

Transverse densities of nucleons and resonances

C. Weiss (JLab), Exploring hadrons with EM probes, JLab, 02-Nov-17
based on work with J.M. Alarcon, A. Hiller Blin, C. Granados



Objective: Explain/predict structure of nucleon and baryon resonances using model-independent methods of strong interaction physics

Chiral effective field theory

large-distance dynamics, controlled accuracy, predictive

Dispersion theory

analyticity, global properties, spectrum \leftrightarrow structure

Large- N_c QCD

parametric expansion, $N \leftrightarrow \Delta$, connection with QCD

Structures

EM form factors and transverse densities $N \rightarrow N$, $N \rightarrow N^*$, $N^* \rightarrow N^*$ \leftarrow

Form factors of scalar, axial, twist-2 operators (GPDs)

$\pi N N^*$ vertices, $NN \rightarrow NN^*$ amplitudes

Granados, Weiss, JHEP 1401, 092 (2014); PRC 92, 025206 (2015); JHEP 1507, 170 (2015); JHEP 1606, 075 (2016).

Alarcon, Hiller Blin, Vicente Vacas, Weiss, NPA 964, 18 (2017)

Alarcon, Weiss, arXiv:1707.07682; arXiv:1710.06430.

- Form factors and transverse densities
- Peripheral transverse densities $b = \mathcal{O}(M_\pi^{-1})$

Dispersive representation

χ EFT calculation

Dispersively Improved χ EFT (DI χ EFT)

Nucleon EM densities

$SU(3)$ flavor and octet baryons

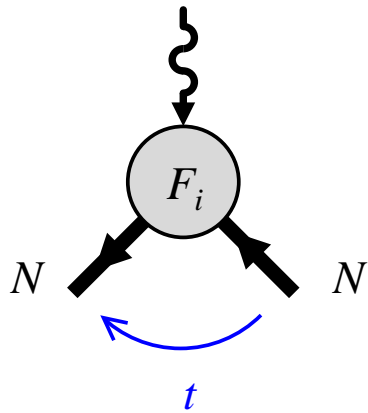
Large- N_c relations

- Resonance form factors and densities

Unstable particles, $N \rightarrow N^*$, $N^* \rightarrow N^*$

- Applications and extensions

Scalar, axial form factor, EM tensor



- Current matrix element

$$\langle N' | J_\mu | N \rangle \rightarrow F_1(t), F_2(t) \quad \text{invariant FFs}$$

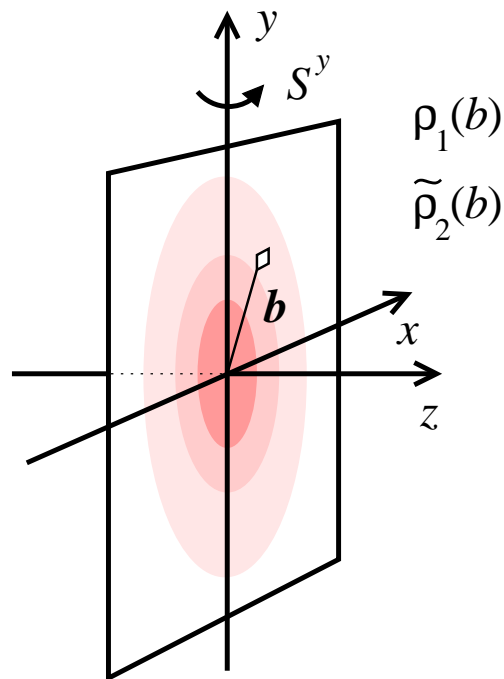
- Transverse densities

Soper 76, Burkardt 00, Miller 07

$$F_{1,2}(t = -\Delta_T^2) = \int d^2b \, e^{i\Delta_T \cdot b} \rho_{1,2}(b) \quad \text{Fourier}$$

