### QCD Master Class 2017

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# Theory of Quarkonium Production

Jianwei Qiu Theory Center, Jefferson Lab June 19-24, 2017

Lecture seven/eight





### The plan for my eight lectures

□ The Goal:

**To understand** the theory of heavy quarkonium production, and strong interaction dynamics in terms of QCD

□ The Plan (approximately):

Inclusive production of a single heavy quarkonium The November Revolution Theoretical models and approximations Surprises and anomalies QCD factorization at the leading and next-to-leading power Five lectures Heavy quarkonium associate and in medium production Quarkonium associate production Quarkonium production in a jet

Quarkonium production in cold/hot medium Three lectures

### NLO theory fits – Butenschoen et al.



### NLO theory fits – Gong et al.

![](_page_3_Figure_1.jpeg)

### NLO theory fits – Chao et al.

![](_page_4_Figure_1.jpeg)

Kang et al. 1702.03287

#### □ Jet fragmentation function:

 $F^{J/\psi}(z_h, p_T) = \frac{d\sigma^{J/\psi}}{dp_T d\eta dz_h} / \frac{d\sigma}{dp_T d\eta}$ Jet cross section with a fully reconstructed J/ $\Psi$  within it  $z_h \equiv p_{J/\psi}^+ / p_{\text{iet}}^+$ 

Production and factorization:

$$\frac{d\sigma^{J/\psi}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_a \otimes f_b \otimes H^c_{ab} \otimes \mathcal{G}_c^{J/\psi}.$$

♦ Semi-inclusive fragmenting jet function:

$$\mathcal{G}_{i}^{J/\psi}(z, z_{h}, p_{\text{jet}}^{+}R, \mu) = \sum_{j} \int_{z_{h}}^{1} \frac{dz'_{h}}{z'_{h}} \mathcal{J}_{ij}(z, z_{h}/z'_{h}, p_{\text{jet}}^{+}R, \mu) \\ \times D_{j}^{J/\psi}(z'_{h}, \mu) + \mathcal{O}(m_{J/\psi}^{2}/(p_{\text{jet}}^{+}R)^{2})$$

♦ Fragmentation function:

$$D_{i \to J/\psi}(z'_h, \mu_0) = \sum_n \hat{d}_{i \to [Q\bar{Q}(n)]}(z'_h, \mu_0) \langle \mathcal{O}^{J/\psi}_{[Q\bar{Q}(n)]} \rangle$$

Kang et al. 1702.03287

#### Jet fragmentation function:

Jet cross section with a fully reconstructed J/ $\Psi$  within it

$$F^{J/\psi}(z_h, p_T) = \frac{d\sigma^{J/\psi}}{dp_T d\eta dz_h} / \frac{d\sigma}{dp_T d\eta}$$
  
Continue with a fully and the second seco

#### **Polarization:**

 $\frac{d\sigma^{J/\psi(\to \ell^+ \ell^-)}}{d\cos\theta} \propto 1 + \lambda_F \cos^2\theta.$ 

$$\lambda_F(z_h, p_T) = \frac{F_T^{J/\psi} - F_L^{J/\psi}}{F_T^{J/\psi} + F_L^{J/\psi}}$$

 $\diamond$  Physical constraint:

 $|\lambda_F(z_h, p_T)| \leq 1$  Since  $F_T^{J/\psi}$  are positive

♦ Expansion in terms of NRQCD LDMEs:

$$F_{T,L}^{J/\psi}(z_h, p_T) \sum_{[Q\bar{Q}(n)]} \hat{F}_{T,L}^{[Q\bar{Q}(n)]}(z_h, p_T, \Lambda) \langle 0 | \mathcal{O}_{[Q\bar{Q}(n)]}^{J/\psi}(\Lambda) | 0 \rangle$$

Linear combination of LDMEs, like p<sub>T</sub> spectrum in NRQCD

Kang et al. 1702.03287

#### □ Fitting values of LDMEs:

	$\langle \mathcal{O}(^{3}S_{1}^{[1]})\rangle$	$\langle \mathcal{O}({}^{1}S_{0}^{[8]})\rangle$	$\langle \mathcal{O}({}^{3}S_{1}^{[8]})\rangle$	$\langle \mathcal{O}({}^{3}P_{0}^{[8]})\rangle$
	$GeV^3$	$10^{-2}~{\rm GeV^3}$	$10^{-2}~{\rm GeV^3}$	$10^{-2}~{ m GeV^5}$
Bodwin	0 <sup>a</sup>	9.9	1.1	1.1
Butenschoen	1.32	3.04	0.16	-0.91
Chao	1.16	8.9	0.30	1.26
Gong	1.16	9.7	-0.46	-2.14

TABLE I.  $J/\psi$  NRQCD LDMEs from four different groups.

