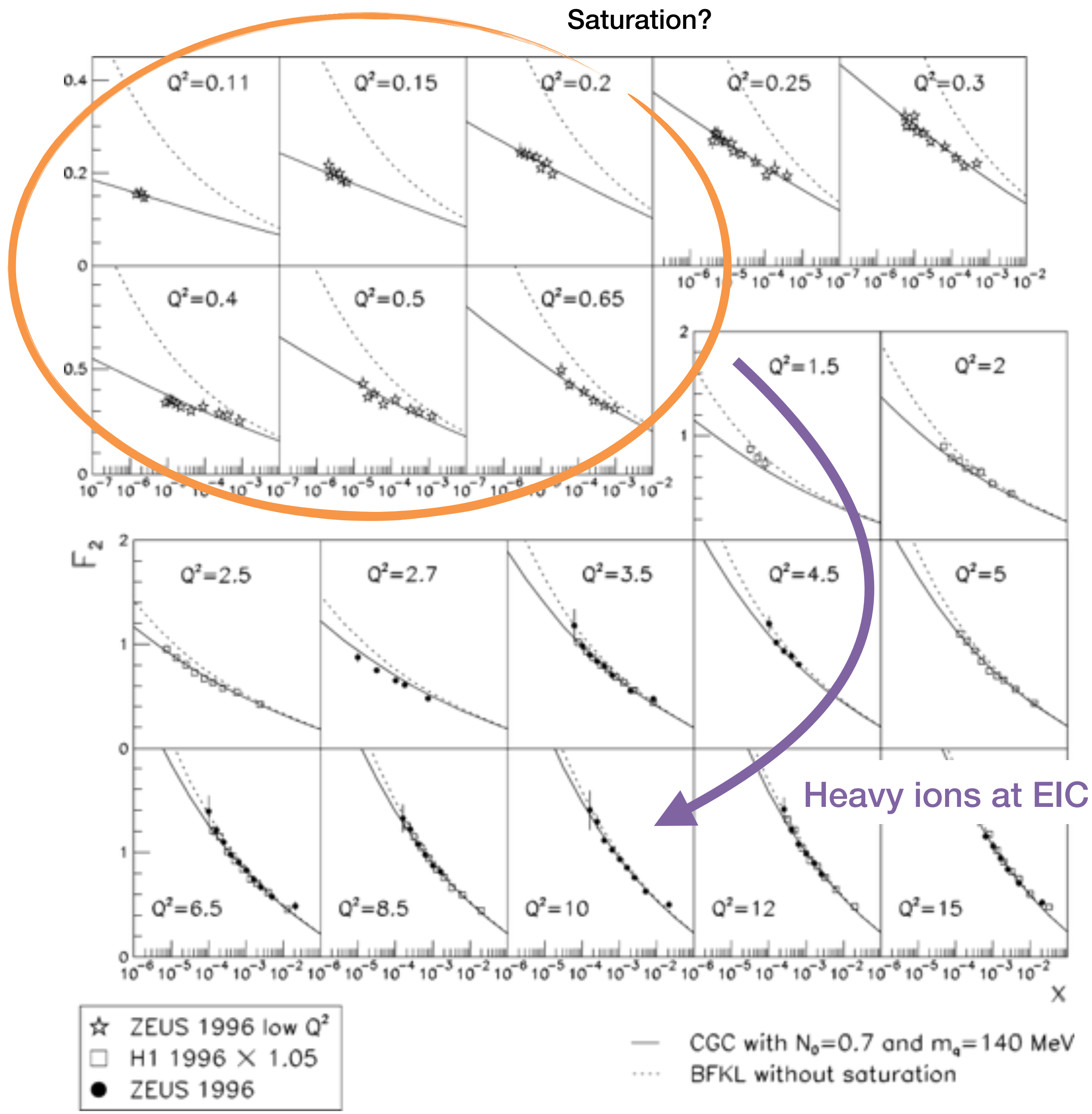
BFKL evolution equation

H. Kowalski, L.N. Lipatov, D.A. Ross, G. Watt
Eur. Phys. J. C (2010) 70, 983

BFKL/BK AND DGLAP

