北京大学高能物理研究中心
Center for High Energy Physics, PKU

[高 瞻远瞩]
站得更高，才能眼界更高
居高临下
以通关全局的视野，审视一切

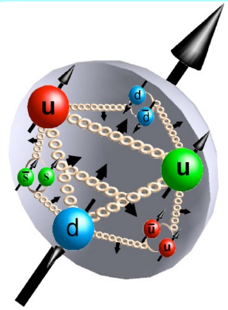
Hadron Structure
from
Lattice QCD Calculations

Jianwei Qiu
Theory Center, Jefferson Lab

Based on work done with
T. Ishikawa, Y.-Q. Ma, K. Orginos, S. Yoshida, …
and work by many others, …

TD Lee Institute and Center for High Energy Physics Joint Workshop on Parton Distributions in Modern Era
14-16 July, 2017, Peking University, China

Hadron structure in QCD



❑ What do we need to know for the structure?

- ✧ **In theory:** $\langle P,S|\mathcal{O}(\overline{\psi},\psi,A^\mu)|P,S\rangle$ – Hadronic matrix elements

  with all possible operators: $\mathcal{O}(\overline{\psi},\psi,A^\mu)$

- ✧ **In fact:** *None of these matrix elements is a direct physical observable in QCD – color confinement!*

- ✧ **In practice:** Accessible hadron structure
  = hadron matrix elements of quarks and gluons, which

  1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
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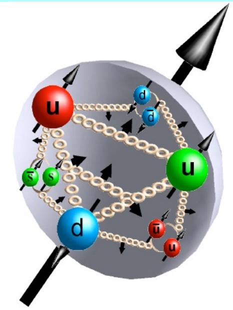

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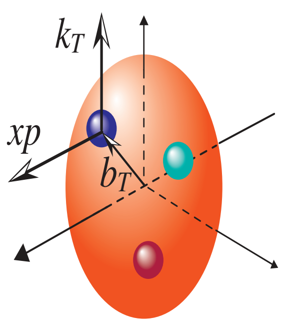
- **Single-parton structure "seen" by a short-distance probe:**
  - ✧ **5D structure:**

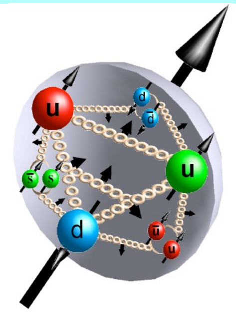


1) $\int d^2 b_T \Rightarrow f(x, k_T, \mu)$ – **TMDs:** ***2D confined motion!***

2) $\int d^2 k_T \Rightarrow F(x, b_T, \mu)$ – **GPDs:** ***2D spatial imaging!***

3) $\int d^2 k_T d^2 b_T \Rightarrow f(x, \mu)$ – **PDFs:** ***Number density!***

Hadron structure in QCD



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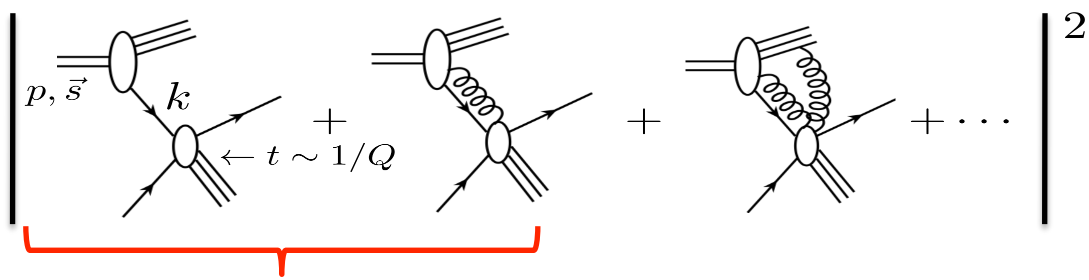
❑ Multi-parton correlations:

$\sigma(Q,\vec{s}) \propto$



$\left(\frac{\langle k_\perp\rangle}{Q}\right)^n$ – Expansion

Quantum interference → 3-parton matrix element – not a probability!

QCD factorization – Approximation

- Cross section with identified hadron(s) is **NON-Perturbative!**

QCD factorization – Approximation

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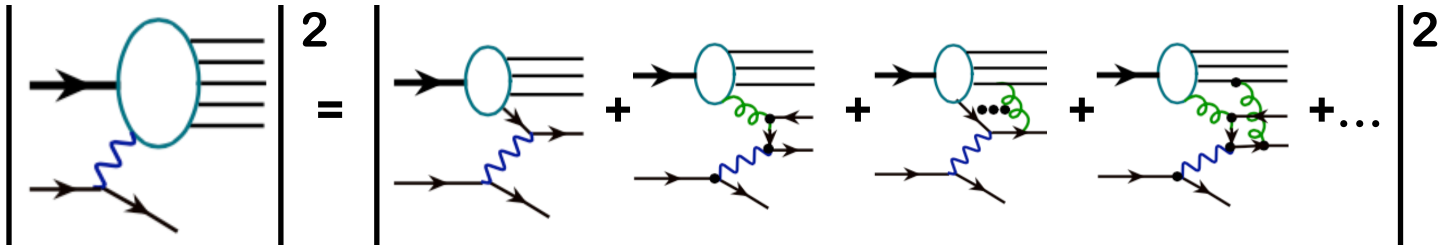
$$\sigma_{\rm DIS}(x, Q^2) =$$



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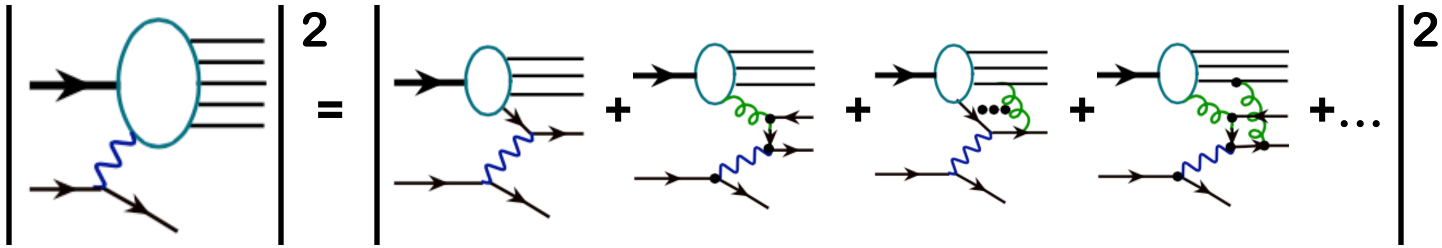

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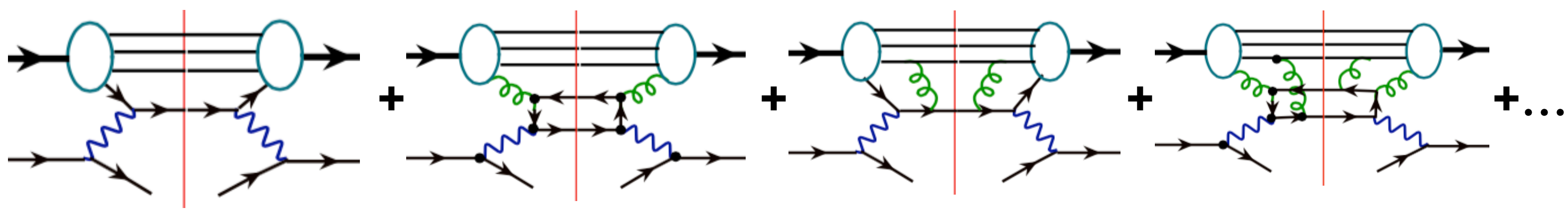

$$= c_q \otimes q(x, Q^2) + c_g \otimes g(x, Q^2) + c_{qg} \otimes T_{qg}(\{x\}, Q^2) + c_{gg} \otimes T_{gg}(\{x\}, Q^2)$$
$$+\mathcal{O}\left(\langle k_T^n \rangle / Q^n, \langle F^{2n} \rangle / Q^n\right) + \ldots$$

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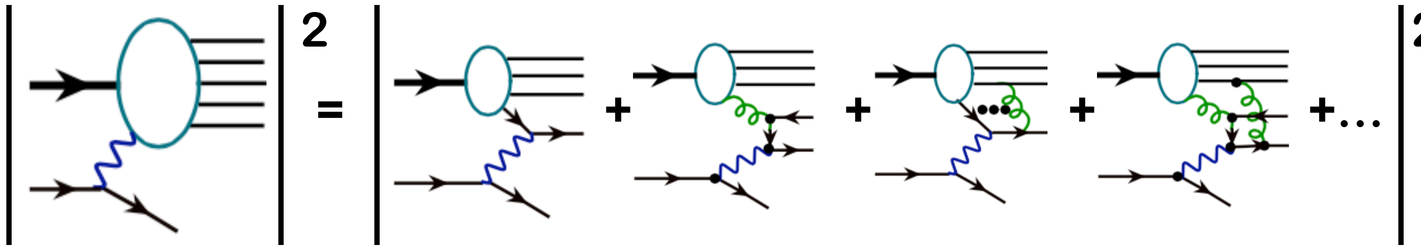

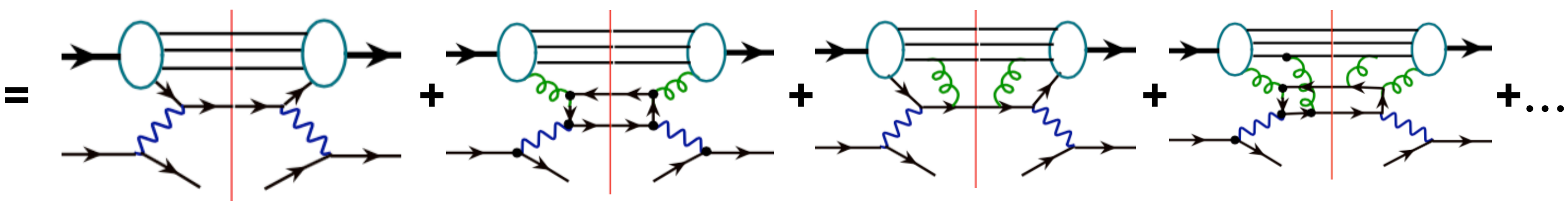

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Leading power
Linear contribution
DGLAP regime
…

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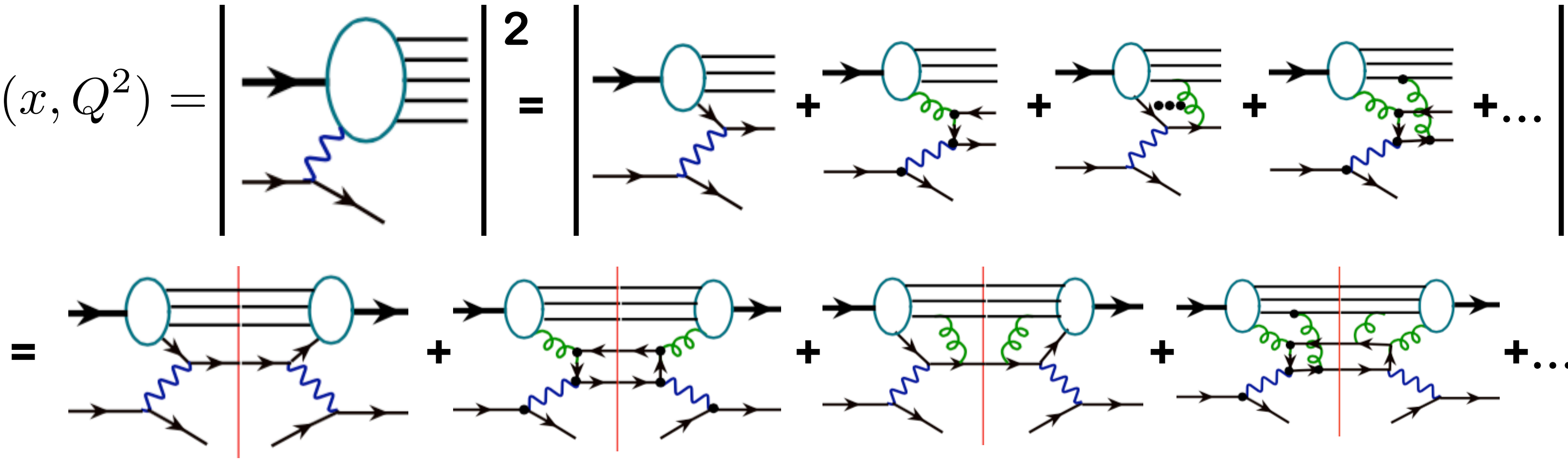

