Towards universal QCD global analysis for nucleon structure distributions

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Physics questions for JLab12

- How does QCD work at energy scales of a few GeV?
- Can we use small coupling techniques?
- Are the factorization theorems valid at such scales?
- How can we model the transition from nonperturbative physics to perturbative physics?



To address these questions we need:

- A solid theoretical framework
- Precise experimental measurements
- Robust data analysis framework

















- Test of universality of collinear framework
- Robust data analysis framework is needed
- Collinear distributions are part of the TMD framework

So far...





4 / 34

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Iterative Monte Carlo analysis of spin-dependent parton distributions

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Data analysis framework:

The goal is to estimate:

$$E[\mathcal{O}] = \int d^{n}a \ \mathcal{P}(\boldsymbol{a}|data) \ \mathcal{O}(\boldsymbol{a})$$
$$V[\mathcal{O}] = \int d^{n}a \ \mathcal{P}(\boldsymbol{a}|data) \ [\mathcal{O}(\boldsymbol{a}) - E[\mathcal{O}]]^{2}$$

- a = (N, a, b, c, d, ...) is a vector of parameters ilic. $f(x, Q_0^2) = Nx^a(1-x)^b P(x)$
- $\mathcal{O}(a)$ is an observable: i.e. PDFs, Δ PDFs, FF, cross sections
- \blacksquare Each flavor increases the number of parameters typically by ~ 5
- The combined PDF, Δ PDF and FF amount to analyzing $30 + 30 + 20 \times 2 \sim 100$ shape parameters

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Maximum Likelihood

Maximize
$$\mathcal{P}(\boldsymbol{a}|data) {
ightarrow} \boldsymbol{a}_0$$

 $\bullet E[\mathcal{O}] \approx \mathcal{O}(\boldsymbol{a}_0)$

• $V[\mathcal{O}] \approx Hessian, \Delta \chi^2 envelope,...$

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Maximum Likelihood

• Maximize $\mathcal{P}(\boldsymbol{a}|data) \rightarrow \boldsymbol{a}_0$

 $\bullet E[\mathcal{O}] \approx \mathcal{O}(\boldsymbol{a}_0)$

Monte Carlo methods

- $\square \mathcal{P}(\boldsymbol{a}|data) \rightarrow \{\boldsymbol{a}_k\}$
- $\operatorname{E}[\mathcal{O}] \approx \frac{1}{N} \sum_{k} \mathcal{O}(\boldsymbol{a}_{k})$
- $V[\mathcal{O}] \approx Hessian, \Delta \chi^2 \text{ envelope,...}$ $V[\mathcal{O}] \approx \frac{1}{N} \sum_k [\mathcal{O}(\boldsymbol{a}_k) E[\mathcal{O}]]^2$

Methodology



Iterative Monte Carlo analysis (IMC)



- Multiple iterations until convergence of posterior distribution
- Keep all the parameters free. No assumptions on the exponents
- Avoid over-fitting by Cross-Validation
- Iterative procedure → Adaptive MC integration (like in Vegas)
- Robust estimation of uncertainties

Fragmentation Functions from SIA data

Technical details

- Use all available e^+e^- data from $Q = 10 \text{ GeV} \rightarrow 91.2 \text{ GeV}$ that includes light and HQ separated samples
- Include recent measurements from Belle and BaBar
- Fit π and K FFs using pQCD @ NLO

$$\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma^{h^{\pm}}}{dz}(z,Q^2) = \frac{2}{\sigma_{\text{TOT}}} \left[\sum_{q} C_q \otimes D_{q^+}(z,Q^2) + C_g \otimes D_g(z,Q^2) \right]$$

- **ZMVS** with input $Q_0^{u,d,s} = 1$ GeV and $Q_0^{c,b} = m_{c,b}$
- z > 0.05 for high energy data and z > 0.1 for low energy data • Traditional ansatz

•
$$D^{\text{favor}}(z) = T_1(z) + T_2(z)$$

$$D^{\text{unfavor}}(z) = T_1(x)$$

$$T_i(z) = \frac{M_i}{B(1+a_i,1+b_i)} z^{a_i} (1-z)^{b_i}$$

24 kaon parameters and 18 pion parameters

π analysis $(\chi^2/N_{ m pts}=1.31)$

- We use BaBar prompt data set
- Belle data set needs 10% normalization
- Good agreement at low z for inclusive data sets
- Data inconsistencies at large z for $Q^2=M_z^2$

K analysis $(\chi^2/N_{ m pts}=1.01)$

- $z_{cut} > 0.2$ for low energies to avoid hadron mass corrections
- z_{cut} > 0.05 for high energies
- Smaller χ^2 than π due to larger errors
- Consistent shapes across all z
- Inconsistencies mostly due to normalizations

Fragmentation functions

- \blacksquare Similar behavior of unfavored $D^{\pi}_{s^+}$ and $D^{K}_{d^+}$
- In contrast the D_g^{π} and D_g^K behave differently
- The charm and bottom FFs become compatible at largez
- Favored $D_{u^+}^{\pi}$ and $D_{s^+}^{K}$ have similar shape at largez
- JAM D^K_{s+} is more similar to DSS. Will it change the sign of ∆s⁺?