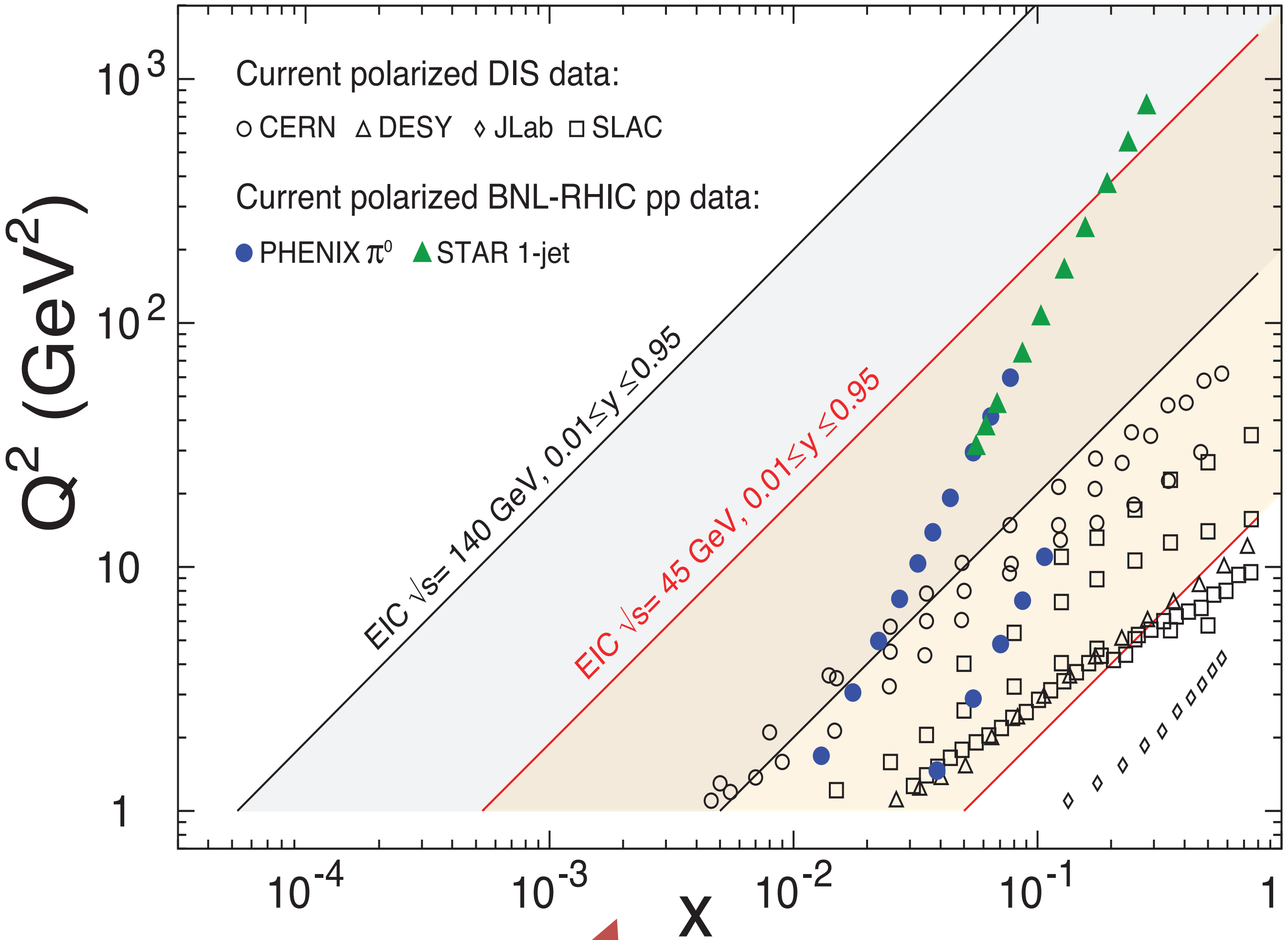
DGLAP fit

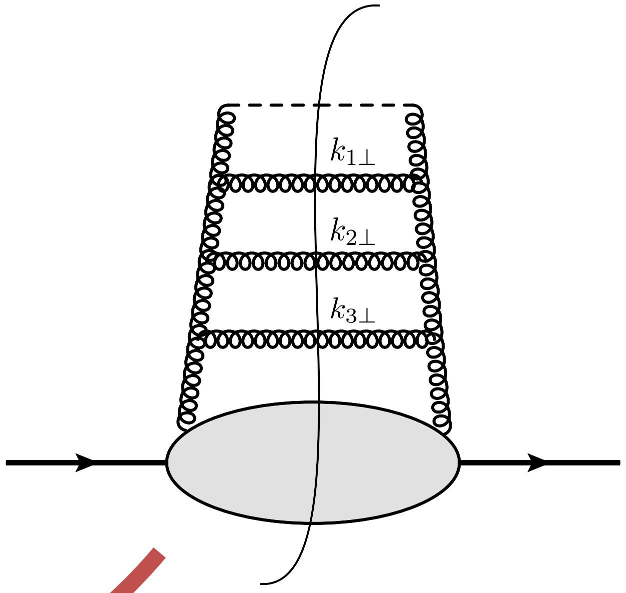




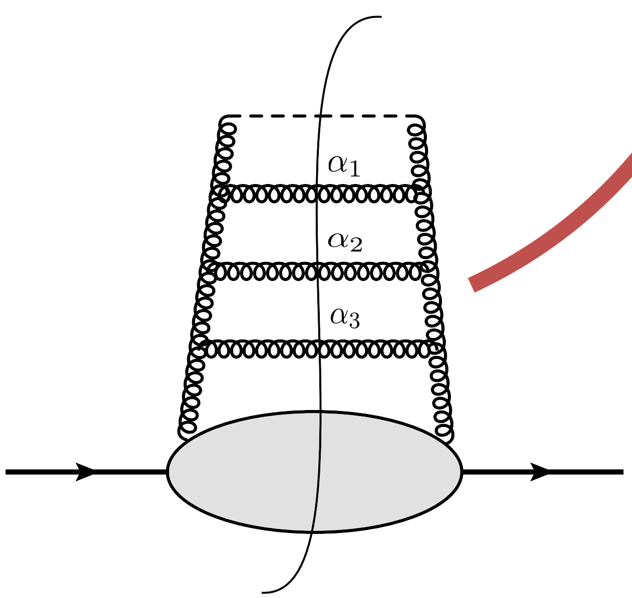
E. Iancu, K. Itakura, S. Munier
Phys. Lett. B 590 (2004) 199