All fit the J/ $\Psi$ 's p<sub>T</sub> distribution

#### **Predictions:**

Kang et al. 1702.03287

![](_page_8_Figure_3.jpeg)

Factor of 10 difference in the predicted rate?!

□ Predictions:

Kang et al. 1702.03287

![](_page_9_Figure_2.jpeg)

Bad LDMEs values: Negative "cross section"?! LHCb has been measuring the J/ $\Psi$  fragmenting function

# **Exclusive production of heavy quarkonium**

![](_page_10_Figure_1.jpeg)

# **Double cc production in e<sup>+</sup>e<sup>-</sup>**

### □ Inclusive production:

$$\sigma(e^+e^- \rightarrow J/\psi c\bar{c})$$
  
Belle:  $(0.87^{+0.21}_{-0.19} \pm 0.17)$  pb  
NRQCD: : 0.07 pb

Kiselev, et al 1994, Cho, Leibovich, 1996 Yuan, Qiao, Chao, 1997

**Ratio to light flavors:** 

$$\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X)$$
  
Belle:  $0.59^{+0.15}_{-0.13} \pm 0.12$ 

Message:

Production rate of  $e^+e^- \rightarrow J/\psi c\overline{c}$  is larger than all these channels:  $e^+e^- \rightarrow J/\psi gg$ ,  $e^+e^- \rightarrow J/\psi q\overline{q}$ , ... combined ?

# **Associated production at B-factory**

![](_page_12_Figure_1.jpeg)

Production rate of a singlet charm quark pair is dominated by the phase space where  $s_3 = (P_1 + P_2 + P_3)^2$  or  $s_4 = (P_1 + P_2 + P_4)^2$  near its minimum

NRQCD formalism does not apply when there are more than one heavy quark velocity involved

Color transfer enhances associated heavy quarkonium production

A heavy quark as a color source to enhance the transition rate for an octet pair to become a singlet pair

Nayak, Qiu, Sterman, PRL 2007

### Soft gluon enhancement – color transfer

#### □ Soft gluons between heavy quarks:

Active pair: P<sub>1</sub>, P<sub>2</sub>; spectators: P<sub>3</sub>, P<sub>4</sub>

![](_page_13_Figure_3.jpeg)

□ There are three heavy quark velocities:  $\beta_{ij} \equiv$ NRQCD approach is not well defined in this region

#### □ Soft gluon between a heavy quark pair:

$$-i g^{2} \int \frac{d^{D}k}{(2\pi)^{D}} \frac{4P_{i} \cdot P_{j}}{[2P_{i} \cdot k + k^{2} + i\epsilon][-2P_{j} \cdot k + k^{2} + i\epsilon][k^{2} + i\epsilon]}$$
$$= \frac{\alpha_{s}}{2\pi} \left[ -\frac{1}{2\varepsilon} \left( \frac{1}{\beta_{ij}} + \beta_{ij} \right) (2\beta_{ij} - i\pi) + \dots \right] \implies i \frac{1}{\varepsilon} \frac{\alpha_{s}}{\beta_{ij}}$$

## Associated production is enhanced

![](_page_14_Figure_1.jpeg)

All other two-loop diagrams give a single pole !

Nayak, Qiu, Sterman, 2007

# **Numerical enhancement from NNLO**

#### □ LO hard parts with color factor:

![](_page_15_Figure_2.jpeg)

Two terms are equally important if  $\beta_s$ : 0.3

Nayak, Qiu, Sterman, 2007

Same feature for heavy quark fragmentation

Kang, et al. 2007

# Heavy quarkonium – puzzles

![](_page_16_Figure_1.jpeg)

# Heavy quarkonium – puzzles

![](_page_17_Figure_1.jpeg)

# Melting a quarkonium in QGP – deconfinement

![](_page_18_Figure_1.jpeg)

![](_page_19_Figure_1.jpeg)

Need a full time-dependent, dynamical model of QGP with heavy quarks!

