Jefferson Lab

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First MC global analysis of nucleon transversity

— with lattice QCD constraints —

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JLab Angular Momentum collaboration

PRL 120, 152502 (2018)

Motivation

- Along with unpolarized distribution $f_1^q = q^\uparrow + q^\downarrow$ and helicity distribution $g_1^q = q^\uparrow - q^\downarrow$, the transversity distribution $h_1^q = q^\rightarrow - q^\leftarrow$ completes the full set of twist-2 PDFs in a spin-$1/2$ hadron, such as a nucleon

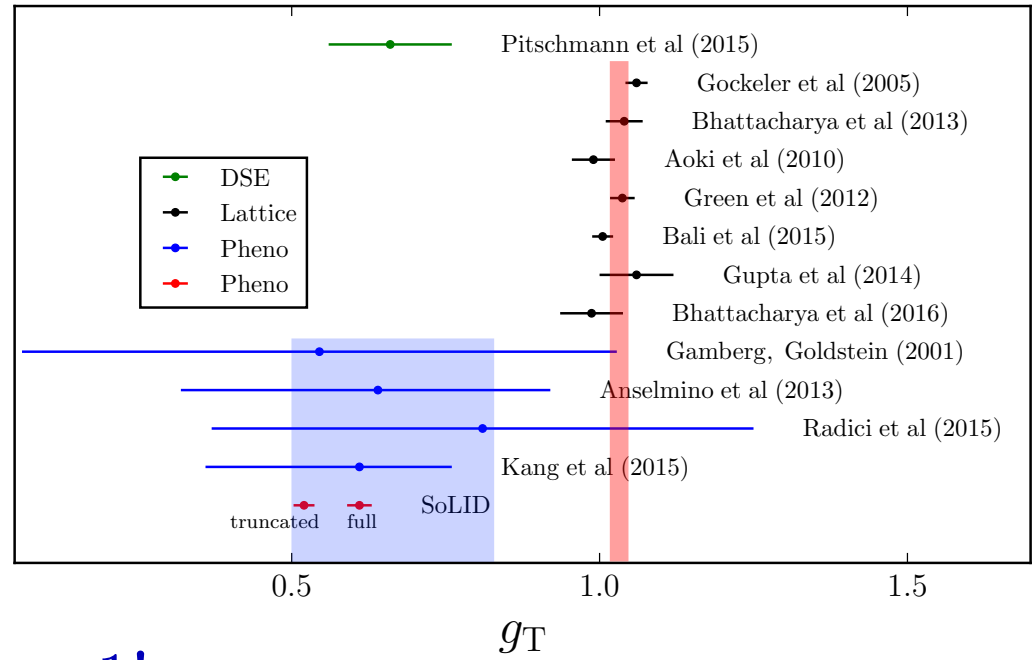
- Since h_1 is chiral-odd, it is difficult to measure experimentally
  $\longrightarrow$ need at least two hadrons in reaction, $e.g.$ SIDIS
  $\longrightarrow$ phenomenology ~ 3 decades behind f_1, ~ 2 decades behind g_1

- Some SIDIS data available from HERMES, COMPASS, …
  — dedicated program under way at Jefferson Lab

- Several global QCD analyses, based on maximum likelihood method (Anselmino *et al.*, Radici *et al.*, Kang *et al.*) have been performed
  $\longrightarrow$ suggest discrepancy with lattice QCD calculations…

Motivation

- Most analyses find tensor charge $g_T = \int dx\,(h_1^u(x) - h_1^d(x))$ with central values ~ 0.6 (with large uncertainty)



Z. Ye, N. Sato et al.
PLB 767, 91 (2017)

- Lattice calculations give $g_T \approx 1$!

  $\longrightarrow$ problem with data? lattice? analysis? transverse spin crisis?

- Perform robust MC analysis of data, with *simultaneous* determination of PDFs & fragmentation functions, to test consistency of experiment and lattice QCD

JAM global QCD analysis

- JAM Collaboration utilizes Bayesian approach to global analysis, with robust fitting methodology based on MC sampling methods (e.g., iterative Monte Carlo, nested sampling, …)

  → allows more flexible parameterizations of input PDFs and fragmentation functions

  → systematic exploration of the parameter space by computing likelihood function directly

  → map likelihood function into an MC-weighted parameter sample, allowing rigorous determination of PDF uncertainties

- Methodology used previously to analyze
  — spin-dependent PDFs *Ethier, Sato, WM PRL 120, 152502 (2018)*
  — fragmentation functions *Sato et al. PRD 94, 114004 (2016)*
  — pion PDFs *Barry, Sato, WM, Ji PRL 121, 152001 (2018)*

