Theory of Quarkonium Production

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June 19-24, 2017

Lecture three/four
The plan for my eight lectures

- **The Goal:**
  
  To understand the theory of heavy quarkonium production, and strong interaction dynamics in terms of QCD

- **The Plan (approximately):**
  
  Inclusive production of a single heavy quarkonium
  
  The November Revolution
  
  Theoretical models and approximations
  
  Surprises and anomalies
  
  QCD factorization at the leading and next-to-leading power
  
  Five lectures

  Heavy quarkonium associate and in medium production
  
  Quarkonium associate production
  
  Quarkonium production in a jet
  
  Quarkonium production in cold/hot medium
  
  Three lectures
Factorization for quarkonium production

- Complexities and Difficulties:
  - Soft gluon interaction could take place between any parts of the scattering at any time – before and/or after the hard collision!
  - Long-range soft gluon interaction could break the universality of non-perturbative distributions – lose the predictive power
  - Factorization for heavy quarkonium production has all the complexities and difficulties that Drell-Yan factorization has, plus more due to nonperturbative formation of final-state quarkonium

- Plan: Drell-Yan factorization first, then quarkonium factorization, ...
Pinch singularity and pinch surface

- “Square” of the diagram with a “unobserved gluon”:
  - “Cut-line” – final-state
  - – in a “cut-diagram” notation

- Pinch singularities “perturbatively” = “surfaces” in k, k’, ...
  - determined by \((p-k)^2=0\), \((p-k-k’)^2=0\), ...
  - “perturbatively”

“Pinched propagators” – “long-lived” partonic states
Drell-Yan factorization in QCD

Why Drell-Yan factorization could be possible?

Soft gluon interaction takes place all the time, but, power suppressed!

– one sentence heuristic argument for believing the factorization

Strength of long range soft gluon interaction:

Leading power & the first subleading power contribution to the cross section could be factorized!
Drell-Yan factorization in QCD

- **Factorization – approximation:**
  - Suppression of quantum interference between short-distance \((1/Q)\) and long-distance \((\text{fm} \sim 1/\Lambda_{\text{QCD}})\) physics
  - Need “long-lived” active parton states linking the two
  - Maintain the universality of PDFs:
    - Long-range soft gluon interaction has to be power suppressed
  - Infrared safe of partonic parts:
    - Cancelation of IR behavior
    - Absorb all CO divergences into PDFs

Leading power: Collins, Soper, Sterman, 1988
1st subleading power: Qiu, Sterman, 1991

\[
\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty
\]

Perturbatively pinched at \(p_a^2 = 0\)

Active parton is effectively on-shell for the hard collision

- on-shell: \(p_a^2, p_b^2 \ll Q^2\)
- collinear: \(p_{aT}^2, p_{bT}^2 \ll Q^2\)
- higher-power: \(p^-_a \ll q^-; \text{ and } p^+_b \ll q^+\)
Leading singular integration regions (pinch surface):

- **Hard:** all lines off-shell by $Q$
- **Collinear:**
  - lines collinear to $A$ and $B$
  - One “physical parton” per hadron
- **Soft:** all components are soft

Collinear gluons:

- Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
- The sum of the effect can be represented by the eikonal lines,

*which are needed to make the PDFs gauge invariant!*
Drell-Yan factorization in QCD

- Trouble with soft gluons:

- Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark’s color and keep it from annihilating with the antiquark of hadron B.

- The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in “too small” region due to the pinch from spectator interaction:

$$k^\pm \sim \frac{M^2}{Q} \ll k_\perp \sim M$$

Need to show that soft-gluon interactions are power suppressed.
Most difficult part of factorization:

- Sum over all final states to remove all poles in one-half plane
  - no more pinch poles
- Deform the $k^\pm$ integration out of the trapped soft region
- Eikonal approximation → soft gluons to eikonal lines
  - gauge links
- Collinear factorization: Unitarity → soft factor = 1

All identified leading integration regions are factorizable!
Factorized Drell-Yan cross section

**TMD factorization ( $q_\perp \ll Q$ ):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) F_{a/A}(x_A, k_{a\perp}) F_{b/B}(x_B, k_{b\perp}) S(k_{s\perp}) + \mathcal{O}(q_\perp/Q)$$

$$x_A = \frac{Q}{\sqrt{s}} e^y \quad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

✧ The soft factor, $S$, is universal, could be absorbed into the definition of TMD parton distribution

✧ CSS resummation in $b_T$-space (conjugate to $k_T$)

**Collinear factorization ( $q_\perp \sim Q$ ):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a f_{a/A}(x_a, \mu) \int dx_b f_{b/B}(x_b, \mu) \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

