PVDIS and nucleon structure

Wally Melnitchouk

Jefferson Lab
**Outline**

- **PVDIS** ($\vec{e}N \rightarrow eX$) is a valuable tool to study flavor and spin dependence of nucleon PDFs
  - establish accurate baseline from which to explore BSM physics

- **PVDIS** at lower $Q^2$ and $W^2$ (e.g. resonance region) provides vital input for estimates of $\gamma Z$ interference corrections
  - (e.g. for Qweak)

- Extraction of PDF information requires various subleading, finite-$Q^2$ corrections to be understood
  - $\alpha_s$ corrections (NLO), $\sigma_L/\sigma_T$ ratio
  - target mass corrections
  - higher twists

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P. Blunden, M. Gorshteyn talks

C. Seng talk
PVDIS at leading twist
Asymmetry between left- and right-handed inclusive electron-nucleon cross sections

\[ A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \]

→ for \( Q^2 \ll M_Z^2 \) numerator sensitive to \( \gamma Z \) interference only

\[ Q^2 = -(q_1 + q_2)^2 \]

→ denominator dominated by electromagnetic component
In terms of structure functions,

\[
A^{PV} = - \left( \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right) \left( g_A^e Y_1 \frac{F_1^\gamma Z}{F_1^\gamma} + g_V^e \frac{Y_3 F_3^\gamma Z}{2 F_1^\gamma} \right)
\]

\[ Y_{1,3} \text{ parameterize dependence on } y = \nu/E \]

\[
Y_1 = \frac{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma Z)) - 2xyM/E}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{1 + R^\gamma Z}{1 + R^\gamma} \right)
\]

\[
Y_3 = \frac{1 - (1 - y)^2}{1 + (1 - y)^2 - y^2(1 - r^2/(1 + R^\gamma)) - 2xyM/E} \left( \frac{r^2}{1 + R^\gamma} \right)
\]

with

\[
r^2 = 1 + \frac{Q^2}{\nu^2} = 1 + \frac{4x^2 M^2}{Q^2}
\]

\[
R^\gamma(\gamma Z) = \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2 \frac{F_2^{\gamma(\gamma Z)}}{2x F_1^{\gamma(\gamma Z)}} - 1
\]
Leading twist electroweak structure functions  
(at leading order in $\alpha_s$)

→ electromagnetic

\[
F_2^\gamma = 2x F_1^\gamma (x) = \sum_q e_q^2 x (q(x) + \bar{q}(x)) \quad V \times V
\]

→ interference

\[
F_2^{\gamma Z} = 2x F_1^{\gamma Z} (x) = \sum_q e_q g_V^q x (q(x) + \bar{q}(x)) \quad V \times V
\]
\[
F_3^{\gamma Z} = 2 \sum_q e_q g_A^q (q(x) - \bar{q}(x)) \quad V \times A
\]

\[
g_V^u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \quad g_A^u = \frac{1}{2} = -g_A^{d,s}
\]
\[
g_V^{d,s} = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W
\]
Leading twist electroweak structure functions

\[ F_2^{\gamma p} = \frac{4}{9} x(u + \bar{u}) + \frac{1}{9} x(d + \bar{d} + s + \bar{s}) + \cdots \]

\[ F_2^{\gamma n} = \frac{4}{9} x(d + \bar{d}) + \frac{1}{9} x(u + \bar{u} + s + \bar{s}) + \cdots \]

\[ F_2^{\gamma Z,p} = \left( \frac{1}{3} - \frac{8}{9} \sin^2 \theta_W \right) x(u + \bar{u}) + \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_W \right) (d + \bar{d} + s + \bar{s}) + \cdots \]

\approx \frac{1}{9} x(u + \bar{u} + d + \bar{d} + s + \bar{s}) + \cdots \quad \text{for } \sin^2 \theta_W \approx 1/4

\[ F_2^{\gamma Z,n} = \left( \frac{1}{3} - \frac{8}{9} \sin^2 \theta_W \right) x(d + \bar{d}) + \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_W \right) (u + \bar{u} + s + \bar{s}) + \cdots \]

\approx \frac{1}{9} x(d + \bar{d} + u + \bar{u} + s + \bar{s}) + \cdots \quad \text{for } \sin^2 \theta_W \approx 1/4

