QCD Master Class 2017

18-24 June 2017
Saint-Jacut-de-la-Mer
Europe/Paris timezone

Theory of Quarkonium Production

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Lecture three/four

Theory Center

Jefferson Lab
EXPLORING THE NATURE OF MATTER

The plan for my eight lectures

- **The Goal:**

  To understand the theory of heavy quarkonium production, and strong interaction dynamics in terms of QCD

- **The Plan (approximately):**

  Inclusive production of a single heavy quarkonium

  The November Revolution

  Theoretical models and approximations

  Surprises and anomalies

  QCD factorization at the leading and next-to-leading power

  Five lectures

  Heavy quarkonium associate and in medium production

  Quarkonium associate production

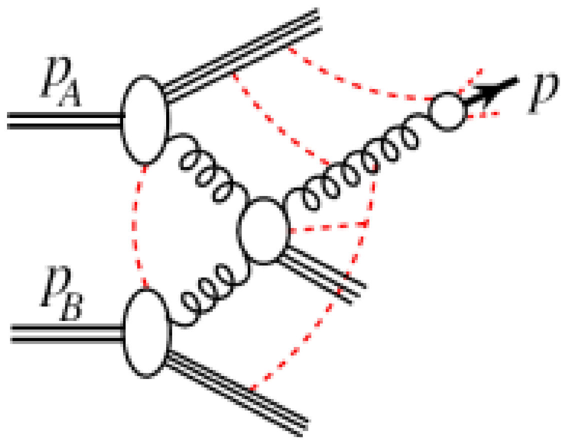
  Quarkonium production in a jet

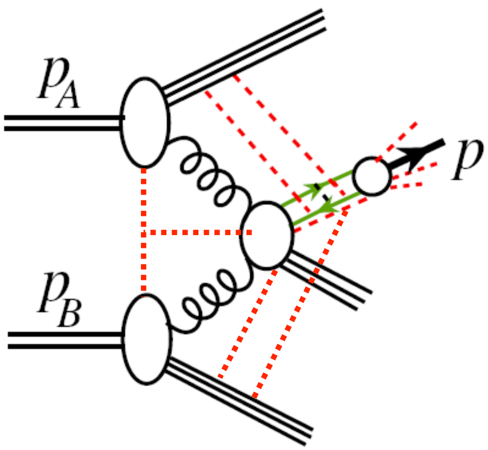
  Quarkonium production in cold/hot medium

  Three lectures

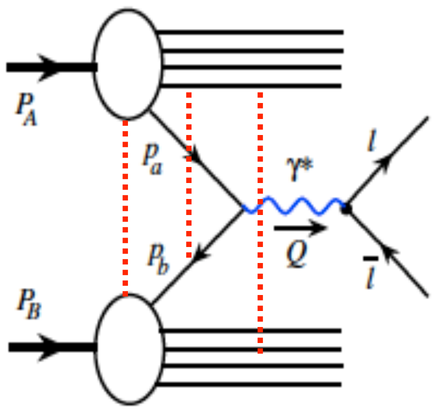
Factorization for quarkonium production

❑ Complexities and Difficulties:





Drell-Yan



✧ Soft gluon interaction could take place between any parts of the scattering at any time – before and/or after the hard collision!

✧ Long-range soft gluon interaction could break the universality of non-perturbative distributions – lose the predictive power

✧ Factorization for heavy quarkonium production has all the complexities and difficulties that Drell-Yan factorization has, plus more due to nonperturbative formation of final-state quarkonium

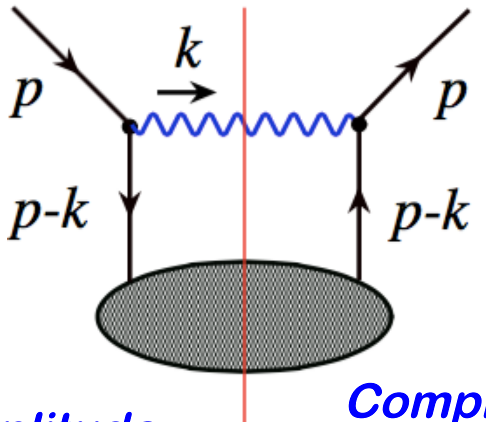
✧ Plan: Drell-Yan factorization first, then quarkonium factorization, …

Pinch singularity and pinch surface

- "Square" of the diagram with a *"unobserved gluon"*:

"Cut-line" – final-state

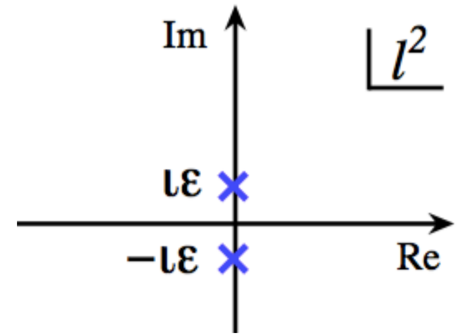
– in a "cut-diagram" notation



$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

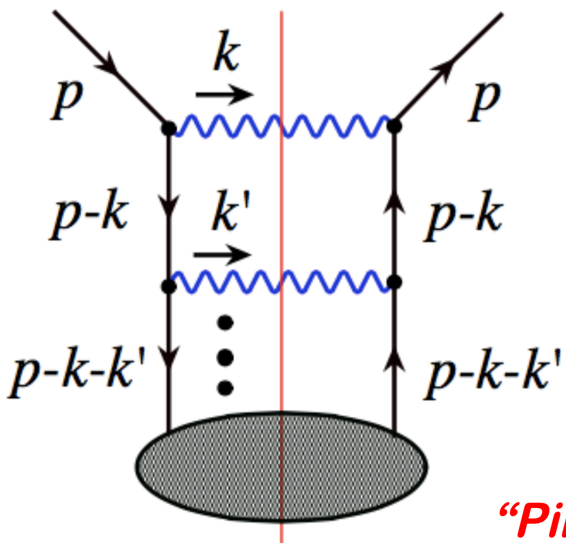
$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Amplitude

Complex conjugate of the Amplitude



Pinch singularities "perturbatively"

Pinch surfaces

= "surfaces" in k, k', …

determined by $(p\text{-}k)^2=0$, $(p\text{-}k\text{-}k')^2=0$, …

"perturbatively"

"Pinched propagators" – "long-lived" partonic states

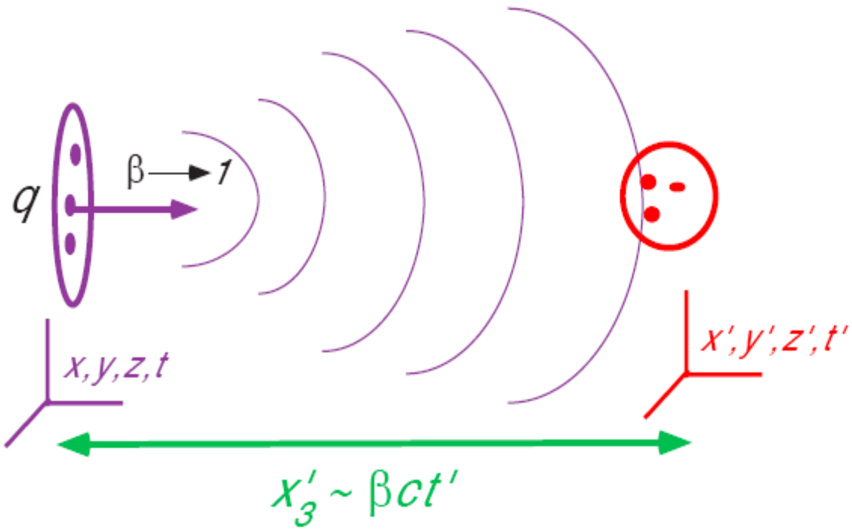
Drell-Yan factorization in QCD

❑ Why Drell-Yan factorization could be possible?

Soft gluon interaction takes place all the time, but, power suppressed!

– one sentence heuristic argument for believing the factorization

❑ Strength of long range soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

Leading power & the first subleading power contribution to the cross section could be factorized!