BFKL/BK AND DGLAP AT EIC

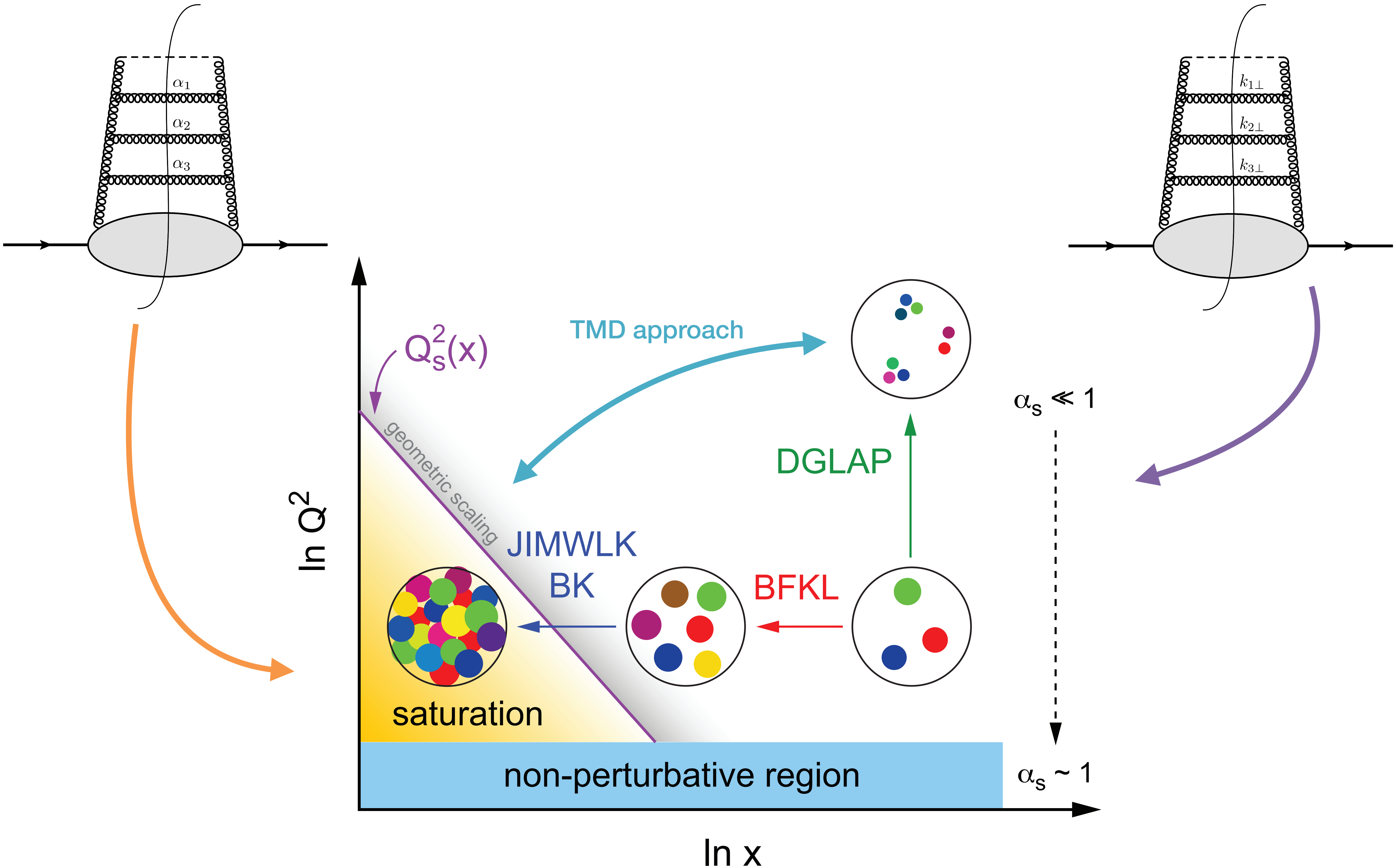




We have to perform simultaneous resummation of both types of emission