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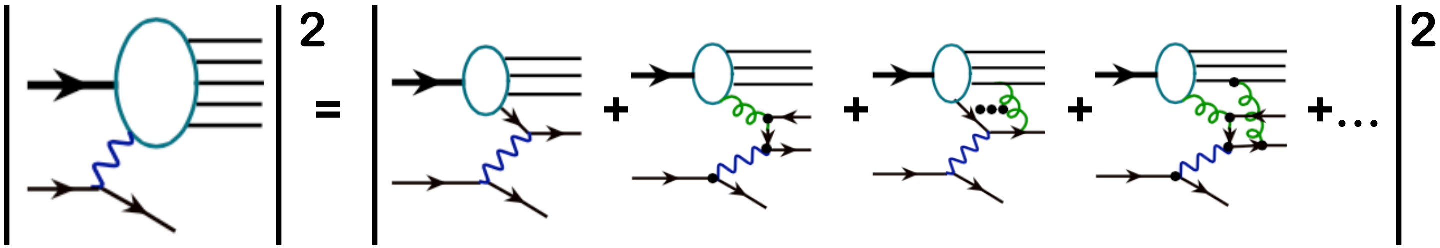

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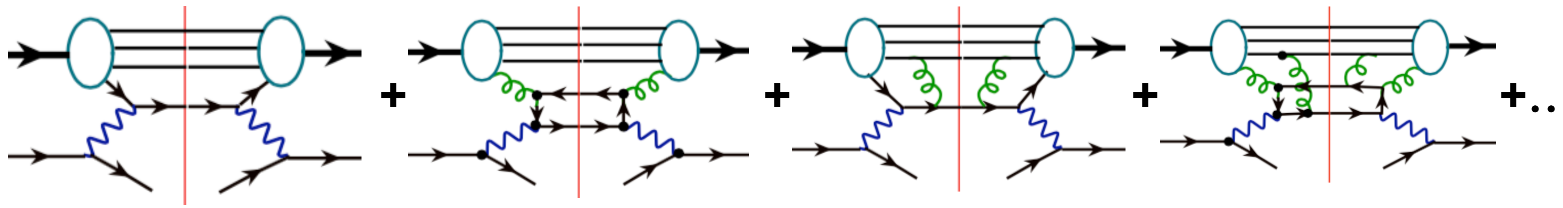

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Approximation – Leading power/twist factorization!

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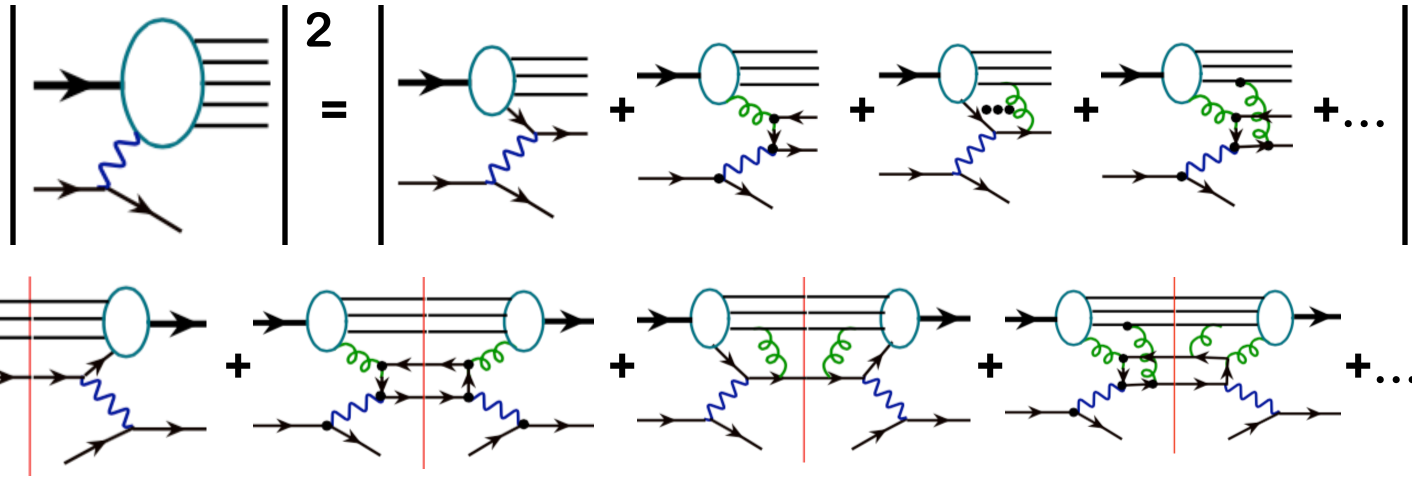

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Non-perturbative physics neglected or in input PDFs!

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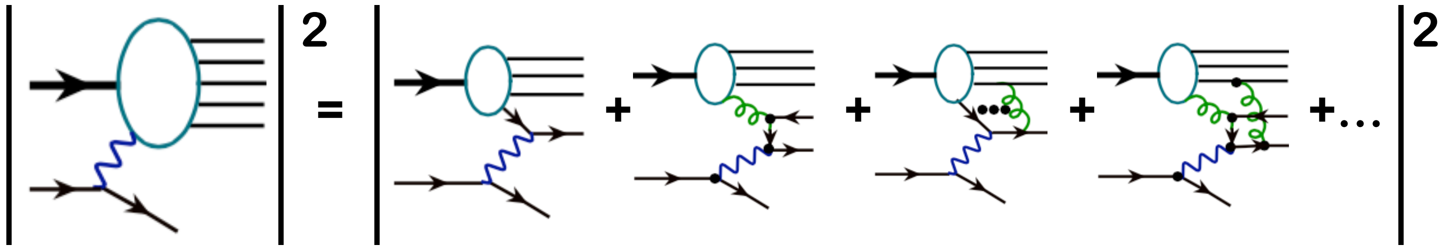

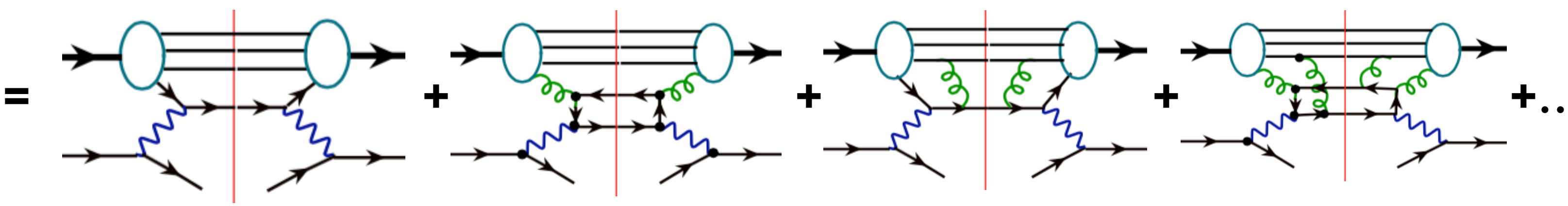

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Probe

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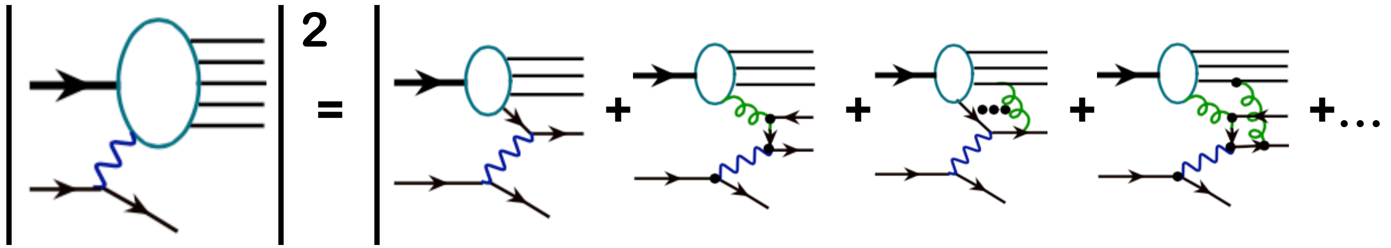
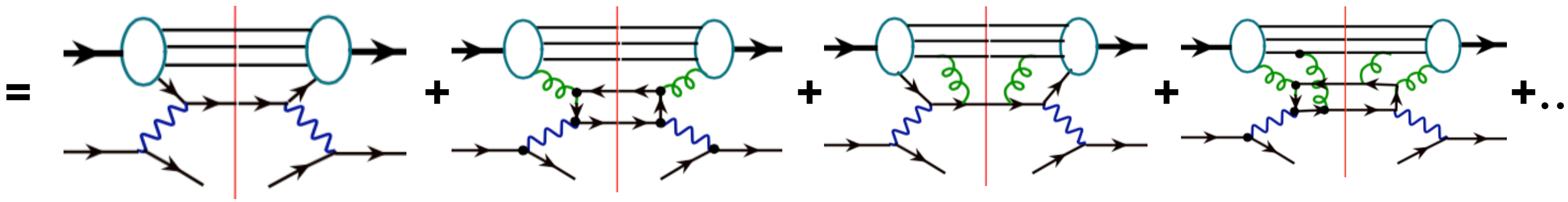
$\sigma_{\rm DIS}(x,Q^2) =$ 



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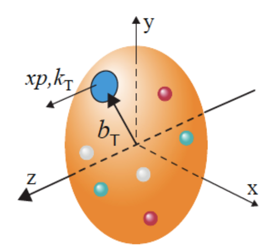
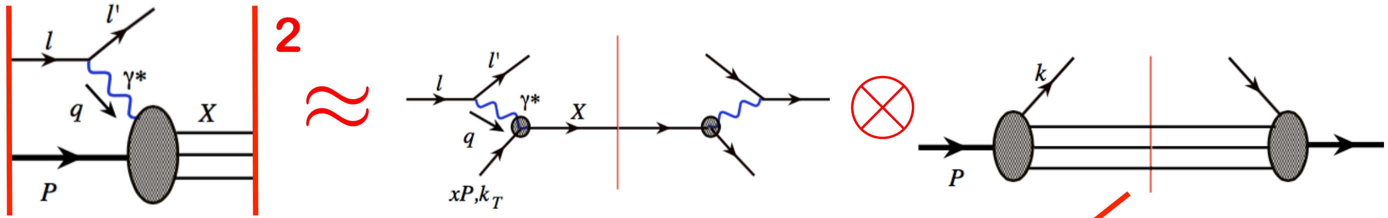
Structure　Structure



Operator definition of PDFs

❑ Definition – from QCD factorization:

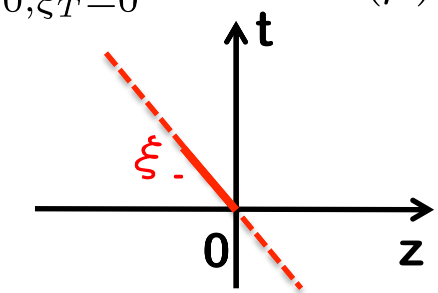


$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

$$\Phi^{[U]}(x;P,\mu)=\int\frac{d\xi^-}{(2\pi)}\,e^{i\,k\cdot\xi}\,\langle P|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P\rangle_{\xi^+=0,\vec{\xi}_T=0}+\mathrm{UVCT}(\mu)$$

✧ Depends on the choice of the gauge link:

$$U(0,\xi)=e^{-ig\int_0^\xi ds^\mu A_\mu}$$



PDFs are not direct physical observables, but, well defined in QCD

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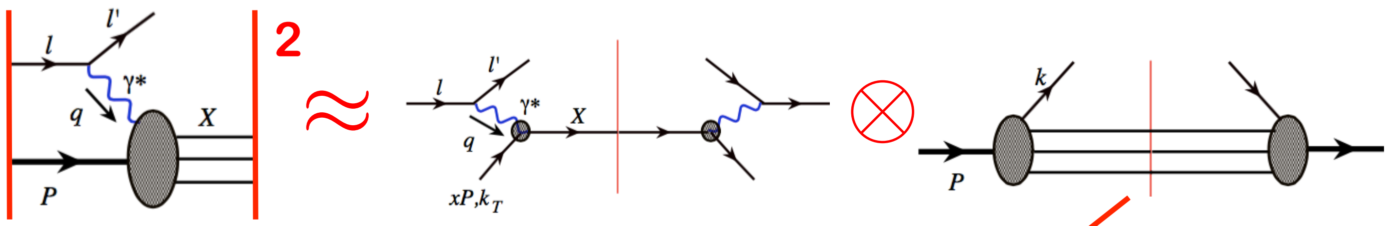