JAM global QCD analysis

- Analysis of data requires estimating expectation values E and variances V of "observables" $\mathcal{O}$ (functions of PDFs) which are functions of parameters

$$E[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\text{data}) \, \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\text{data}) \, \left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

"Bayesian master formulas"

- Using Bayes' theorem, probability distribution $\mathcal{P}$ given by

$$\mathcal{P}(\vec{a}|\text{data}) = \frac{1}{Z} \, \mathcal{L}(\text{data}|\vec{a}) \, \pi(\vec{a})$$

in terms of the likelihood function $\mathcal{L}$

JAM global QCD analysis

- Likelihood function

$$\mathcal{L}(\text{data}|\vec{a}) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

is a Gaussian form in the data, with χ^2 function

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\text{data}_i - \text{theory}_i(\vec{a})}{\delta(\text{data})}\right)^2$$

with priors $\pi(\vec{a})$ and "evidence" Z

$$Z = \int d^n a\, \mathcal{L}(\text{data}|\vec{a})\, \pi(\vec{a})$$

$\longrightarrow$ Z tests if $e.g.$ an n-parameter fit is statistically different from $(n{+}1)$-parameter fit

JAM global QCD analysis

- Standard method for evaluating E, V via maximum likelihood

  $\longrightarrow$ maximize probability distribution

$$\mathcal{P}(\vec{a}|\text{data}) \;\rightarrow\; \vec{a}_0$$

  $\longrightarrow$ if $\mathcal{O}$ is linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0)\,, \quad V[\mathcal{O}(\vec{a})] \rightarrow \text{Hessian} \qquad H_{ij} = \frac{1}{2}\frac{\partial\chi^2(\vec{a})}{\partial a_i \partial a_j}\Big|_{\vec{a}=\vec{a}_0}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method often fails

  $\longrightarrow$ must resort to more robust MC approach

$$E[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \mathcal{O}(\vec{a}_k)\,, \qquad V[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \left[\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]\right]^2$$

Data Analysis

- Collins asymmetry in $e\,p \;\to\; e\,h\,X$

$$A_{UT}^{\sin(\phi_h+\phi_s)} = \frac{2(1-y)}{1+(1-y)^2}\,\frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}}$$

ϕ_h azimuthal angle of h

ϕ_s azimuthal angle of target spin

semi-inclusive structure functions

$$F_{UU} = f_1 \otimes D_1$$

TMD convolution operator

unpolarized TMD FF

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \frac{\hat{h}\cdot\vec{p}_\perp}{z m_h} \otimes h_1 \otimes H_1^\perp$$

transversity TMD PDF

Collins TMD FF

- Factorized parametrization for TMD PDFs

$$f^q(x,k_\perp^2) = f^q(x)\,\mathcal{G}_f^q(k_\perp^2)$$

collinear PDF

$$\mathcal{G}_f^q(k_\perp^2) \;\propto\; \exp\left[-\frac{k_\perp^2}{\langle k_\perp^2\rangle_f^q}\right]$$

