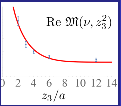

Parton pseudo-distributions and their evolution

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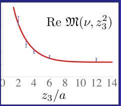
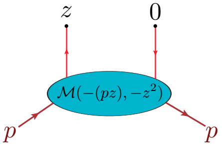


Pseudo-distributions and PDFs



- Basic matrix element (ignoring spin)

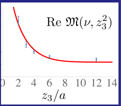
$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz),-z^2)$$

- Lorentz invariance: $\mathcal{M}$ depends on z through $(pz) \equiv -\nu$ and z^2
- Ioffe time ν: $\mathcal{M}(\nu,-z^2) =$ Ioffe-time pseudo-distribution (pseudo-ITD)
- Pseudo $\equiv$ off the light cone
- For any Feynman diagram, for arbitrary z^2 and arbitrary p^2

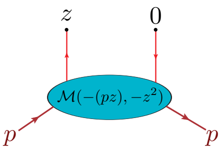
$$\mathcal{M}(\nu,-z^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, \mathcal{P}(x,-z^2)$$

- Limits $-1 \le x \le 1$, negative x correspond to anti-particles
- Pseudo-PDF $\mathcal{P}(x,-z^2)$: Fourier transform of pseudo-ITD with respect to ν for fixed z^2
- On the light cone $z_+ = 0$: usual ITD and usual PDF $\mathcal{P}(x,0) = f(x)$
- If $z^2 \to 0$ limit is singular, regularization (like $\overline{\mathrm{MS}}$) is needed, $f(x) \to f(x,\mu^2)$ and we have $\overline{\mathrm{MS}}$ ITD

$$\mathcal{M}(\nu,0)|_{\mu^2} \equiv \mathcal{I}(\nu,\mu^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, f(x,\mu^2)$$



Quasi-PDFs



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz), -z^2)$$

- Take $z = (0, 0, 0, z_3)$, then $-(pz) \equiv \nu = P z_3$ and $-z^2 = z_3^2$
- Introduce quasi-PDF (Ji,2013)

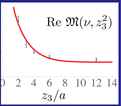
$$Q(y, P) = \frac{P}{2\pi} \int_{-\infty}^{\infty} dz_3 \, e^{-iyPz_3} \, \mathcal{M}(Pz_3, z_3^2)$$

- Quasi-PDF/ pseudo-PDF relation

$$Q(y, P) = \frac{P}{2\pi} \int_{-1}^{1} dx \int_{-\infty}^{\infty} dz_3 \, e^{-i(y-x)Pz_3} \, \mathcal{P}(x, z_3^2)$$

- "Good cross sections". approach (Ma, Qiu, 2014) based on correlators involving currents

$$\langle p|J(0)J(z)|p\rangle$$



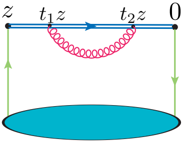
Perturbative corrections and evolution

- In QCD $\mathcal{M}(\nu, z_3^2)$ has logarithmic singularity in z_3^2. At one loop,

$$\mathcal{M}^{\text{hard}}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln(z_3^2) \int_0^1 du\, B(u)\, \mathcal{M}^{\text{soft}}(u\nu, 0)$$

- Generates perturbative evolution. Altarelli-Parisi (AP) evolution kernel

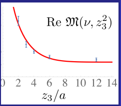
$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$



- One more source of the z^2-dependence of pseudo-ITD: gauge link $\hat{E}(0, z; A)$
- It has specific UV divergences $\sim z_3/a$, $\ln(1 + z_3^2)$, with a = lattice spacing
- Consider reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

- $\mathfrak{M}(\nu, z_3^2)$ has finite $a \to 0$ limit

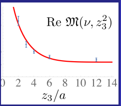


Reduced Ioffe-time pseudo-distribution

- Reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ is a physical observable (like, say, DIS structure functions)
- No need to specify renormalization scheme, scale, etc. for link-related terms
- Pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ is finite for fixed z_3, but is singular in $z_3 \to 0$ limit
- $\ln z_3^2$ terms reflect perturbative evolution
- One-loop relation between $\overline{\text{MS}}$ ITD and reduced pseudo-ITD

$$\mathcal{I}(\nu, \mu^2) = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw \, \mathfrak{M}(w\nu, z_3^2)$$

$$\times \left\{ \frac{1+w^2}{1-w} \left[\ln \left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right] \right\}_+$$

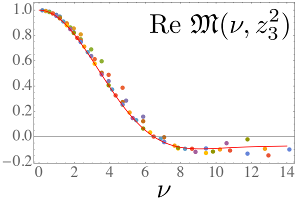


Evolution in lattice data

- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- Real part corresponds to the cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

$$\mathfrak{R}(\nu) \equiv \text{Re}\,\mathfrak{M}(\nu) = \int_0^1 dx\,\cos(\nu x)\,q_v(x)$$

- When plotted as function of ν, data both for real and imaginary parts lie close to respective universal curves