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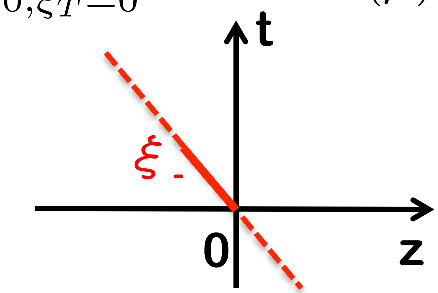

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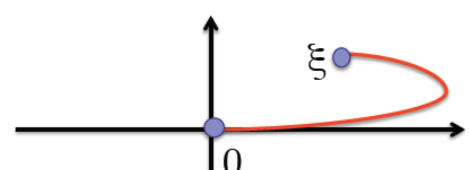
PDFs are not direct physical observables, but, well defined in QCD

❑ Transverse momentum dependent PDFs (TMDs):

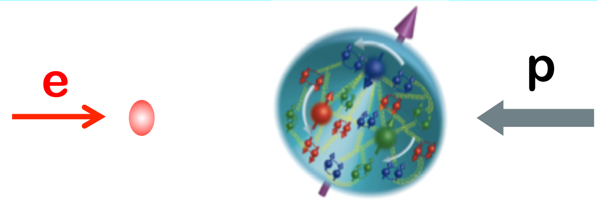
$$\Phi^{[U]}(x,k_T;P,\mu) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,k\cdot\xi}\, \langle P|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P\rangle_{\xi^+=0} + \mathrm{UVCT}(\mu)$$

✧ **General gauge link:**

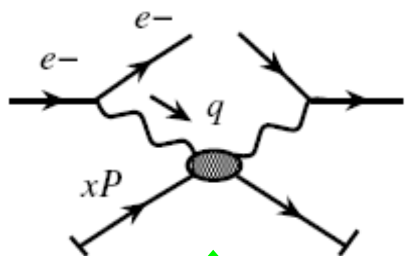


Hard probe and QCD factorization

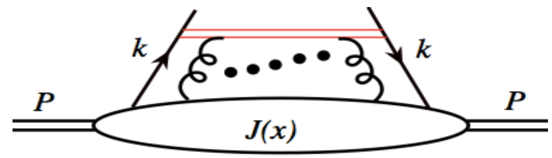
- One hadron:



$$\sigma_{\text{tot}}^{\text{DIS}} :$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Hard-part
Probe

Parton-distribution
Structure

Power corrections
Approximation

Hard probe and QCD factorization

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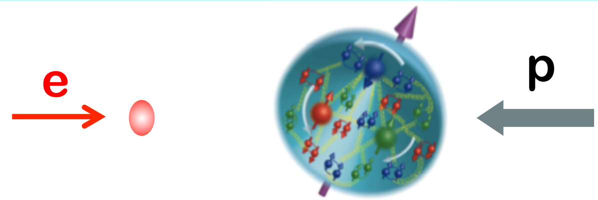

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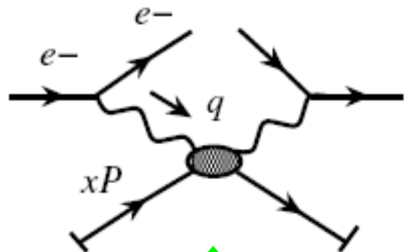

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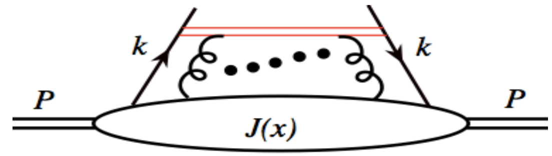
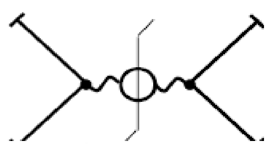


$+ \, O\left(\dfrac{1}{QR}\right)$

Hard-part	Parton-distribution	Power corrections
Probe	Structure	Approximation

- ❑ Two hadrons:

$\sigma_{\text{tot}}^{\text{DY}}$:



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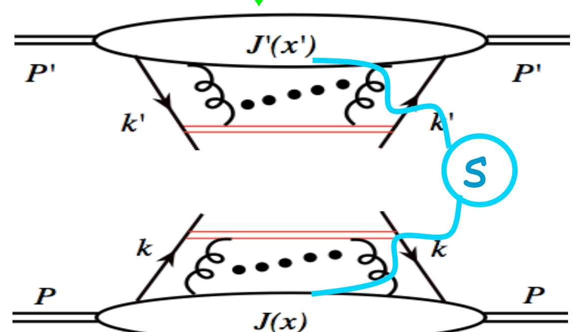

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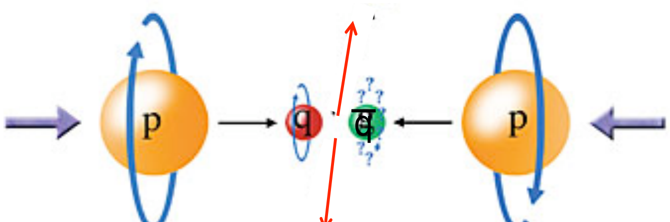

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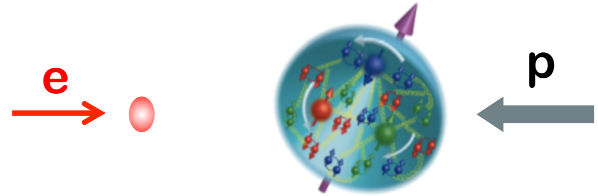

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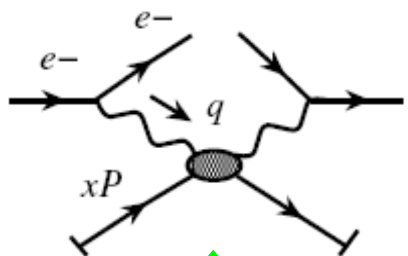

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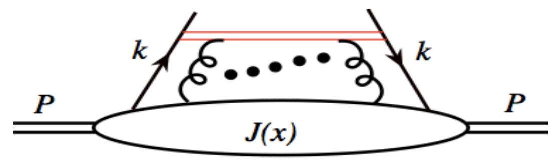

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Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

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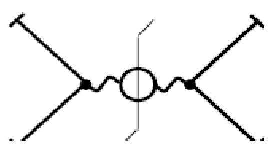

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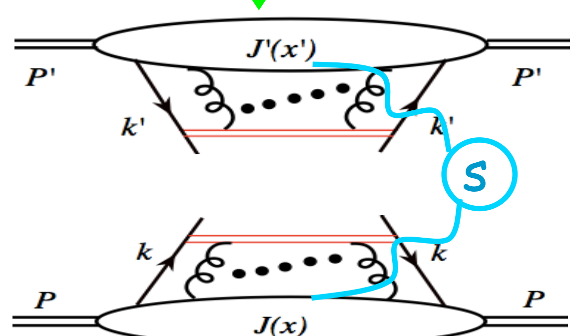

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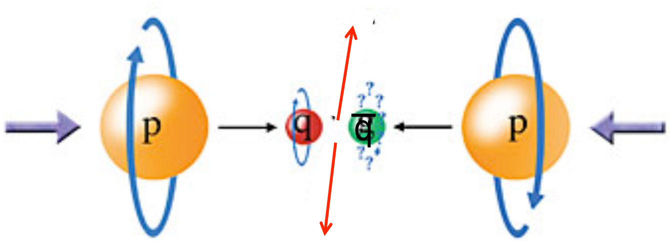

Predictive power:
Universal Parton Distributions

Global QCD analyses – a successful story

- **World data with "Q" > 2 GeV**

  + Factorization:

DIS: $$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

H-H: $$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

+ DGLAP Evolution:

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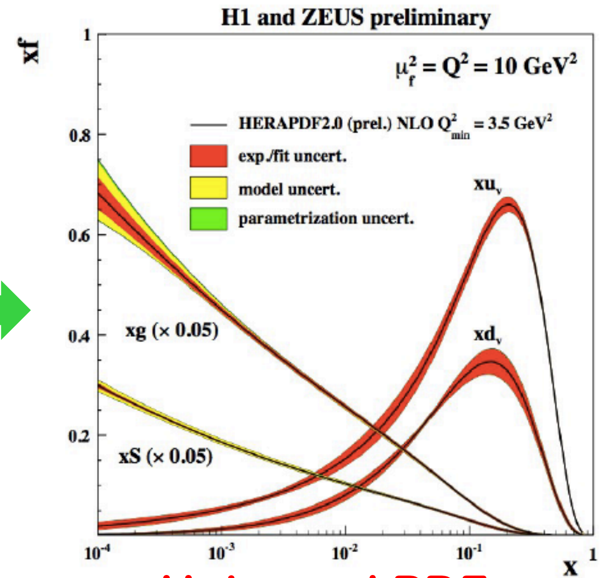

Universal PDFs

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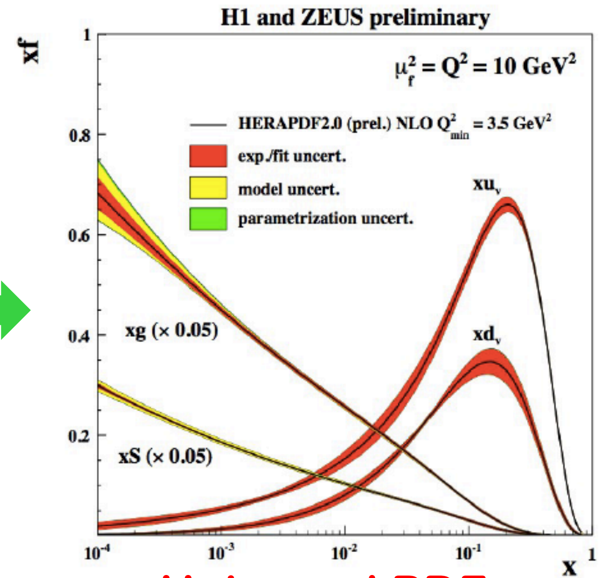
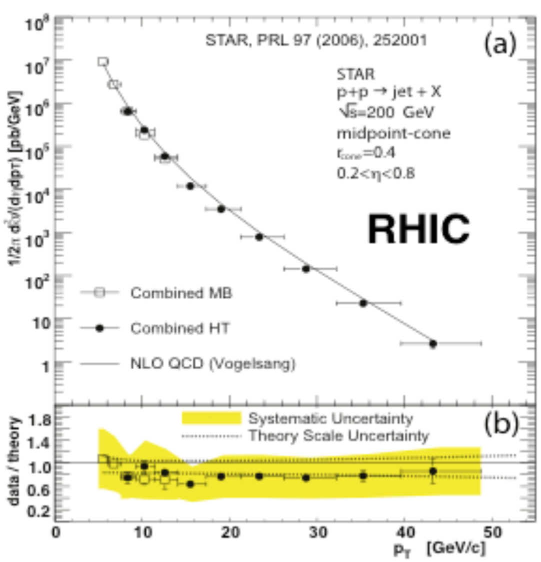
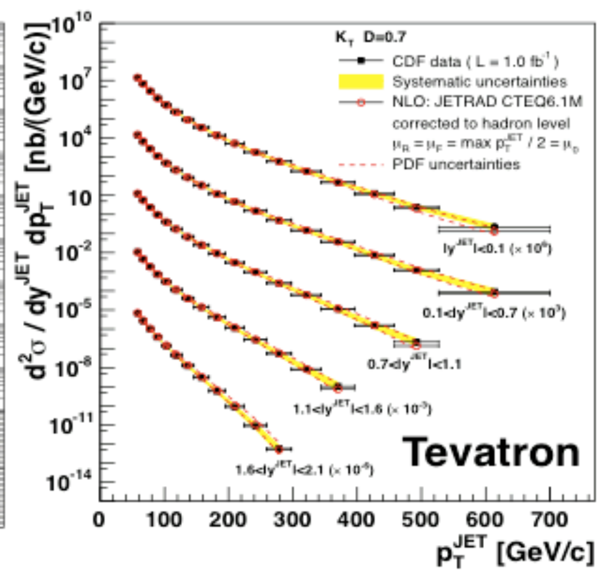
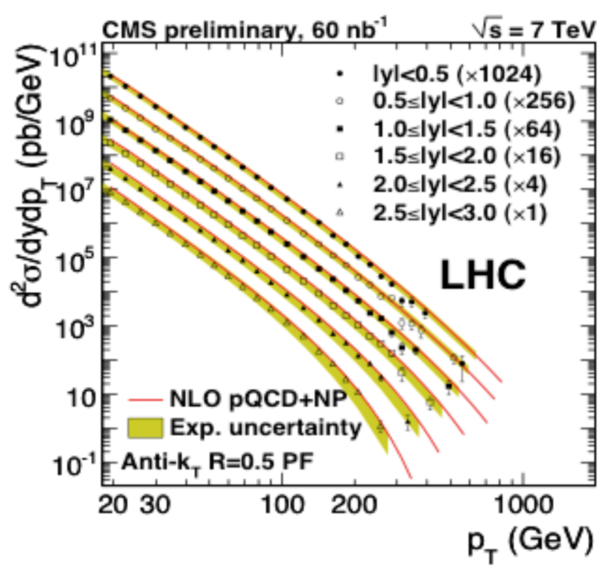