Many model approaches are available, ..., But, not to be discussed here!

![](_page_20_Figure_1.jpeg)

♦ NO QGP (m<sub>Q</sub> >> T)! → Cold nuclear effect for the "production"
 ♦ Nuclei as potential filters of production mechanisms
 ♦ Hard probe (m<sub>Q</sub> >> 1/fm) → quark-gluon structure of nucleus!

Nucleus is not a simple superposition of nucleons!

![](_page_21_Figure_1.jpeg)

![](_page_22_Figure_1.jpeg)

![](_page_23_Figure_1.jpeg)

# **Breaking of factorization in hadronic collisions**

#### □ A-enhanced power corrections, A<sup>1/3</sup>/Q<sup>2</sup>, may be factorizable:

![](_page_24_Figure_2.jpeg)

Total x-section: Factorization argument similar to DIS Collinear power expansion – single scale

# **Multiple scattering in cold nuclear matter**

![](_page_25_Figure_1.jpeg)

OK for pA, but, far off for AA – J/ $\psi$  melting in QGP (MS 1986)?

# **Production with multiple scattering**

**Brodsky and Mueller, PLB 1988** 

□ Production at low  $P_T$  (→0) in p(d)+A collisions:

![](_page_26_Figure_3.jpeg)

# **Production with multiple scattering**

![](_page_27_Figure_1.jpeg)

Non-perturbative formation of  $J/\Psi$  is far outside of nucleus

**Brodsky and Mueller, PLB 1988** 

 $\diamond$  Multiple scattering with incoming parton & heavy quarks, not J/  $\Psi$ 

 $P_A$ 

# **Production with multiple scattering**

### Forward production in p(d)+A collisions:

Q

Q

 $\diamond$  Multiple scattering with incoming parton & heavy quarks, not J/  $\Psi$ 

- Induced gluon radiation energy loss suppression at large y
- ♦ Modified P<sub>T</sub> spectrum transverse momentum broadening
- De-coherence of the pair different QQ state to hadronize lower rate
  - Soft multiple scattering "random walk"

Momentum imbalance – larger invariant mass

Match to the tail of wave function - ``suppression"

**Brodsky and Mueller, PLB 1988** 

### **Suppression in total production rate**

![](_page_29_Figure_1.jpeg)

# A-dependence in rapidity y $(x_F)$ in p+A

#### □ Picture + assumptions:

Arleo, Peigne, 2012 Arleo, Kolevatov, Peigne, 2014

![](_page_30_Figure_3.jpeg)

- Color neutralization nappens on long time scales:  $t_{
  m octet} \gg t_{
  m hard}$
- Medium rescatterings do not resolve the octet cc pair
- Hadronization happens outside of the nucleus:  $t_{\psi} \gtrsim L$
- cc pair produced by gluon fusion

#### □ Model energy loss:

 $\frac{1}{A} \frac{d\sigma_{pA}}{dE}(E,\sqrt{s}) = \int_0^{\varepsilon_{\max}} d\varepsilon \,\mathcal{P}(\varepsilon,E) \,\frac{d\sigma_{pp}}{dE}(E+\varepsilon,\sqrt{s}) \qquad \hat{q}(x) \sim \hat{q}_0 \left(\frac{10^{-2}}{x}\right)^{0.3}$  $\mathcal{P}(\varepsilon,E): \text{ Quenching weight ~ scaling function of } \sqrt{\hat{q}L}/M_\perp \times E$ 

# A-dependence in rapidity $y(x_F)$ in p+A

![](_page_31_Figure_1.jpeg)

### A-dependence in $P_T$ in p+A

Arleo, Peigne, 2012 Arleo, Kolevatov, Peigne, 2014

$$\frac{1}{A} \frac{d\sigma_{\rm pA}^{\psi}}{dE \ d^2 \vec{p}_{\perp}} = \int_{\varepsilon} \int_{\varphi} \mathcal{P}(\varepsilon, E) \frac{d\sigma_{\rm pp}^{\psi}}{dE \ d^2 \vec{p}_{\perp}} \left(E + \varepsilon, \vec{p}_{\perp} - \Delta \vec{p}_{\perp}\right)$$

□ Model:

□ Nuclear A-dependence:  $R_{pA}^{\psi}(y, p_{\perp}) \simeq R_{pA}^{\text{loss}}(y, p_{\perp}) \cdot R_{pA}^{\text{broad}}(p_{\perp})$ 