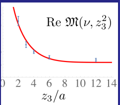


- Overall curve corresponds to the function

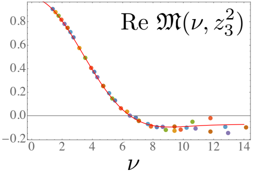
$$f(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

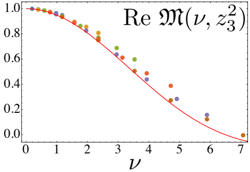
- Obtained by forming cosine Fourier transforms of $x^a(1-x)^b$-type functions and fitting a, b
- Shape is dominated by points with smaller values of Re $\mathfrak{M}(\nu, z_3^2)$

- Data for $\mathfrak{M}(\nu, z_3^2) \equiv \mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$ show no polynomial z_3-dependence for large z_3 though z_3^2/a^2 changes from 1 to ~ 200
- Apparently no higher-twist terms in the reduced pseudo-ITD
- Meaning: $\mathcal{M}(\nu, z_3^2)$ factorizes as $M(\nu)\mathcal{M}(0, z_3^2)$ for large z_3

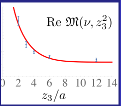


Lattice data for small and large z_3

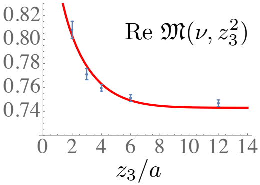




- Points corresponding to $7a \leq z_3 \leq 13a$ values
- Some scatter for points with $\nu \gtrsim 10$
- Otherwise, practically all the points lie on the universal curve based on $f(x)$.
- No z_3-evolution visible in large-z_3 data
- Points in $a \leq z_3 \leq 6a$ region
- All points lie higher than the curve based on the $z_3 \geq 7a$ data
- Perturbative evolution increases real part of the pseudo-ITD when z_3 decreases
- Conjecture that the observed higher values of $\mathrm{Re}\mathfrak{M}$ for smaller-z_3 points may be a consequence of evolution



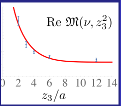
Building $\overline{\mathrm{MS}}$ ITD



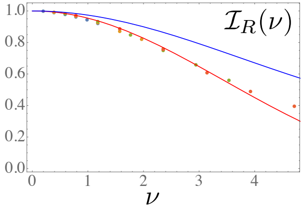
- z_3-dependence of the lattice points for "magic" Ioffe-time value $\nu \equiv z_3 P_3 = 12\pi/16 = 3\pi/4$
- Eye-ball fit line has "Perturbative" $\ln(1/z_3^2)$ behavior for small z_3, and rapidly tends to a constant for $z_3 \gtrsim 6a$
- $\mathfrak{R}(\nu, z_3^2)$ decreases when z_3 increases
- Starts to visibly deviate from a pure logarithmic $\ln z_3^2$ pattern for $z_3 \gtrsim 5a$
- This sets the boundary $z_3 \leq 4a$ on the "logarithmic region"
- $\overline{\mathrm{MS}}$ ITD in terms of reduced pseudo-ITD

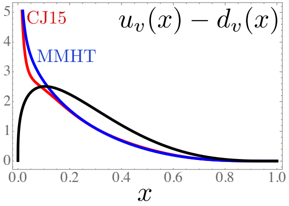
$$\mathcal{I}(\nu, \mu^2) = \mathfrak{M}(\nu, z_3^2) + \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathfrak{M}(w\nu, z_3^2)$$
$$\times \left\{ \frac{1+w^2}{1-w} \left[\ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4\frac{\ln(1-w)}{1-w} - 2(1-w) \right] \right\}_+$$

- $\mathcal{I}(\nu, \mu^2)$ should not depend on z_3
- This happens only if, for some α_s, the $\ln z_3^2$-dependence of the1-loop term cancels actual z_3^2-dependence of the data, visible as scatter in the data



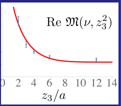
Numerical results for $\overline{\text{MS}}$ ITD





- We choose $\mu = 1/a$ which, at lattice spacing of 0.093 fm is $\approx$ 2.15 GeV
- Using $\alpha_s/\pi = 0.1$ and $z_3 \leq 4a$ data, we generate the points for $\mathcal{I}_R(\nu, (1/a)^2)$
- Upper curve corresponds to the ITD of the CJ15 global fit PDF for $\mu =$2.15 GeV
- Evolved points are close to some universal curve with a rather small scatter
- The curve itself corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$

- $\sim x^{0.35}(1-x)^3$ PDF compared to CJ15 and MMHT global fits for $\mu = 2.15$ GeV
- Unable to reproduce $\sim x^{-0.5}$ Regge behavior
- Possible reasons: large pion mass, quenched approximation



Summary

- Analyzed perturbative structure of pseudo-PDFs using their relation to pseudo-ITDs
- Proposed to use reduced pseudo-ITD
- Studied evolution of exploratory lattice data for reduced pseudo-ITD
- Establlished DGLAP evolution in lattice data