assume Gaussian shape

Data Analysis

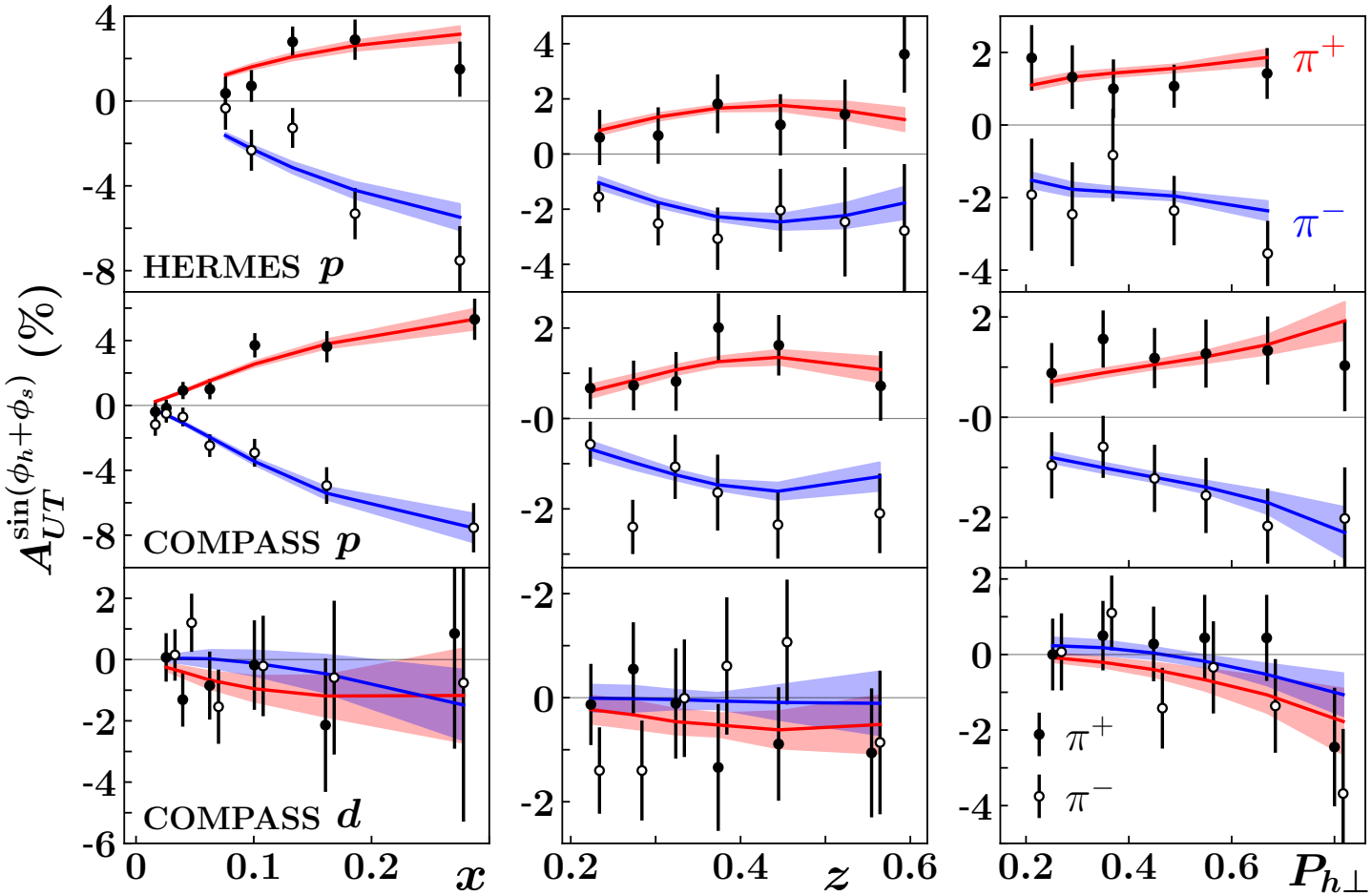
- SIDIS charged-pion data on p & d targets
  — total of 106 data points from HERMES and COMPASS

  $\longrightarrow$ 4 linear combinations of transversity TMD PDFs and Collins TMD fragmentation function:
  — $u, d, \bar{q}$ transversity PDFs $h_1^q(x) = N_q x^{a_q}(1-x)^{b_q}$
  — favored + unfavored Collins FFs
  — respective widths

- World averaged value of lattice tensor charge $g_T^{\rm latt} = 1.01(6)$

- Total of 23 parameters:
  — 19 for F_{UT}
  — 4 for F_{UU} (fitted to 978 points from HERMES π & K production)