![](_page_32_Figure_4.jpeg)

# **Quarkonium pT distribution**

**Quarkonium production is dominated by low**  $p_T$  region

### $\Box$ Low $p_T$ distribution at collider energies:

determined mainly by gluon shower of incoming partons

- initial-state effect Qiu, Zhang, PRL, 2001

 $\Box$  Final-state interactions suppress the formation of J/ $\psi$ :

Also modify the  $p_T$  spectrum – move low pT to high pT – broadening

- Final-state effect

#### Broadening:

- ♦ Sensitive to the medium properties
- ♦ Perturbatively calculable

 $\Box R_{pA}$  at low  $q_T$ :

$$\langle (q_T^2)^n \rangle = \frac{\int dq_T^2 (q_T^2)^n \, d\sigma/dq_T^2}{\int dq_T^2 \, d\sigma/dq_T^2}$$

$$\Delta \langle q_T^2 \rangle = \langle q_T^2 \rangle_{AB} - \langle q_T^2 \rangle_{NN}$$

#### Guo, Qiu, Zhang, PRL, PRD 2002

$$R(A,q_T) = \frac{1}{A} \frac{d\sigma^{hA}}{dQ^2 dq_T^2} \bigg/ \frac{d\sigma^{hN}}{dQ^2 dq_T^2} = A^{\alpha(A,q_T)-1} \approx 1 + \frac{\Delta \langle q_T^2 \rangle}{A^{1/3} \langle q_T^2 \rangle_{hN}} \left[ -1 + \frac{q_T^2}{\langle q_T^2 \rangle_{hN}} \right]$$

### Quarkonium $P_T$ -broadening in p(d)+A

**Broadening:** 

Kang, Qiu, PRD77(2008)

$$\begin{split} \Delta \langle q_T^2 \rangle_{\mathrm{J/\psi}}^{(I)} &= C_A \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} (A^{1/3} - 1) \lambda^2 \right) \approx \Delta \langle q_T^2 \rangle_{\mathrm{J/\psi}}^{(F)} & \begin{array}{c} \text{Calculated in both} \\ & \text{NRQCD and CEM} \\ \lambda^2 &= \kappa \, \ln(Q) \, x^{-\delta} \propto \hat{q}, \quad \kappa = 3.51 \times 10^{-3} \, 1/\mathrm{GeV}^2, \quad \delta = 1.71 \times 10^{-1} \, 1/\mathrm{GeV}^2. \end{split}$$

![](_page_34_Figure_3.jpeg)

### **Final-state multiple scattering - CEM**

#### **Double scattering –** $A^{1/3}$ dependence:

Kang, Qiu, PRD77(2008)

$$\Delta \langle q_T^2 \rangle_{\mathrm{HQ}}^{\mathrm{CEM}} \approx \int dq_T^2 q_T^2 \int_{4m_Q^2}^{4M_Q^2} dQ^2 \frac{d\sigma_{hA \to Q\bar{Q}}^D}{dQ^2 dq_T^2} \Big/ \int_{4m_Q^2}^{4M_Q^2} dQ^2 \frac{d\sigma_{hA \to Q\bar{Q}}}{dQ^2}$$

□ Multiparton correlation:

$$T_{g/A}^{(F)}(x) = T_{g/A}^{(I)}(x) = \int \frac{dy^{-}}{2\pi} e^{ixp^{+}y^{-}} \int \frac{dy_{1}^{-}dy_{2}^{-}}{2\pi} \theta(y^{-} - y_{1}^{-})\theta(-y_{2}^{-})$$

$$\times \frac{1}{xp^{+}} \langle p_{A} | F_{\alpha}^{+}(y_{2}^{-})F^{\sigma+}(0)F^{+}{}_{\sigma}(y^{-})F^{+\alpha}(y_{1}^{-}) | p_{A} \rangle$$

$$= \lambda^{2} A^{4/3} \phi_{g/A}(x)$$

□ Broadening – twice of initial-state effect:

$$\begin{split} \Delta \langle q_T^2 \rangle_{\rm HQ}^{\rm CEM} &= \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \,\lambda^2 A^{1/3} \right) \frac{(C_F + C_A) \sigma_{q\bar{q}} + 2C_A \sigma_{gg}}{\sigma_{q\bar{q}} + \sigma_{gg}} \\ &\approx 2C_A \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \,\lambda^2 A^{1/3} \right) & \text{if gluon-gluon dominates,} \\ &\text{and if } \mathbf{r}_{\rm F} > {\rm R}_{\rm A} \end{split}$$