\rightarrow \quad \text{strange quark sensitivity enhanced relative to e.m. case}
Leading twist electroweak structure functions

\[ F_2^{\gamma p} = \frac{4}{9} x (u + \bar{u}) + \frac{1}{9} x (d + \bar{d} + s + \bar{s}) + \cdots \]
\[ F_2^{\gamma n} = \frac{4}{9} x (d + \bar{d}) + \frac{1}{9} x (u + \bar{u} + s + \bar{s}) + \cdots \]

\[ F_2^{\gamma Z, p} = \left( \frac{1}{3} - \frac{8}{9} \sin^2 \theta_W \right) x (u + \bar{u}) + \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_W \right) (d + \bar{d} + s + \bar{s}) + \cdots \]
\[ \approx \frac{1}{9} x (u + \bar{u} + d + \bar{d} + s + \bar{s}) + \cdots \quad \text{for } \sin^2 \theta_W \approx 1/4 \]
\[ F_2^{\gamma Z, n} = \left( \frac{1}{3} - \frac{8}{9} \sin^2 \theta_W \right) x (d + \bar{d}) + \left( \frac{1}{6} - \frac{2}{9} \sin^2 \theta_W \right) (u + \bar{u} + s + \bar{s}) + \cdots \]
\[ \approx \frac{1}{9} x (d + \bar{d} + u + \bar{u} + s + \bar{s}) + \cdots \quad \text{for } \sin^2 \theta_W \approx 1/4 \]

\[ F_3^{\gamma Z, p} = \frac{2}{3} (u - \bar{u}) + \frac{1}{3} (d - \bar{d} + s - \bar{s}) + \cdots \]
\[ F_3^{\gamma Z, n} = \frac{2}{3} (d - \bar{d}) + \frac{1}{3} (u - \bar{u} + s - \bar{s}) + \cdots \]

→ sensitivity to strange-antistrange asymmetry
PV asymmetry in terms of PDFs

\[ A^{PV} = - \left( \frac{G_F Q^2}{4 \sqrt{2} \pi \alpha} \right) (Y_1 a_1 + Y_3 a_3) \]

(hadronic) vector term

\[ a_1 = \frac{2 \sum_q e_q C_{1q} (q + \bar{q})}{\sum_q e_q^2 (q + \bar{q})} \]

\[ C_{1q} = 2 g_A^e g_V^q \]

(hadronic) axial-vector term

\[ a_3 = \frac{2 \sum_q e_q C_{2q} (q - \bar{q})}{\sum_q e_q^2 (q + \bar{q})} \]

\[ C_{2q} = 2 g_V^e g_A^q \]

→ simplified \( y \) dependence

\[ Y_1 \rightarrow 1 \]

\[ Y_3 \rightarrow \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \]
At large $x$ proton asymmetry sensitive to $d/u$ ratio

$$a_1 = \frac{12C_{1u} - 6C_{1d}}{4 + d/u}$$

$$a_3 = \frac{12C_{2u} - 6C_{2d}}{4 + d/u}$$

$\sim 1\%$ error on $d/u$ expected in SoLID up to $x \sim 0.7$

(note: different PDF uncertainty definitions in different analyses!!)
At large $x$ proton asymmetry sensitive to $d/u$ ratio

$$a_1 = \frac{12C_{1u} - 6C_{1d} d/u}{4 + d/u}$$

$$a_3 = \frac{12C_{2u} - 6C_{2d} d/u}{4 + d/u}$$

→ error reduced by recent $W$ asymmetry data
- but rely on reconstruction from $W \rightarrow \ell \nu$ decays
At smaller $x$, have order of magnitude greater sensitivity of $F^{Z,p}_2$ to strange–quark PDF than of e.m. $F^{p}_2$

$Q^2 = 10 \text{ GeV}^2$

$u/u_{\text{CJ15}}$

$d/d_{\text{CJ15}}$

$s/s_{\text{CJ15}}$

Accardi et al, PRD 93, 114017 (2016)

$\rightarrow$ currently relatively large uncertainties on $s(x)$ at intermediate $x$

– most information obtained from charm production in neutrino–nucleus DIS

Deuteron target – dependence on PDFs cancels at high $Q^2$

\[ a_1 = \frac{6}{5} (2C_{1u} - C_{1d}) \]

\[ a_3 = \frac{6}{5} (2C_{2u} - C_{2d}) \]