New combined analysis of $\Delta DIS + \Delta SIDS + SIA$

Study of FFs in pythia8+DIRE

String effects: PLB261 (1991) (OPAL Collaboration)

- 3 Jets events: $Q\bar{Q}$ and gluon jets. Jets are projected into a plane
- $\psi:$ angle of a given particle relative to the quark jet with the highest energy
- ψ_A : angle between highest energetic jet and gluon jet
- ψ_C : angle between quark jets
- \blacksquare Only events with $\psi_A=\psi_C$ are kept

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- ψ_C : angle between quark jets
- Only events with $\psi_A = \psi_C$ are kept
- Particle flow asymmetry is observed \rightarrow evidence of string effects

Technical details

Simulate e^+e^- at Q = 30,91.2,1000 GeV flavor by flavor

• Fit π and K FFs using pQCD @ NLO

$$\frac{1}{\sigma_{\text{TOT}}} \frac{d\sigma_q^{h^{\pm}}}{dz}(z, Q^2) = \frac{2}{\sigma_{\text{TOT}}} \left[C_q \otimes D_{q^+}(z, Q^2) + C_g \otimes D_g(z, Q^2) \right]$$

• ZMVS with input
$$Q_0 = 11 \text{GeV}$$

Parametrization: $D_{q^+}(z) = N z^{\alpha} (1-z)^{\beta} (1+c_1 z+c_2 z^2+...)$

Pythia8 vs. collinear factorization (preliminary)

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Pythia8+DIRE FFs (preliminary)

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Pythia8+DIRE π FFs and other global analyses

Pythia8+DIRE K FFs and other global analyses

Pythia8+DIRE vs global $e^+e^- \rightarrow \pi + X$

Pythia8+DIRE vs global $e^+e^- \rightarrow K + X$

Beyond 1D distributions...

Intuitive picture

The Breit frame: q = (0, 0, 0, -Q) and $P = (P^0, 0, 0, P_z)$

Small transverse mom. "Intrinsic"

Setup of the calculation

Factorization in TMD observables

$$\Gamma = d\sigma/dq_T q_T = p^h/z$$

$$= \mathbf{T}_{\mathrm{TMD}}\Gamma + [\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma]$$

$$= \underbrace{\mathbf{T}_{\mathrm{TMD}}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\mathrm{coll}}[\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma]}_{\mathbf{Y}} + \mathcal{O}(m^2/Q^2)\Gamma$$

• Region of $q_T \ll Q$

 $\Gamma = \Gamma$

- TMD approx. dominates $\rightarrow~\Gamma\approx \mathbf{T}_{TMD}\Gamma$
- \mathbf{Y} term small

• Region of $q_T \gtrsim Q$

- Collinear approx. dominates $\rightarrow \ \Gamma \approx \mathbf{T}_{coll} \Gamma$
- At large Q , $\mathbf{T}_{\mathrm{TMD}}\Gamma$ is mostly perturbative

$$W = T_{TMD}\Gamma$$
$$FO = T_{coll}\Gamma$$
$$ASY = T_{coll}T_{TMD}\Gamma$$
$$Y = FO - ASY$$

SIDIS (One of the main programs of JLab12)

Cross section and structure functions

$$\frac{d^5 \sigma(S_{\perp})}{dx_B dQ^2 dz_h d^2 P_{h\perp}} = \sigma_0 \Big[F_{UU} + \sin(\phi_h - \phi_s) \ F_{UT}^{\sin(\phi_h - \phi_s)} \\ + \sin(\phi_h + \phi_s) \ \frac{2(1-y)}{1+(1-y)^2} \ F_{UT}^{\sin(\phi_h + \phi_s)} + \dots \Big]$$

CSS formalism

$$F_{UU} = H_{\text{SIDIS}} \frac{1}{z_h^2} \int_0^\infty \frac{db \, b}{(2\pi)} J_0(q_{h\perp}b) \, \widetilde{W}_{UU}(b_*) + Y_{UU}$$

$$F_{UT}^{\sin(\phi_h - \phi_s)} = -H_{\text{SIDIS}} \frac{M_P}{z_h^2} \int_0^\infty \frac{db \, b^2}{(2\pi)} J_1(q_{h\perp}b) \, \widetilde{W}_{UT}^{\sin(\phi_h - \phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h - \phi_s)}$$

$$F_{UT}^{\sin(\phi_h + \phi_s)} = H_{\text{SIDIS}} \frac{M_h}{z_h^2} \int_0^\infty \frac{db \, b^2}{(2\pi)} J_1(q_{h\perp}b) \, \widetilde{W}_{UT}^{\sin(\phi_h + \phi_s)}(b_*) + Y_{UT}^{\sin(\phi_h + \phi_s)}$$