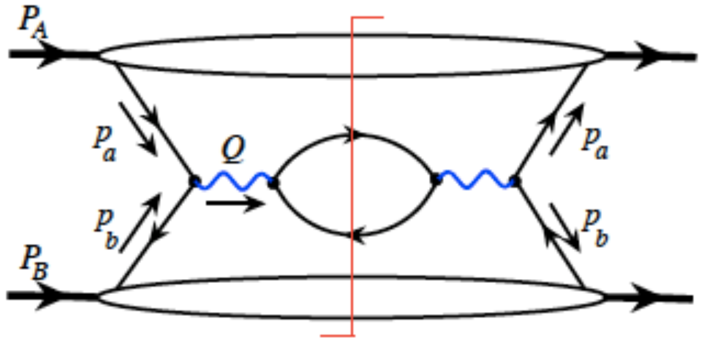
Drell-Yan factorization in QCD

Leading power: Collins, Soper, Sterman, 1988
1st subleading power: Qiu, Sterman, 1991

❑ Factorization – approximation:

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ 1/Λ_{QCD}) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

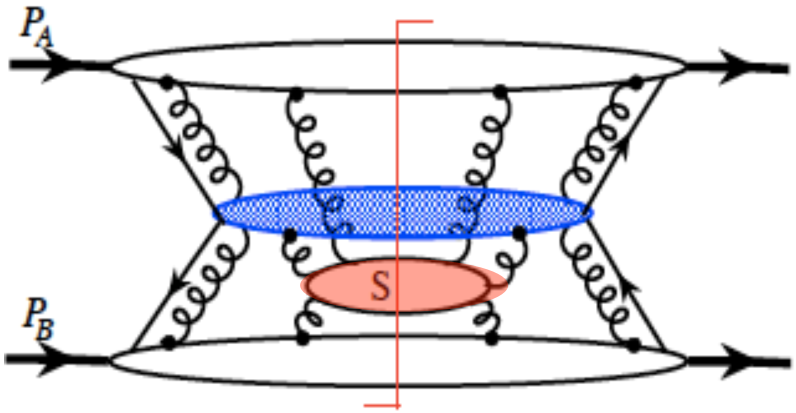
✧ Infrared safe of partonic parts:

Cancelation of IR behavior
Absorb all CO divergences into PDFs

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

Drell-Yan factorization in QCD

❑ **Leading singular integration regions (pinch surface):**



Hard: all lines off-shell by Q

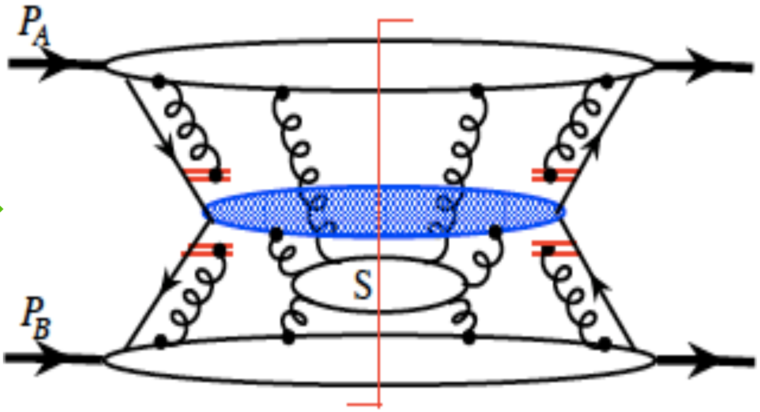
Collinear:

- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

❑ **Collinear gluons:**

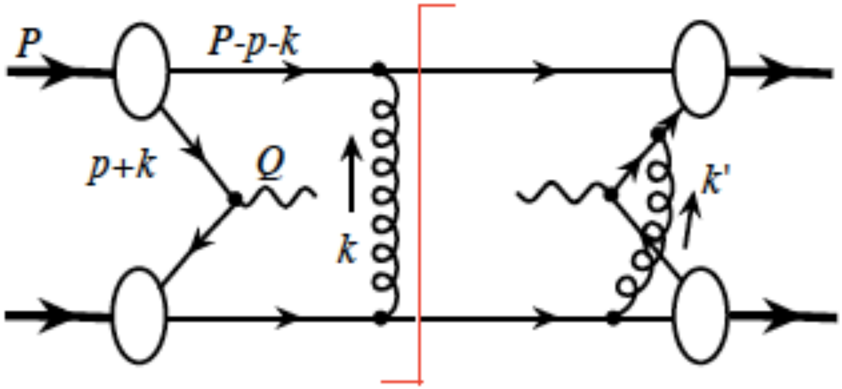
- ✧ Collinear gluons have the polarization vector: $\epsilon^{\mu} \sim k^{\mu}$
- ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan factorization in QCD

❑ Trouble with soft gluons:



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

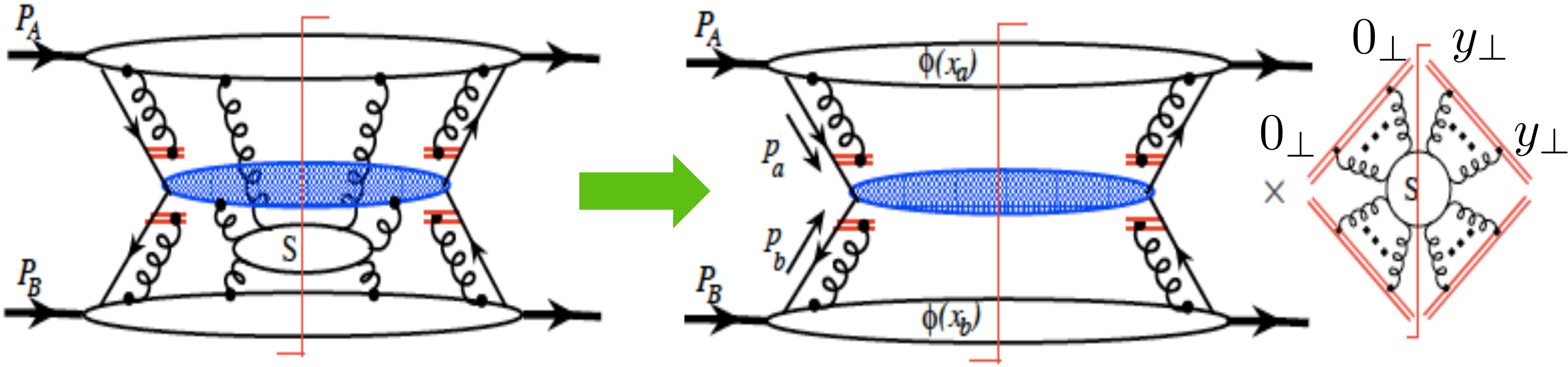
✧ Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B

✧ The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction: $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Drell-Yan factorization in QCD

❑ **Most difficult part of factorization:**



✧ Sum over all final states to remove all poles in one-half plane

– no more pinch poles

✧ Deform the $k^{\pm}$ integration out of the trapped soft region

✧ Eikonal approximation ⟶ soft gluons to eikonal lines

– gauge links

✧ Collinear factorization: Unitarity ⟶ soft factor = 1

All identified leading integration regions are factorizable!

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

  - ✧ The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution
  - ✧ CSS resummation in b_T-space (conjugate to k_T)

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Collinear factorization ($q_\perp \gg Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sum_{a,b,c} \int dz D_{c\to l\bar{l}}(z, \mu) \int dx_a f_{a/A}(x_a, \mu) \int dx_b f_{b/A}(x_b, \mu)$$

$$\times \frac{d\hat{\sigma}_{ab\to c}}{d^4q}(x_a, x_b, z, \alpha_s, \mu) + \mathcal{O}(1/q_\perp)$$

Berger, Qiu, Zhang, 2002

Factorized Drell-Yan cross section

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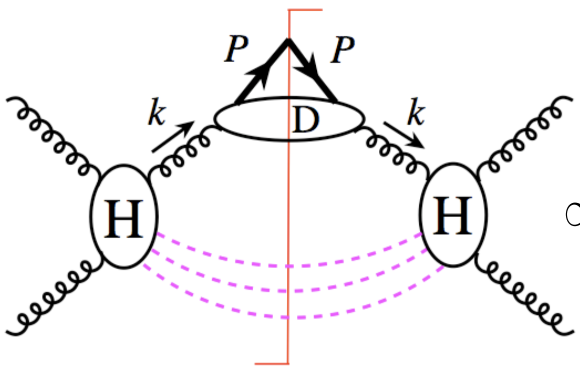
- **Spin dependence:**

The factorization arguments are independent of the spin states of the colliding hadrons

⇒ same formula with polarized PDFs for γ^*,W/Z, H^0...