Universal PDFs







Global QCD analyses – a successful story

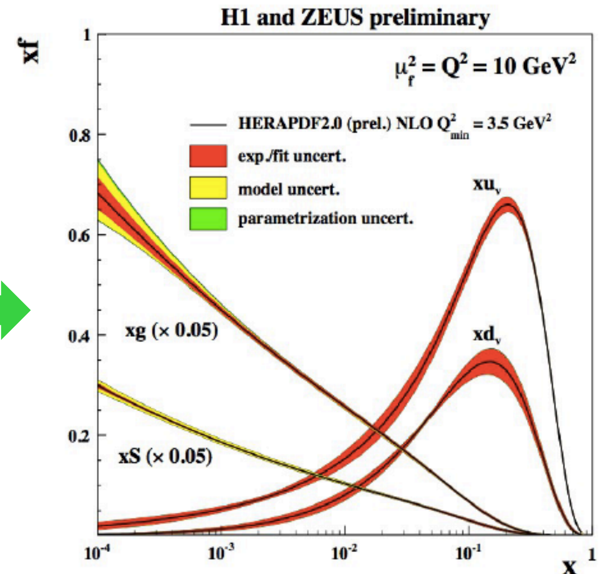
- World data with “Q” > 2 GeV
  + Factorization:

DIS: $$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

H-H: $$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

+ DGLAP Evolution:

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$



Universal PDFs

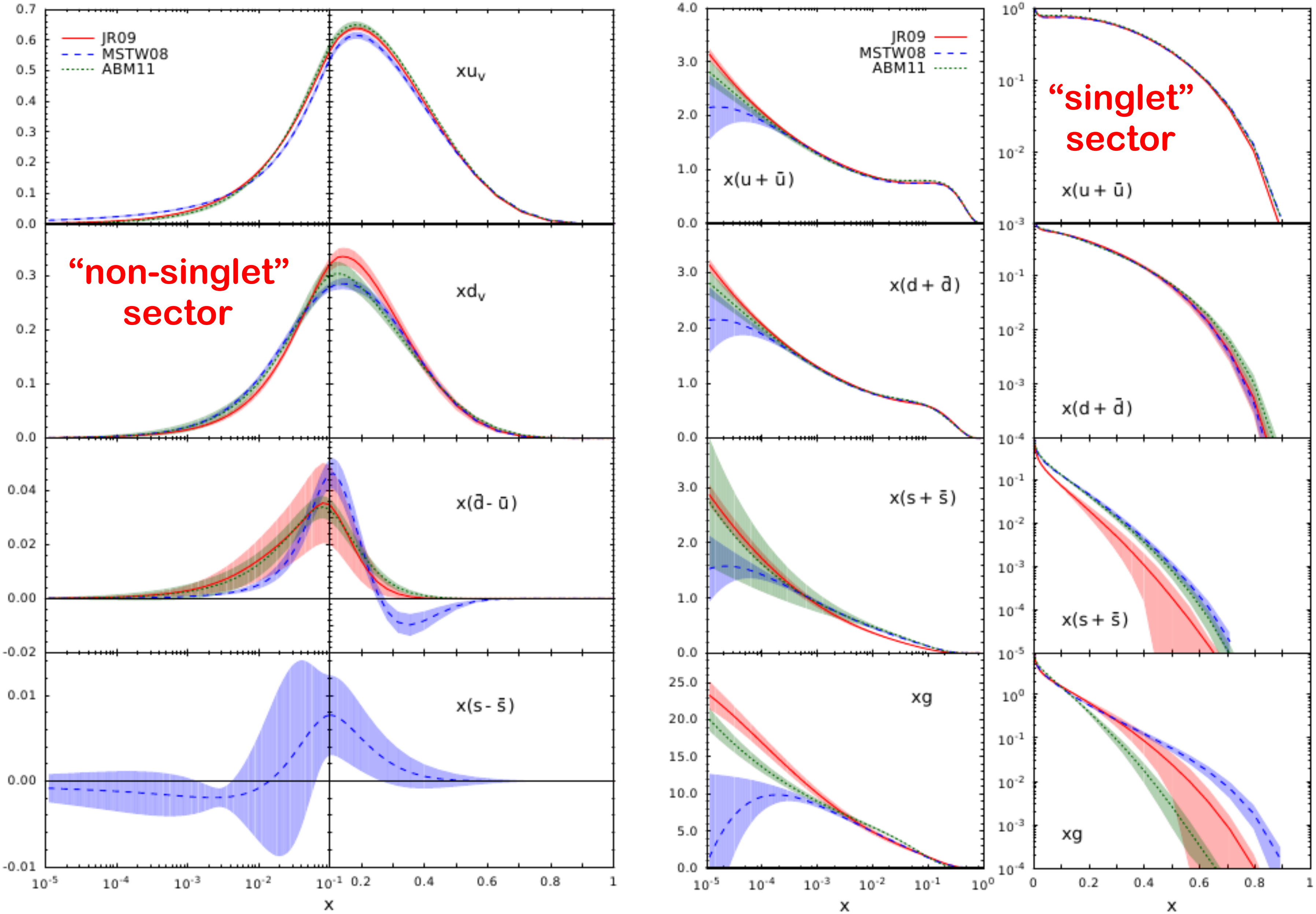
- The “BIG” question(s)

Why these PDFs behave as what have been extracted from the fits?

What have been tested is the evolution from μ_1 to μ_2
But, does not explain why they have the shape to start with!

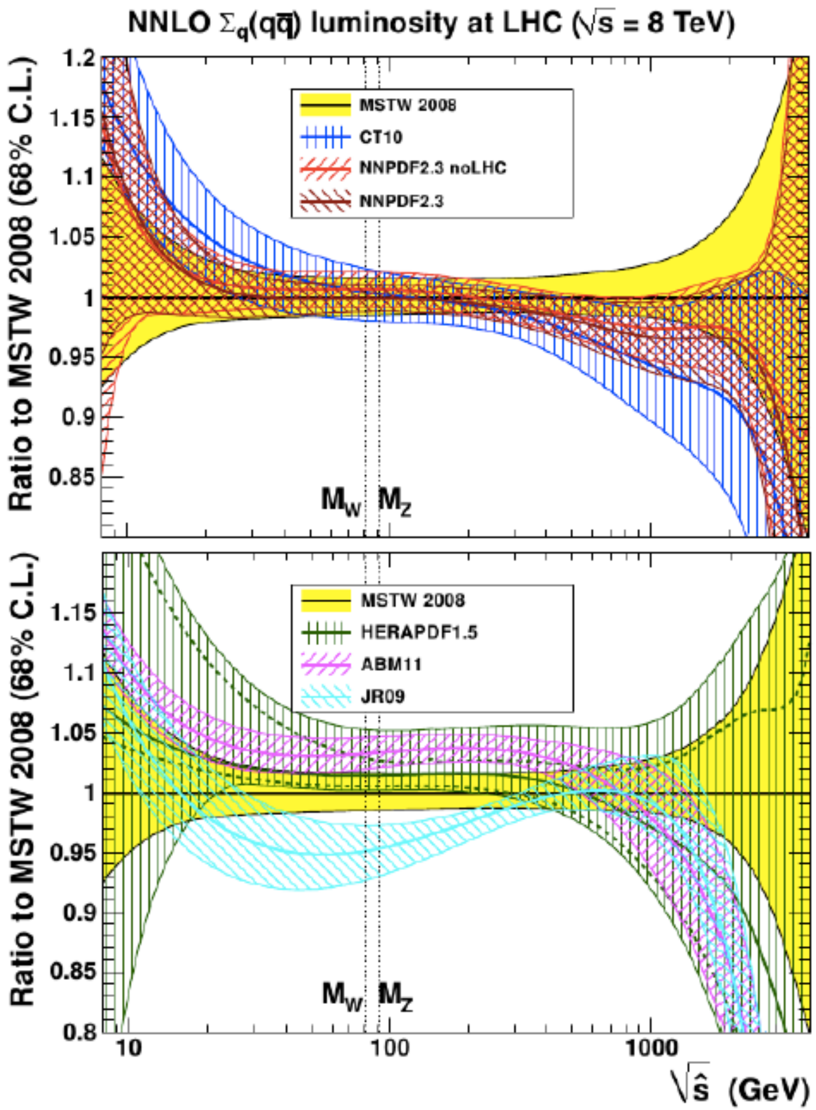
Can QCD calculate and predict the shape of PDFs at the input scale, and other parton correlation functions?

Uncertainties of PDFs

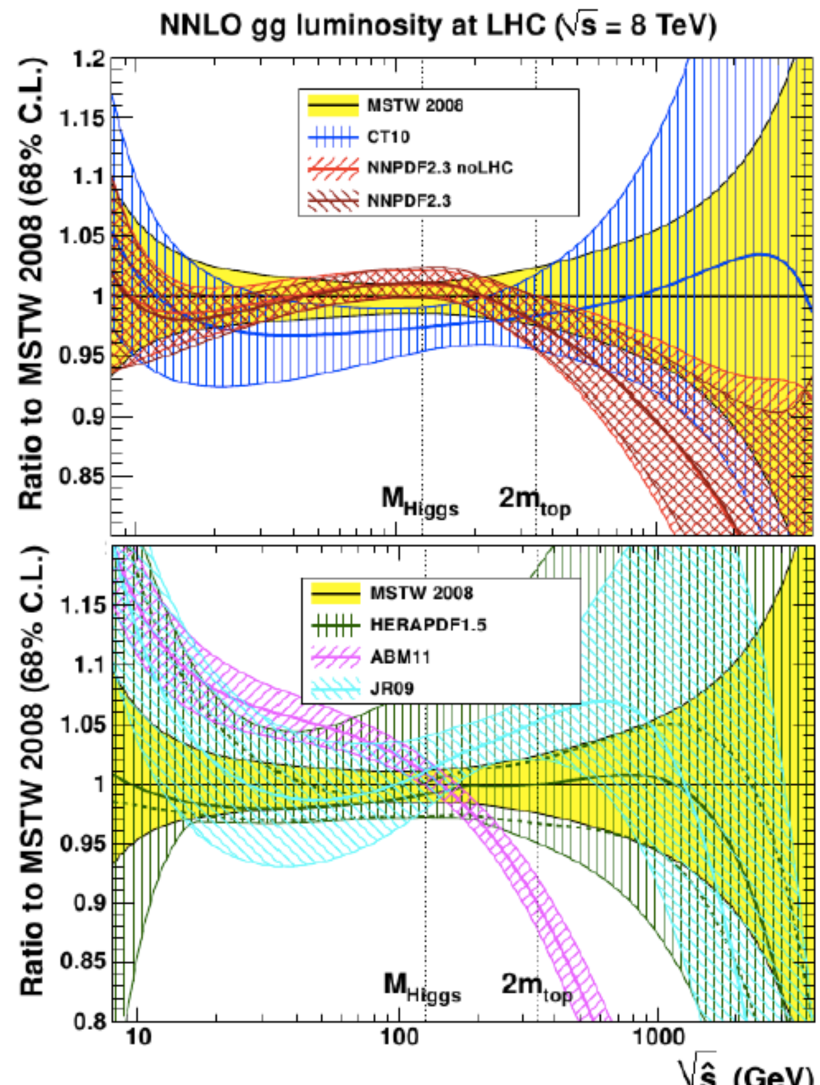


Partonic luminosities

q - qbar



g - g



PDFs at large x

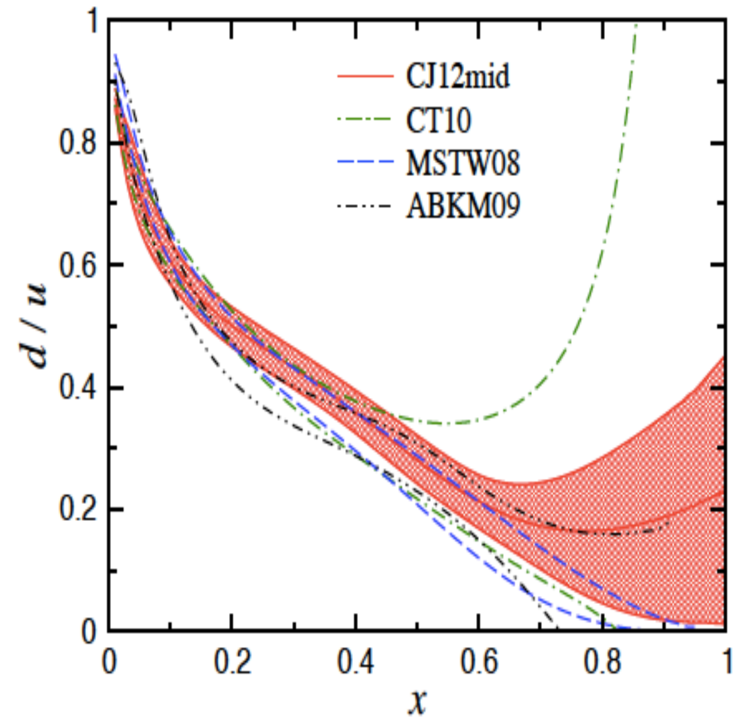
- Testing ground for hadron structure at x → 1:

✧ $d/u \to 1/2$ — SU(6) Spin-flavor symmetry

✧ $d/u \to 0$ — Scalar diquark dominance

✧ $d/u \to 1/5$ — pQCD power counting

✧ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ — Local quark-hadron duality



PDFs at large x

- Testing ground for hadron structure at x → 1:

$\diamond$ $d/u \to 1/2$ — **SU(6) Spin-flavor symmetry** — $\diamond$ $\Delta u/u \to 2/3$, $\Delta d/d \to -1/3$

$\diamond$ $d/u \to 0$ — **Scalar diquark dominance** — $\diamond$ $\Delta u/u \to 1$, $\Delta d/d \to -1/3$

$\diamond$ $d/u \to 1/5$ — **pQCD power counting** — $\diamond$ $\Delta u/u \to 1$, $\Delta d/d \to 1$

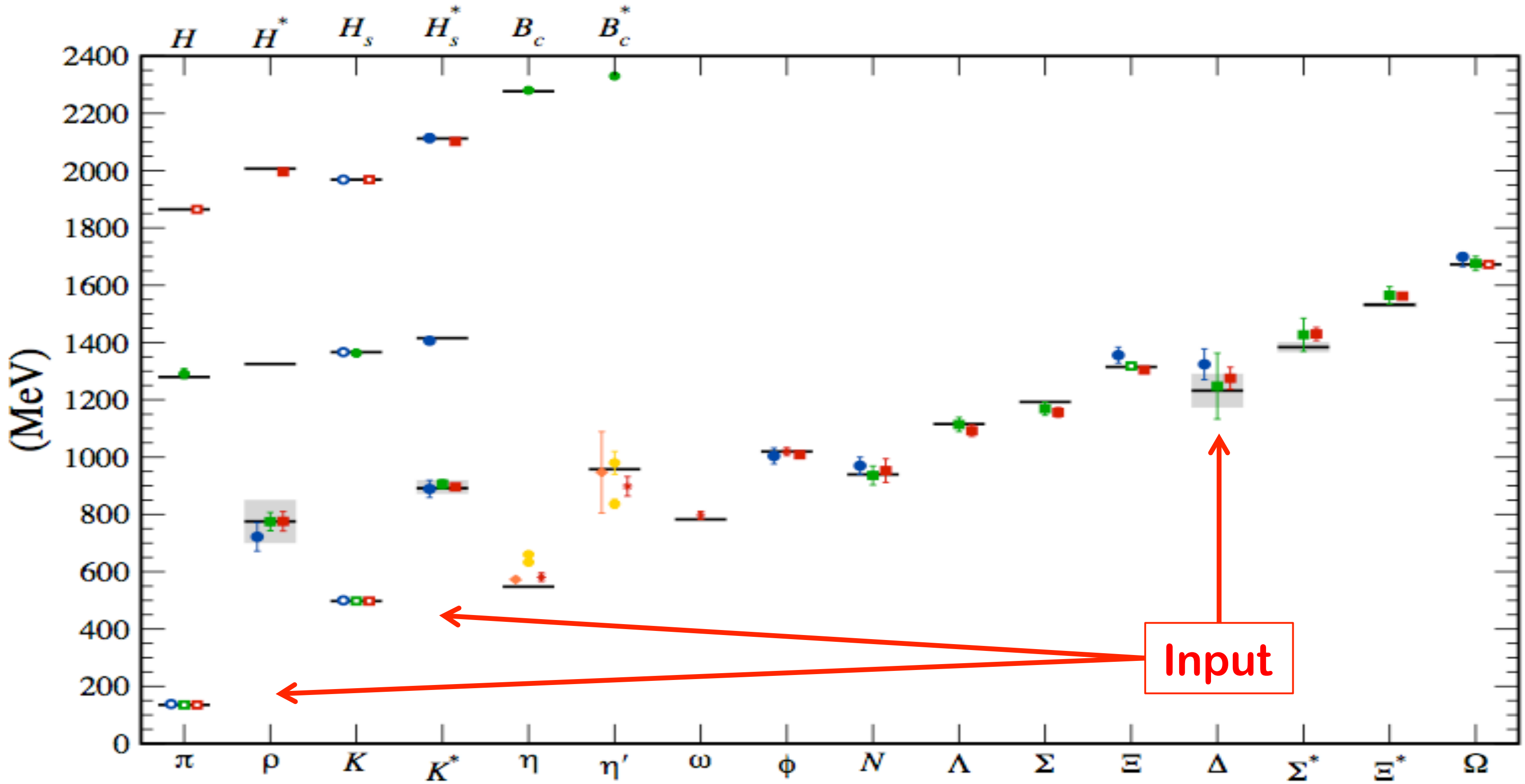
$\diamond$ $d/u \to \dfrac{4\mu_n^2/\mu_p^2 - 1}{4 - \mu_n^2/\mu_p^2} \approx 0.42$ — **Local quark-hadron duality** — $\diamond$ $\Delta u/u \to 1$, $\Delta d/d \to 1$

Can lattice QCD help?

Lattice QCD

- ❑ **Hadron masses:**

Predictions with limited inputs



- ❑ **Lattice "time" is Euclidean:** $\tau = i\,t$

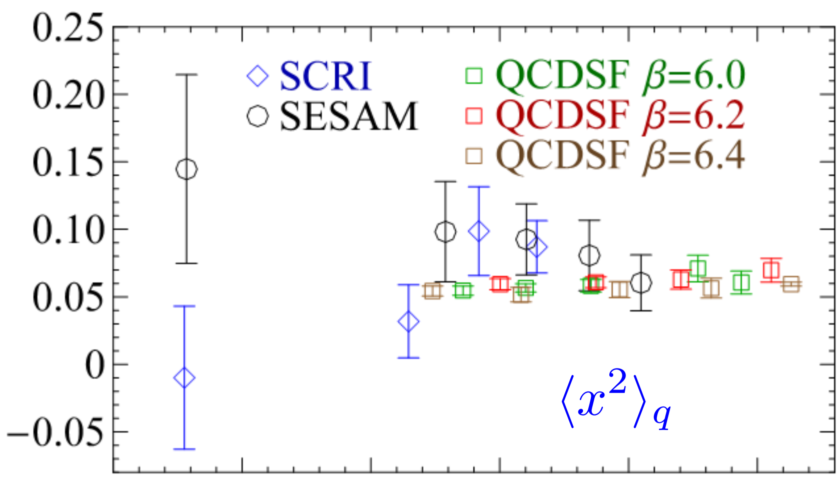
Cannot calculate PDFs, TMDs, …, directly, whose operators are time-dependent

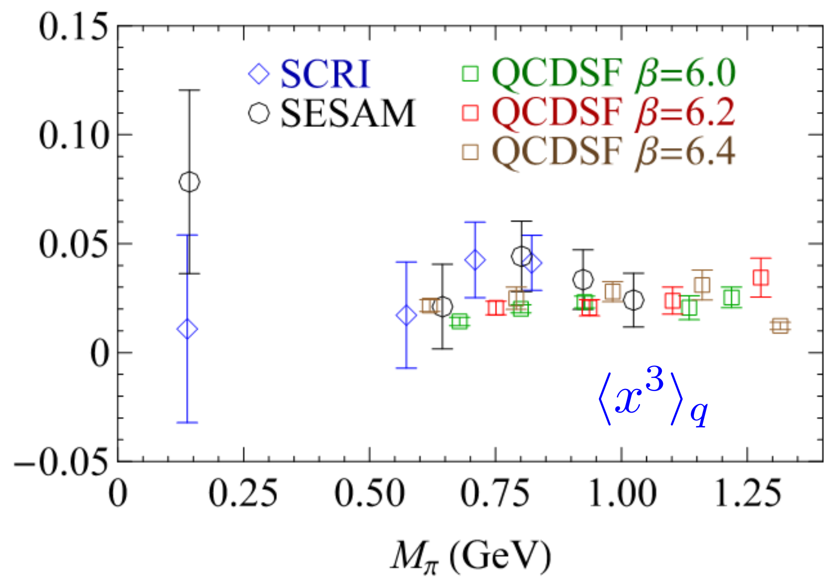
PDFs from lattice QCD

- Moments of PDFs – matrix elements of local operators

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

- Works, but, hard and limited moments:





Dolgov et al., hep-lat/0201021 Gockeler et al., hep-ph/0410187

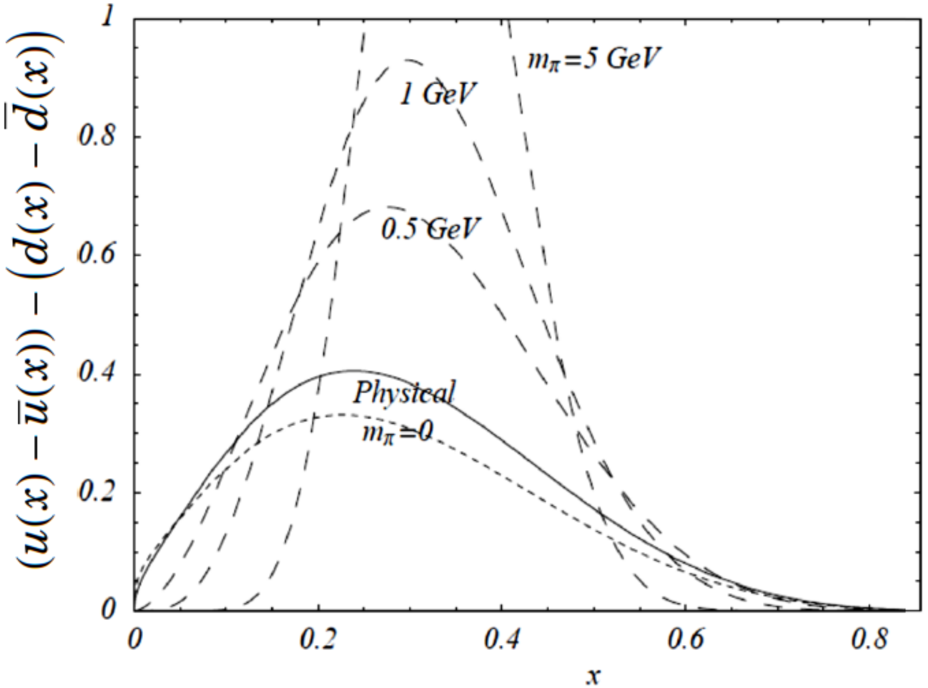
Limited moments – hard to get the full x-dependent distributions!