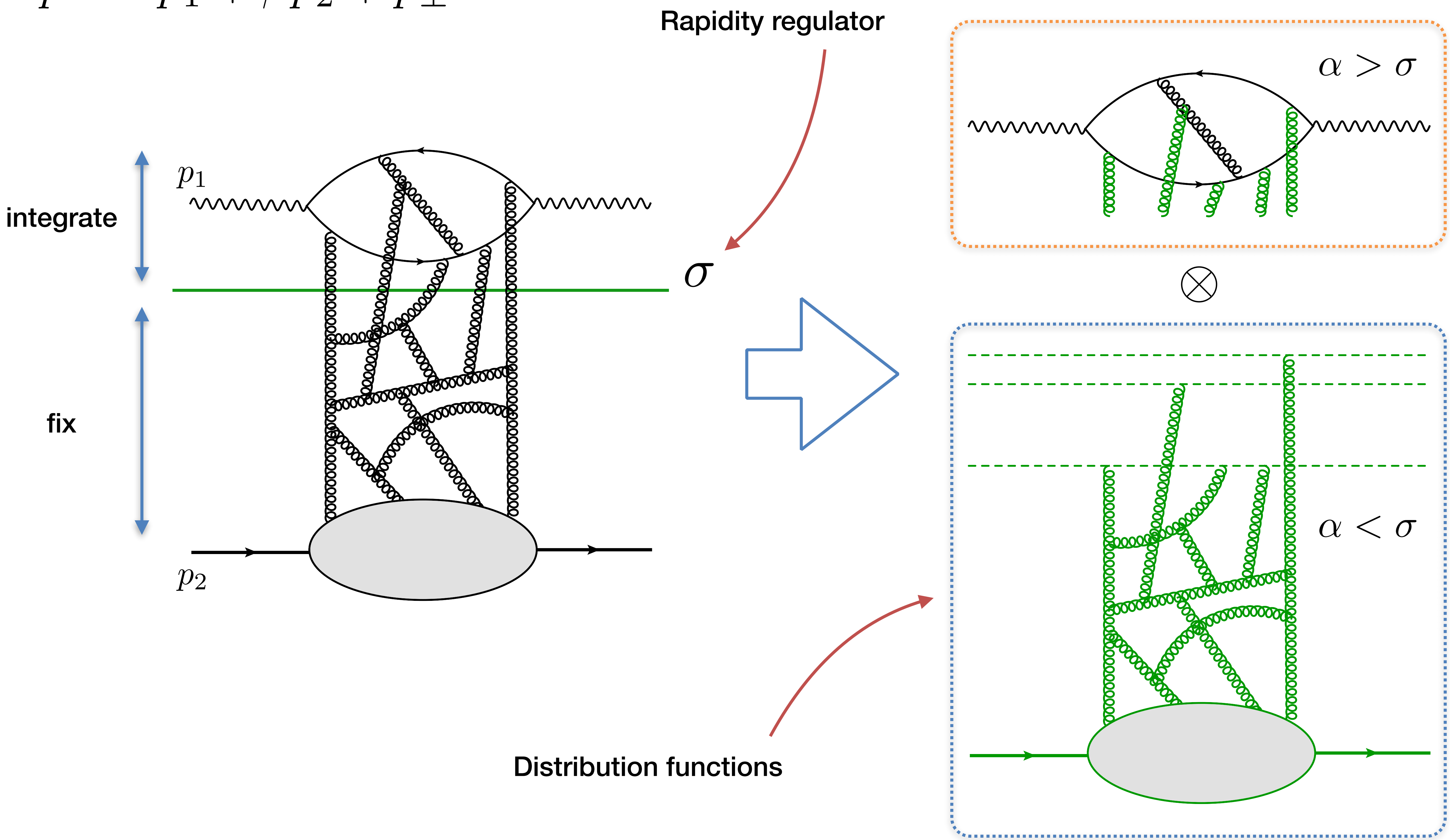


DGLAP vs. BFKL/BK



Rapidity factorization approach

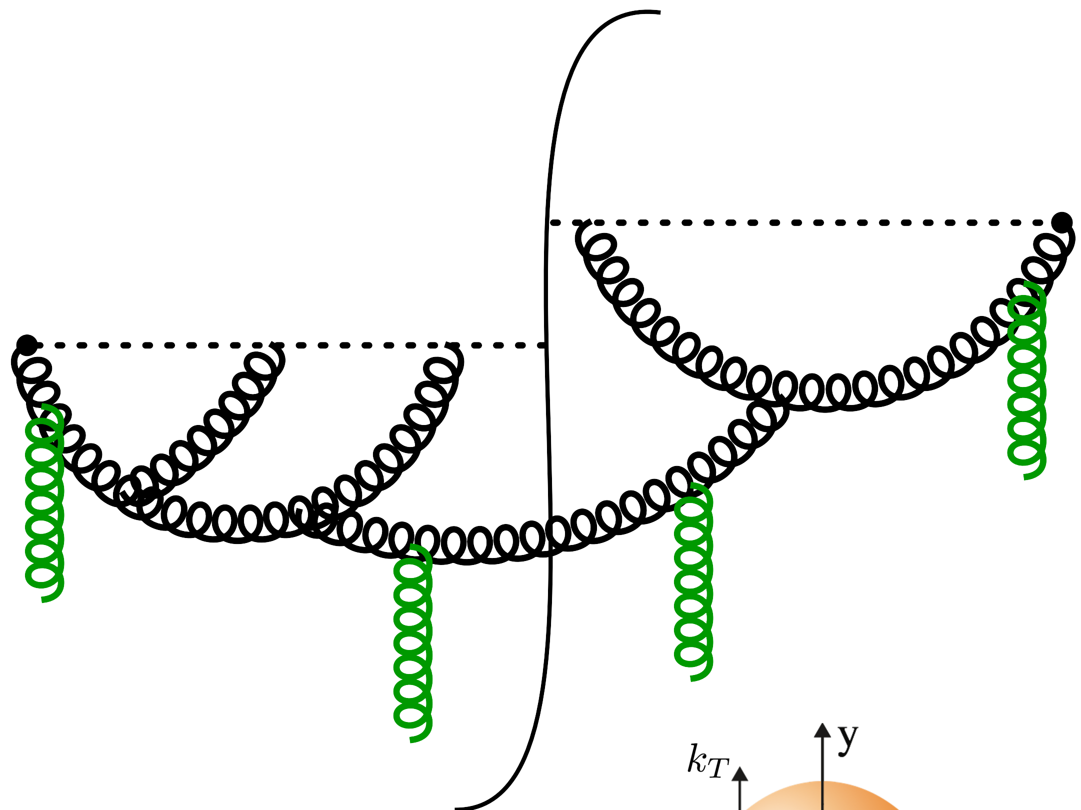
$$p = \alpha p_1 + \beta p_2 + p_\perp$$