Charge/magnetization density, or spin-indep/dependent

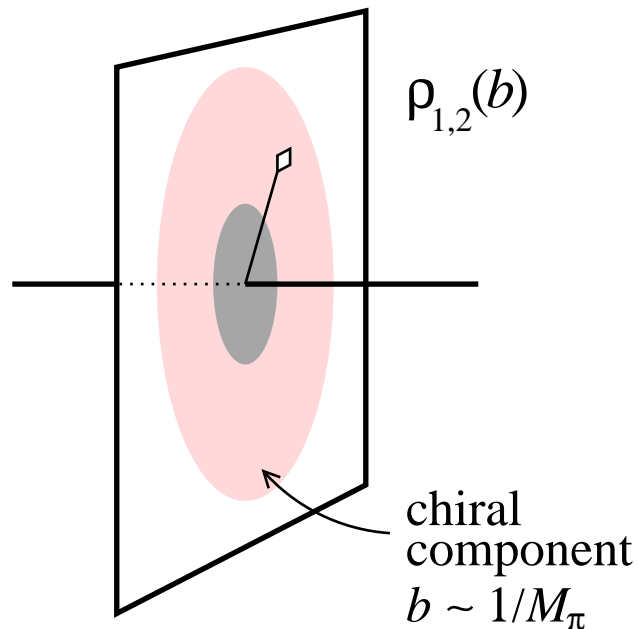
Density at fixed light-front time $x^+ = x^0 + x^3$,
boost-invariant, appropriate for relativistic systems



- Connection with GPDs/QCD

$$\rho_1(b) = \sum_q e_q \int_0^1 dx [q(x, b) - \bar{q}(x, b)]$$

Dual role of transverse densities:
Accessible through low-energy elastic FFs,
interpretable in terms of QCD DoF



- Peripheral densities $b = \mathcal{O}(M_\pi^{-1})$

Governed by chiral dynamics:
universal, model-independent

Calculable using χ EFT + dispersion theory
[Strikman, CW 10](#); [Granados CW 13](#)

- Theoretical interest

Distance as parameter

Proper definition of mesonic component

Space-time picture of chiral dynamics

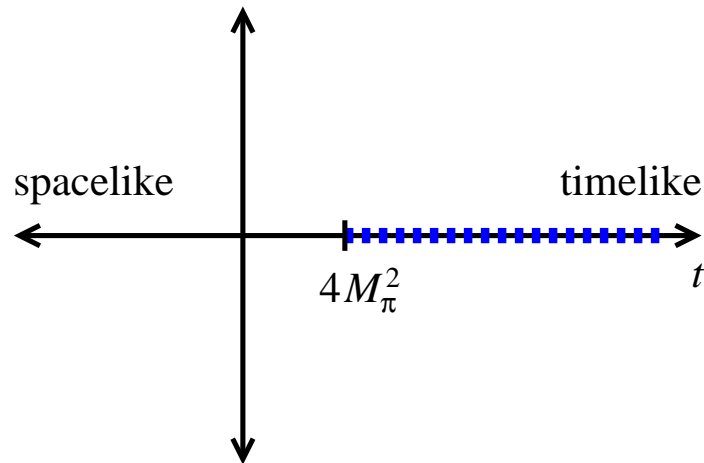
- Practical interest

Low- $|t|$ form factors, proton size

Connection w. peripheral quark/gluon structure

Dispersive representation

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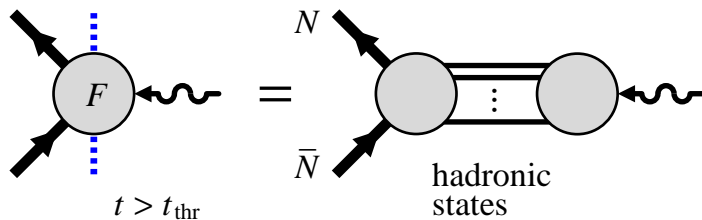
- Dispersive representation of form factor

$$F(t) = \int_{4M_\pi^2}^{\infty} \frac{dt'}{t' - t - i0} \frac{\text{Im } F(t')}{\pi}$$

Process: Current \rightarrow hadronic states $\rightarrow N\bar{N}$

Unphysical region: $\text{Im } F(t')$ from theory

Frazer, Fulco 60; Höhler et al 74; Hoferichter et al 14



- Transverse densities

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{t}b) \frac{\text{Im } F(t)}{\pi}$$