SIDIS: small transverse momentum

• W term formulation in b_T space

$$\begin{split} \widetilde{W}_{UU}(b_{*}) &\equiv e^{-S_{pert}(Q,b_{*}) - S_{\rm NP}^{\rm f1}(Q,b) - S_{\rm NP}^{\rm D1}(Q,b)} \, \widetilde{F}_{UU}(b_{*}) \\ \widetilde{W}_{UT}^{\sin(\phi_{h} - \phi_{s})}(b_{*}) &\equiv e^{-S_{pert}(Q,b_{*}) - S_{\rm NP}^{\rm f1}(Q,b) - S_{\rm NP}^{\rm D1}(Q,b)} \, \widetilde{F}_{UT}^{\sin(\phi_{h} - \phi_{s})}(b_{*}) \\ \widetilde{W}_{UT}^{\sin(\phi_{h} + \phi_{s})}(b_{*}) &\equiv e^{-S_{pert}(Q,b_{*}) - S_{\rm NP}^{\rm h1}(Q,b) - S_{\rm NP}^{\rm H1}(Q,b)} \, \widetilde{F}_{UT}^{\sin(\phi_{h} + \phi_{s})}(b_{*}) \end{split}$$

Small b_T contribution

$$\widetilde{F}_{UU}(b_*) = \sum_q e_q^2 \left(C_{q \leftarrow i}^{f_1} \otimes f_1^i(x_B, \mu_b) \right) \left(\hat{C}_{j \leftarrow q}^{D_1} \otimes D_{h/j}(z_h, \mu_b) \right)$$
$$\widetilde{F}_{UT}^{\sin(\phi_h - \phi_s)}(b_*) = \sum_q e_q^2 \left(C_{q \leftarrow i}^{f_{1T}^\perp} \otimes f_{1T}^{\perp(1)i}(x_B, \mu_b) \right) \left(\hat{C}_{j \leftarrow q}^{D_1} \otimes D_{h/j}(z_h, \mu_b) \right)$$
$$\widetilde{F}_{UT}^{\sin(\phi_h + \phi_s)}(b_*) = \sum_q e_q^2 \left(\delta C_{q \leftarrow i}^{h_1} \otimes h_1^i(x_B, \mu_b) \right) \left(\delta \hat{C}_{j \leftarrow q}^{H_1^\perp} \otimes \hat{H}_1^{\perp(1)j}(z_h, \mu_b) \right)$$

Collinear distribution are important in TMDs

Does it work?

Kinematics $Q^2 = 1.92 \text{ GeV}^2$ x = 0.0318z = 0.375

- Threshold resummation?
- Subleading power corrections?

The unpolarized SIDIS cross sections needs to be ready to interpret upcoming TMD data from JLab 12

SIDIS kinematics analysis Boglione et al (PLB.2017.01.02)

- Can we apply factorization theorems in SIDIS measurements?
- Factorization demands that

$$p_h \cdot k_f = \mathcal{O}(m^2)$$
$$p_h \cdot k_i = \mathcal{O}(Q^2)$$

Define a collinearity parameter

$$R = \frac{(p_h \cdot k_f)}{(p_h \cdot k_i)} = \mathcal{O}(m^2/Q^2)$$

SIDIS kinematics analysis

Summary and outlook

• The collinear master plan:

- New MC fitting methodology
- Combine polarized PDFs and FF analysis \rightarrow sign of ΔS puzzle
- The baseline PDFs, FFs for TMD studies

Open questions:

- Do we understand the shapes of FFs? Especially the gluon FF?
- What governs the low-z FFs?
- Can we trust in the collinear framework for SIDIS at low energies?
- Are the SIDIS measurements in the current region?

