The Isoscalar sector from the lattice QCD Raúl Briceño

















Lattice QCD

- Solution Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \to m_q^{\text{phys.}}$
- a lattice spacing: $a \sim 0.03 0.15$ fm
- finite volume



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The isoscalar, scalar and tensor sectors

Wilson (Royal fellow / Tri	nity)	had	spec
E CHICAN	PRL 118, 022002 (2017) PHYSICA	L REVIEW LETTERS	week ending 13 JANUARY 2017
Dudek (W&M/JLab)	Isoscalar $\pi\pi$ Scattering a	and the σ Meson Resonance from	QCD
	Raul A. Briceño, ^{1,*} Jozef J. Dude	ek, ^{1,2,†} Robert G. Edwards, ^{1,‡} and David J. V	Wilson ^{3,§}
	(for the Had	dron Spectrum Collaboration)	
	¹ Thomas Jefferson National Accelerator Facili ² Department of Physics, College of Wi ³ Department of Applied Mathematics a	lity, 12000 Jefferson Avenue, Newport News, Vir illiam and Mary, Williamsburg, Virginia 23187- and Theoretical Physics, Centre for Mathematica	rginia 23606, USA ·8795, USA al Sciences,
Edwards (JLab)			JLAB-THY-17-2534
	Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD		
	Raul A. Briceño, ^{1, 2, *} Jozef J. (for t	. Dudek, ^{1,3,†} Robert G. Edwards, ^{1,‡} and Dav	rid J. Wilson ^{4,§}
	¹ Thomas Jefferson National Acceler ² Department of Physic ³ Department of Physics, C ⁴ School of M	rator Facility, 12000 Jefferson Avenue, Newport Ne ics, Old Dominion University, Norfolk, VA 23529, College of William and Mary, Williamsburg, VA 2 Iathematics, Trinity College, Dublin 2, Ireland (Dated: August 23, 2017)	ews, VA 23606, USA , USA 23187, USA
	We present the first lattice QC extracted from discrete finite-vo mass corresponding to $m_{\pi} \sim 391$ σ and $f_0(980)$ states, where the to what is seen in experiment, vicinity of the $K\overline{K}$ threshold. For observed as narrow peaks, with the $K\overline{K}$. The presence of all these st of scattering amplitudes, and the	CD study of coupled isoscalar $\pi\pi$, $K\overline{K}$, $\eta\eta$ <i>S</i> - and <i>D</i> plume spectra computed on lattices which have a va 1 MeV. In the $J^P = 0^+$ sector we find analogues of t σ appears as a stable bound-state below $\pi\pi$ thresh the $f_0(980)$ manifests itself as a dip in the $\pi\pi$ cross for $J^P = 2^+$ we find two states resembling the $f_2(127)$ the lighter state dominantly decaying to $\pi\pi$ and the states is determined rigorously by finding the pole sin heir couplings to decay channels are established usin	P-wave scattering lue of the quark the experimental old, and, similar ss section in the 70) and $f'_2(1525)$, the heavier state to ngularity content g the residues of

- Multi-meson ops. are crucial
- Spectrum including a larger basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\overline{\ell}, s\overline{s}\}$



 m_{π} =391 MeV

Isoscalar spectra: S-wave dominant Multi-meson ops. are crucial Spectrum including a larger basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\overline{\ell}, s\overline{s}\}$ 1.00× $[000] A_1^+$ $[100] A_1$ $[110] A_1$ [200]2-3fm $2m\pi \sim 782 \text{ MeV}$ B~37 MeV P 卫 存 0.14 1.9fm 2.4fm 2.9fm Ŧ 0.12 0.10 24 20 16 L/a_s

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Unitarity using all orders perturbation theory:



non-perturbative kernel including all diagrams not shown...

"yep, the left hand cut is there"



Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \mathbf{X} + \mathbf{X} +$$

$$\rho \equiv \frac{p}{8\pi E} \sim \sqrt{s - s_{th}}$$
 square root singularity.

Unitarity using all orders perturbation theory:

$$i\mathcal{M} = \mathbf{X} + \mathbf{X} + \mathbf{X} + \mathbf{X} + \cdots$$



Unitarity using all orders perturbation theory:



Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$$C_L^{2pt.}(P) = \underbrace{V} + \underbrace{V} + \underbrace{V} + \cdots$$



Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$$C_L^{2pt.}(P) = \underbrace{V} + \underbrace{V} + \underbrace{V} + \cdots$$

$$V = \frac{1}{L^3} \sum_{\mathbf{k}} \frac{iB^2}{(2\omega_k)^2} \frac{i}{E - 2\omega_k} + \text{"smooth"}$$

$$= (iB) \left(\left[\frac{1}{L^3} \sum_{\mathbf{k}} -\int \frac{d^3k}{(2\pi)^3} \right] \frac{1}{(2\omega_k)^2} \frac{i}{E - 2\omega_k + i\epsilon} \right) (iB) + i\epsilon \text{ integral}$$

$$\equiv [iB] iF [iB] + i\epsilon \text{ integral}$$

$$= V - \infty + i\epsilon$$

$$F \text{ replaces } \rho$$

Two-particle in finite volume

Consider the finite-volume two-particle correlator (*E*~2*m*):

$$C_L^{2pt.}(P) = \underbrace{V}_{V} + \underbrace{V}_{V-\infty} + \underbrace{V}_{V-\infty} + \underbrace{V}_{V-\infty} + \underbrace{V}_{V-\infty} + \underbrace{V}_{V-\infty} + \cdots$$

$$= "\text{smooth"} + A \frac{i}{F^{-1} + \mathcal{M}} B^{\dagger}$$

poles satisfy:
$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

♀ Lüscher (1986, 1991)

- Rummukainen & Gottlieb (1995)
- 🖗 Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)

Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
 RB (2014)

Multi-channel systems - the cutting edge!

set the *necessary* formalism for doing coupled-channel scattering of

Feng, Li, & Liu (2004) [inelastic scalar bosons]Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]RB (2014) [general 2-body result]

For date, the Hadron Spectrum collaboration is the only one to have extracted coupled-channel scattering amplitude information from QCD

ππ, KK, ηη [isoscalar]:	RB, Dudek, Edwards, Wilson - PRL (2017)
	RB, Dudek, Edwards, Wilson - PRD (2018)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
πη, KK:	Dudek, Edwards, Wilson - PRD (2016)
Dπ, Dη, D _s K:	Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)
$\pi\pi$, KK [isovector]:	Wilson, RB, Dudek, Edwards, Thomas - PRD (2015)



Solution Above $2m_K$, there is not a one-to-one correspondence

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & F_{K\overline{K}}^{-1} + \mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),

Feng, Li, & Liu (2004), Hansen & Sharpe / RB & Davoudi (2012)

- Solution For a set of the set of
- Need that many energy levels at the same energy
- Alternatively, parametrize scattering amplitude and do a global fit

S-wave above $2m_{\pi}$, $2m_K$, and $2m_{\eta}$

Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$









Solution D-wave above $2m_{\pi}$, $2m_K$, and $2m_\eta$

Ansatz $K_{ij}(s) = \frac{g_i^{(1)}g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)}g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$ $\gamma_{ij} = 0$ otherwise



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 $\gamma_{ij} \neq 0$
 $\gamma_{ij} = 0$

 $\gamma_{\eta\eta} \neq 0$ $\gamma_{ij} = 0$ otherwise



Tensor and scalar nonets

First complete determination of the scalar and tensor nonets from LQCD :



Oudek, Edwards - PRL (2017)
Oudek, Edwards - PRD (2017)
ek, Edwards, Thomas, Wilson - PRL (2015)
on, Dudek, Edwards, Thomas - PRD (2015)
ek, Edwards, Wilson - PRD (2016)



Tensor nonets



Scalar nonets



Scattering of spinning particles - $\rho\pi$ scattering

≩the formalism: RB (2014)

 $\frac{1}{2}$ first and only calculation:

 $\rho\pi$ scattering in *I*=2

Woss, Thomas, Dudek, Edwards, Wilson (2018)





Woss

-10-20 $\delta(^3S_1)$ -3010 $\overline{\epsilon}({}^{3}S_{1}|{}^{3}D_{2})$ 50 $\delta(^{3}P_{0})$ -10-20-300 $\delta(^{3}P_{1})$ -10-20-302010 $\delta(^{3}P_{2})$ 0 -10-2010 $\delta(^{3}D_{2})$ 0 -1010 $({}^{3}D_{3})$ 0 -10

50

0.37

0

0.36

0

100

0.38

0.39

150

0.40

200

 $E_{\rm cm} - (m_\pi + m_\rho) / {\rm MeV}$

 $({}^{3}D_{1})$

250

0.41 $a_t E_{cm}$

Woss, Thomas, Dudek, Edwards, Wilson (2018)

THE FUTURE IS OURS TO CREATE.

Structure of states:







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Structure of states:



Beyond two particles



- RB & Davoudi (2013)
- Hansen & Sharpe (2014, '15)
- Hammer, Pang, Rusetsky (2017)

🖗 Mai, Doring (2017,'18)

RB, Hansen & Sharpe (2016,'18)

The team and some references



Scattering amplitudes



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

$$E_L = \text{finite volume}$$

$$E_L = \text{finite volume}$$

$$F = \text{known function}$$

$$\mathcal{M} = \text{scattering amp.}$$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Bernard, Lage, Meißner, Rusetski (2011) [Twisted BCs]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

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