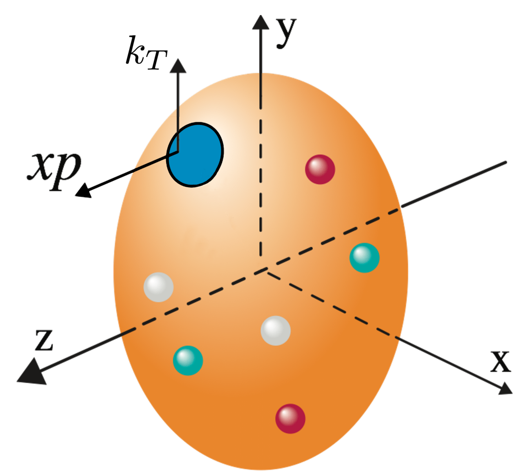


EVOLUTION EQUATION

I. Balitsky, A.T. (2015)

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\langle p|\mathcal{F}_i^a(x_B,x_\perp)\mathcal{F}_j^a(x_B,y_\perp)|p\rangle \\
&= -\alpha_s \mathrm{Tr}\Big\{\langle p|\int {d\!\!\!^-}^2 k_\perp\; L_i{}^\mu(k,x_\perp,x_B)^{\mathrm{light-like}}\theta\Big(1-x_B-\frac{k_\perp^2}{\sigma s}\Big)L_{\mu j}(k,y_\perp,x_B)^{\mathrm{light-like}} \\
&+ 2\mathcal{F}_i(x_B,x_\perp)(y_\perp| - \frac{p^m}{p_\perp^2}\mathcal{F}_k(x_B)(i\overleftarrow{\partial}_l + U_l)(2\delta_m^k\delta_j^l - g_{jm}g^{kl})U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger \\
&+ \mathcal{F}_j(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|y_\perp) \\
&+ 2(x_\perp|U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger(2\delta_i^k\delta_m^l - g_{im}g^{kl})(i\partial_k - U_k)\mathcal{F}_l(x_B)\frac{p^m}{p_\perp^2} \\
&+ \mathcal{F}_i(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|x_\perp)\mathcal{F}_j\big(x_B,y_\perp\big)|p\rangle\Big\} + O(\alpha_s^2)
\end{aligned}
$$





The equation describes the rapidity evolution of gluon TMD operator for any x_B and transverse momenta