MCEG for SIDIS:(new directions)

- Pythia8 validation of Hermes multiplicities
- Extraction of FFs from Pythia8: test of DGLAP and Pythia8's parton shower

Spin PDFs from polarized DIS

Global polarized DIS data

Asymmetries

$$A_{||} = \frac{\sigma^{\uparrow \Downarrow} - \sigma^{\downarrow \Downarrow}}{\sigma^{\uparrow \Downarrow} + \sigma^{\downarrow \Downarrow}} = D(A_1 + \eta A_2)$$

$$A_{\perp} = \frac{\sigma^{\uparrow \Rightarrow} - \sigma^{\downarrow \Rightarrow}}{\sigma^{\uparrow \Rightarrow} + \sigma^{\downarrow \Rightarrow}} = d(A_2 - \xi A_1)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2 x^2}{Q^2}$$

Polarized structure functions

 $g_1(x,Q^2) = g_1^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_1^{\text{T3+TMC}}(D_u, D_d) + g_1^{\text{T4}}(H_{p,n})$ $g_2(x,Q^2) = g_2^{\text{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, \dots) + g_2^{\text{T3+TMC}}(D_u, D_d)$

Leading twist structure functions:

$$\begin{split} g_1^{\text{LT}+\text{TMC}}(x,Q^2) = & \frac{x}{\xi} \frac{g_1^{\text{LT}}(\xi)}{(1+4\mu^2 x^2)^{3/2}} + 4\mu^2 x^2 \frac{x+\xi}{\xi(1+4\mu^2 x^2)^2} \int_{\xi}^{1} \frac{dz}{z} g_1^{\text{LT}}(z) \\ & -4\mu^2 x^2 \frac{2-4\mu^2 x^2}{2(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^{1} \frac{dz}{z} \int_{z'}^{1} \frac{dz'}{z'} g_1^{\text{LT}}(z') \\ g_2^{\text{LT}+\text{TMC}}(x,Q^2) = & -\frac{x}{\xi} \frac{g_1^{\text{LT}}(\xi)}{(1+4\mu^2 x^2)^{3/2}} + \frac{x}{\xi} \frac{(1-4\mu^2 x\xi)}{(1+4\mu^2 x^2)^2} \int_{\xi}^{1} \frac{dz}{z} g_1^{\text{LT}}(z) \\ & + \frac{3}{2} \frac{4\mu^2 x^2}{(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^{1} \frac{dz}{z} \int_{z'}^{1} \frac{dz'}{z'} g_1^{\text{LT}}(z') \end{split}$$

Leading twist quark distributions:

$$g_1^{ ext{LT}}(x) = rac{1}{2} \sum_q e_q^2 \left[\Delta C_{qq} \otimes \Delta q(x) + \Delta C_{qg} \otimes \Delta g(x)
ight]$$

Twist-3 structure functions:

$$g_1^{\text{T3+TMC}}(x,Q^2) = 4\mu^2 x^2 \frac{D(\xi)}{(1+4\mu^2 x^2)^{3/2}} - 4\mu^2 x^2 \frac{3}{(1+4\mu^2 x^2)^2} \int_{\xi}^{1} \frac{dz}{z} D(z)$$
$$+ 4\mu^2 x^2 \frac{2-4\mu^2 x^2}{(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^{1} \frac{dz}{z} \int_{z'}^{1} \frac{dz'}{z'} D(z')$$
$$g_2^{\text{T3+TMC}}(x,Q^2) = \frac{D(\xi)}{(1+4\mu^2 x^2)^{3/2}} - \frac{1-8\mu^2 x^2}{(1+4\mu^2 x^2)^2} \int_{\xi}^{1} \frac{dz}{z} D(z)$$
$$- \frac{12\mu^2 x^2}{(1+4\mu^2 x^2)^{5/2}} \int_{\xi}^{1} \frac{dz}{z} \int_{z'}^{1} \frac{dz'}{z'} D(z')$$

Twist-3 quark distributions:

$$D(x,Q^2) = \frac{4}{9}D_u(x,Q^2) + \frac{1}{9}D_d(x,Q^2)$$

Twist-4 structure function (Nucleon d.o.f.):

$$g_1^{\mathrm{T4}(p,n)}(x,Q^2) = H^{(p,n)}(x)/Q^2$$

Nuclear corrections: \rightarrow nuclear smearing functions

$$g_i^A(x,Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y,\gamma) g_j^N(x/y,Q^2)$$

 $\mbox{Addition constraints:} \rightarrow$ weak neutron and hyperon decay constants

•
$$\Delta u^{+(1)} - \Delta d^{+(1)} = F + D = 1.269(3)$$

• $\Delta u^{+(1)} + \Delta d^{+(1)} - 2\Delta s^{+(1)} = 3F - D = 0.586(31)$

Data vs theory: proton

Data vs theory: proton

Data vs theory: proton JLab eg1-dvcs

Data vs theory: proton JLab eg1b

Data vs theory: ³He

Results

moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.022 ± 0.008	0.022 ± 0.003
d_2^n	-0.004 ± 0.004	-0.004 ± 0.004
h_p	-0.000 ± 0.001	0.000 ± 0.002
h_n	0.001 ± 0.002	0.001 ± 0.003

$$\quad \quad \mathbf{\chi}^2/N_{npts} = 1.07$$

- Sign of τ₃ distributions is the same as τ₂
- Negative ∆s⁺
- ∆g compatible with the most recent DSSV fits
- Moment of Δg not constrained