### **Final-state multiple scattering - NRQCD**

#### **Cross section:**

Kang, Qiu, PRD77(2008)

$$\sigma_{hA\to H}^{\text{NRQCD}} = A \sum_{a,b} \int dx' \phi_{a/h}(x') \int dx \phi_{b/A}(x) \left[ \sum_{n} H_{ab\to Q\bar{Q}[n]} \langle \mathcal{O}^{H}(n) \rangle \right]$$

**Broadening:** 

$$\Delta \langle q_T^2 \rangle_{\rm HQ}^{\rm NRQCD} = \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right) \frac{(C_F + C_A)\sigma_{q\bar{q}}^{(0)} + 2C_A \sigma_{gg}^{(0)} + \sigma_{q\bar{q}}^{(1)}}{\sigma_{q\bar{q}}^{(0)} + \sigma_{gg}^{(0)}}$$

Hard parts:

$$\hat{\sigma}_{q\bar{q}}^{(0)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{16}{27} \delta(\hat{s} - M^2) \langle \mathcal{O}^H({}^3S_1^{(8)}) \rangle \qquad \text{Only color octet} \\ \hat{\sigma}_{q\bar{q}}^{(1)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{80}{27} \delta(\hat{s} - M^2) \langle \mathcal{O}^H({}^3P_0^{(8)}) \rangle \qquad \text{Channel contributes} \\ \hat{\sigma}_{gg}^{(0)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{5}{12} \delta(\hat{s} - M^2) \Big[ \langle \mathcal{O}^H({}^1S_0^{(8)}) \rangle + \frac{7}{m_Q^2} \langle \mathcal{O}^H({}^3P_0^{(8)}) \rangle \Big]$$

#### **Leading features:**

$$\Delta \langle q_T^2 \rangle_{\rm HQ}^{\rm NRQCD} \approx \Delta \langle q_T^2 \rangle_{\rm HQ}^{\rm CEM} \approx (2C_A/C_F) \Delta \langle q_T^2 \rangle_{\rm DY}$$

# Quarkonium $P_T$ -distribution in p(d)+A

□ Nuclear modification – low pT region:

$$\frac{d\sigma_{AB}}{dyd^2p_T} \approx \frac{d\sigma_{AB}}{dy} \left[ \frac{1}{\pi(\langle p_T^2 \rangle_{NN} + \Delta \langle p_T^2 \rangle_{AB})} e^{-p_T^2/(\langle p_T^2 \rangle_{NN} + \Delta \langle p_T^2 \rangle_{AB})} \right]$$

![](_page_37_Figure_3.jpeg)

**ALICE data** 

# Forward quarkonium production in p(d)+A

![](_page_38_Figure_1.jpeg)

# Summary

It has been over 40 years since the discovery of J/ $\Psi$ 

 $\Box$  When  $p_T >> m_o$  at collider energies, earlier model calculations for the production of heavy quarkonia are not perturbatively stable

LO in  $\alpha_s$ -expansion may not be the LP term in  $1/p_{T}$ -expansion

QCD factorization works for both LP and NLP ( $\alpha_s$  for each power)

- $\diamond$  LP dominates:  ${}^{3}S_{1}^{[8]}$  and  ${}^{3}P_{J}^{[8]}$  channels
- $\stackrel{\circ}{\diamond} \text{NLP dominates: } \stackrel{\circ}{1}S_{0}^{[8]} \text{ and } \stackrel{\circ}{3}S_{1}^{[1]} \text{ channels} \\ \stackrel{\circ}{\diamond} \text{ From current data: } \stackrel{\circ}{3}P_{J}^{[8]} \text{ likely to cancel } \stackrel{\circ}{3}S_{1}^{[8]} \\ \text{ the production dominated by } ^{1}S_{0}^{[8]} \\ \end{array}$

There are still a lot of unanswered questions related to quarkonium! □ Nuclear medium could be a good "filter" or a fermi-scale detector

for studying how a heavy quarkonium is emerged from a pair of heavy quarks **Thank you!** 

# **Backup slides**