→ PV asymmetry independent of hadronic structure

\[ A^{PV} = -\left( \frac{3G_F Q^2}{10 \sqrt{2} \pi \alpha} \right) \left( Y_1(2C_{1u} - C_{1d}) + Y_3(2C_{2u} - C_{2d}) \right) \]

→ sensitivity to electroweak couplings

→ alternatively, could probe charge symmetry violation (CSV) in PDFs

1. Cloet talk
Historically, PVDIS was first measured at SLAC (on deuterium) at $Q^2 \sim 1 \text{ GeV}^2$ and $W \sim 3 \text{ GeV}$

→ important for testing SM, but not for PDF determination

Prescott et al, PLB 77, 347 (1978)
More recently, PVDIS measured more precisely at Jefferson Lab E08-011 at lower $W$

→ indication of quark-hadron duality in $\gamma Z$ structure functions in the nucleon resonance region
\( \gamma Z \) structure functions measured at HERA at very high \( Q^2 \) & \( W \)

\[ F_2^{\gamma Z} \]

Transformed to \( Q^2 = 1500 \text{ GeV}^2 \)

\[ xF_3^{\gamma Z} \]

Transformed to \( Q^2 = 1500 \text{ GeV}^2 \)

\[ H1 \] Collaboration, JHEP 09 (2012) 61

\[ \rightarrow \] essentially no proton PVDIS data at lower \( Q^2 \)
Finite-$Q^2$ corrections
hadronic axial-vector term relatively more important at finite $Q^2$

Hobbs, WM, PRD 77, 114023 (2008)
**Sensitivity to** \( R^\gamma = \sigma_L^\gamma / \sigma_T^\gamma \) and \( R^{\gamma Z} = \sigma_L^{\gamma Z} / \sigma_T^{\gamma Z} \)

- relative change to Bjorken limit asymmetry

\[
Q^2 = 5 \text{ GeV}^2
\]

\[x\] relative change to Bjorken limit asymmetry

\[R^\gamma = 0, \ r^2 \neq 1\]
\[R^\gamma = R_{1990}\]
\[R^\gamma = R_{1990} \pm \Delta R\]
\[d/u \text{ uncertainty}^*\]

- uncertainty due to \( R^\gamma \) smaller than \( d/u \text{ error}^* \) at high \( x \)
- need estimate for \( R^{\gamma Z} \) to determine impact on asymmetry uncertainty

* assuming \( d/u \to 0 \) or 0.2 at \( x = 1 \); uncertainty smaller with new \( W \) asymmetry data
What could break $R^{\gamma Z} = R^{\gamma}$?

- **higher order pQCD corrections**
  
  $R^{\gamma(\gamma Z)} = 0$ at leading order (perturbatively);
  at NLO different $\gamma/Z$ couplings induce $R^{\gamma} \neq R^{\gamma Z}$

- **target mass corrections** (“kinematical” $1/Q^2$ corrections)
  
  since TMCs for $F_{L}$ and $F_{T}$ are different, differences in ratios will be different with & without TMC
  (depend on treatment of struck parton – off-shell, collinear, ...)

- **dynamical higher twist**
  
  $\Lambda^2/Q^2$ corrections from multi-parton correlations

- **nuclear effects in deuteron**
Target mass corrections

- **operator product expansion** (Georgi, Politzer)  
  \( \text{(PRD 14, 1829 (1976))} \)
  \( \rightarrow \) standard approach, uses inverse Mellin transform;  
  “threshold problem” at \( x = 1 \)

- **collinear factorization** (Ellis, Furmanski, Petronzio)  
  \( \text{(NP B212, 29 (1983))} \)
  \( \rightarrow \) diagrammatic approach, impose \( x < 1 \) by definition;  
  include parton virtuality & \( k_T \); to \( \mathcal{O}(1/Q^2) \) only

- **collinear factorization** (Accardi, Qiu)  
  \( \text{(JHEP 07, 090 (2008))} \)
  \( \rightarrow \) similar to EFP, but include only 2-leg (cf. 4-leg) diagrams;  
  allow for jet-mass corrections, NLO effects