PDFs from lattice QCD

❑ **How to get x-dependent PDFs with a limited moments?**

- ✧ **Assume a smooth functional form with some parameters**
- ✧ **Fix the parameters with the lattice calculated moments**

$$xq(x) = a\, x^b (1-x)^c (1 + \epsilon\sqrt{x} + \gamma\, x)$$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

Cannot distinguish valence quark contribution from sea quarks

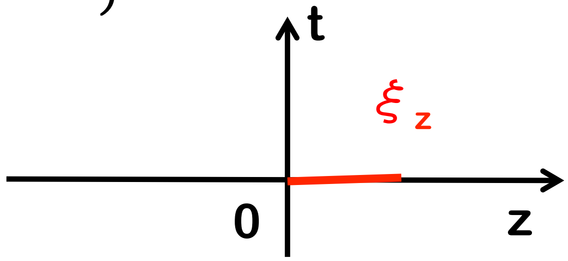
From quasi-PDFs to PDFs (Ji's idea)

Ji, arXiv:1305.1539

- "Quasi" quark distribution (spin-averaged):

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \text{UVCT}(\mu^2)$$

Quasi-PDFs =\= PDFs



- Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

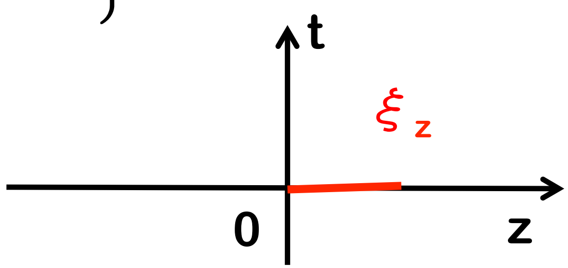
Quasi-PDFs → *Normal PDFs when $P_z \to \infty$?*

From quasi-PDFs to PDFs (Ji's idea)

Ji, arXiv:1305.1539

- "Quasi" quark distribution (spin-averaged):

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \mathrm{UVCT}(\mu^2)$$



Quasi-PDFs =\= PDFs

- Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Quasi-PDFs → *Normal PDFs* *when* $P_z \to \infty$?

- Excellent idea and great potential:

IDEA: Calculate something =\= PDFs, but, carry all the information of PDFs

CHALLENGES:

✧ Quasi-PDFs could be calculated using the lattice QCD method

✧ Extract PDFs from what you can calculate, …

"Quasi-PDFs" have no parton interpretation

❑ **Normal PDFs conserve parton momentum:**

$$M = \sum_q \left[\int_0^1 dx\, x f_q(x) + \int_0^1 dx\, x f_{\bar{q}}(x) \right] + \int_0^1 dx\, x f_g(x)$$
$$= \sum_q \int_{-\infty}^{\infty} dx\, x f_q(x) + \frac{1}{2} \int_{-\infty}^{\infty} dx\, x f_g(x)$$
$$= \frac{1}{2(P^+)^2} \langle P | T^{++}(0) | P \rangle = \text{constant}$$

$T^{\mu\nu}$

Energy-momentum tensor

❑ **"Quasi-PDFs" do not conserve "parton" momentum:**

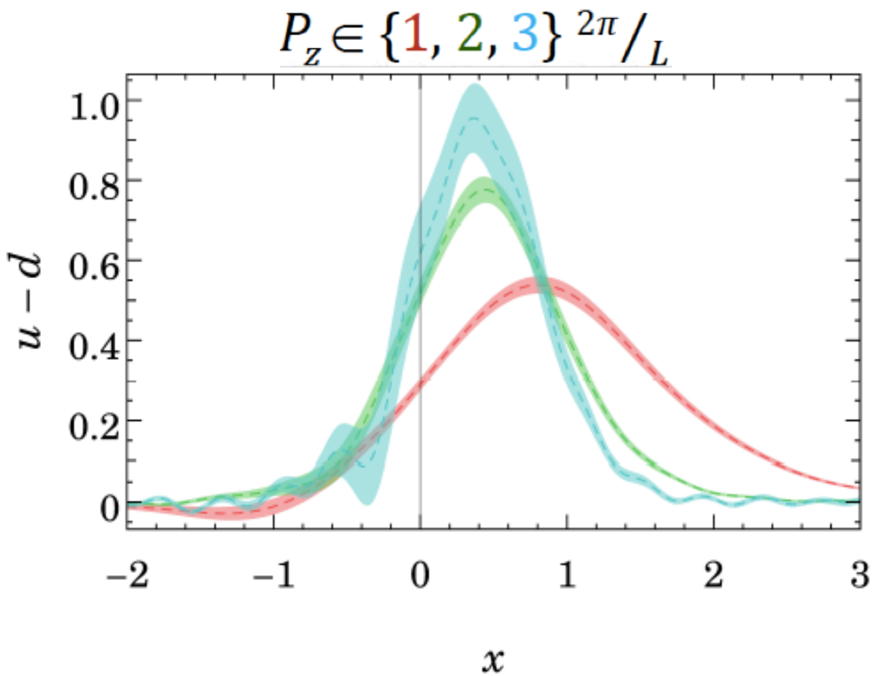
$$\widetilde{\mathcal{M}} = \sum_q \left[\int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_{\bar{q}}(\tilde{x}) \right] + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \sum_q \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \frac{1}{2} \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x})$$
$$= \frac{1}{2(P_z)^2} \langle P | \left[T^{zz}(0) - g^{zz}(\ldots) \right] | P \rangle \neq \text{constant}$$

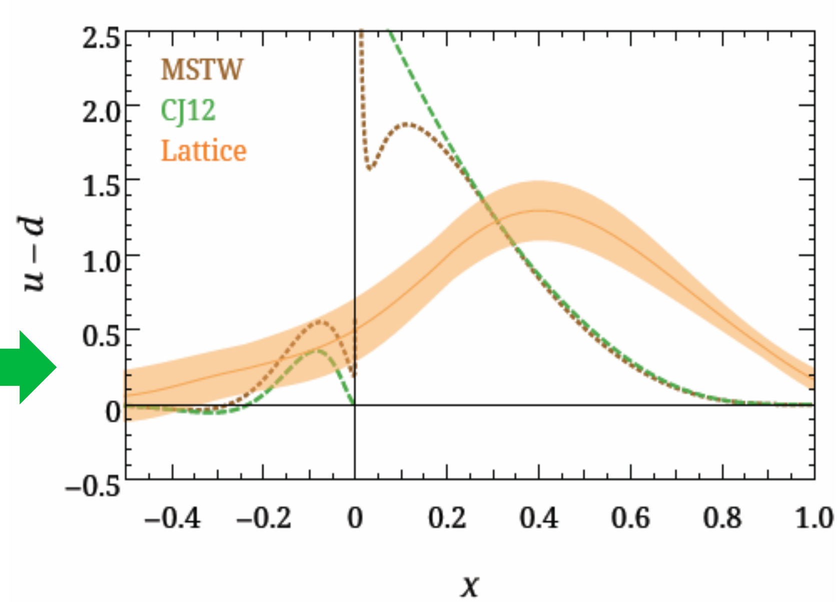
Note: "Quasi-PDFs" are not boost invariant

Lattice calculation of quasi-PDFs

Lin *et al.*, arXiv:1402.1462

❑ Exploratory study:





Quasi-Quark Distribution with different P_z ➡ Predicted quark distribution along with global fitted one

Matching – taking into account:

Target mass: $(M_N/P_z)^2$

High twist: $a+b/P_z^2$

Our observation

- **Quasi-PDFs are NOT defined by "twist-2" operators:**

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \mathrm{UVCT}(\mu^2)$$

Twist = Dimension - Spin

- **Renormalization scale dependence does not obey DGLAP:**

$$\mu^2 \frac{d}{d\mu^2}\widetilde{q}(x,\mu^2,P_z) \neq \mathrm{DGLAP}$$

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Twist = Dimension - Spin

- Renormalization scale dependence does not obey DGLAP:

$$\mu^2 \frac{d}{d\mu^2}\widetilde{q}(x,\mu^2,P_z) \neq \text{DGLAP}$$

- Questions to ask:
  - The operators defining quasi-PDFs renormalizable?
  - The renormalization mix with other operators? within a close set?
  - The renormalized quasi-PDFs and PDFs share the same CO properties?
  - Reliability to extract PDFs from the renormalized quasi-PDFs?
  - Lattice calculation: nonperturbative renormalization?
  - …?
- Extract hadron structure beyond quasi-DPFs?

Our observation

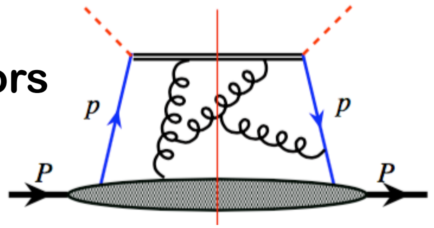
See talks by Zhang, Ji, Ma

❑ Renormalization – different from PDFs:

✧ PDFs – moments – twist-2 operators:

Twist-2 operators

$$\overline{\psi}(\xi^-)\gamma^+\Phi_n(\xi^-,0)\psi(0) = \sum_m \frac{(i\xi^-)^m}{m!}\mathcal{O}^{\mu_1\cdots\mu_m}(0)n_{\mu_1}\ldots n_{\mu_m}$$



Moments of PDFs ⟷ Matrix-elements of twist-2 operators

Renormalization of PDFs ⟷ Renormalization of twist-2 operators

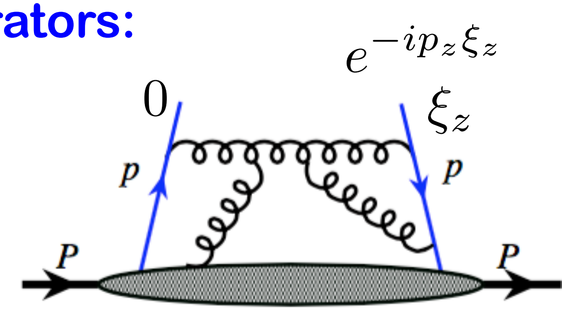
Mixing of all twist-2 operators

✧ Quasi-PDFs – NO moments – NOT by twist-2 operators:

In $A\cdot n_z = 0$, NO gauge link!

Renormalization of QCD in $A\cdot n_z = 0$ gauge

NO guarantee for quasi-PDFs renormalization



✧ Most challenge part of quasi-PDFs renormalization:

Renormalization of the bi-local/composite operators!

Our observation

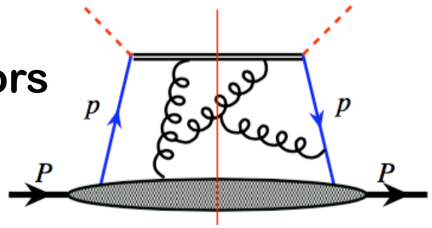
See talks by Zhang, Ji, Ma

❑ Renormalization – different from PDFs:

✧ **PDFs – moments – twist-2 operators:**

Twist-2 operators

$$\overline{\psi}(\xi^-)\gamma^+\Phi_n(\xi^-,0)\psi(0) = \sum_m \frac{(i\xi^-)^m}{m!}\mathcal{O}^{\mu_1\ldots\mu_m}(0)n_{\mu_1}\ldots n_{\mu_m}$$



Moments of PDFs $\longleftrightarrow$ Matrix-elements of twist-2 operators

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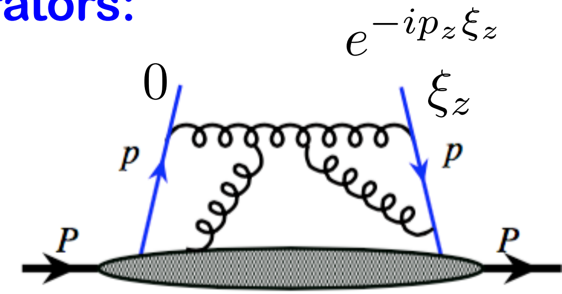


✧ **Most challenge part of quasi-PDFs renormalization:**

Renormalization of the bi-local/composite operators!

❑ Conclusion from arXiv:1707.03107:

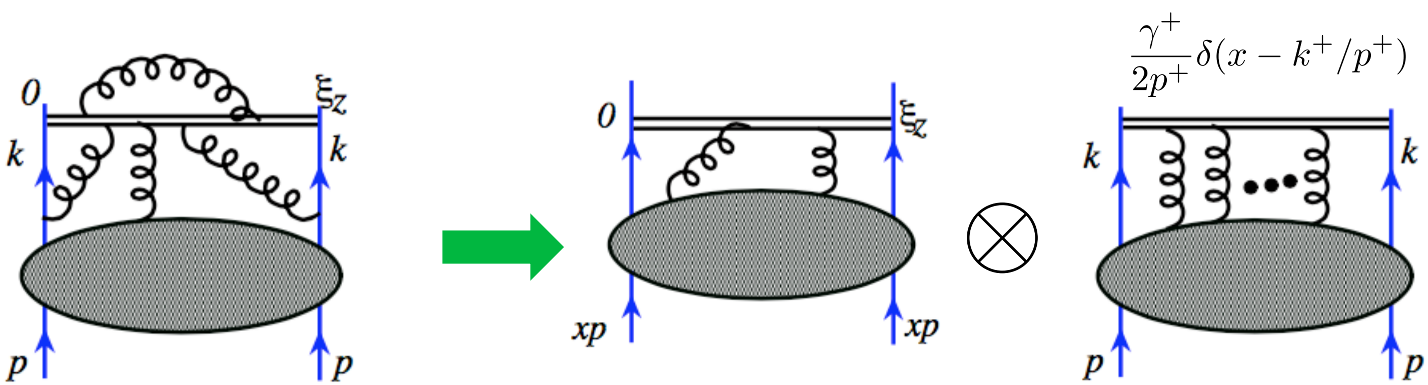
See Ma's talk tomorrow

$$\tilde{F}^R_{i/p}(\xi_z,\tilde{\mu}^2,p_z) = e^{-C_i|\xi_z|}Z^{-1}_{wi}Z^{-1}_{vi}\tilde{F}^b_{i/p}(\xi_z,\tilde{\mu}^2,p_z)$$

No mix with other flavors or gluon!