Single parton fragmentation to a quarkonium

❑ Perturbative pinch singularity:



Dominated by k^2 ~ 0 region

$$\propto \int d^4k\, \mathcal{H}_{gg\to g}(Q,k)\,\frac{1}{k^2+i\epsilon}\,\frac{1}{k^2-i\epsilon}\,\mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

$$\approx \int \frac{dz}{z} d^2k_\perp\, \mathcal{H}_{gg\to g}(Q,k^2=0) \int dk^2 \frac{1}{k^2+i\epsilon}\,\frac{1}{k^2-i\epsilon}\,\mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

$$+\,\mathcal{O}(\langle k^2\rangle/Q^2)$$

Long-lived parton state

❑ Parton model collinear factorization:

$$k^2, k_\perp^2 << Q \qquad z = P\cdot n/k\cdot n$$

Fragmentation function

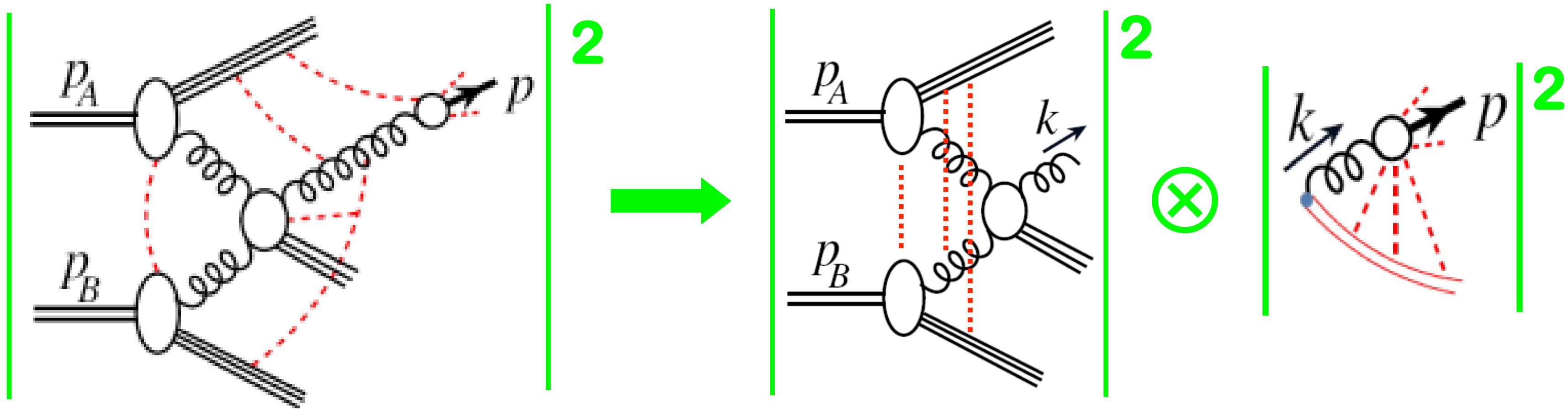
$$\approx \int \frac{dz}{z}\, \mathcal{H}_{gg\to g}(Q, z=P\cdot n/k\cdot n) \int dk^2\, d^2k_\perp\, \frac{1}{k^2+i\epsilon}\,\frac{1}{k^2-i\epsilon}\,\mathcal{D}_{g\to \mathrm{J}/\psi}(k,P)$$

Short-distance part

Single parton fragmentation to a quarkonium

Nayak, Qiu, Stermen, 2005

- **Fragmentation contribution at large P_T**



The rest factorization arguments are the same as Drell-Yan factorization!

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\to i+X}(p_T/z,\mu) \otimes D_{H/i}(z,m_c,\mu) + \mathcal{O}(m_H^2/p_T^2)$$