$K_0 \sim e^{-b\sqrt{t}}$ exponential suppression of large t

Distance b selects masses $\sqrt{t} \sim 1/b$: Filter

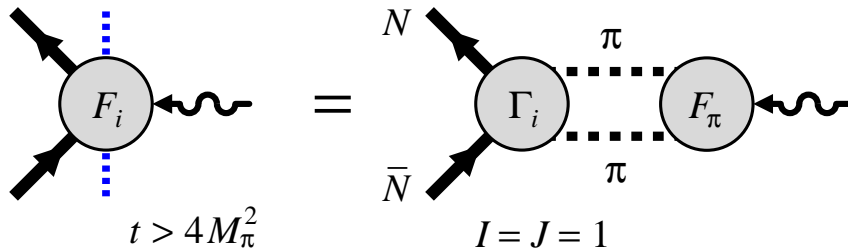
Strikman, CW 10; Miller, Strikman, CW 11

Isovector: $\pi\pi$ (incl. ρ), 4π , ...
 Isoscalar: 3π (incl. ω), $K\bar{K}$ (incl. ϕ), ...

Peripheral densities \longleftrightarrow low-mass states

Spectral functions

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$$\begin{aligned} \text{Im} F_i(t) &= \frac{k_{\text{cm}}^3}{\sqrt{t}} \Gamma_i(t) F_\pi^*(t) \\ &= \frac{k_{\text{cm}}^3}{\sqrt{t}} \underbrace{\frac{\Gamma_i(t)}{F_\pi(t)}}_{\chi\text{EFT}} \underbrace{|F_\pi(t)|^2}_{\text{Data}} \end{aligned}$$

- Elastic unitarity relation

Timelike pion FF $F_\pi(t)$, $\pi\pi \rightarrow N\bar{N}$ partial-wave amplitude Γ_i

Functions have same phase — Watson's theorem

Relation valid up to $t = 16 M_\pi^2$, in practice up to $t \sim 1 \text{ GeV}^2$

Includes ρ as $\pi\pi$ resonance

- New χ EFT-based approach

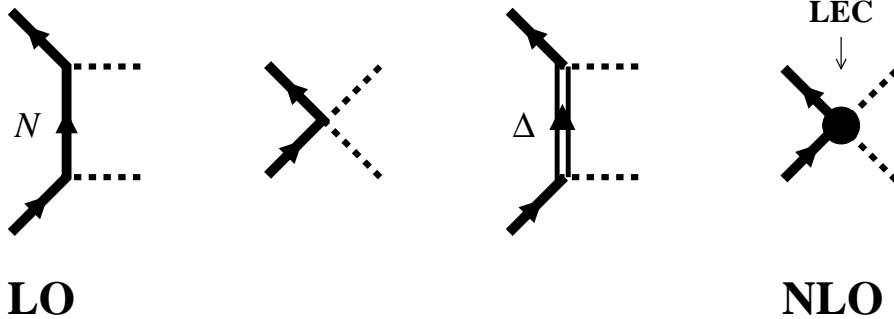
Calculate Γ_i/F_π in χ EFT — free of $\pi\pi$ rescattering, well convergent

Multiply with $|F_\pi|^2$ from e^+e^- data — includes $\pi\pi$ rescattering, ρ resonance

N/D method. Many theoretical advantages. Predictive!