This expression is UV and IR convergent

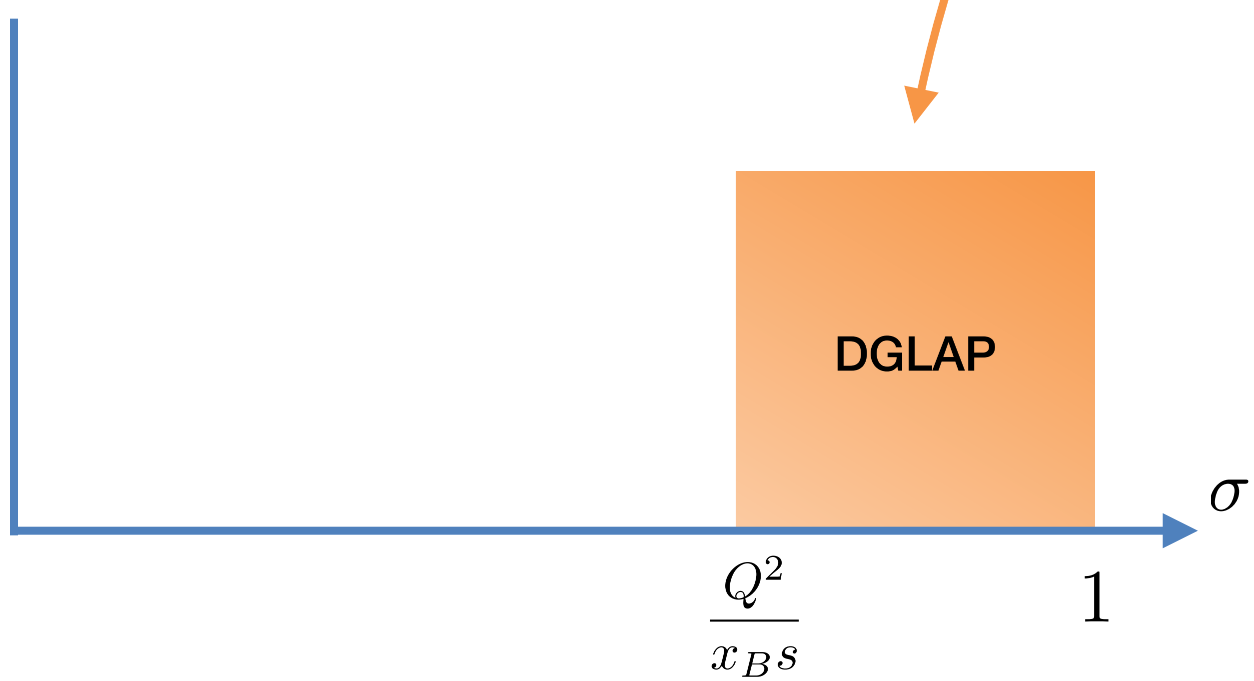
MODERATE X LIMIT

$x_B \sim 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \sim s$

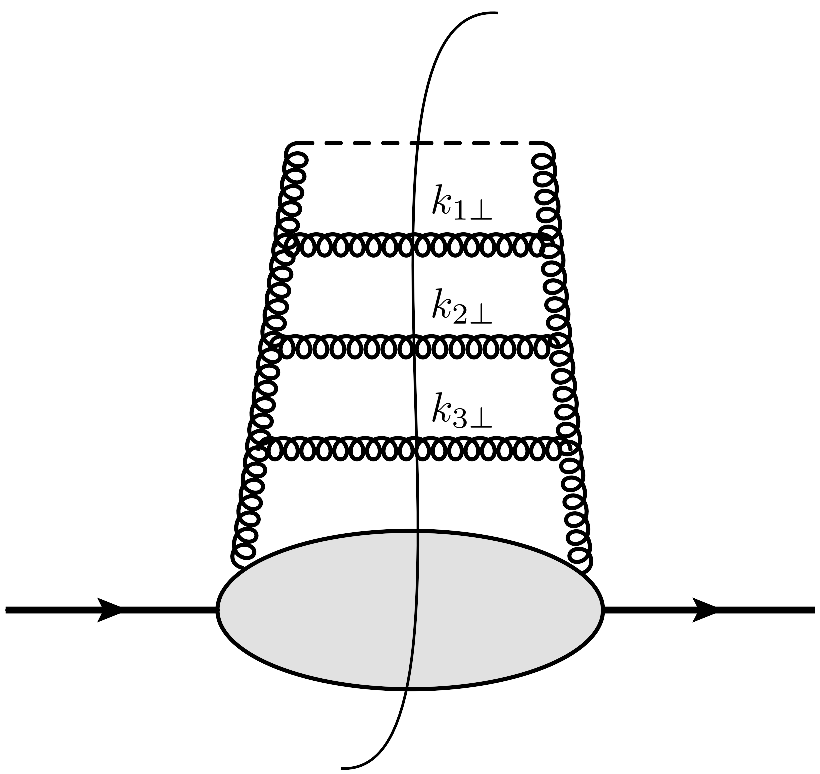
Strong ordering of transverse momenta:

$$k_{1\perp} \gg k_{2\perp} \gg \cdots \gg k_{n\perp}$$

$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) f_g(x_B, \ln\mu^2) = \frac{\alpha_s(\mu)}{\pi} N_c \int_{\beta_B}^{1} \frac{dz'}{z'} \left[\left(\frac{1}{1-z'}\right)_+ + \frac{1}{z'} - 2 + z'(1-z') \right] \alpha_s(\mu) f_g\left(\frac{\beta_B}{z'}, \ln\mu^2\right)$$