**Collinear factorization ( $q_\perp \gg Q$ ):**

$$\frac{d\sigma_{AB}}{d^4q} = \sum_{a,b,c} \int dz D_{c\rightarrow \bar{u}}(z, \mu) \int dx_a f_{a/A}(x_a, \mu) \int dx_b f_{b/A}(x_b, \mu)$$

$$\times \frac{d\hat{\sigma}_{ab\rightarrow c}}{d^4q}(x_a, x_b, z, \alpha_s, \mu) + \mathcal{O}(1/q_\perp)$$

Berger, Qiu, Zhang, 2002
Factorized Drell-Yan cross section

- **TMD factorization** (\( q_\perp \ll Q \)):
  
  \[
  \frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_a \, d^2k_b \, d^2k_s \, \delta^2(q_\perp - k_a - k_b - k_s) \mathcal{F}_{a/A}(x_A, k_a) \mathcal{F}_{b/B}(x_B, k_b) S(k_s) + \mathcal{O}(q_\perp/Q) \]
  
  \[ x_A = \frac{Q}{\sqrt{s}} e^y \quad x_B = \frac{Q}{\sqrt{s}} e^{-y} \]

  ✷ **The soft factor**, \( S \), is universal, could be absorbed into the definition of TMD parton distribution

  ✷ **CSS resummation** in \( b_T \)-space (conjugate to \( k_T \))

- **Collinear factorization** (\( q_\perp \sim Q \)):
  
  \[
  \frac{d\sigma_{AB}}{d^4q} = \int dx_a \, f_{a/A}(x_a, \mu) \int dx_b \, f_{b/B}(x_b, \mu) \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q) \]

- **Spin dependence**:

  The factorization arguments are independent of the spin states of the colliding hadrons

  ✽ same formula with polarized PDFs for \( \gamma^*, W/Z, H^0 \)
Single parton fragmentation to a quarkonium

- **Perturbative pinch singularity:**

  \[ \propto \int d^4k \, \mathcal{H}_{gg\to g}(Q, k) \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - i\epsilon} \mathcal{D}_{g\to J/\psi}(k, P) \]

  \[ \approx \int \frac{dz}{z} d^2k_\perp \mathcal{H}_{gg\to g}(Q, k^2 = 0) \int dk^2 \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - i\epsilon} \mathcal{D}_{g\to J/\psi}(k, P) \]

  \[ + \mathcal{O}(\langle k^2 \rangle / Q^2) \]

- **Parton model collinear factorization:**

  \[ k^2, k_\perp^2 \ll Q \quad z = P \cdot n / k \cdot n \]

  \[ \approx \int \frac{dz}{z} \mathcal{H}_{gg\to g}(Q, z = P \cdot n / k \cdot n) \int dk^2 d^2k_\perp \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - i\epsilon} \mathcal{D}_{g\to J/\psi}(k, P) \]

  **Long-lived parton state**

  **Fragmentation function**

  **Short-distance part**

  **Dominated by** \( k^2 \sim 0 \) **region**
Single parton fragmentation to a quarkonium

- Fragmentation contribution at large $P_T$

The rest factorization arguments are the same as Drell-Yan factorization!

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + \mathcal{O}(m_H^2/p_T^2)$$

- Fragmentation function – gluon to a hadron $H$ (e.g., $J/\psi$):

$$D_{H/g}(z, m_c, \mu) \propto \frac{1}{P^+} \text{Tr}_{\text{color}} \int dy^- e^{-ik^+y^-}$$

$$\times \langle 0 | F^{+\lambda}(0) [\Phi^{(g)}_-(0)]^\dagger a_H(P^+) a_H^\dagger(P^+) \Phi^{(g)}_-(y^-) F^{+\lambda}(y^-) | 0 \rangle$$

Effectively, the same factorization formalism for light hadron production!
Single parton fragmentation to a quarkonium

- Fragmentation contribution at large $P_T$

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + O(m_H^2/p_T^2)$$

- Fragmentation function – gluon to a hadron $H$ (e.g., $J/\psi$):

$$D_{H/g}(z, m_c, \mu) \propto \frac{1}{P_+} \text{Tr}_{\text{color}} \int dy^- e^{-ik^+y^-}$$

$$\times \langle 0|F_+^+(0)[\Phi_-^{(g)}(0)]^+ a_H(P^+)a_H^+(P^+)\Phi_-^{(g)}(y^-)F_+^+(y^-)|0\rangle$$

The rest factorization arguments are the same as Drell-Yan factorization!