Spectral functions II

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$\pi\pi \rightarrow N\bar{N}$ amplitude in χ EFT

- Relativistic χ EFT

Expansion in $(M_\pi, k_\pi)/\Lambda_\chi$

Controlled accuracy,
systematic improvement

π, N, Δ as effective DoF

- Spectral function results

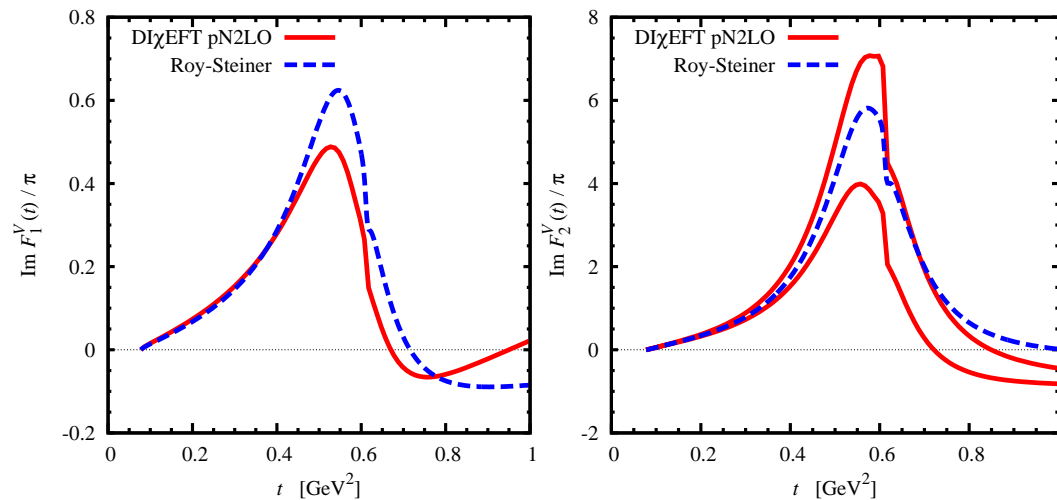
New method includes $\pi\pi$
rescattering, ρ resonance

Dramatic improvement over
conventional χ EFT calculations

Good convergence in higher orders

Possible to compute spectral
functions up to $\sim 1 \text{ GeV}^2$

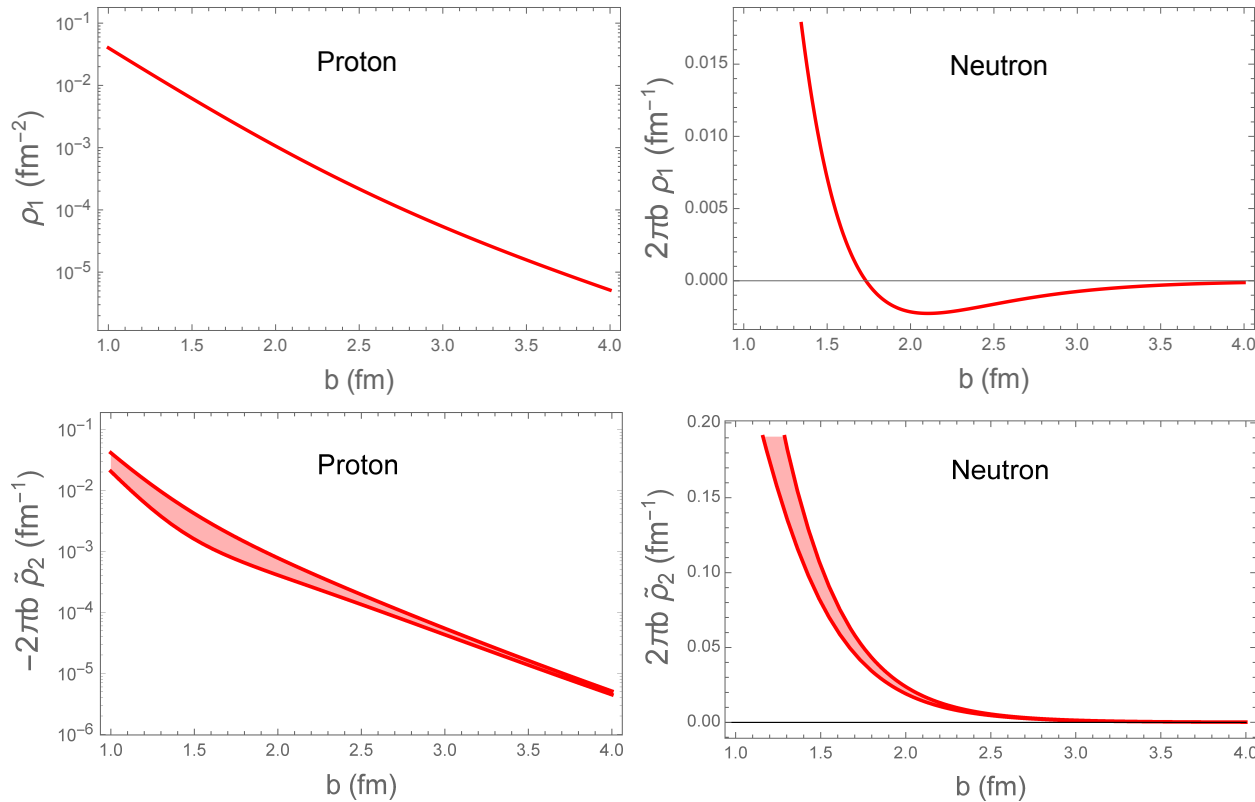
Many applications!