The goal for the rest of my talk

- ❑ Does the renormalized quasi-PDFs and PDFs share the same CO properties?
- ❑ Can we extract PDFs from renormalized quasi-PDFs reliably?



$+$ Corrections

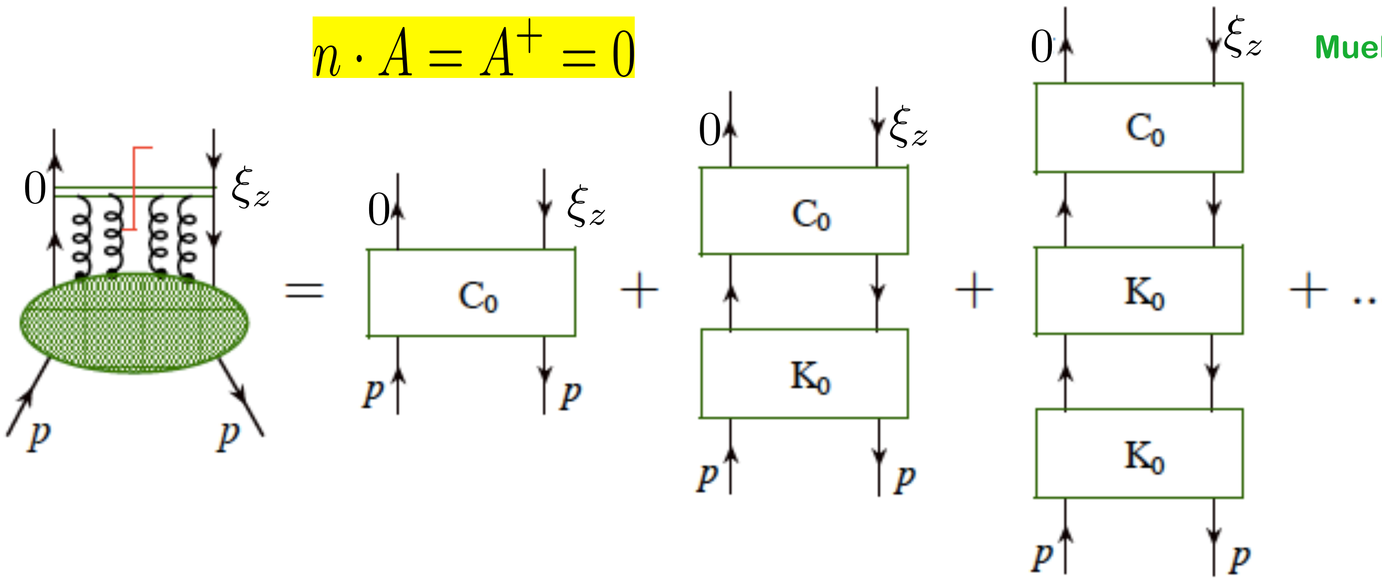
Factorization of CO divergence

Ma and Qiu, arXiv:1404.6860

❑ Generalized ladder decomposition in a physical gauge

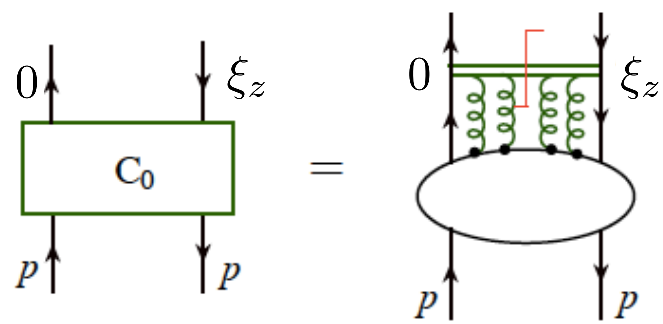
$$n \cdot A = A^+ = 0$$

Mueller, PRD 1974



❑ $C_0, \ K_0$:2PI kernels

✧ Only process dependence:

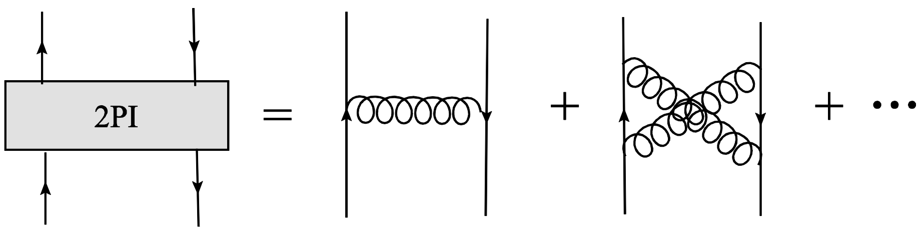


✧ 2PI are finite in a physical gauge for fixed *k* and *p*:

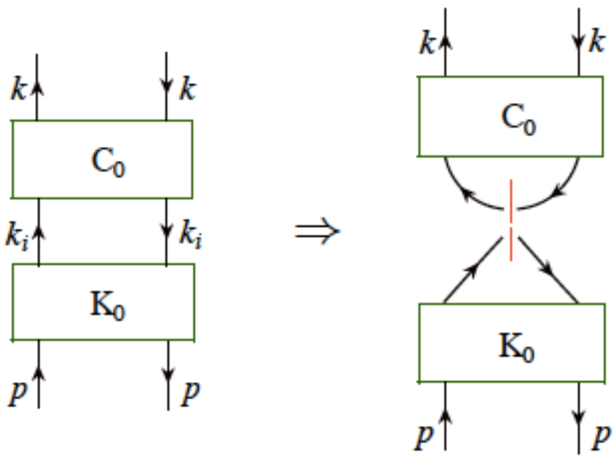
Ellis, Georgi, Machacek, Politzer, Ross, 1978, 1979

Factorization of CO divergence

❑ 2PI kernels – Diagrams:



❑ Ordering in virtuality: $P^2 \ll k^2 \lesssim \tilde{\mu}^2$ – Leading power in $\frac{1}{\tilde{\mu}}$



$\leftarrow \frac{1}{2}\gamma \cdot p$

$\leftarrow \frac{\gamma \cdot n}{2p \cdot n}\, \delta\left(x_i - \frac{k_i \cdot n}{p \cdot n}\right)$

+ power suppressed

Cut-vertex for normal quark distribution
Logarithmic UV and CO divergence

❑ Renormalized kernel - UV & IR safe - parton PDF:

$$K \equiv \int d^4 k_i \, \delta\left(x_i - \frac{k^+}{p^+}\right) \mathrm{Tr}\left[\frac{\gamma \cdot n}{2p \cdot n} K_0 \frac{\gamma \cdot p}{2}\right] + \mathrm{UVCT}_{\mathrm{Logarithmic}}$$

Factorization of CO divergence

❑ Projection operator for CO divergence:

$\widehat{\mathcal{P}}\, K$ Pick up the logarithmic CO divergence of *K*

❑ Factorization of CO divergence:

$$\tilde{f}_{q/p} = \lim_{m\to\infty} C_0 \sum_{i=0}^{m} K^i + \mathrm{UVCT}s$$

If multiplicative

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=0}^{m-1} K^i (1-\widehat{\mathcal{P}})K\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$= \lim_{m\to\infty} C_0 \left[1 + \sum_{i=1}^{m} \left[(1-\widehat{\mathcal{P}})K\right]^i\right]_{\mathrm{ren}} + \tilde{f}_{q/p}\,\widehat{\mathcal{P}}\,K$$

$$\Rightarrow \quad \tilde{f}_{q/P} = \left[C_0 \frac{1}{1-(1-\widehat{\mathcal{P}})K}\right]_{\mathrm{ren}} \left[\frac{1}{1-\widehat{\mathcal{P}}K}\right]$$

Normal Quark distribution

CO divergence free

All CO divergence of quasi-quark distribution

$$\Rightarrow \quad \tilde{f}_{i/h}(\tilde{x}, \tilde{\mu}^2, P_z) \approx \sum_j \int_0^1 \frac{dx}{x}\, \mathcal{C}_{ij}\!\left(\frac{\tilde{x}}{x}, \tilde{\mu}^2, P_z\right) f_{j/h}(x, \mu^2) + \text{Power corrections}$$

One-loop example: quark →quark

Ma and Qiu, arXiv:1404.6860

- **Expand the factorization formula:**

$$\tilde{f}_{i/h}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_j \int_0^1 \frac{dx}{x}\, \mathcal{C}_{ij}(\frac{\tilde{x}}{x},\tilde{\mu}^2,P_z)\, f_{j/h}(x,\mu^2)$$

To order α_s :

$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

$$\Longrightarrow \quad \mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

One-loop example: quark ➔quark

Ma and Qiu, arXiv:1404.6860

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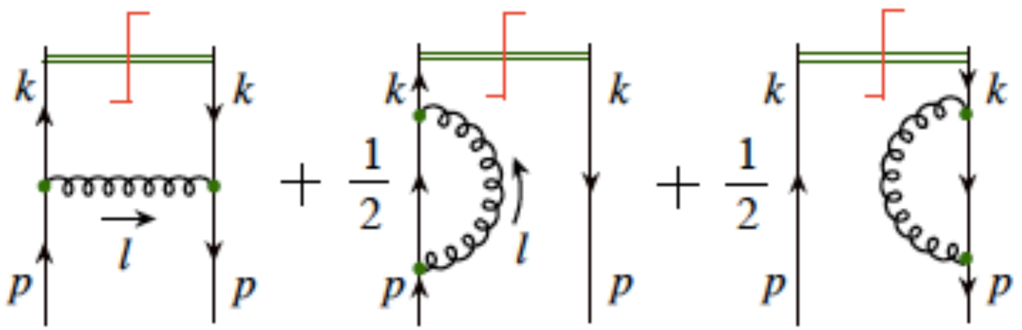
- **Feynman diagrams:**

Same diagrams for both

$\tilde{f}_{q/q}$ and $f_{q/q}$



But, in different gauge: $n_z \cdot A = 0$ for $\tilde{f}_{q/q}$ $\qquad n \cdot A = 0$ for $f_{q/q}$

- **Gluon propagator in n_z . A = 0:**

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2\, l^\alpha l^\beta}{l_z^2}$$

with $n_z^2 = -1$

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta\left(1-\tilde{x}-y\right)-\delta\left(1-\tilde{x}\right)\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

❑ Cancelation of CO divergence:

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta\left(1-\tilde{x}-y\right)-\delta\left(1-\tilde{x}\right)\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

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Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

❑ UV renormalization:

Different treatment for the upper limit of $l_\perp^2$ integration - "scheme"

Here, a UV cutoff is used – other scheme is discussed in the paper

One-loop coefficient functions

Ma and Qiu, arXiv:1404.6860

- MS scheme for $f_{q/q}(x,\mu^2)$:

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

CO, UV IR finite!

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}} = \left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ + \left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}+\frac{\mathrm{Sgn}(t)\Lambda_t}{\Lambda_t+|t|}\right.$$
$$\left. -\frac{1+t^2}{1-t}\left[\mathrm{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\mathrm{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t = \sqrt{\tilde{\mu}^2/P_z^2+t^2} - |t|$, $\mathrm{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise.

- Generalized "+" description: $t = \tilde{x}/x$

$$\int_{-\infty}^{+\infty} dt\left[g(t)\right]_N h(t) = \int_{-\infty}^{+\infty} dt\, g(t)\,[h(t)-h(1)]$$

For a testing function $h(t)$

Explicit verification of the CO factorization at one-loop

One-loop coefficient functions

Ma and Qiu, arXiv:1404.6860

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Explicit verification of the CO factorization at one-loop

Note: $\Lambda_t \to \mathcal{O}\left(\frac{\widetilde{\mu}}{P_Z}\right)$ as $P_Z\to\infty$ the linear power UV divergence!