Reproduce DGLAP equation in the collinear limit



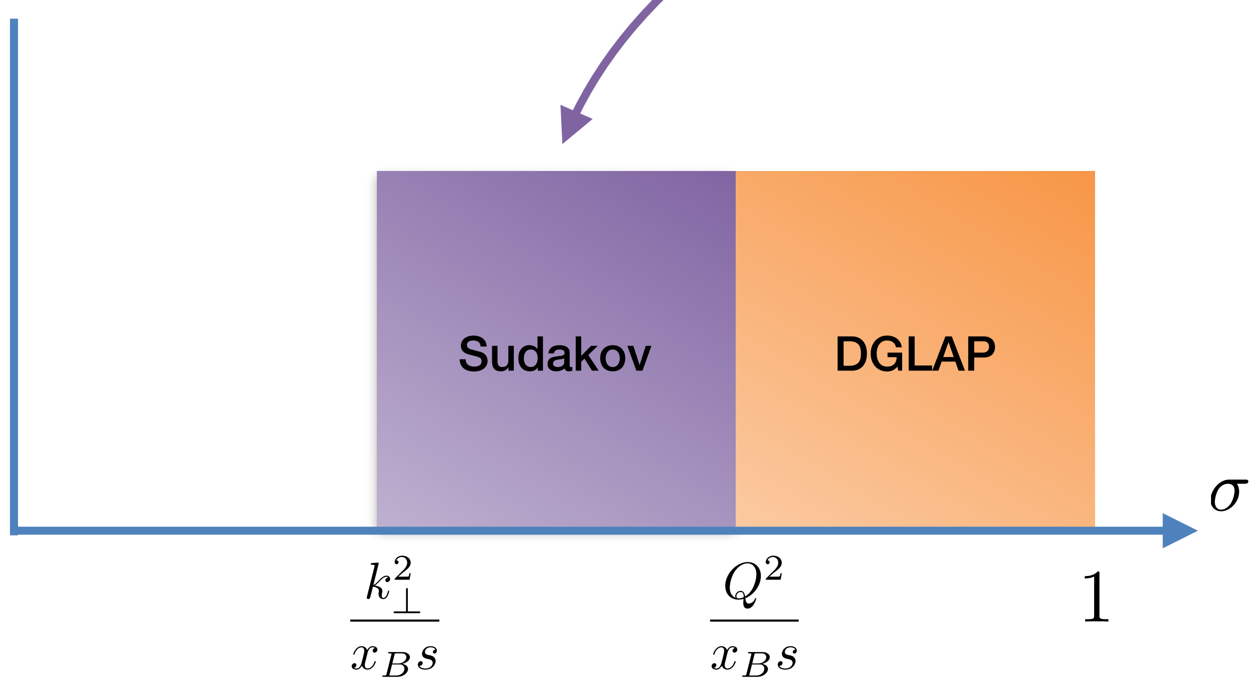
Sudakov evolution

$x_B \sim 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \sim \text{few GeV}^2 \ll s$

We keep evolution in the range of $\frac{k_\perp^2}{x_B s} \ll \sigma \ll \frac{Q_\perp^2}{x_B s}$

No kinematical restriction. Non-linear terms are power suppressed

$$\mathcal{D}(x_B, k_\perp, \ln\sigma) \sim \exp\{-\frac{\alpha_s N_c}{2\pi}\ln^2\frac{\sigma s}{k_\perp^2}\}\mathcal{D}(x_B, k_\perp, \ln\frac{k_\perp^2}{s})$$



Double log evolution

Small x limit

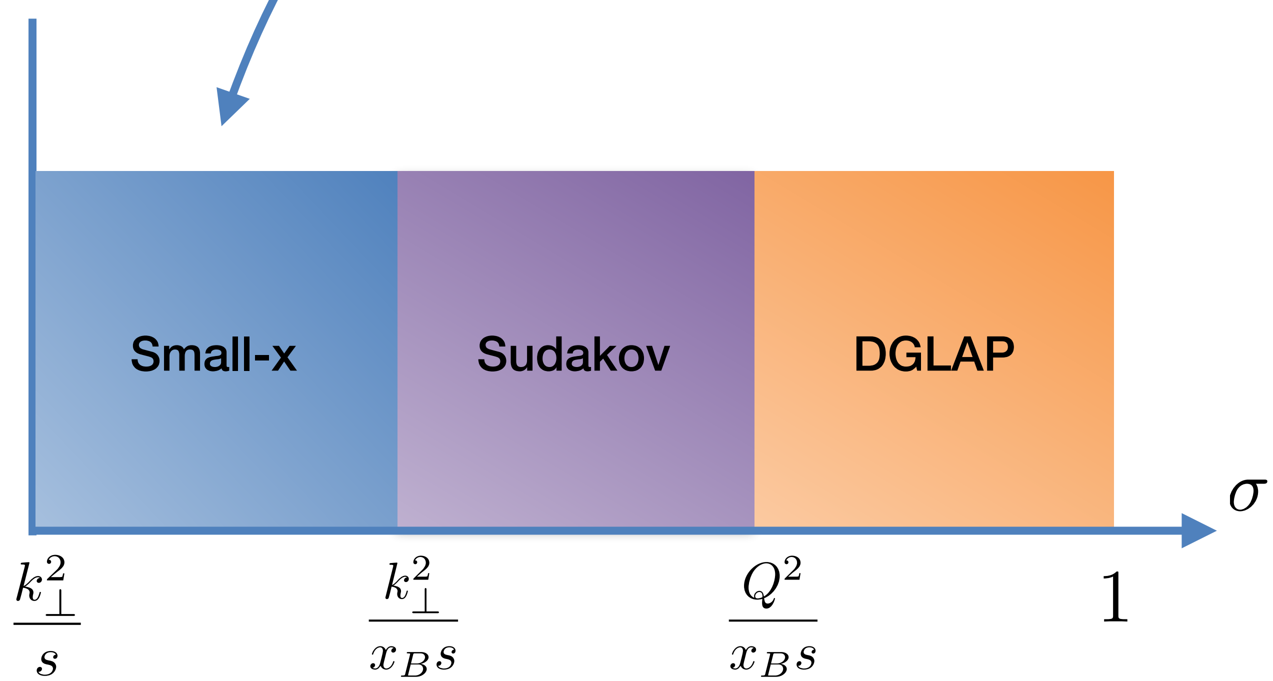
$x_B \ll 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