Alarcon, Blin, Vicente Vacas, Weiss, NPA 964 18 (2017)
NLO: Alarcon, Weiss, arXiv:1710.06430; in progress.

Peripheral densities in nucleon

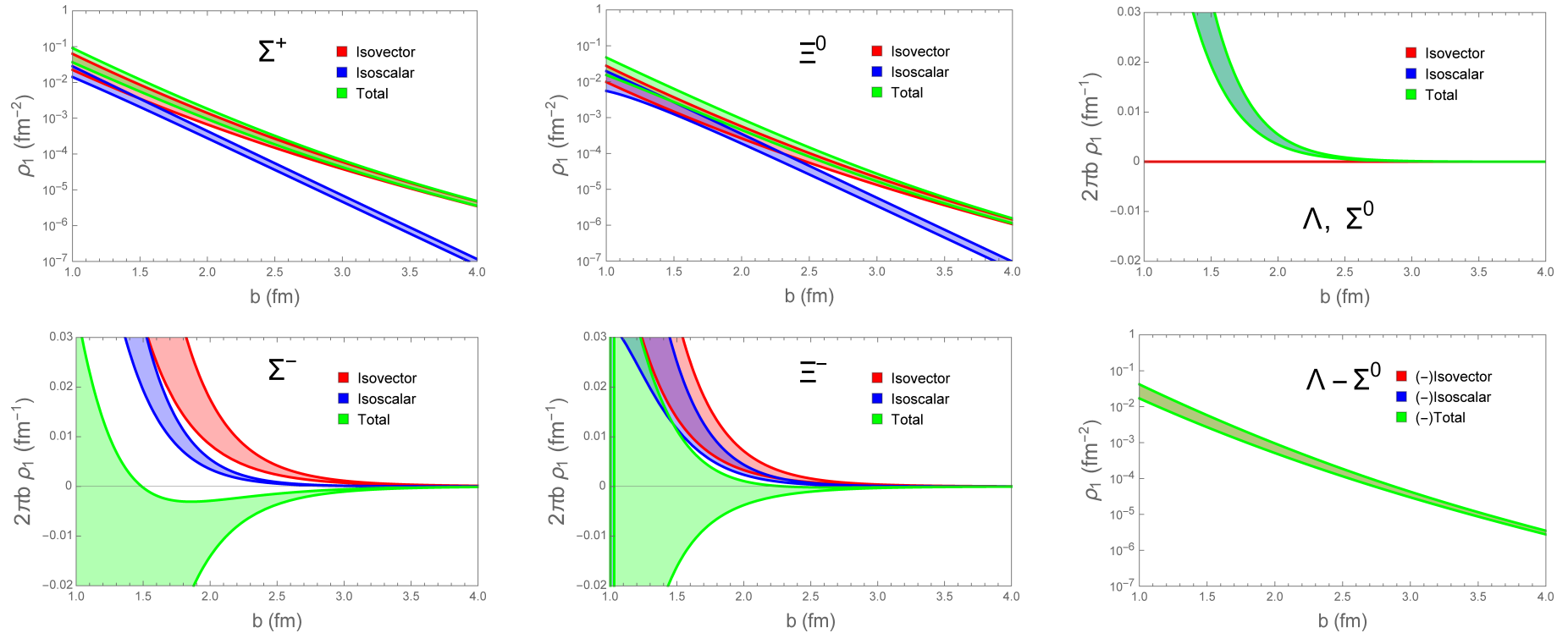
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- Use $\text{D}\chi\text{EFT}$ spectral functions to calculate peripheral transverse densities
- Peripheral isovector densities predicted down to $b \sim 1$ fm with controlled accuracy
Isoscalar densities from empirical parametrization with ω, ϕ
- Peripheral nucleon structure can be computed from first principles!
[Alarcon, Blin, Vicente Vacas, Weiss, NPA 964 18 \(2017\). NLO: Alarcon, Weiss, arXiv:1710.06430; in progress.](#)