- **Fragmentation function – gluon to a hadron H (e.g., J/ψ):**

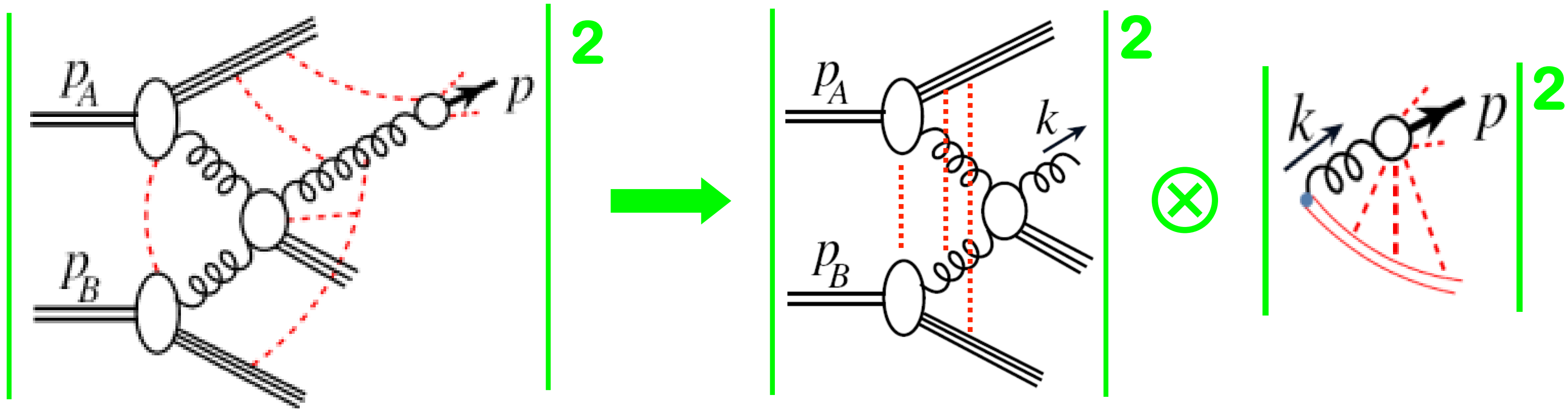
$$D_{H/g}(z,m_c,\mu) \propto \frac{1}{P^+}\mathrm{Tr}_{\mathrm{color}} \int dy^- e^{-ik^+y^-}$$
$$\times \langle 0|F^{+\lambda}(0)[\Phi_-^{(g)}(0)]^\dagger a_H(P^+)a_H^\dagger(P^+)\Phi_-^{(g)}(y^-)F_\lambda{}^+(y^-)|0\rangle$$

Effectively, the same factorization formalism for light hadron production!

Single parton fragmentation to a quarkonium

Nayak, Qiu, Stermen, 2005

- Fragmentation contribution at large P_T



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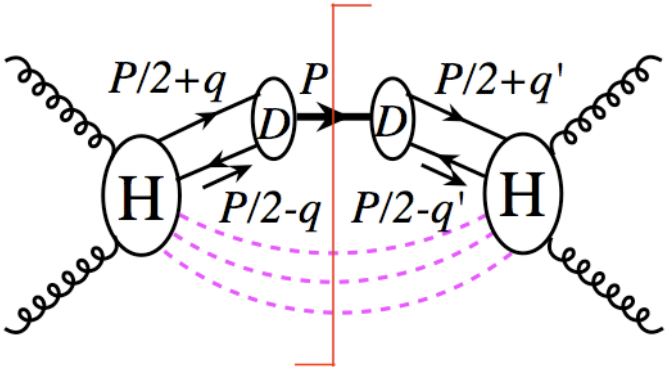
People have been using this formula, although no one has put together the full and consistent arguments

Quark-pair fragmentation to a quarkonium

❑ Perturbative pinch singularity:

Kang, Qiu and Sterman, 2009, 2014



$$P^\mu = (P^+, 4m^2/2P^+, 0_\perp)$$

$$q^\mu = (q^+, q^-, q_\perp)$$

$$q \neq q'$$

$$D_{ij}(P,q) \propto \langle \mathrm{J}/\psi | \psi_i^\dagger(0)\, \chi_j(y) | 0 \rangle$$

✧ Scattering amplitude:

$$\mathcal{M} \propto \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}(P,q,Q)\, \frac{\gamma\cdot(P/2-q)+m}{(P/2-q)^2-m^2+i\epsilon}\, \hat{D}(P,q)\, \frac{\gamma\cdot(P/2+q)+m}{(P/2+q)^2-m^2+i\epsilon}\right]$$

✧ Potential poles:

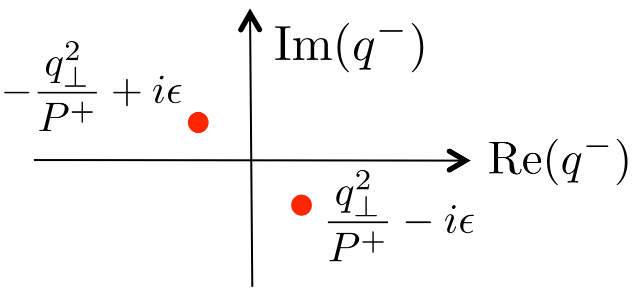
$$q^- = [q_\perp^2 - 2m^2(q^+/P^+)]/(P^+ + 2q^+) - i\epsilon\theta(P^+ + 2q^+) \to q_\perp^2/P^+ - i\epsilon$$

$$q^- = -[q_\perp^2 + 2m^2(q^+/P^+)]/(P^+ - 2q^+) + i\epsilon\theta(P^+ - 2q^+) \to -q_\perp^2/P^+ + i\epsilon$$