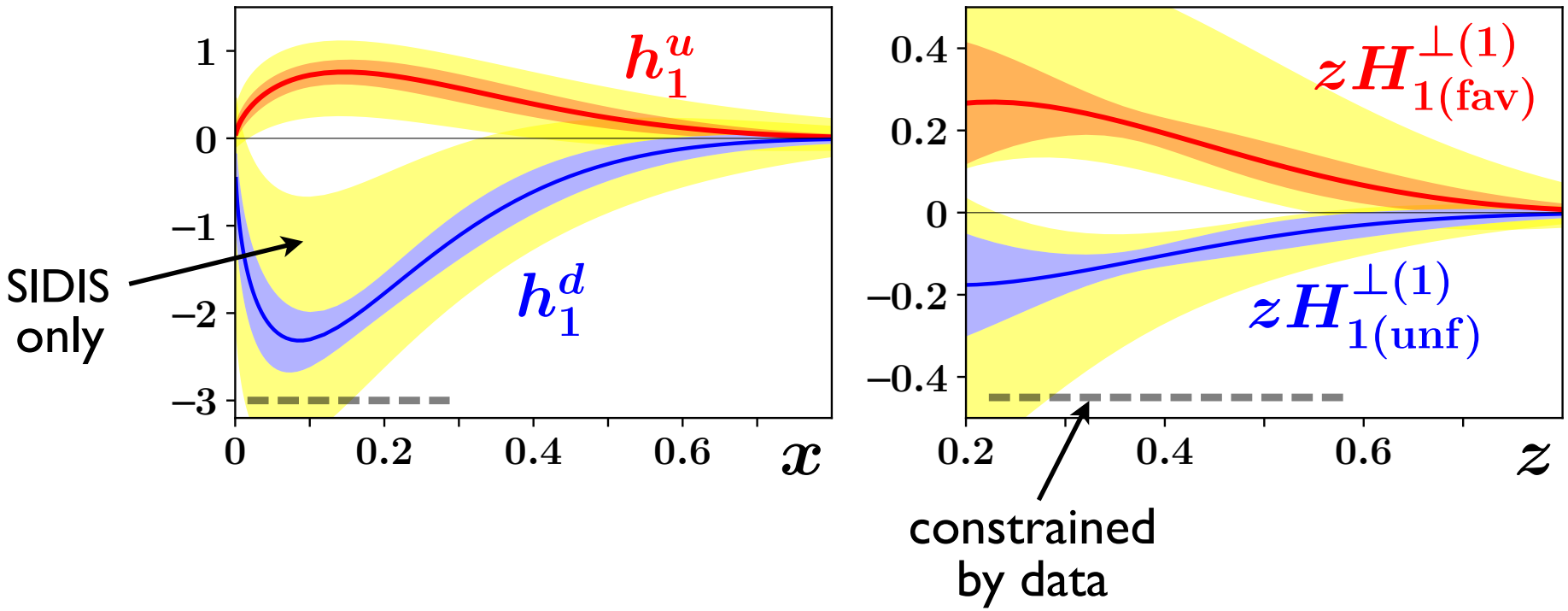
Data Analysis



→ $\chi^2/N_{\rm dat} = 68.9/106 \approx 0.65$

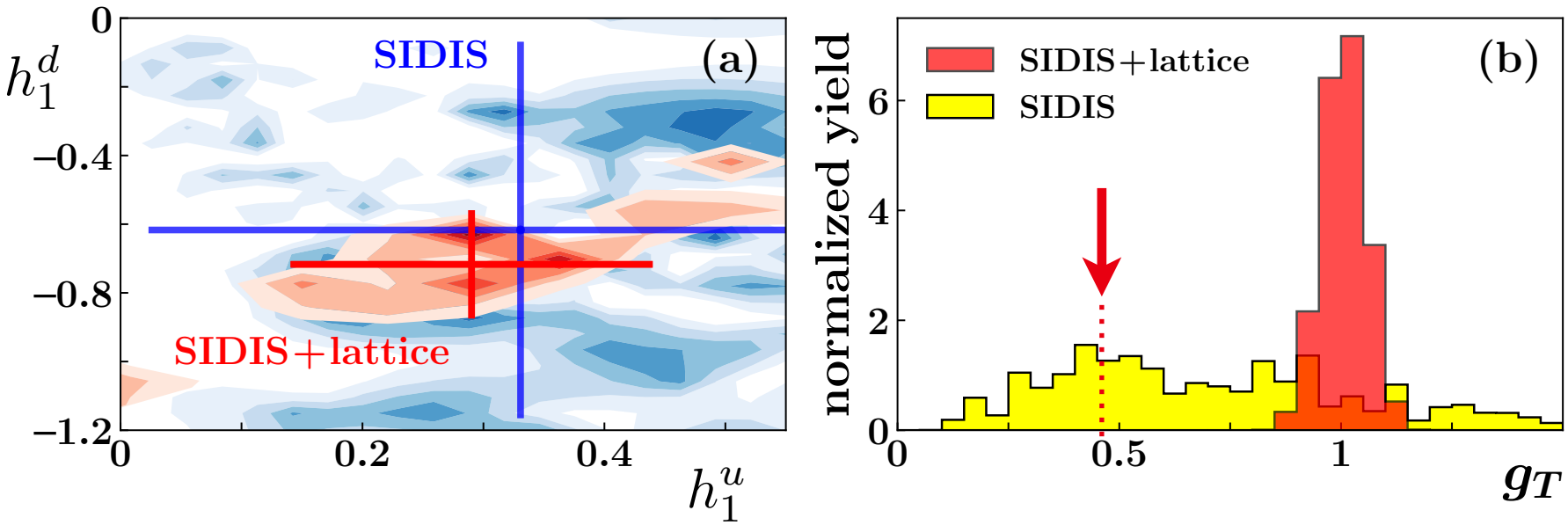
→ for SIDIS-only fit, $\chi^2/N_{\rm dat}$ almost indistinguishable