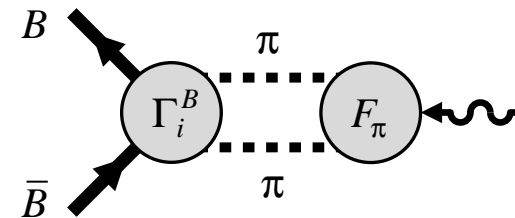
SU3 flavor and octet baryons

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- D χ EFT extended to $SU(3)$ flavor
- $\pi\pi$ spectral functions of octet baryon FFs
 $K\bar{K}$ negligible at peripheral distances
- Peripheral densities of octet baryons,
quark flavor separation $u/d/s$

Alarcon, Blin, Vicente Vacas, CW, 2017



- Study scaling behavior of non-perturbative QCD quantities with N_c :
Meson and baryon masses, current matrix elements, hadronic couplings, ...
'tHooft 73, Witten 79

N_c scaling can be established on general grounds

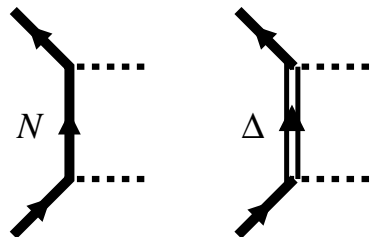
Parametric classification, hierarchy of structures, qualitative insight

- N_c scaling of nucleon transverse densities

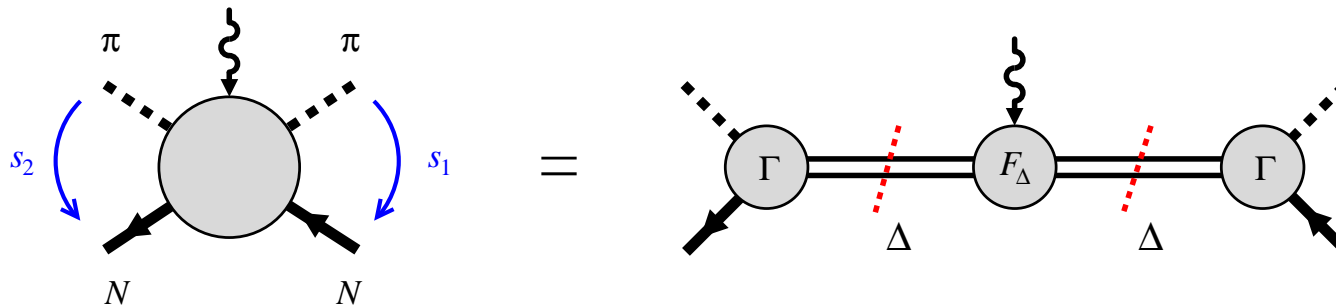
$$\rho_1(b) = \mathcal{O}(N_c^0), \quad \tilde{\rho}_2(b) = \mathcal{O}(N_c), \quad \text{for } b = \mathcal{O}(N_c^0)$$

- χ EFT results have correct N_c scaling if Δ isobar included as dynamical DoF

Cohen, Broniowski 92. Transverse densities and GPDs: Strikman CW 04, 09, 11; Granados, CW 13