- **\( \xi \)-scaling** (Aivazis et al., Kretzer-Reno)  
  \( \text{(PRD 69, 034002 (2004))} \)
  \( \rightarrow \) similar to AQ, but at leading order
Target mass corrections

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ \frac{F_2^{\gamma}}{F_2^{\gamma}(0)} \]

\[ \frac{F_2^{\gamma}}{F_2^{\gamma \text{OPE}}} \]

\[ x \]

- effects large at high \( x \) (and low \( Q^2 \))
- up to \( \sim 10\% \) model dependence in structure functions
Target mass corrections

$Q^2 = 2 \text{ GeV}^2$

$Q^2 = 10 \text{ GeV}^2$

$\rightarrow$ less than $\sim 4\%$ TMC and NLO effects \textit{cf.} $R^\gamma$

$\rightarrow$ reduced to $\sim 2\%$ at $Q^2 = 10 \text{ GeV}^2$
Target mass corrections

\[ R_{\gamma}^{Z} / R_{\gamma} \]

\[ Q^2 = 2 \text{ GeV}^2 \]

\[ R_{d}^{\gamma Z} / R_{N}^{\gamma Z} \]

\[ \rightarrow \] differences between \( R_{\gamma}^{\gamma} \) & \( R_{\gamma}^{\gamma Z} \) small at high \( x \)

\[ \rightarrow \] significant nuclear effect in \( R_{\gamma}^{\gamma Z} \) ratio

\[ \text{reduced with inclusion of TMCs} \]
Dynamical higher twists

- $\Lambda_{QCD}^2/Q^2$ corrections from multi-parton correlations

  $\rightarrow$ for deuterium, only one twist-4 (four-quark) operator

  $$\bar{u}(z)\gamma^\mu u(z)\,d(0)\gamma'^\nu d(0) + (u \leftrightarrow d)$$

  Bjorken, PRD 18, 3239 (1978)
  Wolfenstein, NP B146, 477 (1978)

  Castorina, Mulders, PRD 31, 2760 (1985)

  Mantry, Ramsey-Musolf, Sacco, PRC 82, 065205 (2010)

  Belitsky, Manashov, Schaefer, PRD 84, 014010 (2011)

  C.-Y. Seng talk

- MIT bag model estimate of matrix elements

- extension to $x$ dependence

- estimate using multi-parton light-cone wavefunctions

- matrix elements sensitive to OAM in initial state

  Seng, Ramsey-Musolf, PRC 88, 015202 (2013)

- Alternatively, estimate HT contribution phenomenologically
Dynamical higher twists

In global QCD analyses parametrize structure function as

\[ F_2(x, Q^2) = F_2^{(LT)}(x, Q^2) \left( 1 + \frac{C_{HT}(x)}{Q^2} \right) \]

\[ C_{HT} = h_0 x^{h_1} (1 + h_2 x) \]

→ cannot accommodate \( Q^2 \) dependence of data without power corrections

→ crucial for extracting correct PDFs at high \( x \)

→ can do the same with PVDIS data, especially with sufficient \( Q^2 \) coverage – isolate isospin dependence of higher twists vs. \( x \)!
Polarized PVDIS
PVDIS from a polarized target

- LOI 12-16-007 to JLab PAC44 for “PVDIS from polarized $^3$He”

- Just as with sensitivity to $s$ in $F_2^{\gamma Z}$, the spin-dependent $g_1^{\gamma Z}$ structure function has enhanced sensitivity to $\Delta s$

$$g_1^{\gamma Z,p} = \frac{1}{2} \sum_q e_q g_V^q (\Delta q + \Delta \bar{q}) \approx \frac{1}{9} (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}) + \cdots$$

$\approx g_1^{\gamma Z,n}$

→ Significant uncertainty exists for $\Delta s(x)$, even in sign!

→ Most PDF analyses assume flavor SU(3) symmetry for hyperon decay constants

→ Polarized PVDIS would be first ever test of SU(3) in DIS
Summary

- PVDIS offers unique window on nucleon structure, with unambiguous flavor separation ($u, d$ and $s$)

- Corrections to leading-twist PVDIS from finite $Q^2$ effects identified and quantified

- PVDIS data (unpolarized and polarized) can be included in global QCD analyses ($CJ, JAM$) that are continuously being improved and developed