People have been using this formula, although no one has put together the full and consistent arguments.
Quark-pair fragmentation to a quarkonium

- **Perturbative pinch singularity:**

  - Scattering amplitude:
    \[ \mathcal{M} \propto \int \frac{d^4q}{(2\pi)^4} \text{Tr} \left[ \hat{H}(P,q,Q) \frac{\gamma \cdot (P/2 - q) + m}{(P/2 - q)^2 - m^2 + i\epsilon} \hat{D}(P,q) \frac{\gamma \cdot (P/2 + q) + m}{(P/2 + q)^2 - m^2 + i\epsilon} \right] \]

  - Potential poles:
    \[
    q^- = \left[ q^2_\perp - 2m^2(q^+/P^+) \right]/(P^+ + 2q^+) - i\epsilon \theta(P^+ + 2q^+) \rightarrow q^2_\perp/P^+ - i\epsilon \\
    q^- = -\left[ q^2_\perp + 2m^2(q^+/P^+) \right]/(P^+ - 2q^+) + i\epsilon \theta(P^+ - 2q^+) \rightarrow -q^2_\perp/P^+ + i\epsilon
    \]

  - Condition for pinched poles:
    \[ P^+ \gg q^+(2m^2/q^2_\perp) \geq 2m \quad \text{High } P_T \]
Factorization: fragmentation at next power

- **Heavy quark pair fragmentation:**

  \[ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to [Q\bar{Q}(\kappa)]+X(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes D_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q)} \]

- **Other channels of power corrections:**

  \[ \mathcal{O} \left( \frac{\Lambda_{QCD}^2}{P_T^2} \right) \otimes D_{c\to H} \]

  or

  \[ \mathcal{O} \left( \frac{\Lambda_{QCD}^2}{P_T^2} \right) \otimes \mathcal{D}_{[ff]\to H} \]
Cut vertex = Momentum * Color * Spin:

**Momentum:**
\[
\frac{d^4p_c}{(2\pi)^4} \frac{d^4q_1}{(2\pi)^4} \frac{d^4q_2}{(2\pi)^4} \frac{d^4z}{(2\pi)^4} \delta(z - \frac{P^+}{p_c^+}) \delta(u - \frac{P'^+}{p_c^+}) \delta(v - \frac{P'^+}{p_c^+})
\]
\[
P_Q = \frac{p_c}{2} + q_1, \quad P_Q' = \frac{p_c}{2} - q_1; \quad P_Q'' = \frac{p_c}{2} + q_2, \quad P_Q''' = \frac{p_c}{2} - q_2
\]

**Color:**
\[
C_{ab,cd}^{(1)} = \frac{1}{N_c^2} \delta_{ab} \delta_{cd}
\]
\[
C_{ab,cd}^{(8)} = \frac{4}{N_c^2 - 1} \sum_B (tB)_{ab} (tB)_{cd}
\]

**Spin:**
\[
\mathcal{P}_{ij,lk}^{(v)} = \left[ \frac{1}{4P^+ \gamma \cdot n} \right]_{ij} \left[ \frac{1}{4P^+ \gamma \cdot n} \right]_{lk}
\]
\[
\mathcal{P}_{ij,lk}^{(a)} = \left[ \frac{1}{4P^+ \gamma \cdot n\gamma_5} \right]_{ij} \left[ \frac{1}{4P^+ \gamma \cdot n\gamma_5} \right]_{lk}
\]
\[
\mathcal{P}_{ij,lk}^{(t)} = \frac{1}{2} (-g_\perp^{\alpha\beta}) \left[ \frac{1}{4P^+ \gamma \cdot n\gamma_\alpha} \right]_{ij} \left[ \frac{1}{4P^+ \gamma \cdot n\gamma_\beta} \right]_{lk}
\]

Corresponding projection operators define the hard part.
Factorization for heavy quarkonium production

Factorized cross section:

\[
E \frac{d\sigma_{AB \rightarrow J/\psi}}{d^3 P} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes H^{(2)}_{ab \rightarrow c} \otimes D_{c \rightarrow J/\psi}
\]

\[
+ \sum_{a,b} \phi_{a/A} \otimes \phi_{b/B} \otimes H^{(4)}_{ab \rightarrow Q\bar{Q}} \otimes D^{(4)}_{Q\bar{Q} \rightarrow J/\psi}
\]

\[
+ \sum_{a,b,c} \phi^{(4)}_{a/A} \otimes \phi^{(4)}_{b/B} \otimes H^{(4a)}_{ab \rightarrow c} \otimes D_{c \rightarrow J/\psi}
\]

\[
+ \sum_{a,b,c} \phi^{(4)}_{a/A} \otimes \phi^{(4)}_{b/B} \otimes H^{(4b)}_{ab \rightarrow c} \otimes D_{c \rightarrow J/\psi} + \mathcal{O}\left(\frac{1}{P_T^4}\right)
\]

Expect the first two terms to dominate:

✧ \(H^{(4)}\) are IR safe and free of large logarithms

✧ \(D^{(4)}\) are fragmentation functions of 4-quark operators

New perturbative inputs:

Calculation of \(H^{(4)}\) and evolution of \(D^{(4)}\)
Backup slides