N, Δ cancel in $\rho_1(b)$
amplify $\tilde{\rho}_2(b)$



- Structure of unstable particle

S-matrix theory: Stable-particle amplitude $\pi N \rightarrow \pi N$, Δ as pole in 2-particle channels $s_{1,2} = M_\Delta^2$ complex, residue factorizes

Resonance structure defined at complex pole

Can be implemented in χ EFT

Ledwig et al 10. Alarcon, Blin, CW, in progress

- Form factors and densities of Δ isobar

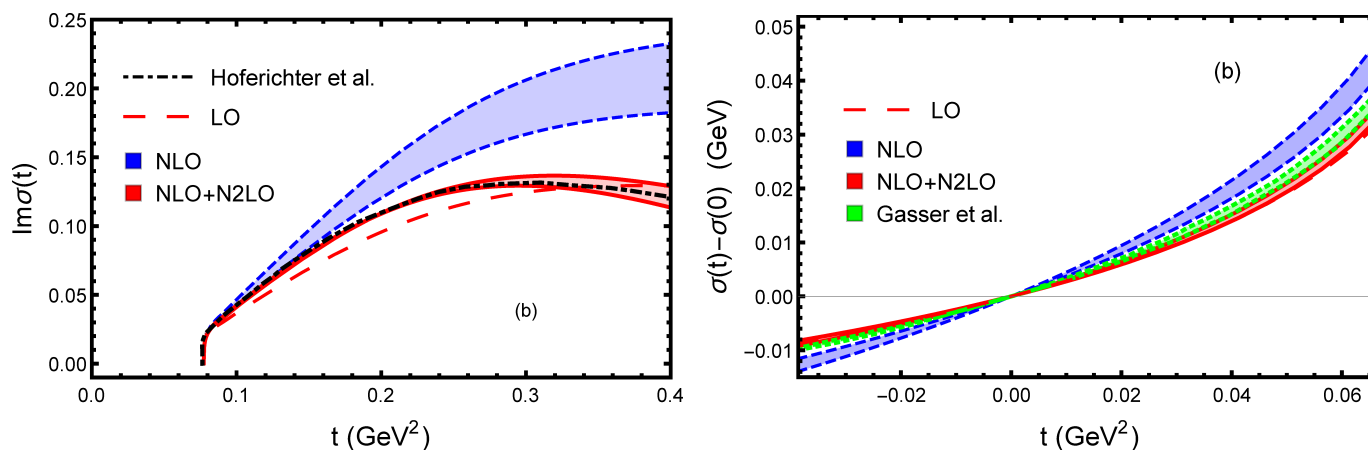
New spin structures because of $S = \frac{3}{2}$

Lorce 09

LQCD results

Alexandrou et al 08; Aubin, Orginos, Pascalutsa, Vanderhaeghe 08

- Electromagnetic form factors at low $t < 0$ and derivatives [Alarcon, CW, arXiv:1710.06430](#)
- Scalar form factor and mass distribution in nucleon [Alarcon, CW, arXiv:1707.07682](#)
- Energy-momentum tensor form factors and angular momentum [Granados, CW 2017](#)
- Peripheral GPDs and high-energy processes [Strikman, CW 04](#)
- Isoscalar-vector form factor from 3-body unitarity [Alarcon, Passemar, Pilloni, CW; in progress](#)



Scalar spectral function and form factor from DIχEFT. Alarcon, CW, arXiv:1707.07682

- Transverse densities are an essential tool for hadron structure studies

Provide objective spatial representation of relativistic system

Connect low-energy dynamics with partonic structure in QCD

- Peripheral transverse densities can be computed with controlled accuracy by combining χ EFT and dispersion theory (Dl χ EFT)

Includes $\pi\pi$ rescattering and ρ resonance

- Numerous applications and extensions

- [• Not covered here: Light-front time-ordered formulation of chiral dynamics, chiral $N \rightarrow \pi N$ wave functions, quantum-mechanical interpretation]

[Granados, Weiss, PRC 92, 025206 \(2015\); JHEP 1507, 170 \(2015\); JHEP 1606, 075 \(2016\).](#)