✧ Condition for pinched poles:

$$P^+ \gg q^+(2m^2/q_\perp^2) \geq 2m$$

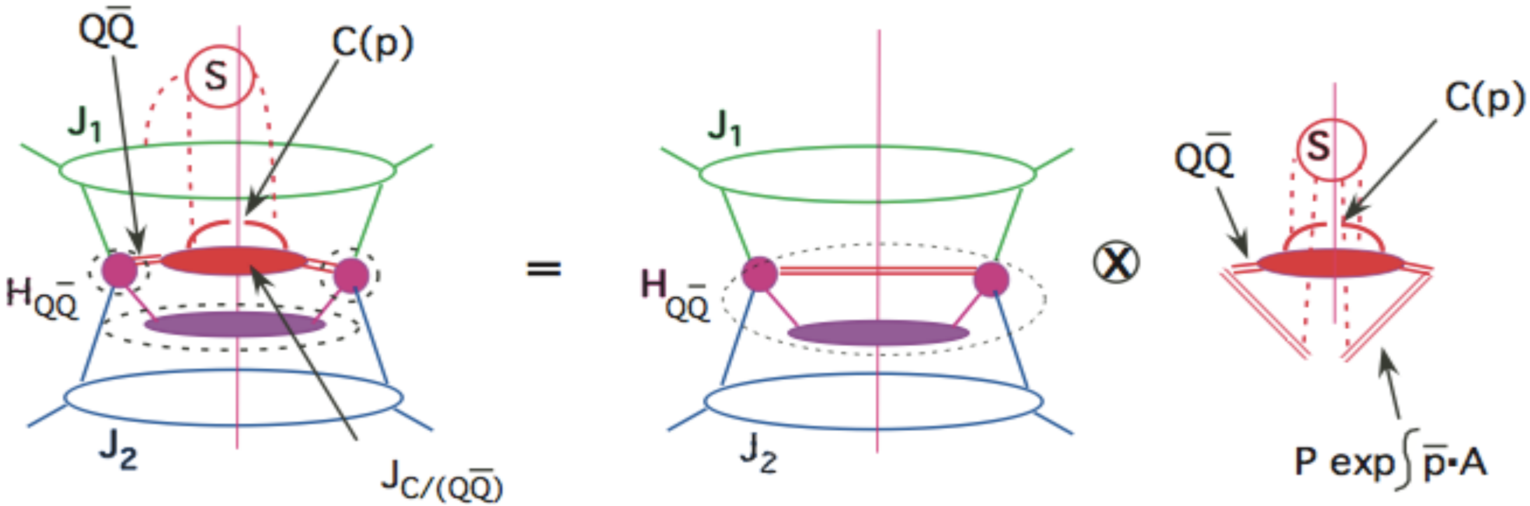
High P_T



Factorization: fragmentation at next power

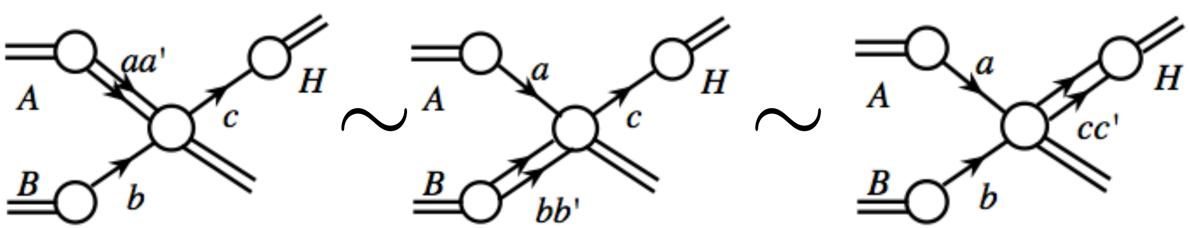
❑ Heavy quark pair fragmentation:

Qiu, Sterman, 1991
Kang, Qiu and Sterman, 2010



$$\sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)$$

❑ Other channels of power corrections:

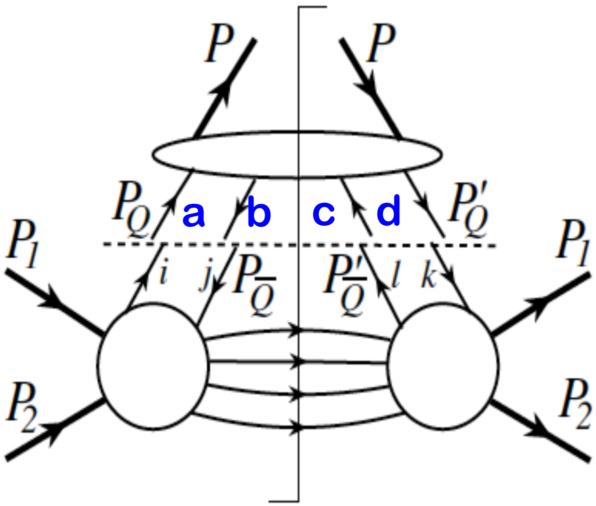


$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

or $$\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$$

Heavy quark pair fragmentation functions

- Cut vertex = Momentum * Color * Spin:



✧ **Momentum:**

$$\frac{d^4p_c}{(2\pi)^4}\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\, z^4\,\delta(z-\frac{P^+}{p_c^+})\delta(u-\frac{P_Q^+}{p_c^+})\delta(v-\frac{P_Q'^+}{p_c^+})$$

$$P_Q=\frac{p_c}{2}+q_1,\, P_{\bar{Q}}=\frac{p_c}{2}-q_1;\, P'_Q=\frac{p_c}{2}+q_2,\, P'_{\bar{Q}}=\frac{p_c}{2}-q_2$$

✧ **Color:**

$$\mathcal{C}^{(1)}_{ab,cd}=\frac{1}{N_c^2}\delta_{ab}\,\delta_{cd}$$

$$\mathcal{C}^{(8)}_{ab,cd}=\frac{4}{N_c^2-1}\sum_B (t^B)_{ab}\,(t^B)_{cd}$$

✧ **Spin:**

$$\mathcal{P}^{(v)}_{ij,lk}=\left[\frac{1}{4P^+}\gamma\cdot n\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\right]_{lk}$$ **Vector**

$$\mathcal{P}^{(a)}_{ij,lk}=\left[\frac{1}{4P^+}\gamma\cdot n\gamma_5\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\gamma_5\right]_{lk}$$ **Axial vector**

$$\mathcal{P}^{(t)}_{ij,lk}=\frac{1}{2}(-g_\perp^{\alpha\beta})\left[\frac{1}{4P^+}\gamma\cdot n\gamma_\alpha\right]_{ij}\left[\frac{1}{4P^+}\gamma\cdot n\gamma_\beta\right]_{lk}$$ **Tensor**

Corresponding projection operators define the hard part

Factorization for heavy quarkonium production

Kang, Qiu and Sterman, 2009

❑ Factorized cross section:

$$
\begin{aligned}
E\frac{d\sigma_{AB\to \mathrm{J}/\psi}}{d^3P} &= \sum_{a,b,c} \phi_{a/A}\otimes\phi_{b/B}\otimes H^{(2)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} \\
&+ \sum_{a,b} \phi_{a/A}\otimes\phi_{b/B}\otimes H^{(4)}_{ab\to Q\bar{Q}}\otimes \mathcal{D}^{(4)}_{Q\bar{Q}\to \mathrm{J}/\psi} \\
&+ \sum_{a,b,c} \phi^{(4)}_{a/A}\otimes\phi_{b/B}\otimes H^{(4a)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} \\
&+ \sum_{a,b,c} \phi_{a/A}\otimes\phi^{(4)}_{b/B}\otimes H^{(4b)}_{ab\to c}\otimes D_{c\to \mathrm{J}/\psi} + \mathcal{O}\left(\frac{1}{P_T^4}\right)
\end{aligned}
$$

❑ Expect the first two terms to dominate:

- ✧ $H^{(4)}$ are IR safe and free of large logarithms
- ✧ $D^{(4)}$ are fragmentation functions of 4-quark operators

Qiu, 1990

❑ New perturbative inputs:

Calculation of $H^{(4)}$ and evolution of $D^{(4)}$

Backup slides