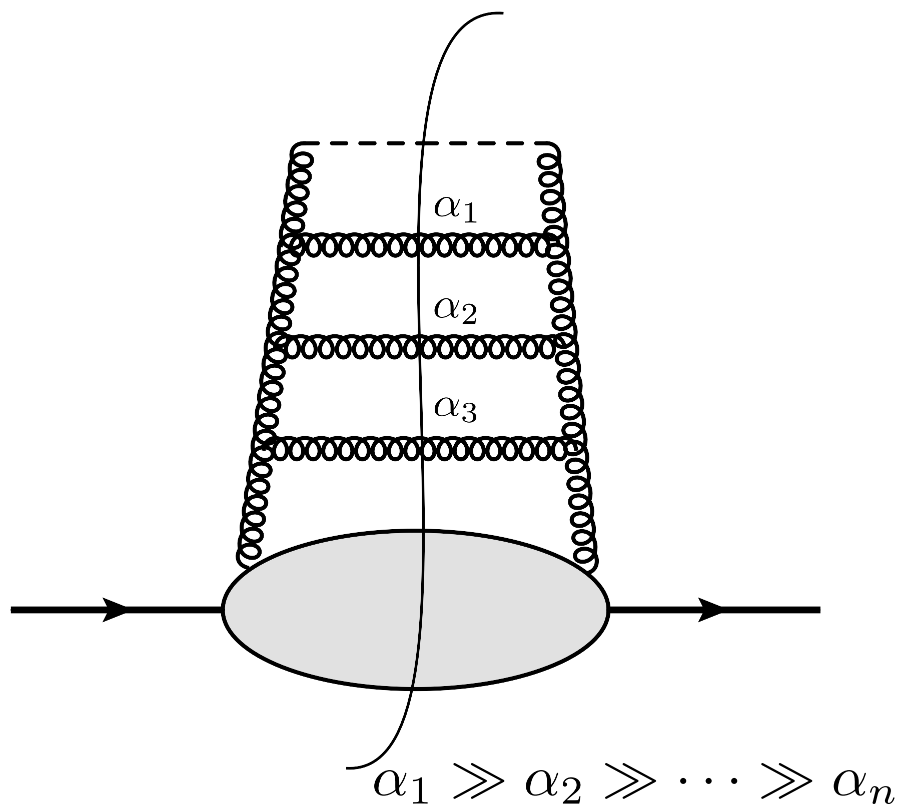
We keep the evolution in the range of $\frac{k_\perp^2}{s} \ll \sigma \ll \frac{k_\perp^2}{x_B s}$

Non-linear evolution of gluon TMDs at small x

I. Balitsky, A.T. (2014)

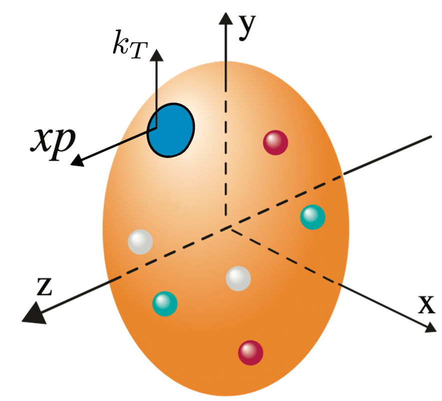
$$\frac{d}{d\ln\sigma}\tilde{U}_i^a(z_1)U_j^a(z_2) = -\frac{g^2}{8\pi^3}\mathrm{Tr}\{(-i\partial_i^{z_1}+\tilde{U}_i^{z_1})\Big[\int d^2z_3(\tilde{U}_{z_1}\tilde{U}_{z_3}^\dagger-1)\frac{z_{12}^2}{z_{13}^2 z_{23}^2}(U_{z_3}U_{z_2}^\dagger-1)\Big](i\overleftarrow{\partial}_j^{z_2}+U_j^{z_2})\}$$





Conclusions

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\langle p|\mathcal{F}_i^a(x_B,x_\perp)\mathcal{F}_j^a(x_B,y_\perp)|p\rangle \\
&= -\alpha_s \mathrm{Tr}\Big\{\langle p|\int d\!\!\bar{}\,^2k_\perp\; L_i{}^\mu(k,x_\perp,x_B)^{\mathrm{light-like}}\theta(1-x_B-\frac{k_\perp^2}{\sigma s})L_{\mu j}(k,y_\perp,x_B)^{\mathrm{light-like}} \\
&+ 2\mathcal{F}_i(x_B,x_\perp)(y_\perp| - \frac{p^m}{p_\perp^2}\mathcal{F}_k(x_B)(i\overleftarrow{\partial}_l + U_l)(2\delta_m^k\delta_j^l - g_{jm}g^{kl})U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger \\
&+ \mathcal{F}_j(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|y_\perp) \\
&+ 2(x_\perp|U\frac{1}{\sigma x_B s + p_\perp^2}U^\dagger(2\delta_i^k\delta_m^l - g_{im}g^{kl})(i\partial_k - U_k)\mathcal{F}_l(x_B)\frac{p^m}{p_\perp^2} \\
&+ \mathcal{F}_i(x_B)\frac{\sigma x_B s}{p_\perp^2(\sigma x_B s + p_\perp^2)}|x_\perp)\mathcal{F}_j(x_B,y_\perp)|p\rangle\Big\} + O(\alpha_s^2)
\end{aligned}
$$



Moderate-x (DGLAP). Linear evolution

$x_B \sim 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \sim 1$

Sudakov evolution

$x_B \sim 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

Small-x. Non-linear evolution

$x_B \ll 1$ and $k_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

The equation describes the rapidity evolution of gluon TMD operator for any x_B and transverse momenta

This expression is UV and IR convergent