Pseudo-PDFs

Radyushkin, 2017

❑ Pseudo-PDFs = generalization of PDFs:

✧ **Definition:** $\xi^2 < 0$

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$
$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi}\, e^{ix\,\nu}\,\frac{1}{2p^+}\mathcal{M}^+(\nu,\xi^2)$$

✧ **Interpretation:**

with $\xi^\mu = (0^+, \xi^-, 0_\perp)$

Off-light-cone extension of PDFs: $f(x) = \mathcal{P}(x, \xi^2 = 0)$

❑ Quasi-PDFs:

$$\xi^\mu = (0, 0_\perp, \xi_z)$$

No longer Lorentz invariant

$$\tilde{q}(x,\mu^2,p_z) = \int \frac{d\nu}{2\pi}\, e^{ix\nu}\,\frac{1}{2p_z}\mathcal{M}^z(\nu = p_z\xi_z, -\xi_z^2)$$

❑ TMDs:

$$\xi^\mu = (0^+, \xi^-, \xi_\perp)$$

$$\mathcal{P}(x, -\xi_\perp^2) \equiv \int d^2k_\perp\, e^{i\vec{k}_\perp\cdot\vec{\xi}_\perp}\,\mathcal{F}(x, k_\perp^2)$$

TMDs with a straight gauge link

Pseudo-PDFs

Orginos, et al, 2017
1706.05373

❑ Pseudo-PDFs:

✧ **Lattice calculation with** $\alpha = 0$:

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$

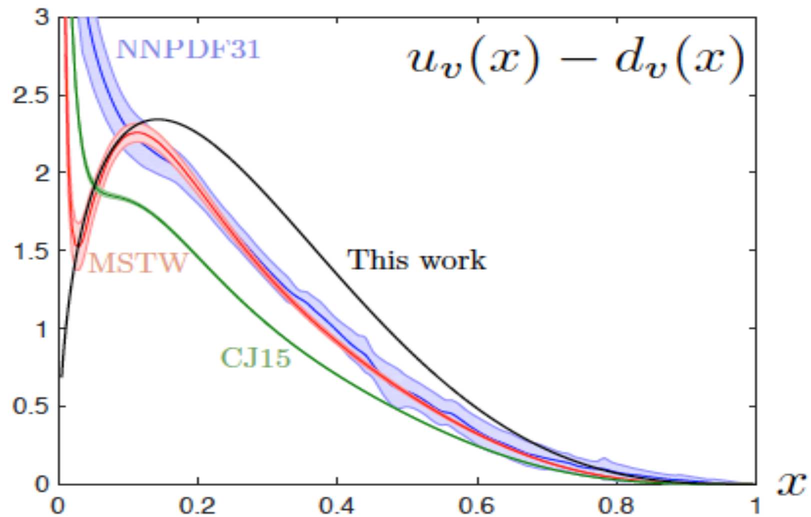
$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

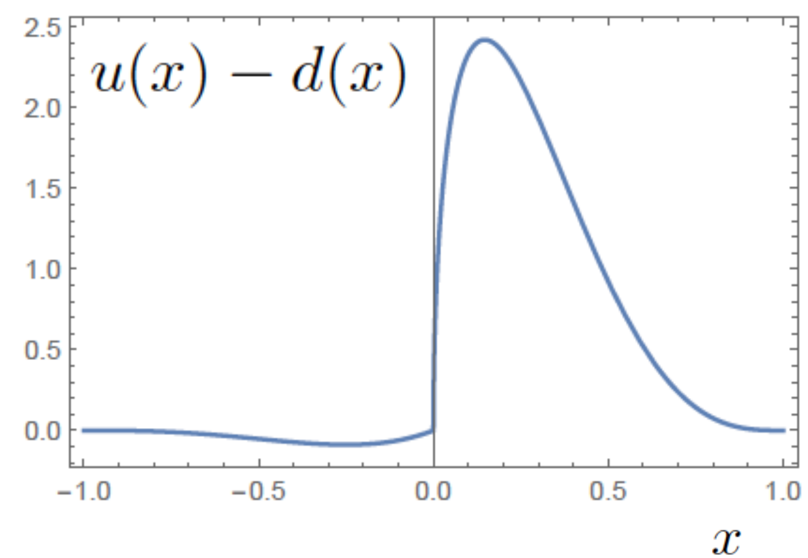
$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi}\, e^{ix\,\nu}\mathcal{M}_{p=p^0}(\nu,\xi^2)/\mathcal{M}_{p=p^0}(0,\xi^2)$$

Remove UV!

✧ **Model quasi-PDFs:** **with** $\xi^\mu = (0, 0_\perp, \xi_z)$

❑ Numerical results:





Go beyond quasi-PDFs?

Ma and Qiu, arXiv:1404.6860

❑ A pQCD factorization approach:

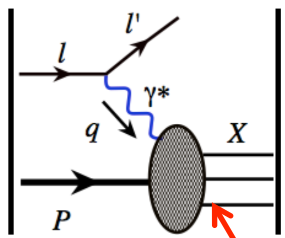
✧ Recall: Collinear factorization of DIS cross section – single hadron

Momentum transfer

CO Factorization

PDFs

$$\sigma^{\rm DIS}(x, Q^2; \sqrt{s}) \propto \left| \vphantom{\frac{1}{1}} \right.$$



$$\left. \vphantom{\frac{1}{1}} \right|^{2} \approx \sum_f \mathcal{C}_f(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_i(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

Collision energy

Single hadron Matrix elements

Perturbative coefficients

Power corrections

Go beyond quasi-PDFs?

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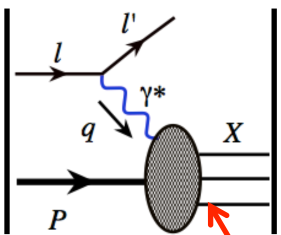

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Collision energy

Single hadron Matrix elements

Perturbative coefficients

Power corrections

✧ Renormalizable + factorizable + lattice calculable "cross section":

PDFs

$$\widetilde{\sigma}_{\rm Lattice}(\tilde{x}, Q, P \sim \sqrt{s}) \approx \sum_f \int \frac{dx}{x} \mathcal{C}_f\left(\frac{x}{\tilde{x}}, \frac{\mu^2}{Q^2}, \alpha_s(\mu); P\right) f(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

Key: controllable hard scale!

DGLAP-Evolution

Go beyond quasi-PDFs?

Ma and Qiu, arXiv:1404.6860

❑ What is lattice "cross section"?

Single hadron matrix elements, with the following properties:

✧ **Lattice calculable:**

Calculable using lattice QCD with an Euclidean time

✧ **UV Renormalizable:**

Ensure a well-defined continuum limit, UV & IR finite!

✧ **CO Factorizable:**

Share the same perturbative collinear divergences with PDFs

Factorizable to PDFs with IR-safe hard coefficients

with controllable power corrections

Go beyond quasi-PDFs?

Ma and Qiu, arXiv:1404.6860

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❑ Key requirement:

A controllable large momentum scale – conjugate to hadron momentum

to define the "collision" dynamics of the "cross section"

to ensure the necessary condition for the factorization

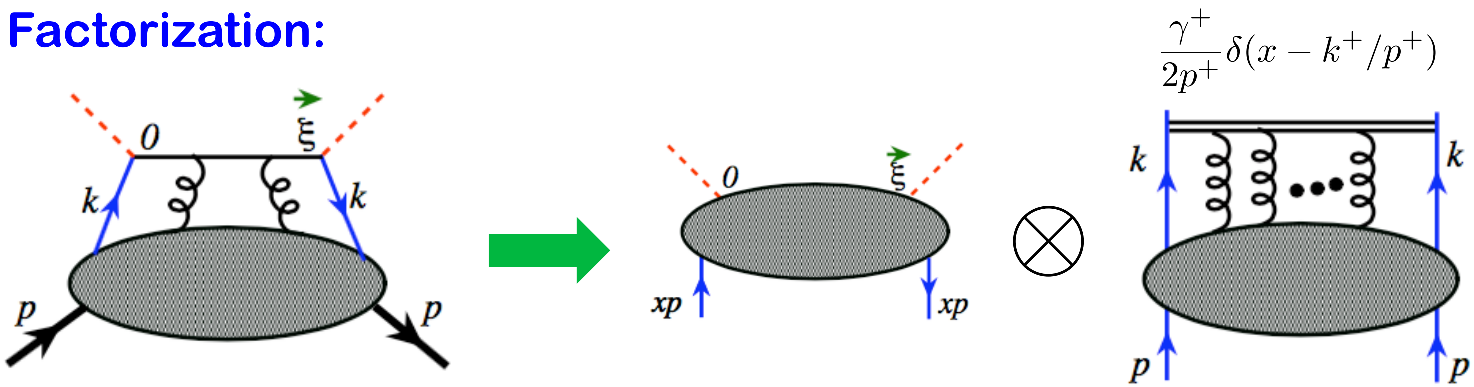
Go beyond quasi-PDFs?

❑ Example: Current correlators

✧ **Coordinate space:**

$$\mathcal{T}_{jj}(p,s,\xi) = \lim_{\xi^0\to 0^+} \langle p,s|T\{\, j_\Gamma(\xi^0,\vec{\xi})\, j_\Gamma(0)\}|p,s\rangle$$

✧ **Factorization:**



$+$ Corrections

Go beyond quasi-PDFs?

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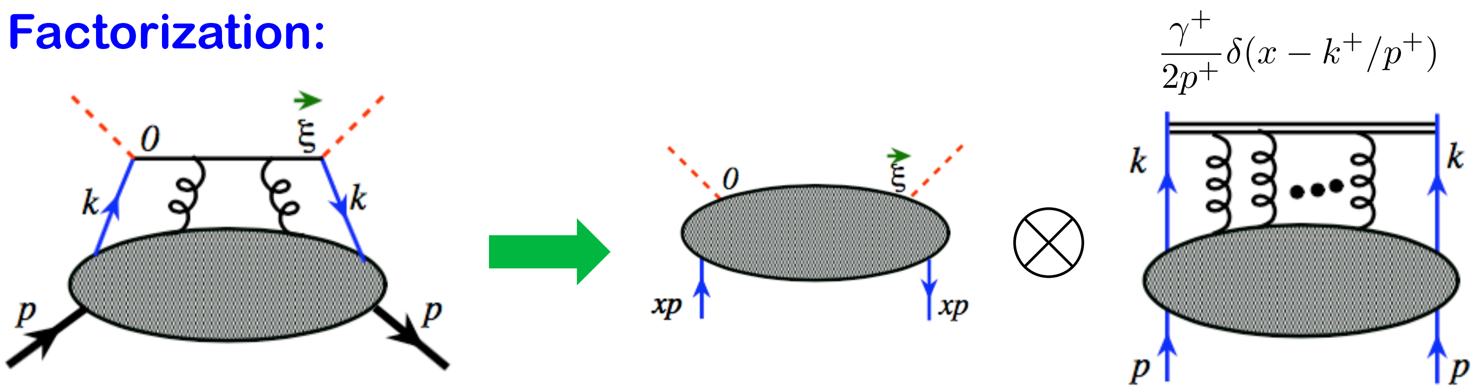

$+$ **Corrections**

❑ Complementarity and advantages:

✧ **Complementary to existing approaches for extracting PDFs, …**

✧ **Go beyond quasi-PDFs with tremendous potentials:**

Neutron PDFs, … (no free neutron target!)
Meson PDFs, such as pion, …
More direct access to gluons – gluonic current, …

Summary and outlook

- "lattice cross sections" = single hadron matrix elements
  calculable in Lattice QCD, renormalizable + factorizable in QCD

Going beyond the quasi-PDFs

- Extract PDFs by global analysis of data on "Lattice x-sections".
  Same should work for other distributions (TMDs, GPDs)

$$\widetilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, \frac{1}{a}, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2) \, \widetilde{\mathcal{C}}_i(\frac{\tilde{x}}{x}, \frac{1}{a}, \mu^2, P_z)$$

- Conservation of difficulties – complementarity:
  High energy scattering experiments
  – less sensitive to large x parton distribution/correlation
  "Lattice factorizable cross sections"
  – more suited for large x PDFs, but limited to large x for now

- Quasi-PDFs are renormalizable & factorizable

- Lattice QCD can be used to study hadron structure, but,
  more works are needed!

Thank you!

BACKUP SLIDES