Transversity distributions



→ significantly reduced uncertainties with lattice constraint

Transversity distributions



→ distributions do not look Gaussian

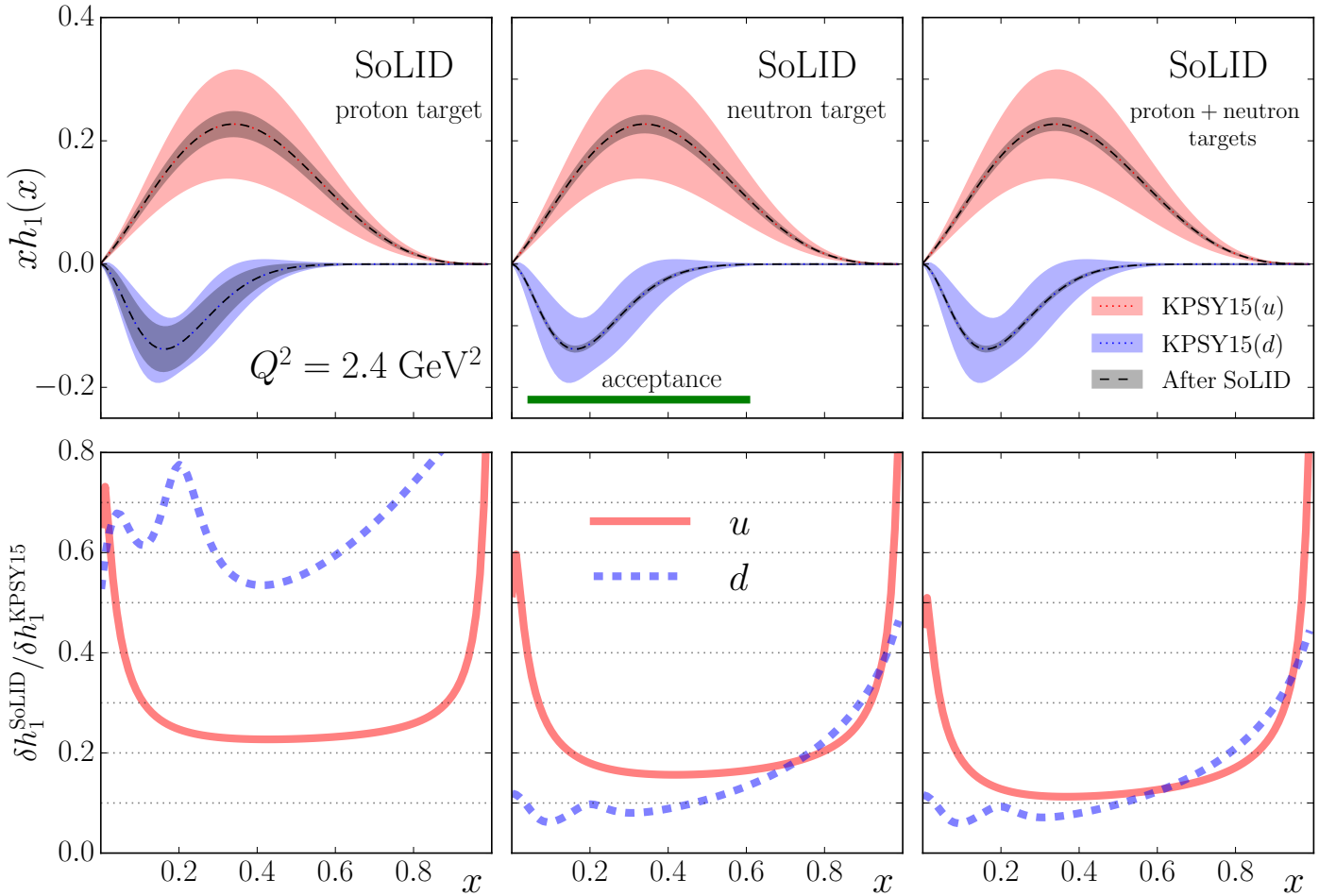
→ full MC analysis gives $g_T^{\rm full} = 1.0(1)$

→ SIDIS-only fit gives $g_T^{\rm SIDIS} = 0.9(8)$, while maximum likelihood analysis would have given $g_T \approx 0.5$

→ no tension with lattice data

Future

- New data expected from SoLID experiment at Jefferson Lab



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