STRESS TESTING TARGET MASS CORRECTIONS IN A SIMPLE FIELD THEORY
QUARK-HADRON DUALITY WORKSHOP SEP 2018

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Introduction

- 12 GeV Experiments at Jlab
  - Unprecedented access to 3-D structure of the nucleon
  - Transition between hadronic and partonic degrees of freedom
  - Perturbative QCD can still be used
  - Target hadron masses don’t satisfy $m \ll Q$
- Target Mass Corrections
Target Mass Corrections (TMCs)

- Different methodologies:
  - Georgi and Pollitzer,, 1976 (GP)
  - Ellis, Furmanski, and Petronzio, 1983 (EFP)
  - Aivazis, Olness, and Tung, 2011 (AOT)
- Some use assumptions about partonic dynamics
- Want to examine a method that is purely kinematical
  - AOT (see Ted’s talk)
- When are these corrections relevant?
Testing in a Simple Field Theory

- Analytical solutions in QCD require approximation
- Factorization methods can be applied to any renormalizable field theory
- Choose a theory for which calculations can be completed without approximation:
  - Calculate exact solution
  - Calculate factorized solution
  - Compare results
Testing in a Simple Field Theory

- Demonstrated in Moffat, Melnitchouk, Rogers, Sato, 2017
- Examined unintegrated low transverse momentum contribution in collinear factorization
- For testing TMCs:
  - Complete the full, integrated factorization
The Simple Field Theory

- Interaction Lagrangian Density:

\[ \mathcal{L}_{\text{int}} = -\lambda \overline{\Psi}_N \psi_q \phi + \text{H.c.} \]

- where:
  - \( \Psi_N \) = Spin-1/2 “nucleon” with mass \( m_p \)
  - \( \psi_q \) = Spin-1/2 “quark” with mass \( m_q \)
  - \( \phi \) = Scalar “di-quark” with mass \( m_s \)
- Nucleon and quark interact with the photon, but di-quark is a spectator
- Coupling constant:

\[ a_\lambda = \frac{\lambda^2}{16\pi^2} \]
DIS Calculations

- Calculate transverse and longitudinal structure functions

\[ F_T = 2x_{bj}F_1 \quad F_L = \rho^2 F_2 - F_T \]

\[ \rho^2 = 1 + \frac{4x_{bj}^2M^2}{Q^2} \]

\[ x_{bj} = \frac{Q^2}{2P \cdot q} \]

- Calculate exact structure functions to order \( a_\lambda \)

- Calculate factorized structure functions to order \( a_\lambda \)

\[ F^{(1)}_{i,\text{fact}} = \hat{F}^{(0)}_{i,flf'} \otimes f^{(1)}_{f'/P} + \hat{F}^{(1)}_{i,flf'} \otimes f^{(0)}_{f'/P} \]
Order $a_\lambda$ Diagrams

(A)

(B)

(C)
TMCs in the Yukawa Theory

- Need target mass $\sim Q$
- If $m_p > m_q + m_s$, particle is unstable
- Larger particle of mass $M$ made up of non-interacting particles

\[
\frac{d\sigma}{d^3l'} = \int dX \frac{d\sigma^p}{d^3l'} \delta(X - c)
\]
TMCs in the Yukawa Theory

- **Exact**
  \[
  F_1(x_N) = \frac{1}{c} F_1^p(x_N^p) \quad F_2(x_N) = \frac{Q^2 (Q^2 - M^2 x_N^2)}{(Q^2 + M^2 x_N^2)^2} F_2^p(x_N^p)
  \]

- **Massless Target Approximation**
  \[
  F_1(x_N) = \frac{1}{c} F_{1, \text{fact}}(x_{bj}/c) \quad F_2(x_N) = F_{2, \text{fact}}(x_{bj}/c)
  \]

- **Target Mass Corrected**
  \[
  F_1(x_N) = \frac{1}{c} F_{1, \text{fact}}(x_N/c) \quad F_2(x_N) = \frac{Q^2 (Q^2 - M^2 x_N^2)}{(Q^2 + M^2 x_N^2)^2} F_{2, \text{fact}}(x_N/c)
  \]

- **Where**
  \[
  x_N = \frac{2 x_{bj}}{1 + \sqrt{1 + \frac{x_{bj}^2 M^2}{Q^2}}} \quad x_{bj} = \frac{Q^2}{2 P \cdot q} \quad x_N^p = \frac{2 x_{bj}^p}{1 + \sqrt{1 + \frac{x_{bj}^2 m_p^2}{Q^2}}} \quad x_{bj}^p = \frac{Q^2}{2 p \cdot q} = \frac{x_N/c}{1 - \frac{x_N^2 M^2}{c^2 Q^2}}
  \]
Results

- $c = 0.8$
- $M = 0.938 \text{ GeV}$
- $m_p = m_q = m_s = 0.0938 \text{ GeV}$
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Results

- \( c = 0.8 \)
- \( M = 0.938 \text{ GeV} \)
- \( m_p = 0.0938 \text{ GeV} \)
- \( m_q = m_s = 0.496 \text{ GeV} \)
Conclusion

- Target mass corrections get larger with increasing $x_{bj}$
- Target masses are relevant when:
  - $x_{bj}$ is large
  - $M \sim Q$
  - $M \gg$ partonic masses